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Non-Identical Synchronization, Anti-Synchronization and Control of Single-Machine Infinite-Bus Power System via Active Control

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Abstract. Since the idea of synchronizing two identical chaotic systems under different initial conditions was first introduced by Pecora and Carroll, the synchronization of chaotic systems has attracted much attention, and the synchronization of non-identical chaotic systems has also been investigated. Single-Machine Infinite-Bus (SMIB) power system has nonlinear behaviour. On account of avoiding undesirable behaviours in power systems such as voltage collapse, the synchronization and control of SMIB power system have considerable importance. This paper presents chaos synchronization and anti-synchronization of SMIB power system to Duffing oscillator by means of active control method. The sum of synchronization and anti-synchronization signals converge asymptotically to zero and achieve the control of SMIB power system. Numerical simulations are used to demonstrate the validity of proposed active control method on the non-identical synchronization, anti-synchronization and control of SMIB power system.

Keywords: Single-Machine Infinite-Bus; Duffing oscillator; synchronization; anti-synchronization; control; active control.

1. Introduction

Due to the complex behaviour, the synchronization and control of chaotic systems have been among the major issues in electrical control engineering. Firstly, Pecora and Carroll introduced a method for synchronizing chaotic systems in 1990 [23]. The aim of synchronization is to use a drive system's output to induce a response system so that the response system's output could follow the drive system's output asymptotically. After the pioneering work of Pecora and Carroll [23], various types of chaos synchronization have been investigated such as anti-synchronization [9, 13, 17, 19, 28], phase synchronization [12], lag synchronization [18], projective synchronization [32], and so on. In anti-synchronization, the response output of synchronized system has the same absolute values but opposite signs. The synchronization and control of nonlinear systems have been extensively studied, some useful methods were developed and applied to numerous chaotic systems. These methods mainly include feedback control [11], active control [1, 3, 9, 21, 27, 28, 34], sliding mode control [7, 14, 26], passive control [2, 16, 36], adaptive control [13, 28], and impulsive control [4]. Synchronization of two identical Lorenz chaotic systems evolving from different initial conditions using active control method was introduced by Bai and Lonngren in 1997 [3]. Thereafter, the synchronization and anti-synchronization have been applied to both identical [1, 21, 27] and non-identical chaotic systems [9, 28, 34] by using active control. Non-identical synchronization deals with synchronizing between two different chaotic systems. Yassen has applied the active control method to realize the chaos synchronization for each pair of Lorenz, Lü and Chen chaotic systems [34]. Emadzadeh and Haeri have implemented the active control method to achieve anti-synchronization of chaos between Lü and Rössler systems [9]. Wang and Shi have concerned with the anti-synchronization

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between Liu and Lorenz chaotic systems by means of active control method with known parameters and adaptive control method with unknown parameters [28]. The other popular method, sliding mode control has been also used for the synchronization and anti-synchronization of identical and non-identical chaotic systems [7, 14, 26]. The synchronization with passive control method has been implemented to Chen system [16], between Rössler and Genesio-Tesi systems [2], hyperchaotic Lü system [36] and many other chaotic systems. The synchronization and control of nonlinear systems will be explored due to its useful applications in a variety of fields including physics, chemistry, ecology, biological systems and secure communication [5, 35].

Power systems are basically a collection of nonlinearly coupled systems and generators which supply electric power to loads. A SMIB power system is a simplified dynamic model of complex power systems. As shown in Fig. 1, it composes of a single synchronous generator connected through a transmission line to a very large grid approximated by an infinite bus. The voltage profile of SMIB is pointed out using current and voltage phasors in Fig. 2. The SMIB power system is also a typical nonlinear dynamical system, the control and synchronization of nonlinear behaviours in electrical power systems have great importance from the management point of view to avoid undesirable behaviours such as voltage collapse [15, 24]. Recently, the control, stability and synchronization of nonlinear behaviours in SMIB have been studied. Chen et al. presented the dynamic behaviours of a SMIB power system with bifurcation diagrams, and controlled its chaos with a feedback controller in 2005 [8]. Ford et al. used nonlinear control technique to examine the transient stability problem for SMIB power systems in 2006 [10]. Shahverdiev et al. applied the chaos synchronization in some simple power models including SMIB power system in 2008 [25]. Yang et al. implemented chaos synchronization in SMIB powers system with sliding mode control and applied to secure communication [33]. Chang et al. designed a fuzzy controller to achieve the strict input passivity and Lyapunov stability for the SMIB power system [6]. Wei and Qin controlled the chaos in SMIB power system with adaptive passive control method in 2011 [29]. Ouassaid et al. developed a new nonlinear observer–controller scheme using sliding mode control method and applied to the SMIB power system in 2012 [22].

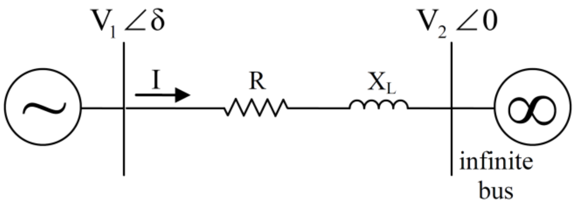


Figure 1. Single-Machine infinite-bus (SMIB) power system

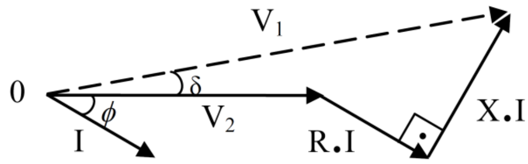


Figure 2. Phasor diagram of SMIB

The aim of this paper is to achieve the non-identical synchronization, anti-synchronization and control of SMIB power system by using active control method. Due to the fact that SMIB power system has second order differential equations, the well-known second order Duffing chaotic system is preferred for synchronization and anti-synchronization. For this purpose, firstly SMIB power system and Duffing oscillator are described and defined as a set of differential equations. Then, the active control, which is a widely-used method for the synchronization of chaotic systems due to its simplicity and success, is applied to these dynamical systems for non-identical synchronization and anti-synchronization. For implementing the control of SMIB power system, synchronization and anti-synchronization signals are summed. Finally, numerical simulations are performed to show the synchronization, anti-synchronization between these two chaotic systems and the control of SMIB power system.

2. System Descriptions

2.1. Single-Machine Infinite-Bus (SMIB) power system

The classical SMIB power system can be defined by the following swing equation as

$$M\ddot{\theta} + D\dot{\theta} + P_{\max}\sin(\theta) = P_m, \tag{1}$$

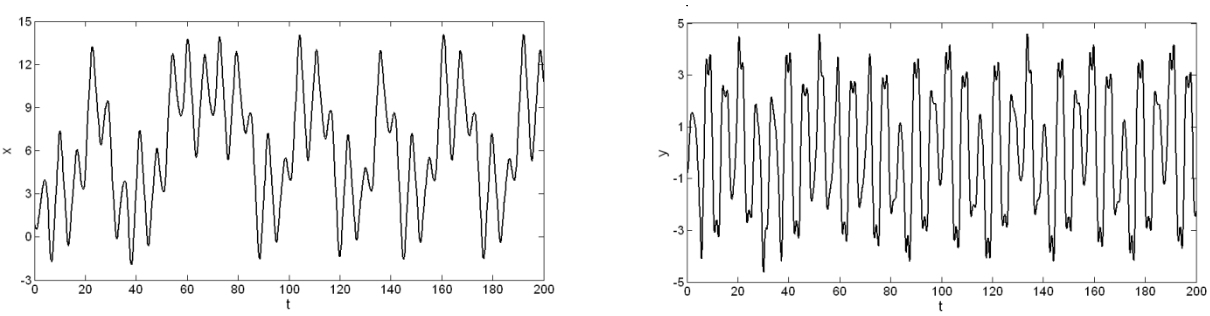
where θ, M, D, $P_{\max}$ and P_m represent angle of generator, moment of inertia, damping constant, maximum power of generator and power of the machine, respectively [31]. Also $P_m = A\sin(\omega t)$ where t is the time variable, A is the amplitude and ω is the angular frequency of the power of the machine [8].

Taking $x = \theta$ and $y = \dot{\theta}$, the equation (1) is equivalent to the following system

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -cy - \beta\sin(x) + f\sin(\omega t), \end{aligned} \tag{2}$$

where $c = D / M$, $\beta = P_{\max} / M$ and $f = A / M$ [8].

As shown in Figs. 3 and 4, when the SMIB power system (2) is at $c = 1$, $\beta = 3$, $f = 5$, $\omega = 1$ parameters values under $x(0) = 1$, $y(0) = -0.5$ initial conditions, it exhibits chaotic behaviour.



(a) (b)

Figure 3. Time series of SMIB power system for (a) *x* signals, and (b) *y* signals

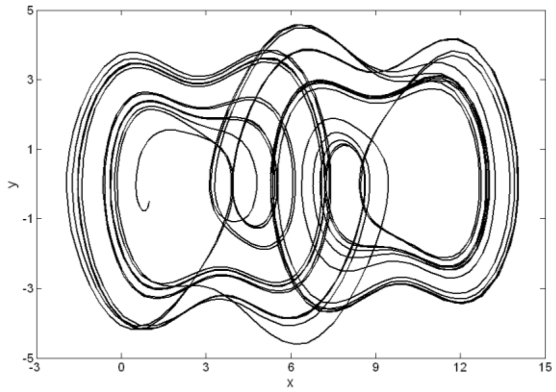


Figure 4. *x-y* phase plane of SMIB power system

2.2. Duffing oscillator

Duffing oscillator is a nonlinear system which describes the hardening spring effect observed in many mechanical problems with a cubic stiffness term. It is one of the extensively studied nonlinear non-autonomous equations, exhibiting various dynamic behaviours, including chaos and bifurcations. The most general forced form of the Duffing equation is

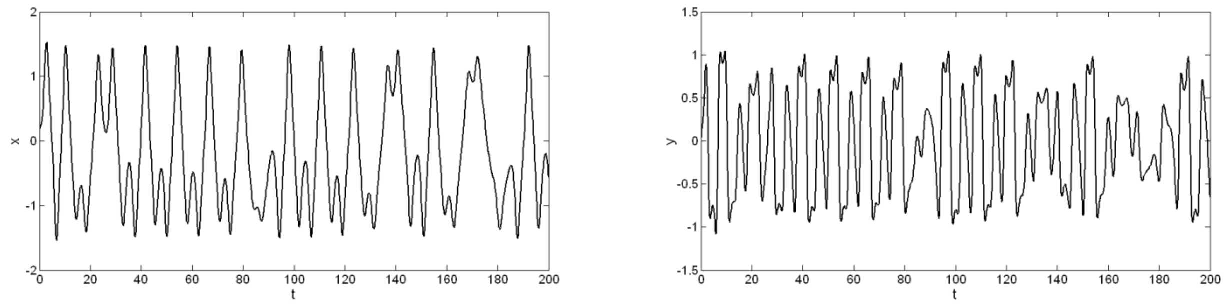
$$\ddot{x} + \delta\,\dot{x} + (\beta\,x^3 \pm \omega_0^2 x) = \mu \sin(\omega t + \phi), \tag{3}$$

where t is the time variable, $\delta > 0$, μ, β, ω and ϕ parameters denote damping coefficient, amplitude of the parametric excitation, stiffness constant, forcing frequency and the clock, respectively [30]. This equation is used in a number of special forms depending on the parameters chosen. For example, with taking $\beta = 1$, $\omega_0 = 1$, resetting the clock so that $\phi = 0$ and using the minus sign, the equation (3) becomes

$$\ddot{x} + \delta\,\dot{x} + (x^3 - x) = \mu \sin(\omega t). \tag{4}$$

The equation (4) can be rewritten as a set of two first-order differential equations [20]:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -x^3 + x - \delta\,y + \mu \sin(\omega t). \end{aligned} \tag{5}$$



a) (b)

Figure 5. Time series of Duffing oscillator for (a) *x* signals, and (b) *y* signals

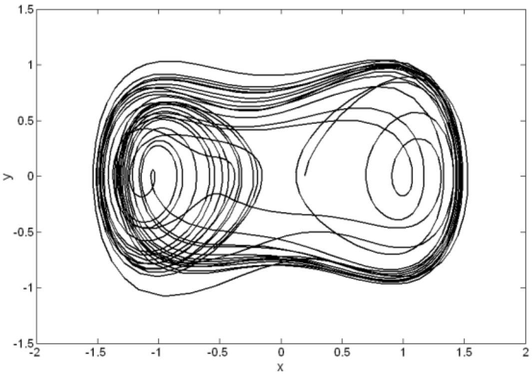


Figure 6. *x-y* phase plane of Duffing oscillator

As shown in Figs. 5 and 6, the Duffing equation (5) represents chaotic behaviour with parameter values $\delta = 0.25$, $\mu = 0.4$, $\omega = 1$ under initial conditions $x(0) = 0.2$, $y(0) = 0$.

3. Non-Identical Synchronization, Anti-Synchronization and Control of SMIB Power System

3.1. Synchronization between SMIB power system and Duffing oscillator

In order to observe the synchronization between SMIB power system and Duffing oscillator, we have two above-mentioned systems where the Duffing drive system denoted by the subscript 1 controls the SMIB response system which is denoted by the subscript 2. The drive system is given by

$$\begin{aligned} \dot{x}_1 &= y_1, \\ \dot{y}_1 &= -x_1^3 + x_1 - \delta\, y_1 + \mu \sin(\omega t), \end{aligned} \tag{6}$$

and the response system is defined as follows:

$$\begin{aligned} \dot{x}_2 &= y_2 + u_1(t), \\ \dot{y}_2 &= -c\, y_2 - \beta \sin(x_2) + f \sin(\omega t) + u_2(t), \end{aligned} \tag{7}$$

where $u_1(t)$ and $u_2(t)$ in system (7) are the control functions to be determined. In order to estimate the control functions for synchronization, we subtract equation (6) from equation (7). We define the error system as the differences between the Duffing oscillator (6) and the SMIB power system (7) that is to be controlled and the controlling system using

$$\begin{aligned} e_1 &= x_2 - x_1, \\ e_2 &= y_2 - y_1. \end{aligned} \tag{8}$$

Subtracting equation (6) from equation (7) and using the notation (8) yields

$$\begin{aligned} \dot{e}_1 &= e_2 + u_1(t), \\ \dot{e}_2 &= -c\, y_2 - \beta \sin(x_2) + x_1^3 - x_1 + \delta\, y_1 \\ &+ \delta\, y_1 + (f - \mu)\sin(\omega t) + u_2(t). \end{aligned} \tag{9}$$

System (9) is called the error system; e_1 and e_2 are the error states. The synchronization problem is to ensure the error system (9) asymptotically stable at the origin. Therefore, we define the active control functions $u_1(t)$ and $u_2(t)$ as follows:

$$\begin{aligned} &u_1(t) = v_1(t), \\ &u_2(t) = c\, y_2 + \beta \sin(x_2) - x_1^3 + \\ &x_1 - \delta\, y_1 - (f - \mu)\sin(\omega t) + v_2(t). \end{aligned} \tag{10}$$

Then, the error system (9) becomes

$$\begin{aligned} \dot{e}_1 &= e_2 + v_1(t), \\ \dot{e}_2 &= v_2(t). \end{aligned} \tag{11}$$

The error system (11) is linear and the convergent solution can be found under appropriate control input $v_1(t)$ and $v_2(t)$ as function of the error states e_1 and e_2. As long as the solutions of the system (11) converge to zero as time t goes to infinity, the synchronization between SMIB power system and Duffing oscillator is realized. There are many possible choices for the control $v_1(t)$ and $v_2(t)$ functions, we take

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, \tag{12}$$

where A is a 2 x 2 constant matrix to be determined. In order to make the closed loop system to be stable, the proper choice of the entries of the matrix A is such that the feedback system must have all of the eigenvalues with negative real parts. Let the matrix A is chosen in the following form

$$A = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}. \tag{13}$$

For this particular choice, the closed loop system (11) has the eigenvalues –1 and –1. This choice of control gains will lead to a stable system in which the error states e_1 and e_2 converge to zero as time t goes to infinity. Hence, it implies that the synchronization between Duffing oscillator and SMIB power system is achieved.

3.2. Anti-synchronization between SMIB power system and Duffing oscillator

In order to observe the anti-synchronization between SMIB power system and Duffing oscillator, as in the previous section, we have two above-mentioned systems where the Duffing drive system denoted by the subscript 1 controls the SMIB response system which is denoted by the subscript 2. The drive and response systems are the same as in equations (6) and (7) with $u_3(t)$ and $u_4(t)$ control functions, respectively.

In this case, we add the drive and response system instead of subtracting:

$$\begin{aligned} e_1 &= x_2 + x_1, \\ e_2 &= y_2 + y_1. \end{aligned} \tag{14}$$

This leads to

$$\begin{aligned} \dot{e}_1 &= e_2 + u_3(t), \\ \dot{e}_2 &= -c\,y_2 - \beta\sin(x_2) - x_1^3 + x_1 - \\ &\delta\,y_1 + (f+\mu)\sin(\omega t) + u_4(t). \end{aligned} \tag{15}$$

The anti-synchronization problem is to ensure the error system (15) asymptotically stable at the origin. Therefore, we define the active control functions $u_3(t)$ and $u_4(t)$ as follows:

$$\begin{aligned} u_3(t) &= v_3(t), \\ u_4(t) &= c\,y_2 + \beta\sin(x_2) + x_1^3 - x_1 + \\ &\delta\,y_1 - (f+\mu)\sin(\omega t) + v_4(t). \end{aligned} \tag{16}$$

Then, this implies

$$\begin{aligned} \dot{e}_1 &= e_2 + v_3(t), \\ \dot{e}_2 &= v_4(t). \end{aligned} \tag{17}$$

The error system (17) of anti-synchronization is reached to exactly the same error system of the synchronization in equation (11). We take $v_3(t)$ and $v_4(t)$ control inputs as before in equation (12). If we choose the A matrix in the form as (13) for anti-synchronization, we will obtain the same negative eigenvalues –1 and –1 for the closed loop system (17). This choice of control gains will lead to a stable system in which the error states e_1 and e_2 converge to zero as time t goes to infinity. Hence, it implies that the anti-synchronization between Duffing oscillator and SMIB power system is achieved.

3.3. Control of SMIB power system

In order to observe the control SMIB power system, we add non-identical synchronization and anti-synchronization signals that are acquired in the previous sections.

The non-identical synchronization of the SMIB power system has the following equation

$$\begin{aligned} \dot{x}_2 &= y_2 + u_1(t), \\ \dot{y}_2 &= -c\,y_2 - \beta\sin(x_2) + f\sin(\omega t) + u_2(t), \end{aligned} \tag{18}$$

where $u_1(t)$ and $u_2(t)$ are the control functions which provide synchronization to Duffing oscillator. From the equations (10), (12), (13) and (8), they can be calculated as follows:

$$\begin{aligned} u_1(t) &= -(x_2 - x_1) - (y_2 - y_1), \\ u_2(t) &= c\,y_2 + \beta\sin(x_2) - x_1^3 + x_1 \\ &- \delta\,y_1 - (f-\mu)\sin(\omega t) - (y_2 - y_1), \end{aligned} \tag{19}$$

where the x_1 and y_1 are the state variables of Duffing oscillator.

The non-identical anti-synchronization of the SMIB power system has the following equation

$$\begin{aligned} \dot{x}_2 &= y_2 + u_3(t), \\ \dot{y}_2 &= -c\,y_2 - \beta\sin(x_2) + f\sin(\omega t) + u_4(t), \end{aligned} \tag{20}$$

where $u_3(t)$ and $u_4(t)$ are the control functions which provide anti-synchronization to Duffing oscillator. From the equations (16), (12), (13) and (14), they can be calculated as follows:

$$\begin{aligned} u_3(t) &= -(x_2 + x_1) - (y_2 + y_1), \\ u_4(t) &= c\,y_2 + \beta\sin(x_2) + x_1^3 - x_1 + \\ &\delta\,y_1 - (f+\mu)\sin(\omega t) - (y_2 + y_1), \end{aligned} \tag{21}$$

where the x_1 and y_1 are the state variables of Duffing oscillator.

So that the error dynamics of non-identical synchronization and anti-synchronization of SMIB power system converge asymptotically to zero, the sum of these functions in equations (18) and (20) will lead to a stable system in which the error states converge to zero as time t goes to infinity. The control signals of SMIB power system becomes

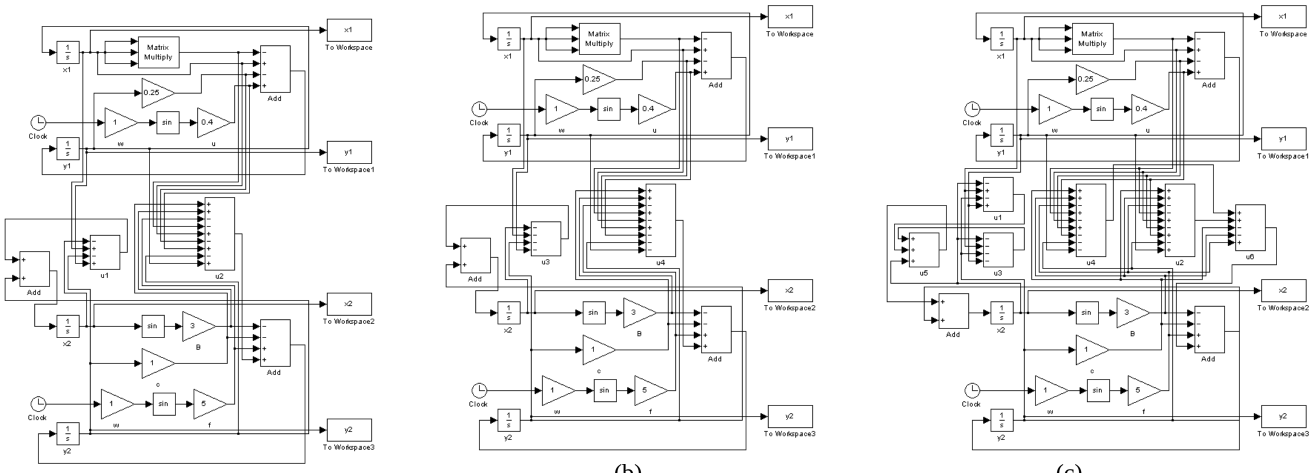
$$\begin{aligned} u_5(t) &= y_2 + u_1(t) + u_3(t), \\ u_6(t) &= -c\,y_2 - \beta\sin(x_2) + f\sin(\omega t) \\ &+ u_2(t) + u_4(t), \end{aligned} \tag{22}$$

where $u_5(t)$ and $u_6(t)$ denote the control functions for the x and y state variables of SMIB power system, respectively.

Hence, it implies that the control of SMIB power system is achieved.

4. Numerical Results

In this section, numerical simulations are performed to show the non-identical synchronization, anti-synchronization and control of SMIB power system. The fourth-order Runge–Kutta method is used in all numerical simulations with variable time step. The parameters of SMIB power system are taken as $c = 1$, $\beta = 3$, $f = 5$, $\omega = 1$ so that the system exhibits chaotic behaviour. The initial values of the SMIB response system are selected as $x\,(0) = 1$ and $y\,(0) = -0.5$. The parameters of drive Duffing oscillator are considered as $\delta = 0.25$, $\mu = 0.4$, $\omega = 1$ to ensure the chaotic behaviour and the initial values of the Duffing system are chosen as $x\,(0) = 0.2$, $y\,(0) = 0$. The controllers are arbitrary activated at $t = 25$ in all simulations. By using Matlab–Simulink program, the mathematical model of the non-identical synchronization, anti-synchronization and control of SMIB power system are constructed and shown in Fig. 7.



(a) (b) (c)

Figure 7. Matlab–Simulink modelling of SMIB power system for (a) non-identical synchronization, (b) non-identical anti-synchronization, and (c) control

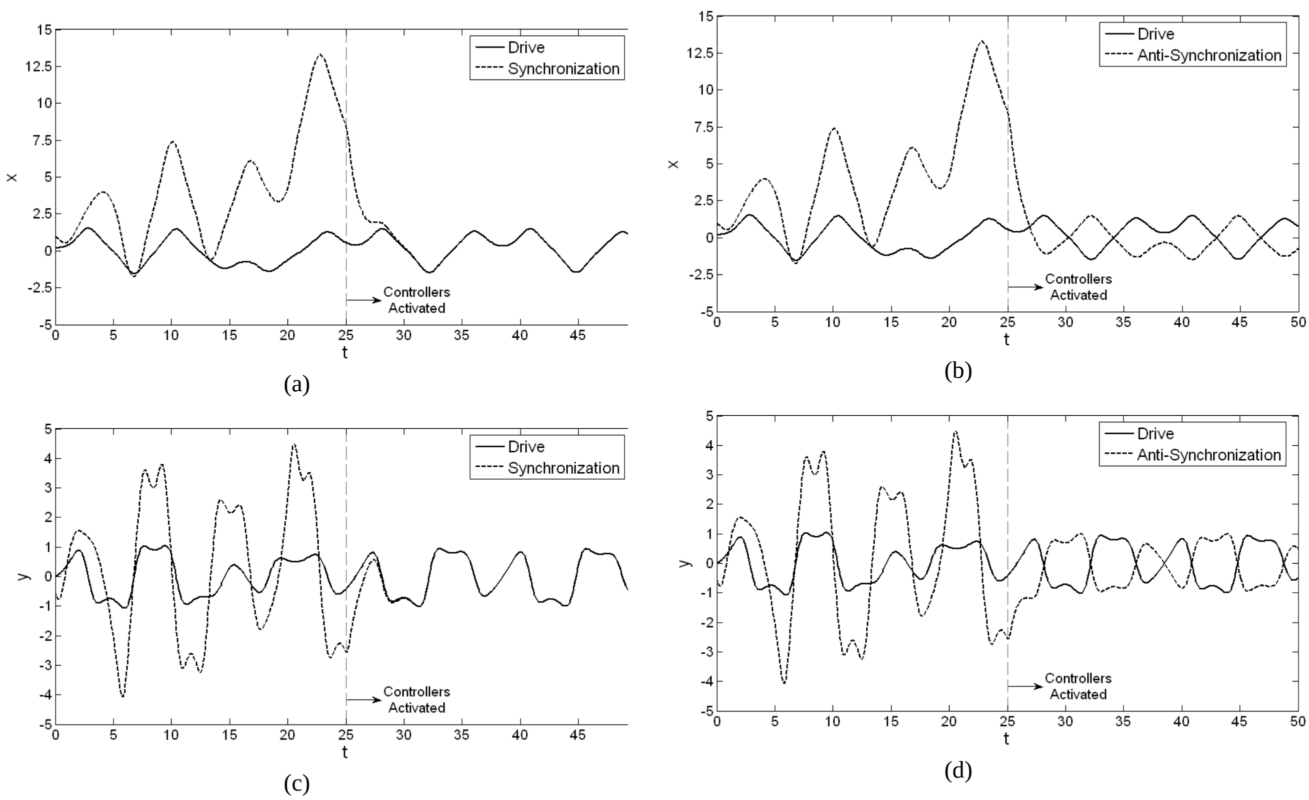
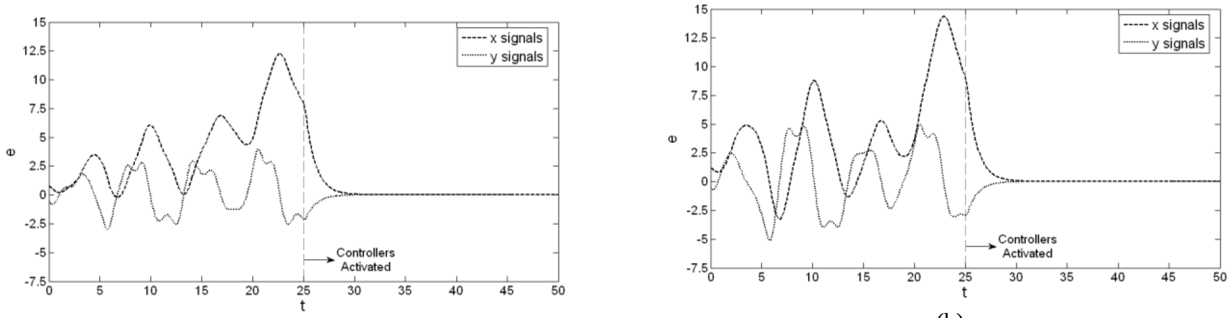
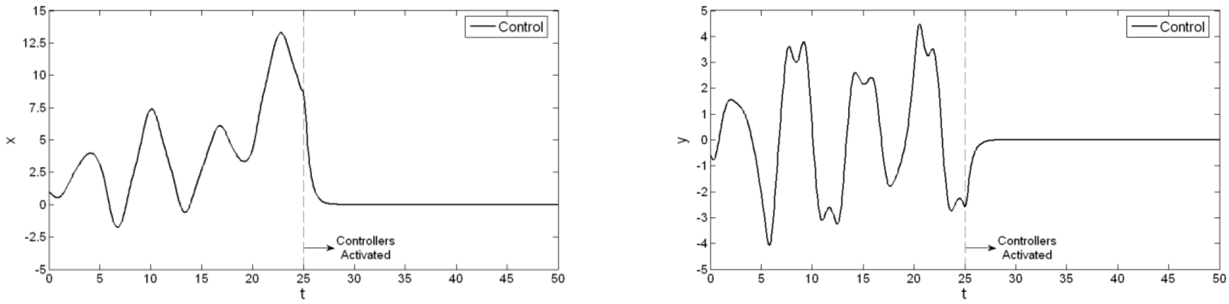


Figure 8. The time response of states for SMIB response system and Duffing drive system with active controllers are activated at $t = 25$ (a) x signals for synchronization, (b) x signals for anti-synchronization, (c) y signals for synchronization, and (d) y signals for anti-synchronization



(a) (b)

Figure 9. The time response of the error signals for SMIB response system and Duffing drive system with active controllers are activated at t = 25 (a) synchronization, and (b) anti-synchronization



(a) (b)

Figure10. The time response of the controlled SMIB power system to $E(0, 0)$ equilibrium point with active controllers are activated at t = 25 (a) x signals, and (b) y signals

The simulation results of the synchronization and anti-synchronization between SMIB power system and Duffing oscillator are shown in Fig. 8: (a) x signals for synchronization, (b) x signals for anti-synchronization, (c) y signals for synchronization, and (d) y signals for anti-synchronization. The error signals between these systems are illustrated in Fig. 9: (a) displays for synchronization, and (b) displays for anti-synchronization.

As expected, the error signals that are shown in Fig. 9 converge asymptotically to zero. Synchronization and anti-synchronization between SMIB power system and Duffing oscillator is observed when $t \geq 29$, which verifies the feasibility of the proposed active control method.

The simulation results for the control of SMIB power system to zero equilibrium point are shown in Fig. 10: (a) displays x signals, and (b) displays y signals.

As expected, the outputs of the SMIB power system converge to the $E(0, 0)$ equilibrium point, after the controllers are activated. As seen in Fig. 10, control is firstly observed when $t \geq 28$ with the active controllers, which confirms the effectiveness of the proposed control method.

Although simplified power system model cannot be analysed basically regarding synchronization and anti-synchronization, with a well-known Duffing chaotic oscillator, the SMIB have been easily synchronized and anti-synchronized non-identically to this system. The related Figs. 8–10 show that the error dynamics converge to zero asymptotically in the non-identical synchronization, anti-synchronization and control of SMIB power system which can be hardly obtained stability condition.

5. Conclusion

In recent years, electrical power systems have been gaining importance, but difficult to operate especially when they exhibit chaotic behaviour under the lack of reactive power and different disturbances. In this paper, it is the first time that the non-identical synchronization and anti-synchronization of SMIB power system is applied. This paper shows that SMIB power system and Duffing oscillator can be master and slave each other. SMIB power system is one of the simple power models which are essentially nonlinear dynamic systems. Duffing oscillator is the well-known second order chaotic system which is appropriate for the synchronization and anti-synchronization of SMIB power system. Based on active control theory, active controllers have been designed for synchronization and anti-synchronization of SMIB power system to Duffing oscillator. The sum of non-identical synchronization and anti-synchronization signals provides the control of chaotic SMIB power system. Numerical simulations show that when the controllers are activated at t = 25, non-identical synchronization and anti-synchronization are observed at $t \geq 29$, and the control of SMIB power system is observed at $t \geq 28$, which validates the robustness of the proposed active control method. As a future work, the other chaos control methods may be applied for the non-identical synchronization and anti-synchronization of SMIB power system.

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