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Reliable Robust Sampled-Data H_∞ Output Tracking Control with Application to Flight Control

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Abstract. This paper is concerned with the problem of robust H_∞ output tracking control for uncertain sampled-data systems with probabilistic actuator failures. By assuming that each actuator fault takes values randomly in a finite set, a new actuator-failure-mode is proposed. Lyapunov-Krasovskii functional combined with the input delay approach as well as the free-weighting matrix approach are employed to establish the H_∞ performance, and the controller design is cast into a convex optimization problem with linear matrix inequality (LMI) constraints. The designed reliable controller can guarantee that the output of the closed-loop sampled-data system tracks the reference signal without steady-state error. An airship model is considered in this paper and its simulation results are given.

Keywords: probabilistic actuator failures; output tracking; sampled-data control; convex polytope; flight control; parameter uncertainty.

1. Introduction

In the past years, output tracking control has received considerable attention due to its wide applications in dynamic processes in industry such as robot control [1], flight control [2-4] and motor control [5, 6]. The main objective of output tracking is to design a controller to guarantee the output of controlled system tracking the reference signal as close as possible, which is more general and more difficult than stabilization. Up to date, many results have been reported on output tracking [7-9].

As is well known, with the fast development of microprocessor and electronic technologies, digital computers are widely used to control continuous-time systems in modern control systems. For example, in a flight control system about airship (see Figure 1), a microcontroller is usually used to sample and quantize a continuous-time measurement signal, and then produce a discrete-time control input signal, which can be further converted into a continuous-time control input signal using a zero-order holder. Such control systems involve both continuous-time and discrete-time signals in continuous-time framework are referred to as sampled-data systems. Considerable research efforts have been made on various aspects of sampled-data systems, such as control systems [10-12] and filtering problems [13-15]. It is worth mentioning that little progress has been made to design controllers for uncertain sampled-data systems to make the output to track the reference signal without steady-state error, although it is of both theoretical significance and practical importance.

In reality, because of the actuators aging, zero shift and electromagnetic interference, actuator failures are unavoidable, which may lead to intolerable performance of the system. Therefore, it is necessary and important to design controllers that can tolerate actuator failures. A common assumption in most of the existing results on reliable control is that the actuator failure model is depicted as an unknown bounded constant [16-18]. It is not difficult to understand that in some situations, however, actuator failures may happen in a random way. Recent works assume that the actuator failures satisfy certain probabilistic distribution on the given intervals [19-21].

Motivated by above discussions, this paper focuses on the controller design for a class of uncertain sampled-data systems with probabilistic actuator failures. The main contributions of this paper are as follows:

1) This is the first paper that a controller is designed to make the output of uncertain sampled-data system to track the reference signal without steady-state error, and the results can be applied to flight control and other areas.

2) A new fault failure mode is established for the first time by assuming that each actuator fault takes values randomly in a finite set, which is more realistic and accurate in some situations.

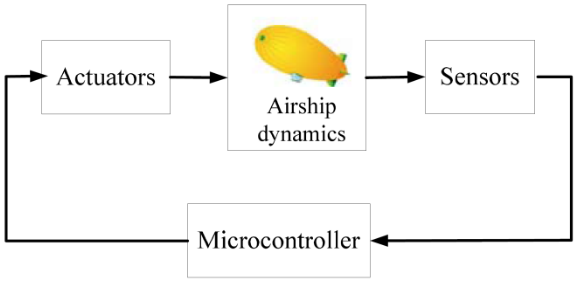

Figure 1. Architecture of airship control system

2. Problem formulation

Consider the following uncertain linear system:

$$\begin{cases} \dot{x}(t) = A(\lambda)x(t) + B(\lambda)u^f(t) + D(\lambda)w(t) \\ \quad y(t) = C_1(\lambda)x(t) + D_1(\lambda)\eta(t) \end{cases} \quad (1)$$

where $x(t) \in R^n$ is the state vector, $u^f(t) \in R^m$ is the actuator output considering possible failure, $y(t) \in R^p$ is the output, $w(t) \in L_2[0,\infty)$ denotes the exogenous disturbance signal, $A(\lambda), B(\lambda), D(\lambda), C_1(\lambda)$ and $D_1(\lambda)$ are system matrices containing uncertain parameters, represented by λ. Assume that $\Omega \triangleq (A(\lambda), B(\lambda), D(\lambda), C_1(\lambda), D_1(\lambda)) \in \mathfrak{R}$, where $\mathfrak{R}$ is a given convex-bounded polyhedral domain described by r vertices

$$\mathfrak{R} \triangleq \left\{\Omega \middle| \Omega = \sum_{i=1}^{r} \lambda_i \Omega_i;\ \sum_{i=1}^{r} \lambda_i = 1, \lambda_i \geq 0\right\} \quad (2)$$

with $\Omega_i \triangleq (A_i, B_i, D_i, C_{1i}, D_{1i}) \in \mathfrak{R}$ denoting the vertices of the polytope.

In this paper, the following actuator failure model will be adopted:

$$u^f(t) = \Theta u(t) = \textstyle\sum_{l=1}^{m} \theta_l \Delta_l u(t), \quad (3)$$

where $\Theta = diag\{\theta_1, \theta_2, \dots, \theta_m\}$, $\theta_l (l = 1, \dots, m)$ are m unrelated random variables and

$$\Delta_l = diag\left\{\underbrace{0, \dots, 0}_{l-1}, 1, \underbrace{0, \dots, 0}_{m-l}\right\}.$$

It is assumed that θ_l takes values in a finite set, that is $\theta_l \in \{\tau_{l1}, \tau_{l2}, \dots, \tau_{lq}\}$. In addition, the process $\{\theta_l\}$ is assumed to be independent and identicallydistributed, with the probabilities given by

$$Prob\{\theta_l = \tau_{lj}\} = \alpha_{lj}, \\ l = 1, \dots, m, j = 1, \dots, q_l \quad (4)$$

where α_{lj} is a positive scalar and $\sum_{j=1}^{q_l} \alpha_{lj} = 1$.

Remark 1. In the most of existing results on reliable control, variable θ_i is an unknown constant with known lower and upper bounds (see, for example [16–18]). Some other results, for example [19–21], assumed that the variable θ_i satisfies a certain probabilistic distribution on the given interval [0; θ], which is more general and practical than the former results in some situations. In many real control systems, however, the type of actuator failures is finite. In this situation, the assumption in (4) can better describe the failure characterization.

Remark 2. In this paper, the random variable θ_i takes values in a finite set. For $\theta_i = 0$, it means complete failure of the ith actuator; for $\theta_i = 1$, it means that the ith actuator is in good work condition; for $0 < \theta_i < 1$, it means partial failure of the ith actuator; for $\theta_i > 1$, it means the actuator-amplifier with forward drift.

It is well known that the tracking error integral action of controller can effectively eliminate the steady-state tracking error. Similar to Ye and Yang [2], and Liao et al. [4], we introduce the following augmented system state-space description of system (1) with actuator failure model:

$$\dot{\varsigma}(t) = \bar{A}(\lambda)\varsigma(t) + \bar{B}(\lambda)\sum_{l=1}^{m} \theta_l \Delta_l u(t) \\ +\bar{D}(\lambda)\bar{w}(t) \quad (5) \\ z(t) = \bar{C}\varsigma(t),$$

where

$$\varsigma(t) = \left[x^T(t) \left(\textstyle\int_0^t e(t)dt\right)^T\right]^T,$$

$$e(t) = r(t) - Sy(t),$$

$$\bar{w}(t) = [w^T(t) \quad \eta^T(t) \quad r^T(t)]^T,$$

$$\bar{A}(\lambda) = \begin{bmatrix} A(\lambda) & 0 \\ -SC_1(\lambda) & 0 \end{bmatrix},$$

$$\bar{B}(\lambda) = \begin{bmatrix} B(\lambda) \\ 0 \end{bmatrix},$$

$$\bar{D}(\lambda) = \begin{bmatrix} D(\lambda) & 0 & 0 \\ 0 & -SD_1(\lambda) & 1 \end{bmatrix},$$

$$\bar{C} = [0 \quad I],$$

$S \in R^{q \times p}$ is a known constant matrix used to form output required to track the reference signal.

The reliable robust sampled-data H_∞ output tracking problem considered in this paper is to design a sampling controller such that:

1) During normal operation, the closed-system is asymptotically stable, and the output $Sy(t)$ tracks the reference signal $r(t)$ without steady-state error, that is $\lim_{t\to\infty} e(t) = 0$. Moreover, the effect of $\bar{w}(t)$ on tracking error integral $z(t)$ is attenuated below a desired level in the H_∞ sense. More specifically, it is required that $\| z(t) \|_2 < \gamma \| \bar{w}(t) \|$ for all nonzero $\bar{w}(t) \in L_2[0,\infty)$ under zero condition, where $\gamma > 0$.

2) In the event of actuator failures, the closedloop system is still stable, and the required output $Sy(t)$ tracks the reference signal $r(t)$ without steadystate error.

For sampled-data control with zero-order holder, the following state-feedback controller is designed for the augmented system [2, 4]:

$$u(t) = u_d(t_k) = K\varsigma(t_k) = [K_1 \quad K_2]\begin{bmatrix} x(t_k) \\ \int_0^{t_k} e(t)dt \end{bmatrix}, \quad (6)$$

$t_k \le t < t_{k+1}, k = 0, 1, 2, \ldots,$

where $u_d(t_k)$ is a discrete-time control signal, t_k denotes the sampling instant. Under control law (6), the closed-loop system is given by

$$\dot{\varsigma}(t) = \bar{A}(\lambda)\varsigma(t) + \bar{B}(\lambda)\sum_{l=1}^{m}\theta_l \Delta_l K\varsigma(t_k)$$
$$+\bar{D}(\lambda)\bar{w}(t), \quad (7)$$
$$z(t) = \bar{C}\varsigma(t),$$
$$t_k \le t < t_{k+1}.$$

Assumption 1. The interval between two consecutive sampling instants is bounded, that is $t - t_k \le h, \forall > 0$:

Similar to Liao et [2], the sampled-data formulation in (7) can be transformed into the following system:

$$\dot{\varsigma}(t) = \bar{A}(\lambda)\varsigma(t) + \bar{B}(\lambda)\sum_{l=1}^{m}\theta_l \Delta_l K\varsigma(\mathrm{t}-\mathrm{d(t)}) + \bar{D}(\lambda)\bar{w}(t)$$
$$z(t) = \bar{C}\varsigma(t), \quad (8)$$

where $d(t) = t - t_k \le h$, $t_k \le t < t_{k+1}$ is piece-wise linear with derivative $d(t) = 1$ for $t \ne t_k$.

Remark 3. *The input delay approach is an effective one for the analysis and design of sampled-data systems which was introduced by Fridman et al. [10] and extensively used by Fridman et al. [11], Gao et al. [12]. This approach can be applied to systems with non-uniform uncertain sampling and system parameter uncertainties, which has been recognized to be a difficult problem for traditional lifting techniques.*

3. H_∞ output tracking performance analysis

In this section, we are concerned with the problem of H_∞ output tracking analysis based on the transformed closed-loop system in (8). More specifically, assuming that the controller gains K_1 and K_2 are known, we shall study the conditions under which the system in (8) achieves H_∞ output tracking performance γ.

To solve the problem with probabilistic actuator failure model in (3), we introduce indicator functions $\pi_{\{\theta_l=\tau_{lj}\}}$ as

$$\pi_{\{\theta_l=\tau_{lj}\}} = \begin{cases} 1, & \theta_l = \tau_{lj}, \\ 0, & \theta_l \ne \tau_{lj}. \end{cases} \quad (9)$$

Thus we obtain

$$\boldsymbol{E}\left\{\pi_{\{\theta_l=\tau_{lj}\}}\right\} = Prob\{\theta_l = \tau_{lj}\} = \alpha_{lj}, \quad (10)$$
$$l = 1, \ldots, m, j = 1, \ldots, q_l.$$

Therefore, the augmented closed-loop system in (8) can be rewritten as

$$\begin{cases} \dot{\varsigma}(t) = \bar{A}(\lambda)\varsigma(t) + \\ \bar{B}(\lambda)\sum_{l=1}^{m}\sum_{j=1}^{q_l}\pi_{\{\theta_l=\tau_{lj}\}}\tau_{lj}\Delta_l L\varsigma(t-d(t)) + \\ \bar{D}(\lambda)\bar{w}(t) \\ z(t) = \bar{C}\varsigma(t). \end{cases} \quad (11)$$

Remark 4. *We introduce indicator functions $\pi_{\{\theta_l=\tau_{lj}\}}$ satisfying Bernoulli distributions to solve the problem with probabilistic actuator failures. To the best of the authors' knowledge, few attempts have been made to utilize it for solving the problem related to probabilistic actuator failures, which is one of the important contributions of this paper.*

Now, we are in a position to present the conditions to achieve H_∞ output tracking performance.

Theorem 1. *Given scalar $h > 0$ and the controller gains K_1 and K_2, the augmented closed-loop system in (11) achieves the H_∞ output tracking performance , if there exist matrices $P > 0$, $Q > 0$, $R > 0$, M_{ji} and N_{ji}, $j = 1, 2, 3, 4$ satisfying*

$$\begin{bmatrix} \Lambda_i & \sqrt{h}M_i & \sqrt{h}N_i & \Xi_{i17} & \Xi_{i18} \\ * & -R & 0 & 0 & 0 \\ * & * & -R & 0 & 0 \\ * & * & * & -R & 0 \\ * & * & * & * & \Xi_{i88} \end{bmatrix} < 0, \quad (12)$$

$i = 1, \ldots, r,$

where

$$\Lambda_i = \begin{bmatrix} \Xi_{i11} & P\bar{B}_i\Theta K - M_{1i} + M_{2i}^T + N_{1i} & M_{3i}^T - N_{1i} & P\bar{D}_i + M_{4i}^T \\ * & -M_{2i} - M_{2i}^T + N_{2i} + N_{2i}^T & -M_{3i}^T - N_{2i} + N_{3i}^T & M_{4i}^T + N_{4i}^T \\ * & * & -Q - N_{3i} - N_{3i}^T & -N_{4i}^T \\ * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\Xi_{i11} = P\bar{A}_i + \bar{A}_i^T P + Q + M_{1i} + M_{1i}^T + \bar{C}_2^T\bar{C}_2,$$

$$\Xi_{i17} = \sqrt{h}[R\bar{A}_i \quad R\bar{B}_i\Theta K \quad 0 \quad R\bar{D}_i]^T,$$

$$\Xi_{i17}=\begin{bmatrix}0 & \sqrt{h\sum_{j=1}^{q_1}\alpha_{1j}\tau_{1j}^2}\,\bar{B}_i\Delta_1 K & 0 & 0\\ 0 & \sqrt{h\sum_{j=1}^{q_2}\alpha_{2j}\tau_{2j}^2}\,\bar{B}_i\Delta_2 K & 0 & 0\\ \vdots & \vdots & \vdots & \vdots\\ 0 & \sqrt{h\sum_{j=1}^{q_m}\alpha_{mj}\tau_{mj}^2}\,\bar{B}_i\Delta_m K & 0 & 0\end{bmatrix}^T,$$

$$M_i=[M_{1i}^T\quad M_{2i}^T\quad M_{3i}^T\quad M_{4i}^T]^T,$$

$$N_i=[N_{1i}^T\quad N_{2i}^T\quad N_{3i}^T\quad N_{4i}^T]^T,$$

$$\bar{\Theta}=\sum_{l=1}^{m}\sum_{j=1}^{q_l}\alpha_{lj}\tau_{lj}\Delta_l.$$

▼**Proof.** Choose a Lyapunov-Krasovskii functional as

$$V(\varsigma_t)=V_1(\varsigma_t)+V_2(\varsigma_t)+V_3(\varsigma_t),$$
$$V_1(\varsigma_t)=\varsigma^T(t)P\varsigma(t),$$
$$V_2(\varsigma_t)=\int_{t-h}^{t}\varsigma^T(s)Q\varsigma(s)ds, \tag{13}$$
$$V_3(\varsigma_t)=\int_{-h}^{0}\int_{t+s}^{t}\dot{\varsigma}^T(\theta)R\dot{\varsigma}(\theta)d\theta\, ds,$$

where $P>0$, $Q>0$, $R>0$ are matrices to be determined. The infinitesimal operator L is defined as

$$LV(\varsigma_t)=\lim_{\Delta\to 0^+}\frac{1}{\Delta}\{\mathbf{E}\{V(\varsigma_t+\Delta)|\varsigma_t\}-V(\varsigma_t)\}. \tag{14}$$

Using the operator (14) for (13), and taking expectation on it, we have

$$\mathbf{E}\{LV_1(\varsigma_t)\}=2\varsigma^T(t)P\left[\bar{A}(\lambda)\varsigma(t)+\bar{B}(\lambda)\sum_{l=1}^{m}\sum_{j=1}^{q_l}\alpha_{lj}\tau_{lj}\Delta_l K\varsigma\big(t-d(t)\big)+\bar{D}(\lambda)\bar{w}(t)\right],$$
$$\mathbf{E}\{LV_2(\varsigma_t)\}=\varsigma^T(t)Q\varsigma(t)-\varsigma^T(t-h)Q\varsigma(t-h), \tag{15}$$
$$\mathbf{E}\{LV_3(\varsigma_t)\}=\mathbf{E}\left\{h\dot{\varsigma}^T(t)R\dot{\varsigma}(t)-\int_{t-h}^{t}\dot{\varsigma}^T(s)R\dot{\varsigma}(s)ds\right\}.$$

From (11), we obtain

$$\mathbf{E}\{h\dot{\varsigma}^T(t)R\dot{\varsigma}(\mathrm{t})\}=\mathbf{E}\left\{h\left[\bar{A}(\lambda)\varsigma(t)+\bar{B}(\lambda)\sum_{l=1}^{m}\sum_{j=1}^{q_l}\pi_{\{\theta_l=\tau_{lj}\}}\tau_{lj}\Delta_l K\varsigma\big(t-d(t)\big)+\bar{D}(\lambda)\bar{w}(t)\right]^T,\right.$$
$$\left.R\left[\bar{A}(\lambda)\varsigma(t)+\bar{B}(\lambda)\sum_{l=1}^{m}\sum_{j=1}^{q_l}\pi_{\{\theta_l=\tau_{lj}\}}\tau_{lj}\Delta_l K\varsigma\big(t-d(t)\big)+\bar{D}(\lambda)\bar{w}(t)\right]\right\}$$
$$=h\xi^T(t)[\bar{A}(\lambda)\bar{\Theta}\bar{B}(\lambda)K\quad 0\quad \bar{D}(\lambda)]^T R\,[\bar{A}(\lambda)\quad \bar{\Theta}\bar{B}(\lambda)K\quad 0\quad \bar{D}(\lambda)]\xi(t) \tag{16}$$
$$-h\xi^T(t)\Theta^2K^T\bar{B}^T(\lambda)R\bar{B}(\lambda)K\xi(t)$$
$$+\varsigma^T\big(t-d(t)\big)\sum_{l=1}^{m}\sum_{j=1}^{q_l}\alpha_{lj}\tau_{lj}^2K^T\Delta_l^T\bar{B}^T(\lambda)hR\bar{B}(\lambda)\Delta_l K\varsigma\big(t-d(t)\big).$$

In addition, by the Newton-Leibniz formula, for any appropriately dimensioned matrices $M(\lambda)=[M_1^T(\lambda)\quad M_2^T(\lambda)\quad M_3^T(\lambda)\quad M_4^T(\lambda)]^T$ and $N(\lambda)=[N_1^T(\lambda)\quad N_2^T(\lambda)\quad N_3^T(\lambda)\quad N_4^T(\lambda)]^T$, we obtain

$$2\big[\varsigma^T(t)M_1(\lambda)+\varsigma^T\big(t-d(t)\big)M_2(\lambda)+\varsigma^T(t-h)M_3(\lambda)-\bar{w}^T(t)M_4(\lambda)\big]$$
$$\times\left[\varsigma(t)-\varsigma\big(t-d(t)\big)-\int_{t-d(t)}^{t}\dot{\varsigma}(\alpha)\,d\alpha\right]$$
$$2\big[\varsigma^T(t)N_1(\lambda)+\varsigma^T\big(t-d(t)\big)N_2(\lambda)+\varsigma^T(t-h)N_3(\lambda)-\bar{w}^T(t)N_4(\lambda)\big]$$
$$\times\left[\varsigma^T\big(t-d(t)\big)-\varsigma^T(t-h)-\int_{t-h}^{t-d(t)}\dot{\varsigma}(\alpha)\,d\alpha\right]=0, \tag{17}$$

where

$$\xi(t)=\big[\varsigma^T(t)\quad \varsigma^T\big(t-d(t)\big)\quad \varsigma^T(t-h)\quad \bar{w}^T(t)\big]^T.$$

Combining (15)-(17), we have

$$\mathbf{E}\{LV(\varsigma t)\} + \mathbf{E}\{z^T(t)z(t)\} - \gamma^2\mathbf{E}\{\bar{w}^T(t)\bar{w}(t)\}$$

$$\leq \xi^T(t)\Bigg[\Lambda + h[\bar{A}(\lambda) \quad \bar{\Theta}\bar{B}(\lambda)K \quad 0 \quad \bar{D}(\lambda)]^T R[\bar{A}(\lambda) \quad \bar{\Theta}\bar{B}(\lambda)K \quad 0 \quad \bar{D}(\lambda)]$$

$$+ h\sum_{l=1}^{m}\sum_{j=1}^{q_l}\alpha_{lj}\tau_{lj}^2[0 \quad \bar{B}(\lambda)\Delta_l K \quad 0 \quad 0]^T R[0 \quad \bar{B}(\lambda)\Delta_l K \quad 0 \quad 0] + hM(\lambda)R^{-1}M^T(\lambda)$$

$$+ hN(\lambda)R^{-1}N^T(\lambda)\Bigg]\xi(t) - h\xi^T(t)\bar{\Theta}^2K^T\bar{B}^T(\lambda)R\bar{B}(\lambda)K\xi(t)$$

$$- \int_{t-d(t)}^{t}[\xi^T(t)M(\lambda) + \dot{\varsigma}^T(s)R]R^{-1}[M^T(\lambda)\xi(t) + R\dot{\varsigma}(s)]ds$$

$$- \sum_{t-h}^{t-d(t)}[\xi^T(t)N(\lambda) + \dot{\varsigma}^T(s)R]\,R^{-1}[N^T(\lambda)\xi(t) + R\dot{\varsigma}(s)]ds, \tag{18}$$

where

$$\Lambda = \begin{bmatrix} \Xi_{11} & P\bar{B}(\lambda)\bar{\Theta}K - M_1(\lambda) + M_2^T(\lambda) + N_1(\lambda) & M_3^T - N_1(\lambda) & PD(\lambda) + M_4^T(\lambda) \\ * & -M_2(\lambda) - M_2^T(\lambda) + N_2(\lambda) + N_2^T(\lambda) & -M_3^T(\lambda) - N_2(\lambda) + N_3^T(\lambda) & M_4^T(\lambda) + N_4^T(\lambda) \\ * & * & -Q - N_3(\lambda) - N_3^T(\lambda) & -N_4^T(\lambda) \\ * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\Xi_{11} = P\bar{A}(\lambda) + \bar{A}^{-T}(\lambda)P + Q + M_1(\lambda)M_1^T(\lambda) + \bar{C}_2^T\bar{C}_2.$$

By Schur complement, inequalities (12) guarantee

$$\Lambda_i + h[\bar{A}_i \quad \bar{\Theta}\bar{B}_iK \quad 0 \quad \bar{D}_i]^T R\,[\bar{A}_i \quad \bar{\Theta}\bar{B}_iK \quad 0 \quad \bar{D}_i]$$

$$+h\sum_{l=1}^{m}\sum_{j=1}^{q_l}\alpha_{lj}\tau_{lj}^2[0 \quad \bar{B}_i\Delta_lK \quad 0 \quad 0]^T\ R\,[0 \quad \bar{B}_i\Delta_lK \quad 0 \quad 0]$$

$$+hM_iR^{-1}M_i^T + hN_iR^{-1}N_i^T < 0. \tag{19}$$

According to the inner property of polytopic uncertain systems, and considering the form $\bar{A}(\lambda) = \sum_{i=1}^{r}\lambda_iA_i$, $\bar{B}(\lambda) = \sum_{i=1}^{r}\lambda_i\bar{B}_i$, $\bar{D}(\lambda) = \sum_{i=1}^{r}\lambda_i\bar{D}_i$, $M(\lambda) = \sum_{i=1}^{r}\lambda_iM_i$, $N(\lambda) = \sum_{i=1}^{r}\lambda_iN_i$, we obtain from (19) that

$$\Lambda + h[\bar{A}(\lambda) \quad \bar{\Theta}\bar{B}(\lambda)K \quad 0 \quad \bar{D}(\lambda)]^T R[\bar{A}(\lambda) \quad \bar{\Theta}\bar{B}(\lambda)K \quad 0 \quad \bar{D}(\lambda)]$$

$$+h\sum_{l=1}^{m}\sum_{j=1}^{q_l}\alpha_{lj}\tau_{lj}^2[0 \quad \bar{B}(\lambda)\Delta_lK \quad 0 \quad 0]^T\ R\,[0 \quad \bar{B}(\lambda)\Delta_lK \quad 0 \quad 0]$$

$$+ h\,M(\lambda)R^{-1}M^T(\lambda) + hN(\lambda)R^{-1}N^T < 0. \tag{20}$$

Note that $R > 0$, thus the last three terms of (18) are negative. Therefore, we have

$$\mathbf{E}\{LV(\varsigma_t)\} + \mathbf{E}\{z^T(t)z(t)\} - \gamma^2\mathbf{E}\{\bar{w}^T(t)\bar{w}(t)\} < 0 \tag{21}$$

for all nonzero $\bar{w}(t) \in L_2[0,\infty)$. Under zero conditions, we have $V(0) = 0$ and $V(\infty) \geq 0$. Integrating both sides of (21) yields $\| z(t) \|_2 < \gamma \| \bar{w}(t) \|$ for all nonzero $\bar{w}(t) \in L_2[0,\infty)$, and the H_∞ output tracking performance is established.

This completes the proof. ▲

Remark 5. *In deriving the H_∞ output tracking performance conditions in Theorem 1, Lyapunov-Krasovskii functional plus free weighting matrix techniques are utilized to analyze transformed delay system in (12). It is worth noting that not only the sampling interval h but also the actuator failure probabilities α_{lj} have been incorporated into the conditions presented in Theorem 1. When h and α_{lj} are known, the conditions are LMIs over the decision variables to be determined.*

4. H_∞ output controller design

In this section, the problem of H_∞ output tracking controller design will be solved based on Theorem 1.

Theorem 2. *Given scalar $h > 0$, there exists a state-feedback controller in the form of (6) such that the augmented closed-loop system in (11) achieves the H_∞ output tracking performance γ if there exist matrices $\hat{P} > 0$, $\hat{Q} > 0$, $\hat{R} > 0$, $\hat{M}_{ij}$ and $\hat{N}_{ij}$ $j = 1, 2, 3, 4$, and $\hat{K}$, satisfying*

$$\begin{bmatrix} \hat{\Lambda}_i & \sqrt{h}\hat{M}_i & \sqrt{h}\hat{N}_i & \hat{\Xi}_{i17} & \hat{\Xi}_{i18} & \hat{P}\hat{C}_2^T \\ * & -\hat{R} & 0 & 0 & 0 & 0 \\ * & * & -\hat{R} & 0 & 0 & 0 \\ * & * & * & \hat{R}-2\hat{P} & 0 & 0 \\ * & * & * & * & \hat{\Xi}_{i88} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0,$$

$$i = 1, \dots, r \tag{22}$$

where

$$\hat{\Lambda}_i = \begin{bmatrix} \hat{\Xi}_{i11} & \bar{B}_i\bar{\Theta}\hat{K} - \hat{M}_{1i} + \hat{M}_{2i}^T + \hat{N}_{1i} & \hat{M}_{3i}^T - \hat{N}_{1i} & \bar{D}_i\hat{P} + \hat{M}_{4i}^T \\ * & -\hat{M}_{2i} - \hat{M}_{2i}^T + \hat{N}_{2i} + \hat{N}_{2i}^T & -\hat{M}_{3i}^T - \hat{N}_{2i} + \hat{N}_{3i}^T & \hat{M}_{4i}^T + \hat{N}_{4i}^T \\ * & * & -Q - \hat{N}_{3i} - \hat{N}_{3i}^T & -\hat{N}_{4i}^T \\ * & * & * & -\gamma^2 I \end{bmatrix},$$

$\hat{\Xi}_{i11} = \bar{A}_i\hat{P} + \hat{P}\bar{A}_i^T + \hat{Q} + \hat{M}_{1i} + \hat{M}_{1i}^T$,

$\hat{\Xi}_{i11} = \sqrt{h}[\bar{A}_i\hat{P} \quad \bar{B}_i\bar{\Theta}\hat{K} \quad 0 \quad \bar{D}_i\hat{P}]^T$,

$$\hat{\Xi}_{i11} = \begin{bmatrix} 0 & \sqrt{h\sum_{j=1}^{q_1}\alpha_{lj}\tau_{lj}^2}\,\bar{B}_i\Delta_1\hat{K} & 0 & 0 \\ 0 & \sqrt{h\sum_{j=1}^{q_2}\alpha_{2j}\tau_{2j}^2}\,\bar{B}_i\Delta_2\hat{K} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \sqrt{h\sum_{j=1}^{q_m}\alpha_{mj}\tau_{mj}^2}\,\bar{B}_i\Delta_m\hat{K} & 0 & 0 \end{bmatrix}^T,$$

$\hat{M}_i = [\hat{M}_{1i}^T \quad \hat{M}_{2i}^T \quad \hat{M}_{3i}^T \quad \hat{M}_{4i}^T]^T$,

$\hat{N}_i = [\hat{N}_{1i}^T \quad \hat{N}_{2i}^T \quad \hat{N}_{3i}^T \quad \hat{N}_{4i}^T]^T$,

$$\bar{\Theta} = \sum_{l=1}^{m}\sum_{j=1}^{q_l}\alpha_{lj}\tau_{lj}\Delta_l.$$

Moreover, if the conditions have a feasible solution, the gain matrix of a desired controller in the form of (3) is given by

$$[K_1 \quad K_2] = \hat{K}\hat{P}^{-1}. \tag{23}$$

▼**Proof.** By noticing $\hat{R} > 0$, we have

$$(\hat{R} - \hat{P})\hat{R}^{-1}(\hat{R} - \hat{P}) > 0,$$

which is equivalent to $-\hat{P}\hat{R}^{-1}\hat{P} \le \hat{R} - 2\hat{P}$. Performing a congruence transformation to (12) by

$$diag\{P^{-1}, P^{-1}, P^{-1}, I, P^{-1}, P^{-1}, I, \underbrace{I, \dots, I}_{m}\},$$

and define

$\hat{P} = P^{-1}$, $\hat{Q} = P^{-T}QP^{-1}$,

$\hat{R} = P^{-T}RP^{-1}$,

$\hat{M}_{ni} = P^{-T}M_{ni}P^{-1}$, $n = 1, 2, 3$,

$\hat{N}_{ni} = P^{-T}N_{ni}P^{-1}$, $n = 1, 2, 3$,

$\hat{M}_{3i} = M_{3i}P^{-1}$, $\hat{N}_{4i} = N_{4i}P^{-1}$,

$K = KP^{-1} = [K_1 \quad K_2]P^{-1}$,

we obtain (22) by Schur complement. This completes the proof.▲

Remark 6. *Theorem 2 provides LMI conditions for the existence of desired reliable H_∞ output tracking controller. The scalar γ can be included as an optimization variable to obtain a reduction of the guaranteed H_∞ performance bound. Then the minimal γ can be found by solving the following convex optimization problem: minimize γ subject to (22) over $\hat{P} > 0$, $\hat{Q} > 0$, $\hat{R} > 0$, $\hat{M}_{ij}$ and $\hat{N}_{ij}$ $j = 1, 2, 3, 4$, and $\hat{K}$.*

5. Simulation example

In this section, an example of output tracking control for a linear airship model is given to show the effectiveness of the proposed method. The linearized dynamics of an autonomous airship (Altitude: $300m$, Speed: $7m/s$) in the vertical plane is given by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Dw(t) \\ y(t) = Cx(t) \end{cases} \tag{25}$$

where

$$A = \begin{bmatrix} -0.0516 & -0.0069 & 0.0296+\lambda_1 & 0.0695+\lambda_2 \\ 0 & -0.2865 & 3.22 & 0 \\ 0.0005 & 0.0088 & -0.0037 & -0.0886 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

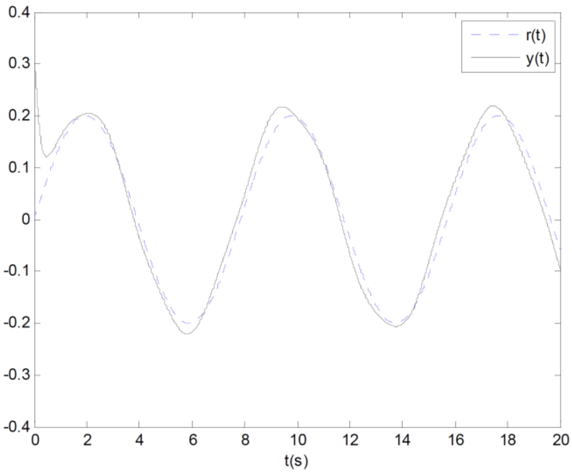

Figure 2. $r(t)$ and $y(t)$ without actuator failures

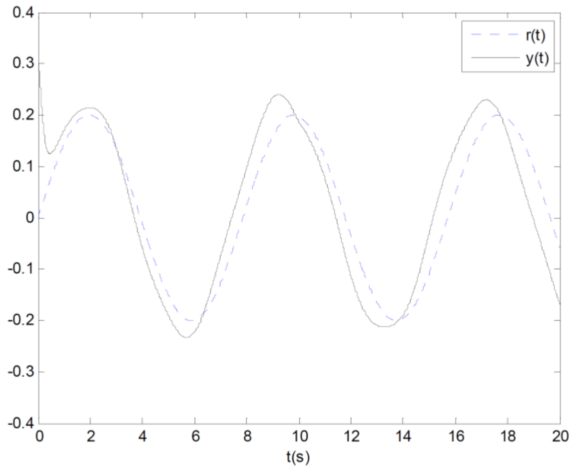

Figure 3. $r(t)$ and $y(t)$ with probabilistic actuator failures

$$B = \begin{bmatrix} 0.0001 & 0.0186 \\ 0 & -0.3178 \\ 0 & -0.0237 \\ 0 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.001 \\ 0.001 \\ 0.001 \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}^T,$$

where $x(t) = [u \quad w \quad q \quad \theta]$ is the state vector which is composed of linear velocities u and w (along X and Z body axes, respectively), angular velocities q (around Y body axes) and pitch angle θ; $u(t) = [n \quad \delta_e]$ is the control input which is composed of engine speed n and elevator angle δ_e; $y(t) = \theta$ is the output; $w(t) \in L_2[0, \infty)$ is the exogenous disturbance. λ_1 and λ_2 are system parameter uncertainties satisfying $|\lambda_1| \leq \bar{\lambda}_1 = 0.012$ and $|\lambda_2| \leq \bar{\lambda}_2 = 0.016$. Then, the system (25) can be represented by a four-vertex polytopic system.

For simulation, we assume $S = 1$, $h = 10ms$, $w(t) = 0.05 \sin 3t$ and $r(t) = 0.2 \sin 0.8t$. In addition, the initial state of the longitudinal airship system is assumed to be $[0.1 \quad 0 \quad 0 \quad 0.3]^T$.

Case 1. Without considering the actuator failures, that is $\theta_1 = \theta_2 = 1$. By solving the convex optimization problem formulated in Remark 5, the obtained minimum guaranteed H_∞ tracking performance is min $\gamma_{\min} = 2.6204$ and the admissible controller gain matrices are as follows:

$$K_1 = \begin{bmatrix} 0.0101 & -3.4961 & 18.7102 & -2.9051 \\ -0.0347 & -2.8593 & 30.4816 & -0.0387 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 1.5190 \\ 2.9768 \end{bmatrix}.$$

The output $y(t)$ and the reference output signal $r(t)$ are shown in Fig. 2, from which we can see that $y(t)$ tracks $r(t)$ well with parameter uncertainties.

Case 2. Considering the probabilistic actuator failures, setting $\theta_1 \in \{0.4, 1, 1.3\}$ and $\theta_2 \in \{0.5, 1, 1.6\}$ with probabilities given by

$$\begin{aligned} &Prob\{\theta_1 = 0.4\} = Prob\{\theta_2 = 0.5\} = 0.1, \\ &Prob\{\theta_1 = 1\} = Prob\{\theta_2 = 1\} = 0.8, \qquad (26) \\ &Prob\{\theta_1 = 1.3\} = Prob\{\theta_2 = 1.6\} = 0.1. \end{aligned}$$

By solving the corresponding optimization problem, the obtained minimum guaranteed H_∞ tracking performance is $\gamma_{\min} = 2.9731$ and the admissible controller gain matrices are given by

$$K_1 = \begin{bmatrix} 0.0419 & -8.4961 & 7.5827 & -6.9247 \\ 0.0019 & -4.8255 & 13.7493 & -4.8461 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.4937 \\ 1.2864 \end{bmatrix}.$$

The output $y(t)$ and the reference output signal $r(t)$ are shown in Fig. 3. From Fig. 3, it can be seen that $y(t)$ tracks $r(t)$ well with both probabilistic actuator failures and parameter uncertainties.

6. Conclusions

In this paper, the problem of robust H_∞ output tracking control for uncertain sampled-data systems with probabilistic actuator failures has been investigated. By assuming that each actuator fault takes values randomly in a finite set, a new actuator-failuremodel has been proposed. The system with sampling measurements has been transformed into a time delay system and polytopic parameter uncertainty has been utilized to characterize the real uncertain situation. Then a reliable output tracking controller design scheme has been proposed. An example of airship flight control has been considered and the simulation results have been given to illustrate the effectiveness of proposed method.

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