ISSN 1392–124X (print), ISSN 2335–884X (online) INFORMATION TECHNOLOGY AND CONTROL, 2014, T. 43, Nr. 4

# Predictor-based Self-tuning Control of Pressure Plants

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crossref http://dx.doi.org/10.5755/j01.itc.43.4.8742

**Abstract**. A digital predictor-based self-tuning control with constraints for the pressure plants, which is able to cope with minimum-phase and nonminimum-phase plant models is presented in this paper. We determined that applying polynomial factorization for such models the characteristic polynomials of closed-loops are changed. Therefore, the on-line identification of the models' parameters is performed in a way that ensures stable closed-loops. A choice of the sampling period in digital control typically impacts a control quality of the plant, thus we propose a method for optimization of a sampling period in the digital predictor-based self-tuning control system. The impact of the selection of the sampling period and input signals' constraints – amplitude boundaries and the change rate - on the control quality of the pressure plant is experimentally analysed.

**Keywords**: predictor-based self-tuning control, minimum-phase and nonminimum-phase model, factorization, on-line identification, closed-loop stability, sampling period optimization, pressure plant.

## 1. Introduction

At present, the majority of various physical nature processes are still continuous-time plants, but are frequently controlled by digital controllers [2, 3, 6].

One of the classes of such – continuous-time – plants is a "balls-in-tubes" system. Perreira and Boyles [14] analysed the "balls-in-tubes" system that had only one tube with two types of the balls – a ping pong ball and styrofoam ball. Experimental analysis of two control laws, proportional-integral-derivative (PID) and fuzzy-PID, showed that the control quality of the system was more effective with fuzzy-PID as compared to conventional PID control. Quijano et al. [15] investigated the four-tube "balls-in-tubes" system in terms of resource allocation, proposing two possible strategies of resource allocation: juggler strategy and dynamic proportioning strategy. "Balls-in-tubes" system was regulated by PID controllers. Ziwei et al. [16] analysed the four-tube "balls-in-tubes" system with PID and fuzzy-PID controllers, showing that nonlinear characteristics of the system were caused by plant's dynamical properties and demonstrating effectiveness of fuzzy-PID control as compared to conventional PID control for the plant. Coelho and Pessôa [5] proposed a digital control system for the one-tube "balls-in-tubes" system that was described by nonlinear model, where identification of model parameters were performed by applying intellectual methods: genetic algorithms and evolutionary programming. Caro and Quijano [4] analysed a modular version of conventional "balls-in-tubes" system by Quijano et al. [15], where the whole system was constructed from independent one-dimensional modules of tubes. The joint four-tube "balls-in-tubes" system was regulated by four independent fuzzy controllers.

The plant analysed in this paper is a two-dimensional "balls-in-tubes" system - referred to as pressure plant - of two parallelly-connected tubes mounted on a shared air inlet tank that supplies an air for both tubes, where the balls are lifted or lowered and kept suspended at predetermined heights in each tube.

In our previous papers [11-13], the digital self-tuning PID control with optimization of closed-loop parameters and sampling period was being developed for pressure plant control. Therefore, as an alternative to self-tuning PID control for pressure plant, the predictor-based self-tuning control with constraints [8, 9] for the plant is analysed in this paper.

## 2. The pressure plant

The scheme of pressure plant is depicted in Fig. 1.

The plant consists of four main components: shared air inlet and outlet tanks, two air chambers and two tubes with balls in them. The air from the inlet tank flows to air channels through air chambers and leaves the equipment through the upper outlet tank. The distance to balls is measured using ultrasound distance sensors. The fans are used to create pressure in the air channels in order to lift the balls in tubes. The air chambers are utilized for the purpose to stabilize oscillations of the pressure in each tube. The momentum of the fan, the inductance of the fan motor, air turbulence in the tube leads to complex physics governing ball behaviour. Slightly different weights of the balls and the location of the air feeding vent additionally impact the behaviour of ball in the tubes.

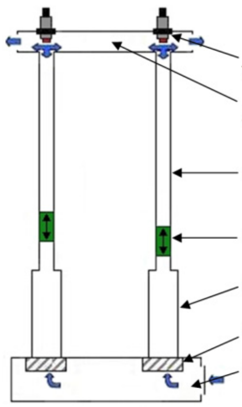



**Figure 1.** Scheme of the pressure plant

The control signals (inputs) of the process are the voltage values for each fan from the range 0 to 10V. The intermediate values of voltage affect the power of the fan proportionately. The control responses (outputs) are the distances between the balls and the bottom of their tubes from the range 20 to 90 in centimetres. The control problem is to regulate the speed of the fan supplying the air into the tube so as to keep the ball suspended at some pre-determined level in the tube.

Analysis of dynamic characteristics of each tube of the pressure plant showed that when the balls are lifted those are close to the integral process and when are lowered are close to the aperiodic process of the second order system. Also their dependency on input signals (balls are lifted) or height (balls are lowered) levels have been determined. Therefore, it is reasonable to apply adaptive control techniques for the pressure plant.

## 3. Predictor-based Self-tuning Control with Constraints

Each tube of the plant is defined by discrete linear second order input-output model

$$A^{(i)}\left(z^{-1}\right)y_t^{(i)} = B^{(i)}\left(z^{-1}\right)u_t^{(i)} + \xi_t^{(i)}, \tag{1}$$

$$\begin{aligned} A^{(i)}\left(z^{-1}\right) &= 1 + a_1^{(i)} z^{-1} + a_2^{(i)} z^{-2}, \\ B^{(i)}\left(z^{-1}\right) &= b_1^{(i)} z^{-1} + b_2^{(i)} z^{-2}, \end{aligned} \tag{2}$$

where $A^{(i)}(z^{-1})$, $B^{(i)}(z^{-1})$ are the model polynomials, $i = 1, 2$ is the index of the tube of pressure process, $y_t^{(i)} = y^{(i)}(tT_0)$, $u_t^{(i)} = u^{(i)}(tT_0)$ are output and input signals with sampling period $T_0$, respectively, $\xi_t^{(i)}$ is a white noise of the $i$th tube with a zero mean and finite variance and $z^{-1}$ is a backward-shift operator ( $z^{-1}y_t^{(i)} = y_{t-1}^{(i)}$ ). Mathematical model of the second order (1)-(2) was chosen because of the fact that such order models are utilized for building PID control laws [11].

A predictor-based digital control law for each tube of the plant is synthesized by minimizing minimum variance control quality criterion in an admissible domain [8, 9]:

$$u_{t+1}^{(i)*} : \; Q_t^{(i)}\left(u_{t+1}^{(i)}\right) \rightarrow \min_{u_{t+1}^{(i)} \in \Omega_u^{(i)}}, \quad i = 1,2 \tag{3}$$

$$Q_t^{(i)}\left(u_{t+1}^{(i)}\right) = \mathrm{M}\left(y_{t+2}^{(i)*} - y_{t+2}^{(i)}\right)^2, \tag{4}$$

$$\Omega_u^{(i)} = \left\{ u_{t+1}^{(i)} : \begin{array}{c} u_{\min}^{(i)} \le u_{t+1}^{(i)} \le u_{\max}^{(i)} \\ \left| u_{t+1}^{(i)} - u_t^{(i)*} \right| < \delta_t^{(i)} \end{array} \right\}, \tag{5}$$

where $y_t^{(i)*}$ is a reference signal of the $i$th tube, $u_{\min}^{(i)}$, $u_{\max}^{(i)}$ are the control signal boundaries of the $i$th tube, $\delta_t^{(i)} > 0$ is the restriction value for the change rate of the control signal, $\mathrm{M}$ is a mathematical expectation symbol.

Then, solving the minimization problem (3)-(5) for the model (1)-(2), the control law of the $i$th tube is described by equations

$$u_{t+1}^{(i)*} = \begin{cases} \min\left\{u_{\max}^{(i)}, u_t^{(i)} + \delta_t^{(i)}, \tilde{u}_{t+1}^{(i)}\right\}, & \text{if } \tilde{u}_{t+1}^{(i)} \ge u_t^{(i)} \\ \max\left\{u_{\min}^{(i)}, u_t^{(i)} - \delta_t^{(i)}, \tilde{u}_{t+1}^{(i)}\right\}, & \text{otherwise} \end{cases}, \tag{6}$$

$$\tilde{B}_t^{(i)}\left(z^{-1}\right)\tilde{u}_{t+1}^{(i)} = \left[-L_t^{(i)}\left(z^{-1}\right)y_t^{(i)} + zy_{t+1}^{(i)*}\right], \tag{7}$$

$$\tilde{B}_t^{(i)}\left(z^{-1}\right) = \tilde{b}_{0t}^{(i)} + \tilde{b}_{1t}^{(i)} z^{-1} + \tilde{b}_{2t}^{(i)} z^{-2}, \tag{8}$$

$$\begin{aligned} L_t^{(i)}\left(z^{-1}\right) &= l_{0t}^{(i)} + l_{1t}^{(i)} z^{-1} = \\ &= \left[\left(\hat{a}_{1t}^{(i)}\right)^2 - \hat{a}_{2t}^{(i)}\right] + \left[\hat{a}_{1t}^{(i)}\hat{a}_{2t}^{(i)}\right] z^{-1}, \end{aligned} \tag{9}$$

where

$$\tilde{b}_{0t}^{(i)} = \begin{cases} \hat{b}_{1t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\ \hat{b}_{2t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1 \\ -\hat{a}_{1t}^{(i)}\hat{b}_{1t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\ -\hat{a}_{1t}^{(i)}\hat{b}_{2t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1 \end{cases}, \tag{10}$$

$$\tilde{b}_{1t}^{(i)} = \begin{cases} \hat{b}_{2t}^{(i)} - \hat{a}_{1t}^{(i)}\hat{b}_{1t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\ \hat{b}_{1t}^{(i)} - \hat{a}_{1t}^{(i)}\hat{b}_{2t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1 \\ \hat{b}_{1t}^{(i)} - \hat{a}_{1t}^{(i)}\hat{b}_{2t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\ \hat{b}_{2t}^{(i)} - \hat{a}_{1t}^{(i)}\hat{b}_{1t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1 \end{cases}, \qquad (11)$$

$$\tilde{b}_{2t}^{(i)} = \begin{cases} -\hat{a}_{1t}^{(i)}\hat{b}_{2t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\ -\hat{a}_{1t}^{(i)}\hat{b}_{1t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1 \\ \hat{b}_{2t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\ \hat{b}_{1t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1 \end{cases}, \qquad (12)$$

where $z$ is a forward-shift operator ( $zy_t^{(i)*} = y_{t+1}^{(i)*}$ ).

Coefficients of polynomial $L_t^{(i)}\left(z^{-1}\right)$ are found from equation

$$1 = \hat{A}_t^{(i)}\left(z^{-1}\right)F_t^{(i)}\left(z^{-1}\right) + z^{-2}L_t^{(i)}\left(z^{-1}\right), \qquad (13)$$

where

$$F_t^{(i)}\left(z^{-1}\right) = 1 + f_{1t}^{(i)}z^{-1} = 1 - \hat{a}_{1t}^{(i)}z^{-1}. \qquad (14)$$

The coefficients (10)-(12) are obtained by applying factorization method [1] to polynomial

$$\tilde{B}_t^{(i)}\left(z^{-1}\right) = \overline{B}_t^{(i)}\left(z^{-1}\right)F_t^{(i)}\left(z^{-1}\right), \qquad (15)$$

where

$$\overline{B}_t^{(i)}\left(z^{-1}\right) = \hat{b}_{1t}^{(i)} + \hat{b}_{2t}^{(i)}z^{-1}. \qquad (16)$$

In each expression of the coefficients (10)-(12), the first and the third conditions correspond to minimum-phase model, while the second and the fourth - to nonminimum-phase. Model is called to be a minimum-phase model if polynomial

$$\overline{B}_t^{(i)}(z) = z\overline{B}_t^{(i)}\left(z^{-1}\right) \qquad (17)$$

root

$$\left|z_t^{(i)}\right| \le 1, \quad z_t^{(i)} = -\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}, \qquad (18)$$

otherwise, model is called to be a nonminimum-phase model.

The transfer function of closed-loop of the $i$th tube for a control law (6)-(12) is as follows

$$W_{cl,t}^{(i)}\left(z^{-1}\right) = \begin{cases} 1, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\ \dfrac{\hat{B}_t^{(i)}\left(z^{-1}\right)}{D_t^{(i)}\left(z^{-1}\right)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1 \\ \dfrac{1}{D_t^{(i)}\left(z^{-1}\right)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\ \dfrac{\hat{B}_t^{(i)}\left(z^{-1}\right)}{D_t^{(i)}\left(z^{-1}\right)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1 \end{cases}, \qquad (19)$$

where characteristic polynomial of closed-loop of the $i$th tube is defined by expressions

$$D_t^{(i)}\left(z^{-1}\right) = \overline{B}_t^{(i)}\left(z^{-1}\right) + \hat{A}_t^{(i)}\left(z^{-1}\right)F_t^{(i)}\left(z^{-1}\right) \times \times\left[\overline{B}\bullet_t^{(i)}\left(z^{-1}\right) - \overline{B}_t^{(i)}\left(z^{-1}\right)\right], \quad \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1, \qquad (20)$$

$$D_t^{(i)}\left(z^{-1}\right) = 1 + \hat{A}_t^{(i)}\left(z^{-1}\right) \times \left[F\bullet_t^{(i)}\left(z^{-1}\right) - F_t^{(i)}\left(z^{-1}\right)\right], \quad \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1, \qquad (21)$$

$$D_t^{(i)}\left(z^{-1}\right) = \overline{B}_t^{(i)}\left(z^{-1}\right) + \hat{A}_t^{(i)}\left(z^{-1}\right) \times \times\left[\overline{B}\bullet_t^{(i)}\left(z^{-1}\right)F\bullet_t^{(i)}\left(z^{-1}\right) - \overline{B}_t^{(i)}\left(z^{-1}\right)F_t^{(i)}\left(z^{-1}\right)\right], \quad \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1, \qquad (22)$$

where

$$\overline{B}\bullet_t^{(i)}\left(z^{-1}\right) = \hat{b}_{2t}^{(i)} + \hat{b}_{1t}^{(i)}z^{-1}, \quad F\bullet_t^{(i)}\left(z^{-1}\right) = -\hat{a}_{1t}^{(i)} + z^{-1}. \qquad (23)$$

In order to ensure the stability of closed-loop of such a control system, all the roots of each polynomial

$$D_t^{(i)}(z) = z^{n_{dt}^{(i)}} D_t^{(i)}\left(z^{-1}\right), \quad i = 1, 2 \qquad (24)$$

must be inside the unit disc at each control step $t$. This property is ensured by the algorithm of on-line identification of the model (1)-(2) [11], which is obtained by minimizing identification quality criterion at the stability region of closed-loop [7]

$$\hat{\Theta}_t^{(i)}: \; Q_t^{(i)}\left(\Theta^{(i)}\right) \to \min_{\Theta^{(i)} \in \Omega_\Theta^{(i)}}, \quad i = 1, 2 \qquad (25)$$

$$Q_t^{(i)}\left(\Theta^{(i)}\right) = \sum_{k=1}^{t} \varphi^{t-k}\left[y_k^{(i)} - y_{k|k-1}^{(i)}\left(\Theta^{(i)}\right)\right]^2, \qquad (26)$$

$$\Omega_\Theta^{(i)} = \left\{\Theta_t^{(i)}: \; \left|z_j^{(i)}\left(\Theta_t^{(i)}\right)\right| < 1, \quad j = 1, \ldots, n_{dt}^{(i)}\right\}, \qquad (27)$$

where

$$y_{t|t-1}^{(i)}\left(\Theta_{t-1}^{(i)}\right) = z\left[1 - A_{t-1}^{(i)}\left(z^{-1}\right)\right]y_{t-1}^{(i)} + B_{t-1}^{(i)}\left(z^{-1}\right)u_t^{(i)} \qquad (28)$$

is a one-step-ahead output prediction of the $i$th tube,

$$\hat{\Theta}_t^{(i)T} = \left[\hat{b}_{1t}^{(i)}, \hat{b}_{2t}^{(i)}, \hat{a}_{1t}^{(i)}, \hat{a}_{2t}^{(i)}\right] \qquad (29)$$

are the estimates of model (1)-(2) parameters, $z_j^{(i)}\left(\Theta_t^{(i)}\right)$ are the roots of characteristic polynomial (24), $0 < \varphi \le 1$ is a weighing constant. The scheme of predictor-based self-tuning controller with constraints is illustrated in Figure 2.

## 4. Sampling period optimization

Applying a digital control for continuous-time plant, there is always an issue how to select a proper value of sampling period $T_0$. On one hand, selecting a relatively

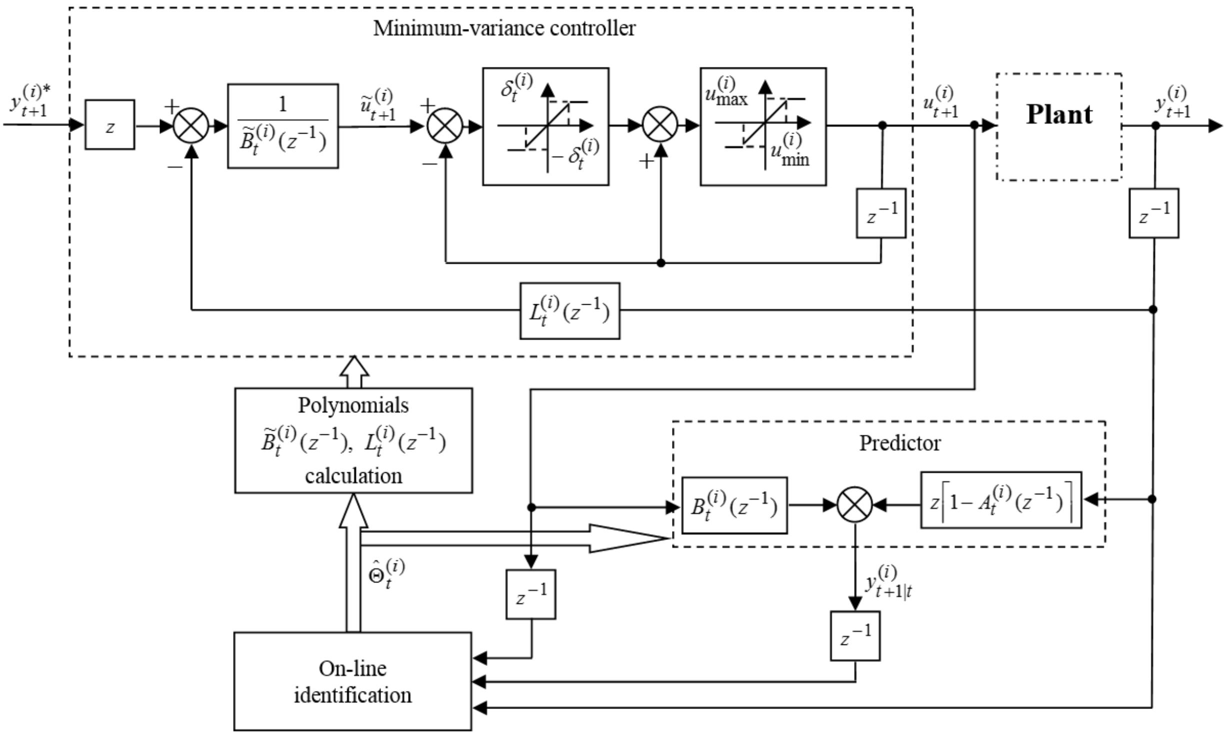



**Figure 2.** Scheme of predictor-based self-tuning controller with constraints for the $i$th tube

short sampling period $T_0$, discrete models (1)-(2) of continuous-time plant move towards their stability boundaries, consequently, complicating on-line identification of discrete model (1)-(2) parameters in closed-loop [7]. On the other hand, choosing a relatively long sampling period $T_0$, control performance of the digital control is significantly decreased. Therefore, some authors [2, 6, 10] proposed to choose a sampling period $T_0$ from certain intervals, which were generally based on the second order characteristic polynomial of closed-loop for PID control.

Sampling period $T_0$ can also be chosen by optimizing the criterion of control quality

$$T_0^*: \quad J_T(T_0) \to \min_{T_0}, \tag{30}$$

$$J_T(T_0) = \frac{1}{N}\sum_{t=1}^{N}\left\{\left(y_t^{(1)*} - y_t^{(1)}\right)^2 + \left(y_t^{(2)*} - y_t^{(2)}\right)^2\right\}, \tag{31}$$

where $N$ is the number of observations.

In order to find an optimal value $T_0^*$ of sampling period in terms of optimization problem (30)-(31), the golden section algorithm has been used [12, 13].

The scheme of the digital predictor-based self-tuning control of the pressure plant is depicted in Fig. 3.

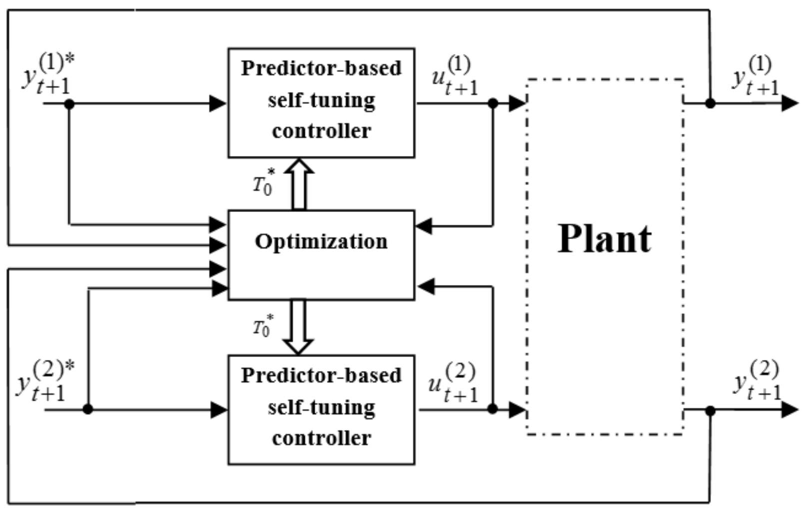



**Figure 3.** Scheme of digital predictor-based self-tuning control of the pressure plant

## 5. Experimental analysis

Realization of the digital predictor-based self-tuning control is performed by employing an industrial Beckhoff BK9000 programmable logic controller (PLC). The PLC controller is configured and controlled by TwinCat software.

The digital predictor-based self-tuning control by optimizing its sampling period $T_0$ and investigating impact of the input signal constraints – amplitude boundaries $u_{\min}^{(i)}, u_{\max}^{(i)}$ and change rate $\delta_t^{(i)}$ - on the control quality of the pressure plant was experimentally analysed. The selected interval for the optimization of the sampling period was $T_0 \in [0.005\text{s}, 2.0\text{s}]$ and the impact of the input signal constraints was investigated in 4 cases:

$$u_{\min}^{(i)} = 3.5\text{V}, u_{\max}^{(i)} = 10\text{V}, \delta_t^{(i)} = 0.5\text{V}, \qquad (32)$$

$$u_{\min}^{(i)} = 3.5\text{V}, u_{\max}^{(i)} = 10\text{V}, \delta_t^{(i)} = 2\text{V}, \qquad (33)$$

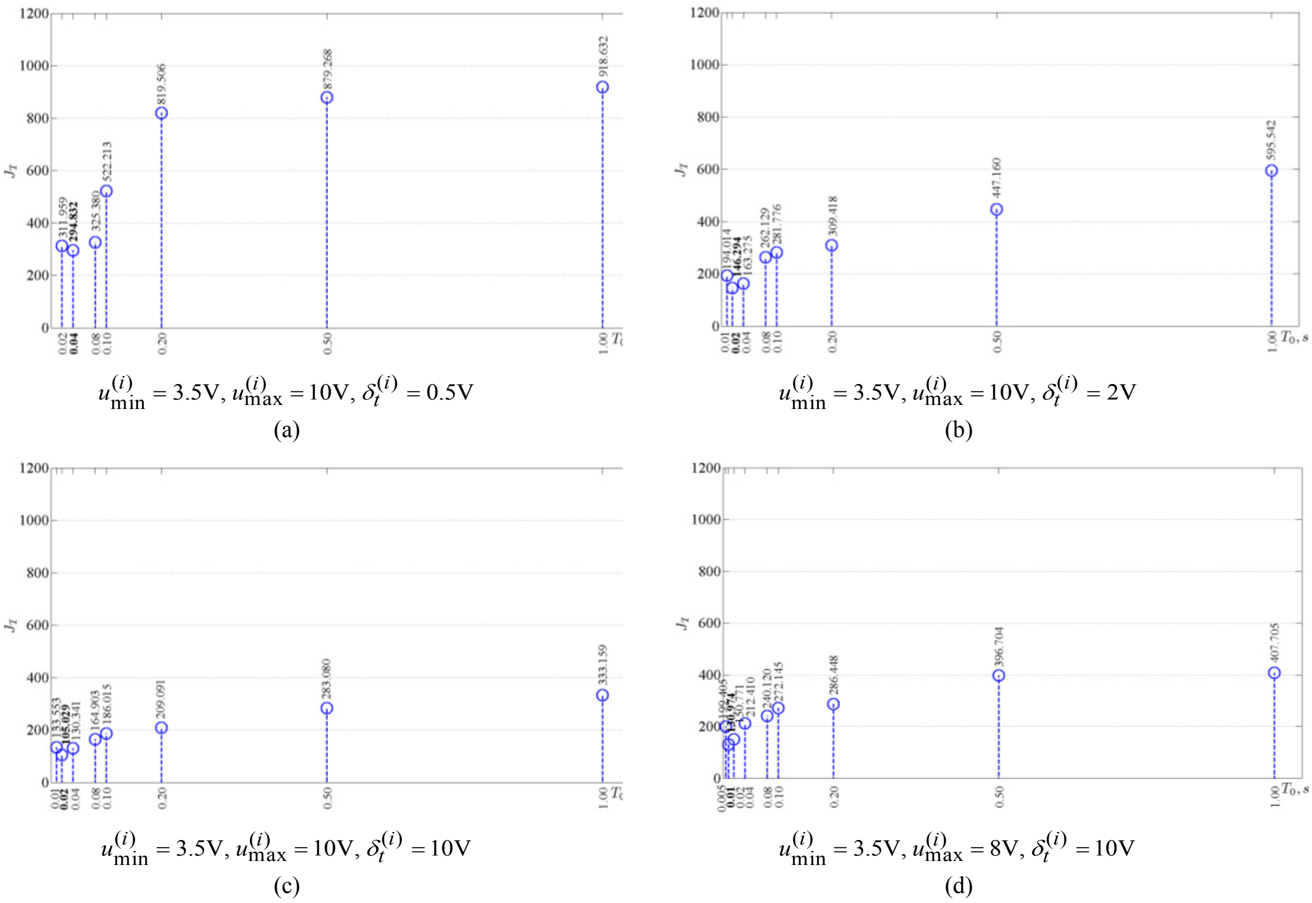
$$u_{\min}^{(i)} = 3.5\text{V}, u_{\max}^{(i)} = 10\text{V}, \delta_t^{(i)} = 10\text{V}, \qquad (34)$$

$$u_{\min}^{(i)} = 3.5\text{V}, u_{\max}^{(i)} = 8\text{V}, \delta_t^{(i)} = 10\text{V}. \qquad (35)$$

The same step-shape reference signal for both tubes has been applied with repeatable values of 75 and 40. The observation time of each signal is 1000s, but only the last 800 seconds are used in criterion calculations, in order to eliminate the influence of initial adaptation process.

The results of sampling period optimization of the digital predictor-based self-tuning control for the pressure plant, in terms of control quality criterion (31), showed that optimal sampling periods are: $T_0^* = 0.04\text{s}$ ( $J_T^* = 294.83$ ) with input signal constraints (32); $T_0^* = 0.02\text{s}$ ( $J_T^* = 146.29$ ), with input signal constraints (33); $T_0^* = 0.02\text{s}$ ( $J_T^* = 105.03$ ), with input signal constraints (34); $T_0^* = 0.01\text{s}$ ( $J_T^* = 130.97$ ), with input signal constraints (35). We obtain that optimal values of sampling period are outside of the intervals [2, 6, 10], which are based on the second order characteristic polynomial of closed-loop for PID control, i.e. optimal values are significantly lower than the lower boundaries of these intervals.

Search process of the optimal sampling period $T_0$ in the digital predictor-based self-tuning control system with various input signal constraints for the pressure plant is depicted in Figure 4.




$u_{\min}^{(i)} = 3.5\text{V}, u_{\max}^{(i)} = 10\text{V}, \delta_t^{(i)} = 0.5\text{V}$

(a)

$u_{\min}^{(i)} = 3.5\text{V}, u_{\max}^{(i)} = 10\text{V}, \delta_t^{(i)} = 2\text{V}$

(b)

$u_{\min}^{(i)} = 3.5\text{V}, u_{\max}^{(i)} = 10\text{V}, \delta_t^{(i)} = 10\text{V}$

(c)

$u_{\min}^{(i)} = 3.5\text{V}, u_{\max}^{(i)} = 8\text{V}, \delta_t^{(i)} = 10\text{V}$

(d)

**Figure 4.** Search process of the optimal sampling period $T_0$ in the digital predictor-based self-tuning control system with various input signal constraints for the pressure plant

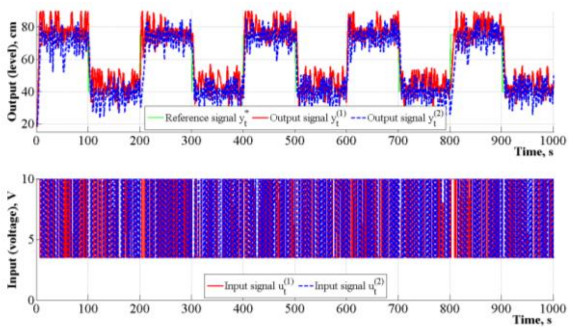
**Table 1.** Control quality criterion (31) dependency on sampling period $T_0$ and various input signals constraints

| $T_0$, s | **Input signals constraints** | | | |
|---|---|---|---|---|
| | **(32)** | **(33)** | **(34)** | **(35)** |
| | $J_T$ | | | |
| 0.005 | - | - | - | 199.41 |
| 0.01 | - | 194.014 | 133.55 | 130.97 |
| 0.02 | 311.96 | 146.29 | 105.03 | 150.77 |
| 0.04 | 294.83 | 163.28 | 130.34 | 212.41 |
| 0.08 | 325.38 | 262.13 | 164.90 | 240.12 |
| 0.1 | 522.21 | 281.78 | 186.02 | 272.15 |
| 0.2 | 819.51 | 309.42 | 209.09 | 286.45 |
| 0.5 | 879.27 | 447.16 | 283.08 | 396.70 |
| 1.0 | 918.63 | 595.54 | 333.16 | 407.71 |

The dependency of control quality criterion (31) on different sampling periods and various input signals constraints is presented in Table 1.

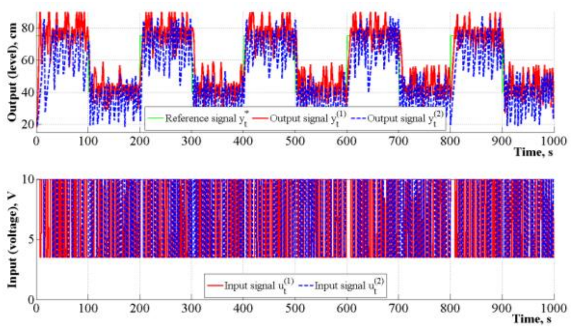
Control performances of the digital predictor-based self-tuning control with input signals constraints (33) for the pressure plant on various sampling periods - $T_0 - (0.02\text{s}, 0.08\text{s}, 0.2\text{s}, 1.0\text{s})$ - are illustrated in Figure 5. It is seen that the selection of sampling period significantly impacts the control quality of the pressure plant.

Control performances of the digital predictor-based self-tuning control with input signals constraints (34) for the pressure plant on various sampling periods - $T_0 - (0.02\text{s}, 0.08\text{s}, 0.2\text{s}, 1.0\text{s})$ - are illustrated in Figure 6. Notice that a high variation of the input signals occur in all of the cases (Figure 5 and Figure 6).



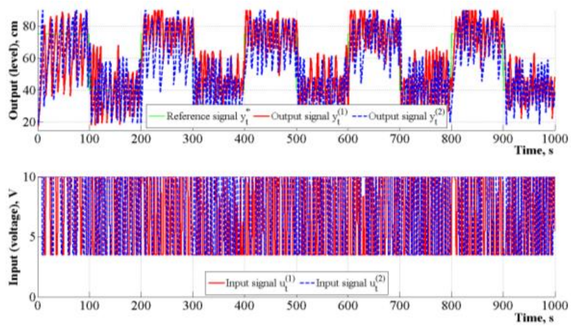

$T_0^* = 0.02\text{s},\ J_T^* = 146.29$

(a)



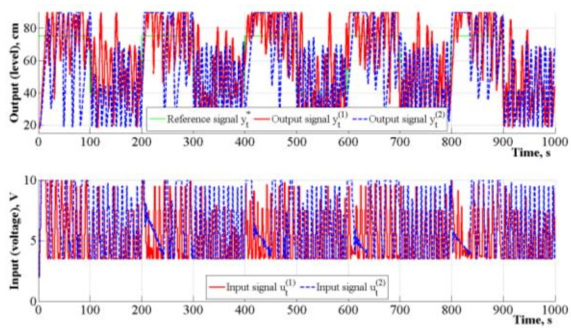

$T_0 = 0.08\text{s},\ J_T = 262.13$

(b)




$T_0 = 0.2\text{s},\ J_T = 309.42$

(c)




$T_0 = 1.0\text{s},\ J_T = 595.54$

(d)

**Figure 5.** Control performance of predictor-based self-tuning control with input signals constraints (33) for the pressure plant

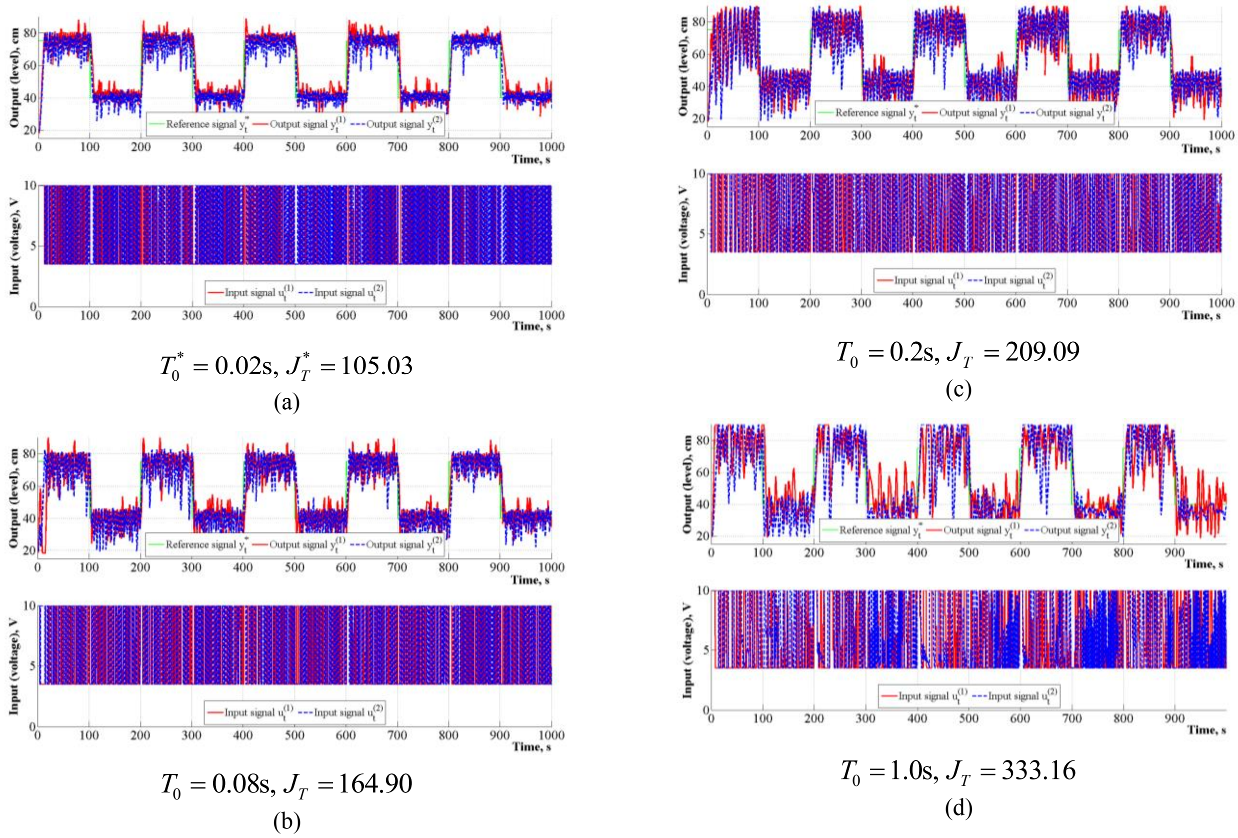



**Figure 6.** Control performance of predictor-based self-tuning control with input signal constraints (34) for the pressure plant

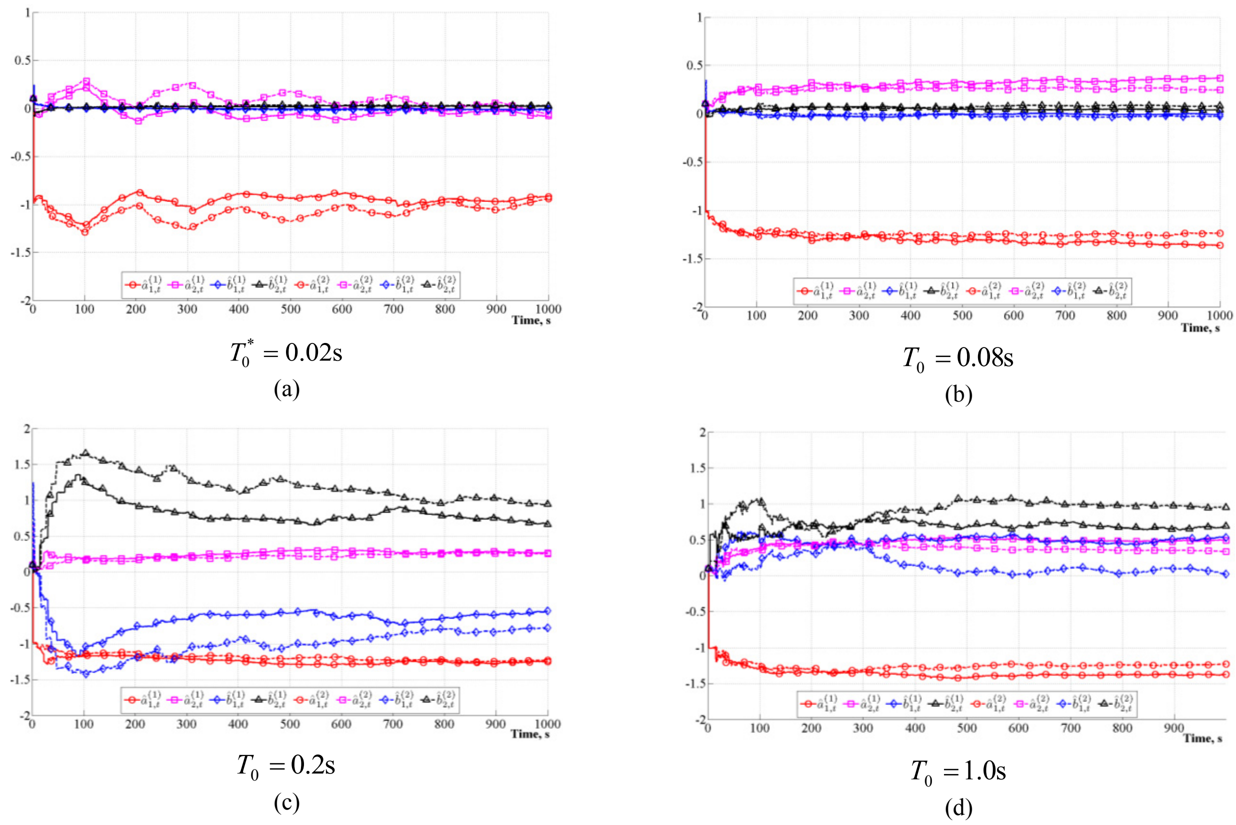



**Figure 7.** On-line identification of models' parameters of predictor-based self-tuning control with input signals constraints (34)

Considering the values of control quality criterion (31), it can be seen that with selected input signals constraints (34) (Figure 6) those values are up to 1.5-2 times smaller / better as compared to criterion values with selected input signals constraints (33) (Figure 5).

On-line identification of models' parameters of predictor-based self-tuning control with input signals constraints (34) and various sampling periods $T_0 - (0.02\text{s}, 0.08\text{s}, 0.2\text{s}, 1.0\text{s})$ is demonstrated in Figure 7. Notice that the choice of sampling period also significantly impacts the estimates of models parameters, especially $\hat{b}_{1t}^{(i)}$, $\hat{b}_{2t}^{(i)}$.

## 6. Conclusions

A digital predictor-based self-tuning control with constraints system for pressure plant has been developed, where on-line identified model of each plant's tube can feature both - minimum-phase and nonminimum-phase.

We showed that applying polynomial factorization for minimum-phase and nonminimum-phase plant models the closed-loops of the control system are changed, and expressions of the transfer functions of closed-loops have been obtained.

In order to ensure the stabilities of closed-loop systems, which models' parameters are obtained via on-line identification, whether all the roots of characteristic polynomials of closed-loops are inside the unit disc at each control step are necessary to track.

We have proposed a method for optimization of sampling period in the digital predictor-based self-tuning control system of the pressure plant.

The impact of the selection of the sampling period and input signals constraints – amplitude boundaries and the change rate - on the control quality of the pressure plant has been analysed. We demonstrated that the predictor-based controller is particularly sensitive to the limitation of the change rate of the input signals. Optimization of sampling period significantly improves the control quality of the pressure plant.

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Received November 2014.