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Synchronization of Chaos in Nonlinear Finance System by means of Sliding Mode and Passive Control Methods: A Comparative Study

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Abstract. In this paper, two different control methods, namely sliding mode control and passive control, are investigated for the synchronization of two identical chaotic finance systems with different initial conditions. Based on the sliding mode control theory, a sliding surface is determined. A Lyapunov function is used to prove that the passive controller provides global asymptotic stability of the system. Numerical simulations validate the synchronization of chaotic finance systems with the proposed sliding mode and passive control methods. The synchronization performance of these two methods is compared and discussed.

Keywords: Chaotic finance system, chaos synchronization, sliding mode control, passive control.

1. Introduction

Financial system dynamics have a significant role in micro- and macroeconomics [6, 10, 39]. The financial and economic systems become more complicated and economic growth changes from low to high financial markets. Based on multiple variables, the process of economical development and growth is more complex. They have some nonlinear factors such as interest rate, the price of goods, investment demand, and stock [25]. Even if an economical system possesses deterministic characteristics, a chaotic behaviour can occur in the financial system. Chaotic systems have sensitive dependence on initial conditions. Because of slight errors, chaotic dynamical systems can lead to completely different trajectories. Hence, the synchronization of chaos in the financial systems is required. It has great importance from the management point of view to avoid undesirable trajectories and make the precise economic adaptation and prediction possible.

The synchronization of chaos has recently received much attention due to its complex behaviour and potential applications in information processing such as secure communication [13, 28, 36], and it becomes one of the major issues in the control engineering area. Many methods have been used in synchronization of chaotic systems including active control [18], sliding mode control [14, 21, 29], adaptive control [22], passive control [31, 32, 34], impulsive control [2], and backstepping design [24]. Among them, the active control method is popular due to its simplicity in implementation and configuration; and it has been applied in synchronization of chaotic finance systems [40]. The sliding mode control is one of the other well-known control methods, and its dynamic performance is determined by the prescribed manifold or sliding surface where a switching structure maintains the control. This method provides discontinuous control by enforcing the system states to stay on the sliding surface [19]. Recently, the sliding mode control has been used to synchronize many chaotic systems [14, 21, 29]. Nowadays, applying the synchronization using only one state controller is preferred due to its considerable significance in reducing the cost and complexity [33, 37]. The passive control method has been gaining importance in synchronization and control of chaotic systems on account of using only a single controller. The main idea of passivity theory is to keep the system internally stable with implementing a controller which renders the closed loop system passive upon the properties of the system. In recent years, the passive control method has been successfully implemented for the synchronization of hyperchaotic Lorenz [31], unified [32], Rikitake [34], and other chaotic systems. The methodology of sliding mode and passive control is studied in many papers [14, 19, 21, 29, 31, 32, 34].

In the last decade, some chaotic finance systems were introduced [3, 8, 25]. The dynamic behaviours of the chaotic finance systems such as equilibrium points, stability, topological structure, Lyapunov exponents and Hopf bifurcation analysis were investigated in detail [1, 10, 25–27, 38, 39]. The control of the chaotic finance systems was implemented with effective speed feedback control [5, 8, 35, 38], linear feedback control [5, 30, 35, 38], adaptive control [5], the selection of gain matrix control [35], the revision of gain matrix control [35], passive control [9], and time-delayed feedback control [6, 11, 39] methods. The control of fractional-order chaotic finance system has been realized using a sliding mode control method [7]. Active controllers [17, 40], nonlinear feedback controllers [4, 16], adaptive controllers [15], and a single controller based on Lyapunov stability theory and linear matrix inequality [20] are employed for synchronizing the chaotic finance systems. To the knowledge of the authors, neither sliding mode control nor passive control approach for the synchronization of the chaotic finance systems exist in the literature.

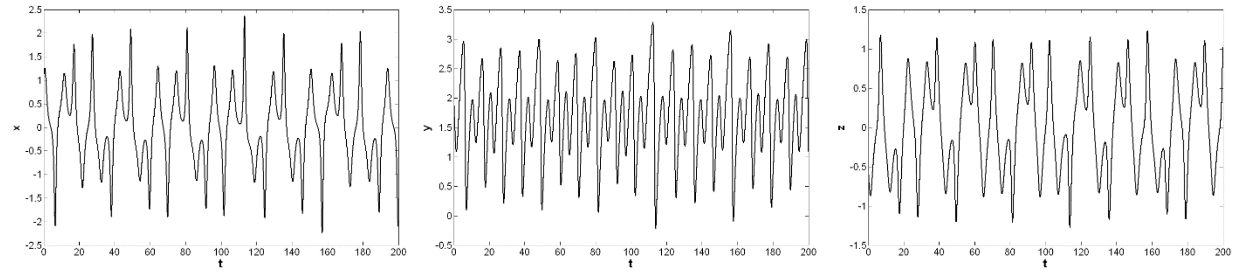
In this study, further investigations on the synchronization of chaotic finance system have been explored. First, a brief description of the chaotic finance system is given. Then, sliding mode controllers are employed for achieving the synchronization of two identical chaotic finance systems. Based on the property of passivity theory, a single passive controller is designed for synchronization of this nonlinear system. Afterwards, numerical simulations are performed for the synchronization of the chaotic finance systems to show the effectiveness of the proposed sliding mode and passive control methods. Finally, the advantages and disadvantages are discussed.

2. Chaotic Finance System

Financial systems consist of enterprise units and markets that interact, generally in a complex manner, for the purpose of economic growth within investment and the demand of commercials. In this study, the considered finance model defines the time variations of three state variables: x is the interest rate, y is the investment demand, and z is the price exponent. The interest rate is the amount charged, expressed as a percentage of principal by a lender to a borrower for the use of assets. Investment demand can be defined as the desired or planned capitals and inventories by the firms. It has a negative relation between investment expenditures and the interest rate. Price exponent determines the variance of the price distribution. The chaotic finance system is described by the set of three first-order differential equations as follows

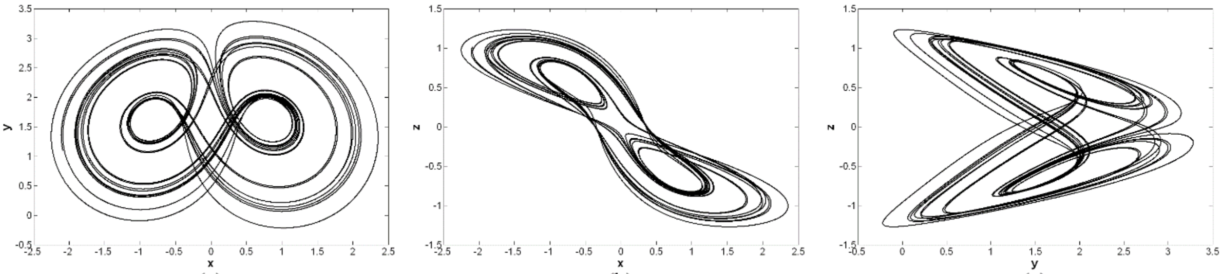
$$\begin{aligned} \dot{x} &= z + (y-a)x, \\ \dot{y} &= 1 - by - x^2, \\ \dot{z} &= -x - cz \end{aligned} \tag{1}$$

where a, b, c are positive constant parameters, and represent the saving amount, the per-investment cost, and the elasticity of demands of commercials, respectively [25]. In a financial system, saving amount means that enterprise unit increases its gross financial. Per-investment cost is defined as the ratio of original cost less distribution received from target funds. The elasticity of demands of commercials is a measure of the relationship between a change in the quantity demanded of a particular good and a change in its price. The nonlinear finance system exhibits chaotic behaviour when the parameter values are taken as $a = 0.9$, $b = 0.2$, and $c = 1.2$ [35]. The time series of the chaotic finance system under the initial conditions $x(0) = 1$, $y(0) = 2$, and $z(0) = -0.5$ are shown in Fig. 1, the 2D phase portraits are shown in Fig. 2, and the 3D phase plane is shown in Fig. 3.



(a) (b) (c)

Figure 1. Time series of chaotic finance system for (a) *x* signals, (b) *y* signals, (c) *z* signals



(a) (b) (c)

Figure 2. Phase portraits of chaotic finance system in (a) *x–y* phase plane, (b) *x–z* phase plane, (c) *y–z* phase plane

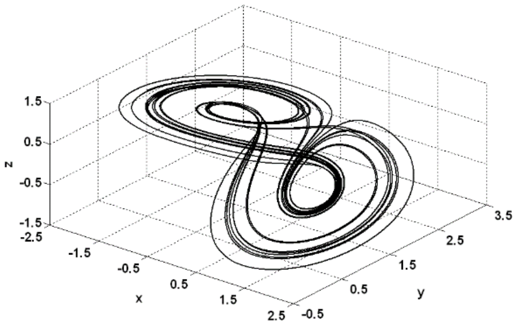


Figure 3. 3D phase plane of chaotic finance system

3. The Synchronization of Chaotic Finance Systems using Sliding Mode Control

The parameters a, b and c are taken in a range to ensure the system (1) will display chaotic behaviour. In order to observe the synchronization, it is assumed that two chaotic finance systems are taken where the drive system controls the response system. The initial position on the drive system is different from that of the response system. The drive system is denoted by subscript 1 and the response system is denoted by subscript 2. The drive system is given by:

$$\begin{aligned} \dot{x}_1 &= z_1 + (y_1 - a)x_1, \\ \dot{y}_1 &= 1 - by_1 - {x_1}^2, \\ \dot{z}_1 &= -x_1 - cz_1, \end{aligned} \tag{2}$$

and the response system is defined as:

$$\begin{aligned} \dot{x}_2 &= z_2 + (y_2 - a)x_2 + u_1(t), \\ \dot{y}_2 &= 1 - by_2 - {x_2}^2 + u_2(t), \\ \dot{z}_2 &= -x_2 - cz_2 + u_3(t) \end{aligned} \tag{3}$$

where $u_1(t)$, $u_2(t)$, and $u_3(t)$ in Eq. (3) are the sliding mode control functions to be determined. The drive system is subtracted from response system to obtain the control function for synchronization. The e_1, e_2, and e_3 state errors between finance system (3) that is to be controlled and the controlling finance system (2) are defined as

$$\begin{aligned} e_1 &= x_2 - x_1, \\ e_2 &= y_2 - y_1, \\ e_3 &= z_2 - z_1. \end{aligned} \tag{4}$$

Thus, the error dynamics become

$$\begin{aligned} \dot{e}_1 &= e_3 - ae_1 + x_2 y_2 - x_1 y_1 + u_1(t), \\ \dot{e}_2 &= -be_2 - {x_2}^2 + {x_1}^2 + u_2(t), \\ \dot{e}_3 &= -e_1 - ce_3 + u_3(t). \end{aligned} \tag{5}$$

The error dynamics (5) can be regularized in matrix notation as

$$\dot{e} = Ae + \eta(x, y) + u \tag{6}$$

where

$$A = \begin{bmatrix} -a & 0 & 1 \\ 0 & -b & 0 \\ -1 & 0 & -c \end{bmatrix}, \quad \eta(x, y) = \begin{bmatrix} x_2 y_2 - x_1 y_1 \\ -x_2^2 + x_1^2 \\ 0 \end{bmatrix}, \quad u = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}. \tag{7}$$

According to the sliding mode control methodology, the control signal u is defined as [29]:

$$u(t) = -\eta(x, y) + Bv(t) \tag{8}$$

where v is a control signal, and B is a matrix. B is chosen so that (A, B) will be controllable. Therefore, B is taken as

$$B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}. \tag{9}$$

The sliding surface must be selected so that the system dynamics can remain stable. In order to acquire the sliding surface, the system is transformed into regular form and the sliding surface coefficients are evaluated by using regular form [23]. If Eq. (8) is substituted into Eq. (6), the system alters to the following linear form:

$$\dot{e} = Ae + Bv, \tag{10}$$

where $A \in R^{nxn}$, $B \in R^{nxm}$, $e \in R^n$, and $v \in R^m$. The error dynamics of system (10) are separated into two subsystems and one of them includes a control signal. In order to transform the system into its regular form, a non-singular transformation can be used as follows:

$$z = Te, \tag{11}$$

where T is a non-singular transformation matrix. When Eq. (11) is substituted into the linear form (10), the following alternative system, which consists of two subsystems, is revealed as

$$\begin{aligned} \dot{z}_1 &= A_{11}z_1 + A_{12}z_2, \\ \dot{z}_2 &= A_{21}z_1 + A_{22}z_2 + Lv, \end{aligned} \tag{12}$$

where L is a gain matrix. Then, the sliding surface design is considered as

$$s(t) = Sz = S_1 z_1 + S_2 z_2 = 0, \tag{13}$$

where $S_1 \in R^{1x(n-m)}$, and $S_2 \in R^1$. Solving for z_1 in Eq. (13) and substituting z_1 into Eq. (12) yields

$$\dot{z}_1 = [A_{11} - A_{12}S_2{}^{-1}S_1]\,z_1, \tag{14}$$

which renders the ideal sliding motion. Since the dynamics of z_2 depend on z_1, the stabilization of z_1 stabilizes z_2. According to the dynamics of z_1, the eigenvalues of the expression $A_{11} - A_{12}S_2^{-1}S_1$ should be in the left-half s-plane so that the dynamics of z_1 are asymptotically stable. In order to find $S_2^{-1}S_1$, pole replacement and optimal control techniques can be used. S_2 may be arbitrary selected on condition that it is not singular. After that, S_1 is calculated according to S_2. Now, the sliding surface equation becomes

$$s(t) = Sz = STe = Ce. \tag{15}$$

This implies

$$C = ST. \tag{16}$$

The eigenvalues of $A_{11} - A_{12}S_2^{-1}S_1$ have been placed in the left-half s-plane. Then S_2^{-1} has been selected as identity matrix and so S_1 is calculated [12]. From Eq. (16), the sliding surface vector C has been determined as [–1.75 2.75 0]. Then, the sliding mode state equation gives asymptotically stable behaviour, when the sliding mode variable is designed as

$$s = Ce = [-1.75 \quad 2.75 \quad 0]e = -1.75e_1 + 2.75e_2. \tag{17}$$

From the property of the sliding mode control theory [14]:

$$v(t) = -(CB)^{-1}[C(kI + A)e + q\text{sign}(s)]. \tag{18}$$

where k and q are the sliding mode control parameters. A large value of k can cause chattering; an appropriate value of q reduces chattering and the time to reach the sliding surface.

Now, the $v(t)$ control signal becomes

$$\begin{aligned} &v(t) = 1.75(k-a)e_1 - 2.75(k-b)e_2 + \\ &1.75e_3 - q\text{sign}\,(-1.75e_1 + 2.75e_2). \end{aligned} \tag{19}$$

Then, the required sliding mode control signal is obtained as Eq. (8) where $\eta(e)$ and B are described as in Eqs. (7) and (9), respectively:

$$\begin{aligned} u_1(t) &= -x_2 y_2 + x_1 y_1 + v(t), \\ u_2(t) &= x_2^2 - x_1^2 + v(t), \\ u_3(t) &= 0. \end{aligned} \tag{20}$$

The synchronization of chaotic finance system (3) by using the sliding mode control method is completed with Eqs. (19) and (20). Hence, the synchronization of two identical chaotic finance systems by means of sliding mode control is achieved.

4. The Synchronization of Chaotic Finance Systems using Passive Control

The drive system is again taken to be:

$$\begin{aligned} \dot{x}_1 &= z_1 + (y_1 - a)x_1, \\ \dot{y}_1 &= 1 - by_1 - x_1{}^2, \\ \dot{z}_1 &= -x_1 - cz_1, \end{aligned} \tag{21}$$

and the response system is defined as:

$$\begin{aligned} \dot{x}_2 &= z_2 + (y_2 - a)x_2 + u(t), \\ \dot{y}_2 &= 1 - by_2 - x_2{}^2, \\ \dot{z}_2 &= -x_2 - cz_2 \end{aligned} \tag{22}$$

where $u(t)$ in Eq. (22) is the passive control function to be determined. As in the sliding mode control, the drive system is subtracted from response system to obtain the synchronization error. Then,

$$\begin{aligned} \dot{e}_1 &= e_3 - ae_1 + x_2 y_2 - x_1 y_1 + u(t), \\ \dot{e}_2 &= -be_2 - x_2{}^2 + x_1{}^2, \\ \dot{e}_3 &= -e_1 - ce_3 \end{aligned} \tag{23}$$

where e_1, e_2, and e_3 are the state errors and system (23) is called the error system.

One term of system (23) can be written as

$$-x_2{}^2 + x_1{}^2 = (x_1 + x_2)(x_1 - x_2) = -(x_1 + x_2)e_1. \tag{24}$$

So, error system (23) can be rewritten in the following form:

$$\begin{aligned} \dot{e}_1 &= e_3 - ae_1 + x_2 y_2 - x_1 y_1 + u(t), \\ \dot{e}_2 &= -be_2 - (x_1 + x_2)e_1, \\ \dot{e}_3 &= -e_1 - ce_3. \end{aligned} \tag{25}$$

The purpose is to determine the passive controller $u(t)$ for stabilizing error system (25) at a zero equilibrium point. By assuming that the state variable e_1 is the output of the system and supposing $Y = e_1$, $Z_1 = e_2$, $Z_2 = e_3$, $z = [Z_1\ Z_2]^T$, then system (25) can be denoted by normal form:

$$\begin{aligned}\dot{Z}_1 &= -bZ_1 - (x_1 + x_2)Y,\\ \dot{Z}_2 &= -Y - cZ_2,\\ \dot{Y} &= Z_2 - aY + x_2 y_2 - x_1 y_1 + u(t).\end{aligned} \tag{26}$$

The passive control theory has the following generalized form [31]:

$$\begin{aligned}\dot{Z} &= f_0(Z) + p(Z,Y)Y,\\ \dot{Y} &= b(Z,Y) + a(Z,Y)u,\end{aligned} \tag{27}$$

and according to system (26):

$$\begin{aligned} f_0(Z) &= \begin{bmatrix} -bZ_1 \\ -cZ_2 \end{bmatrix},\\ p(Z,Y) &= \begin{bmatrix} -(x_1+x_2) \\ -1 \end{bmatrix},\\ b(Z,Y) &= Z_2 - aY + x_2 y_2 - x_1 y_1,\\ a(Z,Y) &= 1.\end{aligned} \tag{28}$$

As in [31], let the storage function is chosen as

$$V(Z,Y) = W(Z) + \frac{1}{2}(Y^2), \tag{29}$$

where $W(Z) = \frac{1}{2}(Z_1^2 + Z_2^2)$ is a Lyapunov function of $f_0(Z)$ with $W(0) = 0$. Then,

$$\begin{aligned}\frac{d}{dt}V(Z,Y) &= \frac{\partial W(Z)}{\partial Z}\dot{Z} + Y\dot{Y}\\ &= \frac{\partial W(Z)}{\partial Z} f_0(Z) + \frac{\partial W(Z)}{\partial Z} p(Z,Y)Y\\ &+ Yb(Z,Y) + Ya(Z,Y)u.\end{aligned} \tag{30}$$

According to Eq. (28), by taking the derivative of $W(Z)$

$$\begin{aligned}\dot{W}(Z) &= \frac{d}{dt}W(Z) = \frac{\partial W(Z)}{\partial Z} f_0(Z) = \begin{bmatrix} Z_1 & Z_2 \end{bmatrix}\begin{bmatrix} -bZ_1 \\ -cZ_2 \end{bmatrix}\\ &= -bZ_1{}^2 - cZ_2{}^2.\end{aligned} \tag{31}$$

Since $W(Z) \ge 0$ and $\dot{W}(Z) \le 0$, it can be concluded that $W(Z)$ is the Lyapunov function of $f_0(Z)$ and that $f_0(Z)$ is globally asymptotically stable [34]. The controlled system (25) is equivalent to a passive system and can be asymptotically globally stabilized at its zero equilibrium by the following state feedback controller [32]:

$$\begin{aligned} u(t) &= a(Z,Y)^{-1}\left[-b^T(Z,Y) - \frac{\partial W(Z)}{\partial Z} p(Z,Y) - \alpha Y + v \right]\\ &= 1^{-1}\left[-(Z_2 - aY + x_2 y_2 - x_1 y_1) - \begin{bmatrix} Z_1 & Z_2 \end{bmatrix}\begin{bmatrix} -(x_1+x_2) \\ -1 \end{bmatrix} - \alpha Y + v \right]\\ &= aY - x_2 y_2 + x_1 y_1 + (x_1 + x_2)Z_1 - \alpha Y + v\end{aligned} \tag{32}$$

where α is a positive constant, and v is an external input signal. By noting $Z_1 = e_2$, $Z_2 = e_3$ and $Y = e_1$ conversions, the passive control function becomes

$$u(t) = ae_1 - x_2 y_2 + x_1 y_1 + (x_1 + x_2)e_2 - \alpha e_1 + v. \tag{33}$$

The synchronization of chaotic finance system (22) by using the passive control method is completed with Eq. (33). Therefore, the synchronization of two identical chaotic finance systems by means of passive control is achieved.

5. Numerical simulations

In this section, numerical simulations are performed using MATLAB™ to demonstrate the synchronization of two identical chaotic finance systems. The fourth-order Runge–Kutta method with fixed step size being equal to 0.001 is used to simulate the system. The parameter values of nonlinear finance systems are taken as $a = 0.9$, $b = 0.2$, and $c = 1.2$ to ensure chaotic behaviour [35]. The initial values are chosen as $x_1(0) = 1$, $y_1(0) = 2$, $z_1(0) = -0.5$, $x_2(0) = -1$, $y_2(0) = 1.7$, $z_2(0) = 0.5$. For reducing the chattering, the sliding mode control coefficient q is considered as 0.1. The passive control coefficient v is needed for controlling a chaotic system to its non-zero equilibrium points. Since the synchronization is stabilizing the errors between drive and response system towards to zero, v has to be 0. In order to determine the proper values of k and α control coefficients, they are varied from 1 to 7 with 2 increments. Figs. 4 and 5 show the synchronization error signals for k and α coefficients when the controllers are activated at $t = 25$.

As seen in Figs. 4 and 5, when the sliding mode coefficient k and the passive control coefficient α are greater then 1, the synchronization errors are not changing so much. Bigger k and α choices give slightly better results, but they can cause some difficulties in realization. As a consequence, k and α coefficients are taken as 5 in the simulations. When the sliding mode controllers and the passive controller are activated at $t = 20$, $t = 25$, and $t = 30$, the observed simulation results for the synchronization of two identical chaotic finance systems are shown in Figs. 6–8, respectively. The error signals of synchronization are demonstrated in Figs. 9–11, respectively.

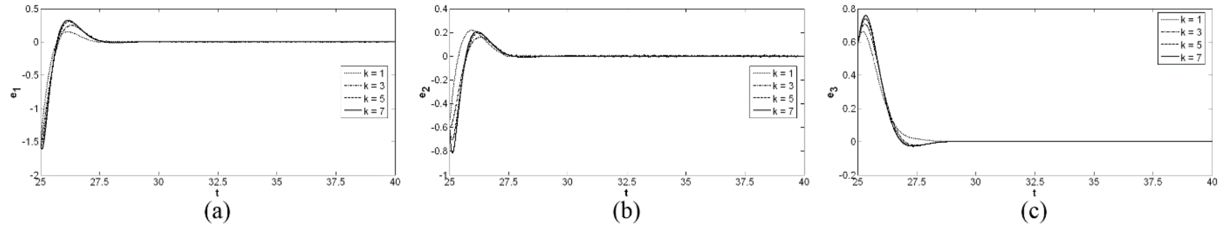


Figure 4. The effect of *k* coefficient to synchronization errors when the sliding mode controllers are activated at t = 25 (a) e_1 signals, (b) e_2 signals, (c) e_3 signals

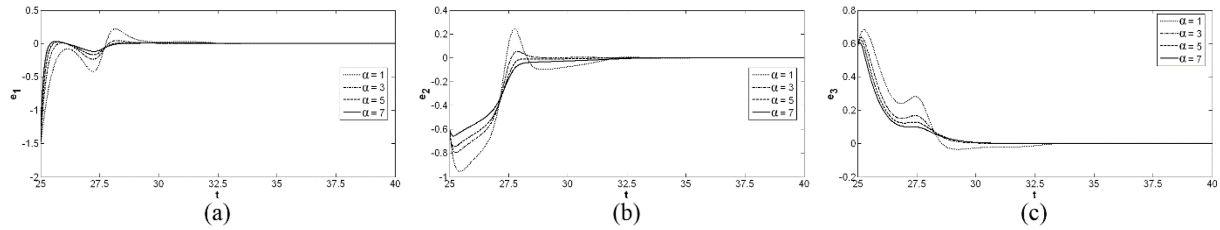


Figure 5. The effect of *α* coefficient to synchronization errors when the passive controller is activated at t = 25 (a) e_1 signals, (b) e_2 signals, (c) e_3 signals

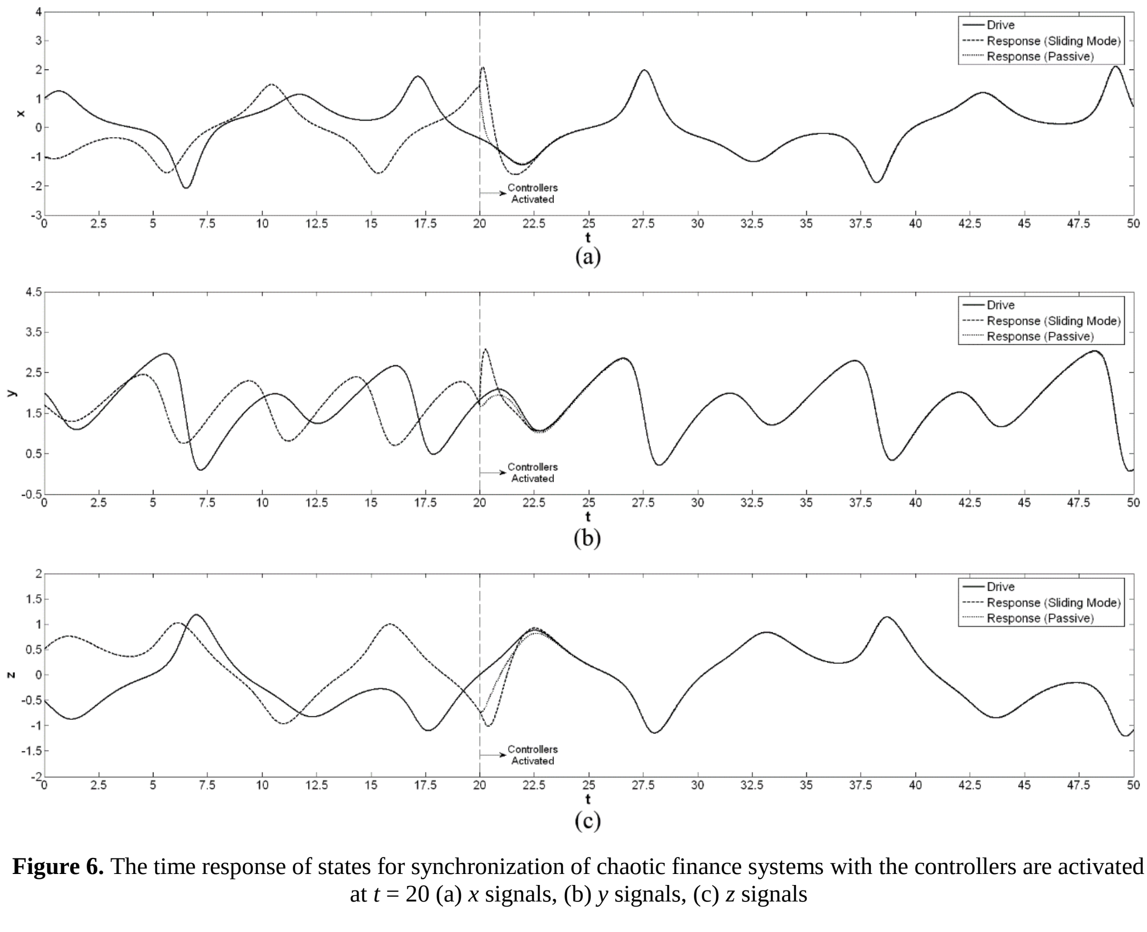
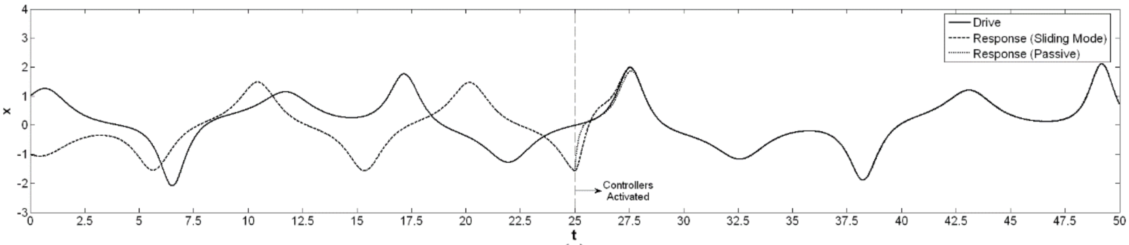
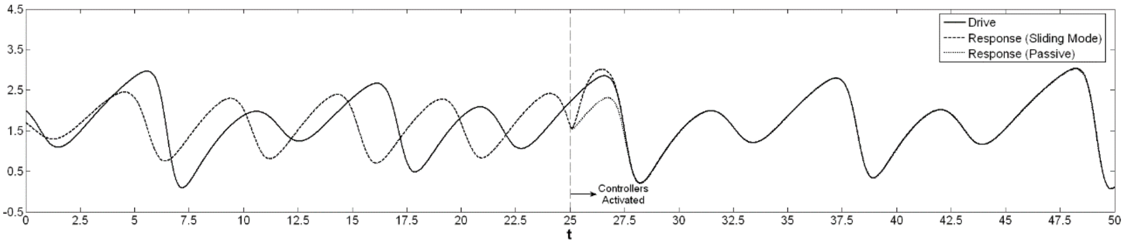


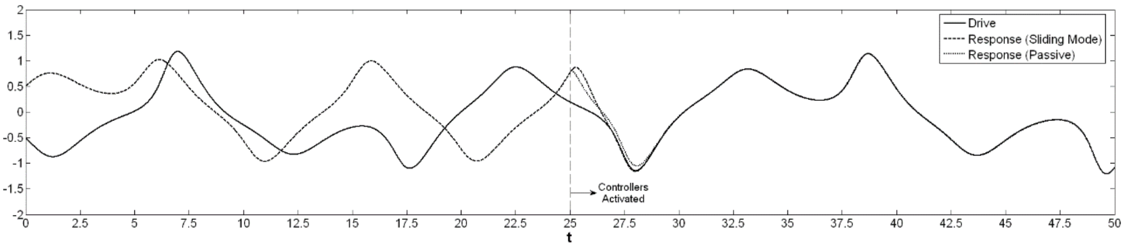
Figure 6. The time response of states for synchronization of chaotic finance systems with the controllers are activated at t = 20 (a) x signals, (b) y signals, (c) z signals



(a)

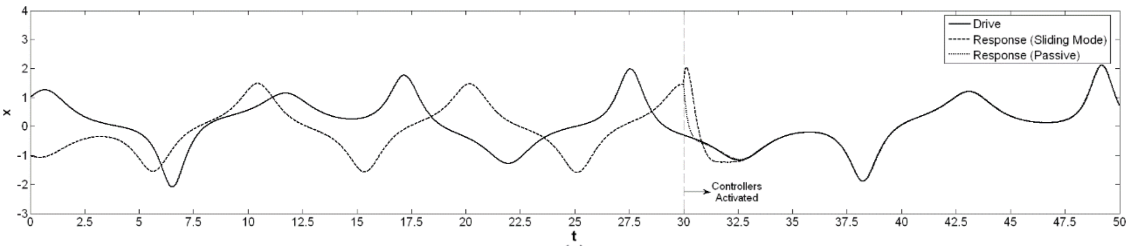


(b)

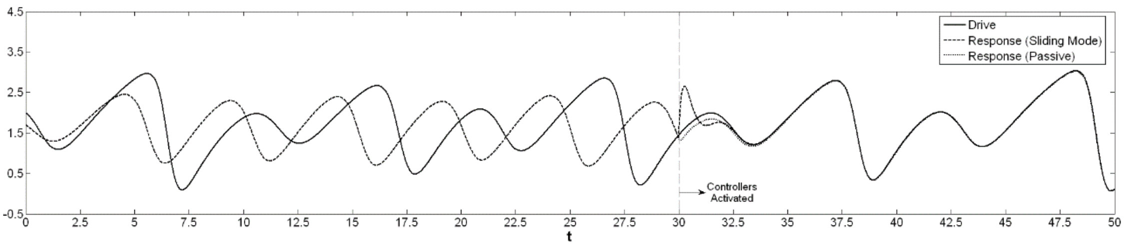


(c)

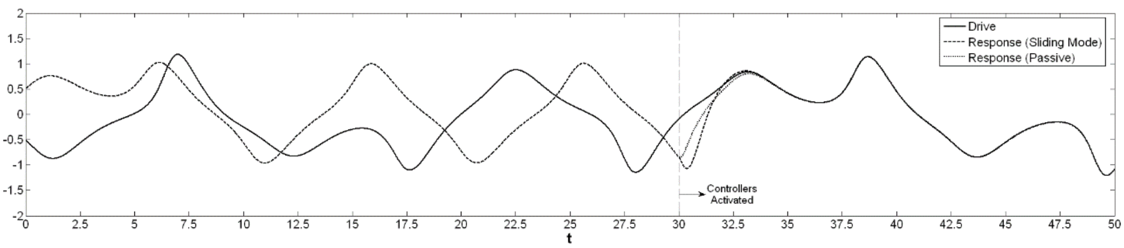
Figure 7. The time response of states for synchronization of chaotic finance systems with the controllers are activated at t = 25 (a) x signals, (b) y signals, (c) z signals



(a)

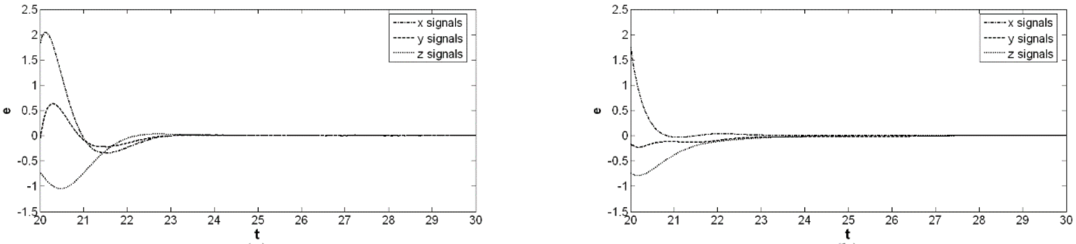


(b)



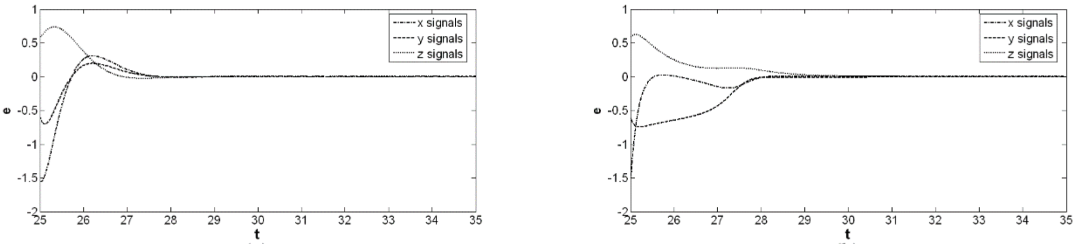
(c)

Figure 8. The time response of states for synchronization of chaotic finance systems with the controllers are activated at t = 30 (a) x signals, (b) y signals, (c) z signals



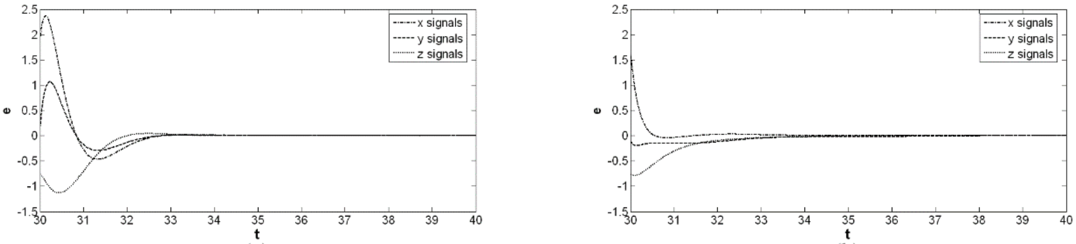
(a) (b)

Figure 9. The time response of the error signals for synchronization of chaotic finance systems with the controllers activated at t = 20 (a) sliding mode controllers, (b) passive controller



(a) (b)

Figure 10. The time response of the error signals for synchronization of chaotic finance systems with the controllers activated at t = 25 (a) sliding mode controllers, (b) passive controller



(a) (b)

Figure 11. The time response of the error signals for synchronization of chaotic finance systems with the controllers activated at t = 30 (a) sliding mode controllers, (b) passive controller

As expected, the related Figs. 6–8 outputs show that both the sliding mode controllers and the passive controller have achieved synchronization of chaotic finance systems with an appropriate time period. The error signals that are shown in Figs. 9–11 converge asymptotically to zero. The figures include comparative results for the synchronization of chaotic finance systems. While synchronization is provided at $t \geq 24$ by using the sliding mode control, it is reached when $t \geq 28$ with the passive control when the controllers are activated at t = 20. Also, the synchronization is first observed with the sliding mode controllers when the controllers are activated at t = 25, and t = 30. Therefore, these comparisons show that the sliding mode control method performs better than the passive control method for the synchronization of two identical chaotic finance systems. The sliding mode control method realizes the synchronization using two controllers while the passive control method requires only one controller. Multiple controllers appear to reduce the synchronization time period, whereas a single controller provides simplicity in implementation.

The passive control method achieves synchronization by adding or subtracting a value only to the interest rate which is dependent on the saving amount, interest rates and investment demands. It does not need any changes in the investment demand and price exponent, so it is simpler to implement. On the other hand, sliding mode control method achieves synchronization by altering the interest rate and investment demand. It calculates the quantity of changes by using the saving amount, per-investment cost, interest rates, investment demands and price exponents. Both methods do not require the elasticity of demands of commercials for synchronization. The sliding mode control method appears to have some advantages in synchronization speed, but by comparison with the passive control, it is more difficult to apply.

6. Conclusions

The aim of this paper is to investigate the synchronization of chaos in a nonlinear finance system. Synchronization provides that a low dimensional financial system adapts to the global financial system. Instant variations such as price and interest rate are the main factors of demand and volume changes. They lead to nonlinearity in a system. Synchronization to the global finance system utilizes some benefits to economic growth on account of obtaining the same interest rate, investment demand and price exponent. Also, it can reduce the asymmetrical economic risks.

Based on sliding mode and passive control theory, two sliding mode controllers and a single passive controller have been designed for synchronization of chaos in two identical chaotic finance systems. Numerical simulations show all the theoretical analyses of the proposed control methods are succeeded in synchronizing the two chaotic financial systems. Sliding mode controllers regulate the synchronization of chaotic finance systems more effectively than the passive controller in all cases that are shown in Figs. 6–11, so the sliding mode method is more appropriate. The advantage of the passive control method is to achieve the synchronization of chaotic finance systems with only one controller which provides simplicity in implementation. While the sliding mode control realizes synchronization by altering the interest rate and investment demand, the passive control only alters the interest rate.

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