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The Robust Asymmetric Minimum Cost Consensus Models with Interval-type Opinions

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Existing consensus models primarily rely on precise opinions from decision-makers in a predefined context, neglecting the dynamics of expert opinion adjustments. To address this limitation, we introduce the interval-type opinions and explores the group decision consensus model under asymmetric adjustment cost from an uncertainty perspective. Then, the robust optimization theory is applied to adress the uncertainty in adjustment costs of decision-making individuals. The robust asymmetric cost consensus model of interval opinions under three uncertain scenarios is built. Finally, the validity of proposed model is verified by numerical calculations, and a sensitivity analysis and comparative study are performed. The results show that: (1) Utilizing interval opinions can significantly reduce consensus costs when compared to precise opinions;(2) Comprehensively comparing the three proposed robust models, the consensus model with budget asymmetric cost has the best performance.

KEYWORDS: Interval Opinions; Asymmetric Cost; Group Decision Making; Robust Optimization; Consensus Model.

1. Introduction

As an important component of group decision-making, the consensus process [6,15] has become a hot research topic over the world. This process aims to achieve a satisfactory or acceptable result for most of the group members through continuous consultation and adjustment of opinions on a practical decision-making problem. Consensus often requires a compensatory payment by the moderator (the so-called “consensus cost”) to prompt the decision maker (DM) to make adjustments. For this reason, Ben-Arieh et al. [1] pioneered the concept of consensus cost, which has been widely applied to solve problems such as P2P bargaining [19], demolition and relocation negotiation [4], and transboundary water pollution control [3].

However, in practical decision problems, due to the limitations of information acquisition and the complexity of the decision environment, decision makers often find it difficult to give a precise opinion and prefer to express their preferences in terms of interval-opinions. Guo et. al. [8] introduced uncertainty theory to describe the uncertain preference of decision makers, and studied the minimum cost consensus model based on linear uncertainty distribution. Tan et al. [18] express the decision maker’s opinion preferences using interval functions and random distributions to construct a consensus model for cost minimization and individual utility maximization. However, there are significant limitations in expressing decision makers’ preferences only in terms of interval values and functions.

The research framework developed by Ben-Arieh et al. [1] assumes implicitly that the unit adjustment cost required to modify individual opinions to consensus opinions is symmetric, which may not hold in reality, where the distribution of consensus cost may be asymmetric. Cheng et al. [3] argued that experts often consider asymmetric cost functions to account for dissimilar adjustment costs relating to positive and negative deviations when modifying their initial opinions. Based on this premise, they proposed a MCCM that accounts for asymmetric unit adjustment costs and used it to solve the cross-regional water pollution management problem in the Taihu Lake basin. Qu et al. [17] adopted data-driven methods and developed robust consensus models for three asymmetric cost consensus frameworks, each incorporating uncertainty with four distinct uncertainty sets. These frameworks were also subject to direction constraints, compromise limits, and no-cost thresholds, respectively. Li et al. [12] devised a new decomposition algorithm to study stochastic scenarios for a two-stage asymmetric cost consensus model.

In addition, Cheng et al.’s [3] modeling approach considers a deterministic decision environment, based on unit upward/downward adjustment costs, initial opinions, compromise limits, and no-cost thresholds that are fixed values. This approach does not account for the potential influence of experts’ educational background, knowledge, and experience on the decision-making process, nor does it address the impact of the decision environment, which can result in significant uncertainties. Despite the existing research on this topic, there is still a lack of study on considering both interval opinions and asymmetric cost consensus under uncertain environment [21, 11], which makes this research direction have broad prospects.

Methods for dealing with uncertainty in group decision-making research generally include fuzzy interval analysis [10, 16], uncertainty theory[5,7], stochastic optimization [18, 13], and robust optimization [17, 20]. Although these methods partially address the impact of uncertain parameters and provide useful insights for group decision-making, they may have some drawbacks and may not always be practical. For instance, the fuzzy interval method struggles to handle complex decision problems. In stochastic optimization, complete historical data or information on the precise probability distribution of unknown parameters is difficult to access. Additionally, the chance constraint can sometimes reduce the model’s convexity and make it challenging to solve the original problem. The upper exact bounds in uncertainty theory are not always easily attainable, thus limiting their application in GDM. In light of these considerations, we opted to use robust optimization to address uncertain parameters.

Robust optimization (RO) was famous for handle uncertainty when the precise distribution of parameters is unknown, but the boundary information was available. By constructing an uncertainty set, RO discovered an optimal solution that fulfills all restrictions and enhances the model objective function in the worst-case scenario. Bertsimas et al. [2] proposed parametric adjustment of the conservativeness of robust models and applied RO methods to discrete scenarios, contributing to the maturation of the RO theory [9, 14].

This paper presents several principal contributions and innovations, including:

1 Construct a consensus decision model for interval opinions and asymmetric cost by considering both interval-type opinion and asymmetric adjustment cost in MCCM.

2 Robust optimization methods are introduced to describe the uncertainty of unit adjustment cost, build a consensus model for interval-type robust asymmetric costs, and obtain the robust counterpart, respectively.

3 The constructed model is applied to the case study of "The Grains to Greens Program (GTGP)", comparing and analyzing the existing consensus decision-making models to verify the rationality and effectiveness of the proposed model.

4 Application of the proposed model to an offshore oil spill rescue case study for comparing and analyzing the existing cost consensus models in uncertain decision environments, validating the rationality and validity of the proposed model.

The rest of this paper is organized as follows:

A description of the problem is given in Section 2.

And then, we construct the asymmetric minimum cost consensus model with interval opinion in Section 3.

In Section 4, the robust optimization theory is applied to construct three uncertainty scenarios, box, ellipsoid and budget, to express the uncertainty of the unit adjustment cost in the asymmetric cost consensus model of interval-type opinions.

Section 5 is a case study with sensitivity analysis and comparative study, and finally, conclusions and future research directions.

2. Model Description

Consider a group decision-making system consisting of a moderator and n decision makers. Let $o_i(i=1,...,n)$ represent the initial expert opinions and o' denote the consensus opinion. Most of the existing consensus modeling studies set the decision maker's opinions as an initially determined value. However, due to the asymmetry of information and the limitation of expert level, the initial opinions of decision makers often have fuzzy characteristics.

Here, we assume that the initial opinions of the DM is $o_i=[o_{il},o_{ir}](0<o_{il}<o_{ir})$, and introduce the interval preference coefficient $\beta(0\le\beta\le1)$. Then the interval opinions can be expressed as $o_i=o_{il}+\beta(o_{ir}-o_{il})$.

Obviously, when the coefficient β is determined, the initial opinions degenerates to a precise value. Using interval values to express the initial opinions of decision-making individuals can maximize the retention of the initial opinions of experts, and is closer to the practical GDM problems, which is easy to be understood and accepted by DMs. When the opinions of all decision-making individuals converge to the value o', the consensus opinions of the group are reached.

Here, an asymmetric minimum cost consensus model of interval opinions will be developed.

Assuming a linear relationship between the adjustment and bias of decision experts' opinions, the cost of convincing the ith DM to change his or her opinions is c_i. In the interest of moderator, the cost of consensus should be minimized in the interest of efficient consensus. Let the unit costs of upward and downward adjustments of individual decision-making opinions in the consensus process be c_i^U and c_i^D, respectively. Then, the asymmetric minimum cost consensus model for interval opinions is as follows:

$$\begin{aligned}
\min\quad & TC=\sum_{i\in L}c_i^U(o'-[o_{il}+\beta(o_{ir}-o_{il})]) \\
& +\sum_{i\in L}c_i^D(o_i-[o_{il}+\beta(o_{ir}-o_{il})]) \\
s.t\quad & o'\in O \\
& 0\le\beta_i\le1.
\end{aligned} \tag{1}$$

where $L=\{i|o_i\le o',i\in N\}$ is a set of expert opinions below consensus and $H=\{i|o_i\ge o',i\in N\}$ is a feasible set of expert opinions over the consensus opinions.

To solve this complex nonlinear problem, introduce positive and negative deviations $\delta_i^+=\max\{o_i-o',0\}$, $\delta_i^-=\max\{o'-o_i,0\}$ and rewrite Equation (1) as follows:

$$\begin{aligned}
\min\quad & TC=\sum_{i=1}^{n}(c_i^D\delta_i^+ + c_i^U\delta_i^-) \\
s.t\quad & o'+\delta_i^+-\delta_i^-=o_{il}+\beta(o_{ir}-o_{il}) \\
& \delta_i^+\ge0,\ \ \delta_i^-\ge0 \\
& o'\in O, i\in N \\
& 0\le\beta_i\le1.
\end{aligned} \tag{2}$$

In the consensus negotiation process, the experts generally do not revise their opinions endlessly, so the range of opinions adjustments is denoted by γ. The γ-MCCM-DC can be described as:

$$\begin{aligned} &\min \quad TC=\sum_{i=1}^{n}(c_i^D\delta_i^+ + c_i^U\delta_i^-) \\ &s.t \quad o'+\delta_i^+ - \delta_i^- = o_{il}+\beta(o_{ir}-o_{il}) \\ &\quad \delta_i^+ \geq 0,\ \ \delta_i^- \geq 0 \\ &\quad \delta_i^+,\delta_i^- \leq \gamma_i \\ &\quad 0\leq \beta_i \leq 1. \end{aligned} \tag{3}$$

In GDM, many individual decision-makers may be willing to modify their opinions within a particular range to reach an efficient consensus result promptly. This can lead to the attainment of an efficient consensus quickly, with no expenditure of moderator resources within this threshold range. However, if the opinions deviate beyond this limit, the cost of opinions modification should be appropriately compensated. We assume that the no-cost threshold for each expert is φ_i. Then, we get the threshold-based consensus model of interval opinions:

$$\begin{aligned} &\min \quad TC=\sum_{i=1}^{n}(c_i^U u_i^- + c_i^D v_i^+) \\ &s.t \quad u_i^+ - u_i^- = o_{il}+\beta(o_{ir}-o_{il}) - o' + \varphi_i \\ &\quad v_i^+ - v_i^- = o_{il}+\beta(o_{ir}-o_{il}) - o' - \varphi_i \\ &\quad o', u_i^+, u_i^-, v_i^+, v_i^- \geq 0 \\ &\quad 0\leq \beta_i \leq 1. \end{aligned} \tag{4}$$

where

$$u_i^+ = (o_i - o' + \varphi_i)^+, u_i^- = (o' - o_i - \varphi_i)^+$$
$$v_i^+ = (o_i - o' - \varphi_i)^+, v_i^- = (o' - o_i + \varphi_i)^+.$$

and we have

$$u_i^+ \in [o'-\varphi_i, o'], u_i^- \in (0, o'-\varphi_i)$$
$$v_i^+ \in [o'+\varphi_i, +\infty), v_i^- \in (o', o'+\varphi_i)$$
$$u_i^+ \cdot u_i^- = 0, v_i^+ \cdot v_i^- = 0.$$

3. Robust Asymmetric Cost Consensus Model with Interval- Opinions

Inspired by Cheng et. al. [7], this section will explore three asymmetric cost consensus models based on interval opinions in an uncertain environment, namely, direction constraint, compromise limit and threshold-based.

3.1. Robust Directional Constrained Cost Consensus Model of Interval Opinions

In GDM, it is difficult for the moderator to provide a clear and definite unit consensus cost to the decision-making experts due to the influence of decision scenarios. Therefore, the optimal solution of the model obtained by expressing the unit consensus cost using deterministic values is limited or even infeasible. This urges us to explore a method that is not affected by data uncertainty, i.e., whose results are "robust". Based on the RO theory, a robust equivalent formula is constructed to solve the problem (1). where the uncertain set can be expressed as:

$$\mathbb{Z}=\left\{[c^U;c^D]=[c^U;c^D]+\sum_{j=1}^{L}\xi_j[c_j^U;c_j^D]:\xi\in\Xi\right\}. \tag{5}$$

Theorem 1. If the uncertain set of Equation (5) is defined as a box set

$$\mathbb{Z}^{Box}=\left\{\xi\in R^L:\|\xi\|_\infty\leq\tau\right\}. \tag{6}$$

where τ is an uncertain parameter, the expression of the box type directional constrained cost consensus model of interval opinions (Box-RMCCM-DC) model is:

$$\begin{aligned} &\min_{o',\delta_i^+,\delta_i^-} \quad c_0^U\delta^- + c_0^D\delta^+ + \gamma\|v\|_1 \\ &s.t. \quad o'+\delta_i^+ - \delta_i^- = o_{il}+\beta_i(o_{ir}-o_{il}) \\ &\quad -v_j \leq (c_j^U)^T\delta^- + (c_j^D)^T\delta^+ \leq v_j \\ &\quad o'\in O, \delta_i^+,\delta_i^- \geq 0, i=1,...,n, j=1,...,L. \end{aligned} \tag{7}$$

where the vector value of up (down) adjustment cost and the vector value of positive and negative deviation are expressed in matrix form.

$$c^U=(c_1^U,...,c_n^U), c^D=(c_1^D,...,c_n^D)$$
$$\delta^+=(\delta_1^+,...,\delta_n^+), \delta^-=(\delta_1^-,...,\delta_n^-)$$

Proof. Considering the uncertainty of the unit consensus cost. According to the expression of the box set (6) under the robust worst case. For $\forall\|\xi\|_\infty\leq\tau$, we have

$$c_0^U\delta^- + c_0^D\delta^+ + \sum_j^L \xi_j(c_j^U\delta^- + c_j^D\delta^+) \le B$$

$$\max_{\|\xi\|_\infty \le \tau} \sum_j^L \xi_j(c_j^U\delta^- + c_j^D\delta^+) \le B - c_0^U\delta^- - c_0^D\delta^+$$

Considering the optimal solution of the model in the robust worst case, maximizing the left end of the inequality, we get

$$\tau\sum_j^L \xi_j(c_j^U\delta^- + c_j^D\delta^+) \le B - c_0^U\delta^- - c_0^D\delta^+$$

Then, we can get the following linear inequality expression:

$$\begin{cases} -v_j \le c_j^U\delta^- + c_j^D\delta^+ \le v_j, j = 1,..., L \\ c_0^U\delta^- + c_0^D\delta^+ + \tau\sum_{j=1}^L v_j \le B \end{cases}.$$

In summary, Theorem 1 can obtain.

Theorem 2. If the uncertain set Equation (5) is defined as an ellipsoid type

$$\mathbb{Z}^{Epd} = \left\{\xi \in R^L : \|\xi\|_2 \le \Omega\right\}. \quad (8)$$

where Ω is the ellipsoid radius, then the expression of the direction constrains ellipsoid set consensus model of interval opinions (Epd-RMCCM-DC) is:

$$\begin{aligned} \min_{o',\delta_i^+,\delta_i^-} \quad & c_0^U\delta^- + c_0^D\delta^+ + \Omega\|v\|_2 \\ s.t. \quad & o' + \delta_i^+ - \delta_i^- = o_{il} + \beta_i(o_{ir} - o_{il}) \\ & -v_j \le (c_j^U)^T\delta^- + (c_j^D)^T\delta^+ \le v_j \\ & o' \in O, \delta_i^+, \delta_i^- \ge 0, i = 1,..., n, j = 1,..., L \end{aligned} \quad (9)$$

Proof. Similarly, according to the expression of the ellipsoid set (8), for $\forall\|\xi\|_2 \le \Omega$,

$$c_0^U\delta^- + c_0^D\delta^+ + \sum_j^L \xi_j(c_j^U\delta^- + c_j^D\delta^+) \le B$$

$$\max_{\|\xi\|_\infty \le \Omega} \sum_j^L \xi_j(c_j^U\delta^- + c_j^D\delta^+) \le B - c_0^U\delta^- - c_0^D\delta^+$$

Considering the optimal solution of the model in the worst case of robustness, we get

$$\Omega\sqrt{\sum_j^L \xi_j(c_j^U\delta^- + c_j^D\delta^+)^2} \le B - c_0^U\delta^- - c_0^D\delta^+$$

Then, we can get the following linear inequality system:

$$\begin{cases} -v_j \le c_j^U\delta^- + c_j^D\delta^+ \le v_j, j = 1,..., L \\ c_0^U\delta^- + c_0^D\delta^+ + \Omega\sqrt{\sum_{j=1}^L v_j^2} \le B \end{cases}$$

In summary, Theorem 2 can get.

Theorem 3. If the uncertain set of Equation (5) is defined as a budget type

$$\mathbb{Z}^{Bud} = \left\{\zeta \in R^L : \|\zeta\|_1 \le 1, \|\zeta\|_\infty \le \Gamma, 1 \le \Gamma \le L\right\}. \quad (10)$$

The budget type asymmetric consensus model with interval-opinions (Bud-RMCCM-DC) is expressed as follows:

$$\begin{aligned} \min_{o',\delta_i^+,\delta_i^-} \quad & c_0^U\delta^- + c_0^D\delta^+ + \|z_j\|_1 + \pi\|w_j\|_\infty \\ s.t. \quad & o' + \delta_i^+ - \delta_i^- = o_{il} + \beta_i(o_{ir} - o_{il}) \\ & (c_j^U)^T\delta^- + (c_j^D)^T\delta^+ = -(z_j + w_j), j = 1,...L \\ & o' \in O, \delta_i^+, \delta_i^- \ge 0, i = 1,..., n \end{aligned} \quad (11)$$

According to Ben-Tal et al. [28], the equivalent dual cone form of the budgeted type Equation (10) is

$$\mathbb{Z}^{Bud} = \left\{\zeta \in \mathbb{R}^L : P_1\xi + p_1 \in K^1, P_2\xi + p_2 \in K^2\right\}.$$

where K^1, K^2 are non-empty convex cones.

- $P_1\xi = [\xi;0], p_1 = [0_{L\times1};1],$

$$K^1 = \left\{[z;t] \in \mathbb{R}^L \times \mathbb{R} : \|z\|_\infty \le t\right\},$$

$$K_*^1 = \left\{[z;t] \in \mathbb{R}^L \times \mathbb{R} : \|z\|_1 \le t\right\}$$

- $P_2\xi = [\xi;0], p_2 = [0_{L\times1};\pi],$

$$K^2 = K_*^1 = \left\{[z;t] \in \mathbb{R}^L \times \mathbb{R} : \|z\|_1 \le t\right\}, K_*^2 = K^1$$

letting $y_1 = [z;\tau_1], y_2 = [\omega;\tau_2]$, whence 1-dimensional τ and L-dimensional z, ω, then the following set of constraints can be obtained:

$$\begin{aligned} & (c_j^U)^T\delta^- + (c_j^D)^T\delta^+ + \tau_1 + \pi\tau_2 + \le B, \\ & (c_j^U)^T\delta^- + (c_j^D)^T\delta^+ = (z + \omega)_j \\ & \|z\|_1 \le \tau_1, \\ & \|w\|_\infty \le \tau_2 \\ & j = 1,..., L. \end{aligned} \quad (12)$$

Applying (12) to model (11), Bud-RMCCM-DC can be found.

3.2. Robust Compromise Limit Cost Consensus Model with Interval Opinions

In this part, combined with the consensus model of compromise limit and the three robust uncertainty sets, the compromise limit cost consensus model of interval opinions under box set, ellipsoid set and budget uncertainty set are obtained in turn.

Case I. Assuming the uncertain set is box type, and without loss of generality, we can represent the box type compromise limits consensus model of interval opinions (γ-RMCCM-Box) as follows:

$$\begin{aligned} \min_{o',\delta_i^+,\delta_i^-} \quad & c_0^U\delta^- + c_0^D\delta^+ + \gamma\|v\|_1 \\ s.t. \quad & o' + \delta_i^+ - \delta_i^- = o_{il} + \beta_i(o_{ir} - o_{il}) \\ & -v_j \le (c_j^U)^T\delta^- + (c_j^D)^T\delta^+ \le v_j \\ & \delta_i^+, \delta_i^- \ge 0, \delta_i^+, \delta_i^- \le \gamma_i, i \in N \\ & o' \in O, j = 1, \ldots, L. \end{aligned} \tag{13}$$

Case II. Let the unit adjustment cost be the ellipsoid feasible centralized disturbance, then, the ellipsoid-type compromise limits consensus model of interval opinions (γ-RMCCM-Epd) can be expressed as follows:

$$\begin{aligned} \min_{o',\delta_i^+,\delta_i^-} \quad & c_0^U\delta^- + c_0^D\delta^+ + \Omega\|v\|_2 \\ s.t. \quad & o' + \delta_i^+ - \delta_i^- = o_{il} + \beta_i(o_{ir} - o_{il}) \\ & -v_j \le (c_j^U)^T\delta^- + (c_j^D)^T\delta^+ \le v_j \\ & \delta_i^+, \delta_i^- \ge 0, \delta_i^+, \delta_i^- \le \gamma_i, i \in N \\ & o' \in O, j = 1, \ldots, L. \end{aligned} \tag{14}$$

Case III. If the uncertain set is budget type, then the budget-type compromise limits consensus model of interval opinions (γ-RMCCM-Bud) is shown below:

$$\begin{aligned} \min_{o',\delta_i^+,\delta_i^-} \quad & c_0^U\delta^- + c_0^D\delta^+ + \|z_j\|_1 + \pi\|\omega_j\|_\infty \\ s.t. \quad & o' + \delta_i^+ - \delta_i^- = o_{il} + \beta_i(o_{ir} - o_{il}) \\ & -v_j \le (c_j^U)^T\delta^- + (c_j^D)^T\delta^+ \le v_j \\ & \delta_i^+, \delta_i^- \ge 0, \delta_i^+, \delta_i^- \le \gamma_i, i \in N \\ & o' \in O, j = 1, \ldots, L. \end{aligned} \tag{15}$$

3.3. Robust Threshold-based Cost Consensus Model with Interval Opinions

In this section, based on the threshold-based MCCM of interval opinions and robust optimization theory, we get three robust-type threshold-based consensus models with interval opinions in turns.

Case I. Consider the uncertainty set is box type, then the box type threshold-based consensus model with interval opinions (TB-RMCCM-Box) can be expressed as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n}(c_0^U u_i^- + c_0^D v_i^+) + \gamma\|v\|_1 \\ s.t. \quad & -v_j \le (c_j^U)^T\delta^- + (c_j^D)^T\delta^+ \le v_j \\ & u_i^+ - u_i^- = o_{il} + \beta_i(o_{ir} - o_{il}) \quad - o' + \varphi_i \\ & v_i^+ - v_i^- = o_{il} + \beta_i(o_{ir} - o_{il}) \quad - o' - \varphi_i \\ & o', \ u_i^+, u_i^-, v_i^+, v_i^- \ge 0 \\ & o' \in O, i \in N, j = 1, \ldots, L. \end{aligned} \tag{16}$$

Case II. Consider the uncertainty set is (8), without loss of generality, the ellipsoid type threshold-based consensus model of interval opinions (TB-RMCCM-Epd) can be expressed as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n}(c_0^U u_i^- + c_0^D v_i^+) + \Omega\|v\|_2 \\ s.t. \quad & -v_j \le (c_j^U)^T\delta^- + (c_j^D)^T\delta^+ \le v_j \\ & u_i^+ - u_i^- = o_{il} + \beta_i(o_{ir} - o_{il}) \quad - o' + \varphi_i \\ & v_i^+ - v_i^- = o_{il} + \beta_i(o_{ir} - o_{il}) \quad - o' - \varphi_i \\ & o', \ u_i^+, u_i^-, v_i^+, v_i^- \ge 0 \\ & o' \in O, i \in N, j = 1, \ldots, L. \end{aligned} \tag{17}$$

Case III. If the uncertainty set is budget type, then the budget type threshold-based consensus model of interval opinions (TB-RMCCM-Bud) can be expressed as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n}(c_0^U u_i^- + c_0^D v_i^+) + \|z_j\|_1 + \pi\|\omega\|_\infty \\ s.t. \quad & (c_j^U)^T\delta^- + (c_j^D)^T\delta^+ = -(z_j + \omega_j) \\ & u_i^+ - u_i^- = o_{il} + \beta_i(o_{ir} - o_{il}) \quad - o' + \varphi_i \\ & v_i^+ - v_i^- = o_{il} + \beta_i(o_{ir} - o_{il}) \quad - o' - \varphi_i \\ & o', \ u_i^+, u_i^-, v_i^+, v_i^- \ge 0 \\ & o' \in O, i \in N, j = 1, \ldots, L. \end{aligned} \tag{18}$$

4. Case Study

A series of ecological problems such as serious soil erosion, land desertification, salinization, and frequent sandstorms have seriously threatened people's healthy life. "The Grains to Greens Program (GTGP)" is one of the important policies in China's implementation of the western development strategy, which aims to repair and improve the ecological environment. Over the past 20 years of implementation, remarkable achievements have been made, which has significantly improved the source of rivers and lakes and the coastal ecological environment in China. GTGP is mainly based on the creation of ecological forests, relying on government grants to provide quota subsidies to farmers who respond to the policy according to the area of land that has ceased to be cultivated. It is a program with a large investment and a high level of mass participation. In the process of consensus negotiation, local governments hope that villages will support the implementation of policies, return as much arable land as possible to plant trees, and reduce the compensation cost (consensus cost) for the standard of returned land area as low as possible. However, due to different geographical locations, landforms, and labor composition, villages have different ability to return farmland. They need to adjust their opinions according to their own development and allowances (including food subsidies, planting subsidies, and living subsidies). When the area of cultivated land returned by the village is too small, it will bear the great pressure of the local government and public opinions. On the contrary, it may face a variety of problems such as developing new industries, promoting economic development, and solving labor employment. Here, we define the local government as the moderator, the farmer representative as the decision maker, and construct the group decision-making system. According to their level of development, the DM individuals estimate the minimum and maximum area of returning farmland to forest in the form of interval. Assuming that there are four village representatives and one local government representative participating in consultations on consensus decision-making problem on the GTGP. Assume that the initial opinions of the four decision experts are $o_1=[17,20]$, $o_2=[16,21]$, $o_3=[22,24]$, $o_4=[18,23]$ .The unit consensus cost is divided into upward and downward adjustment costs according to the direction, which are $(c_1^U,c_2^U,c_3^U,c_4^U)=(5,4,2,5)$, $(c_1^D,c_2^D,c_3^D,c_4^D)=(3,2,3,2)$. For experts whose initial opinions are higher than consensus opinions, the unit upward adjustment cost will not affect the results of group decision-making, but a small change in the unit downward adjustment cost may lead to a gigantic change in the consensus results, and vice versa.

For upward adjustment costs the negative deviation is taken as

$$c^U=\begin{pmatrix} -0.1 & -0.1 & -0.1 & -0.1 \\ -0.06 & -0.06 & -0.06 & -0.06 \\ -0.08 & -0.08 & -0.08 & -0.08 \\ -0.2 & -0.2 & -0.2 & -0.2 \end{pmatrix}$$

For the downward adjustment cost, the positive deviation is taken as

$$c^D=\begin{pmatrix} 0.2 & 0.2 & 0.2 & 0.2 \\ 0.05 & 0.05 & 0.05 & 0.05 \\ 0.15 & 0.15 & 0.15 & 0.15 \\ 0.1 & 0.1 & 0.1 & 0.1 \end{pmatrix}$$

Here we assume that all the uncertainty parameter of the model is 2. We used Python 3.7 to call the RSOME toolbox to solve the model.

By solving models (7), (9), and (11), the results of the robust consensus model with direction constraints are obtained: the optimal consensus is $o'_{Box}=o'_{Epd}=o'_{Bud}=18.39$, thus the positive and negative deviations are $\delta^+_{Box}=\delta^+_{Epd}=\delta^+_{Bud}=(0,1.2,2.2,3.6)$, $\delta^-_{Box}=\delta^-_{Epd}=\delta^-_{Bud}=(1.5,0,0,0)$, the interval coefficient are $(\beta_1,\beta_2,\beta_3,\beta_4)=(0.632,0.741,0.837,0.263)$. The total costs of the three models are 30.81, 27.13 and 26.14, respectively.

In the robust consensus model with compromise limits, each decision-maker is assumed to have limited compromises and tolerance behaviors. Let $(\gamma_1,\gamma_2,\gamma_3,\gamma_4)=(2,4,3,4)$ The total cost of three robust-type consensus models with compromise limits is 41.61, 38.73, 37.35. The optimal consensus opinions at this time is $o'_{Box}=o'_{Epd}=o'_{Bud}=19.51$, the corresponding positive and negative deviations are $\delta^+_{Box}=\delta^+_{Epd}=\delta^+_{Bud}=(0,0,1.4,2.1)$, $\delta^-_{Box}=\delta^-_{Epd}=\delta^-_{Bud}=(2.3,1.2,0.4,0)$, $(\beta_1,\beta_2,\beta_3,\beta_4)=(0.541,\ 0.132,0.631,0.844)$.

Next, we consider the case of cost-free thresholds, where each decision expert has its threshold. Since the model of threshold-based is derived from the consensus model of direction constraints, we continue to extend the data from the model in direction constraints for experiments. Suppose that the expert's no-cost threshold is $(\varphi_1,\varphi_2,\varphi_3,\varphi_4)=(2,3,0.5,1)$. By solving the models(16), (17), (18), the minimum total cost

Table 1
The Positive and negative deviation of threshold-based model

Type	$(u_1^+, u_1^-, v_1^+, v_1^-)$	$(u_2^+, u_2^-, v_2^+, v_2^-)$	$(u_3^+, u_3^-, v_3^+, v_3^-)$	$(u_4^+, u_4^-, v_4^+, v_4^-)$
Box	(0,0,4.7,0)	(0,2.2,2.8,0)	(0,1.1,0,0)	(0,4.3,0,0)
Epd	(0,0,5.6,0)	(0,1.7,2.8,0)	(0,1.9,0,0)	(0,3.6,0,0)
Bud	(0,0,3.1,0)	(0,2.2,3.7,0)	(0,3.6,0,0)	(0,2.4,0,0)

of the robust consensus model threshold-based is 19.31, 17.53, 16.42. The optimal consensus opinions is $o'_{Box} = o'_{Epd} = o'_{Bud} = 17.14$. The positive and negative deviation are shown in the following table. The interval coefficient is $(\beta_1, \beta_2, \beta_3, \beta_4) = (0.921, 0.233, 0.512, 0.452)$.

By comparing the model results of the above three different robust types asymmetric cost consensus models of interval opinions. It is found that the box-type model has the largest consensus cost, while the budget type model has the minimum cost and strong robustness.

5. Model Discussion

Sensitivity analysis is usually used to analyze the sensitivity of model (or system) parameters or surrounding conditions to model output, which is helpful for model parameter correction.

5.1. Sensitivity Analysis

5.1.1. Effect of Uncertain Parameters

When applying robust optimization methods to address uncertainty problems, the uncertainty parameter serves as an indicator of the extent to which the uncertainty is being perturbed. In the context, we are comparing the consensus cost of the model from the perspective of uncertain parameters.

Figure 1 depicts a comparative analysis of robust asymmetric cost consensus models of interval opinions. The presented graphs plot the variation of total costs for the direction constraints model, the compromise limits model, and threshold-based consensus

Figure 1
Comparison of consensus cost of different robust models

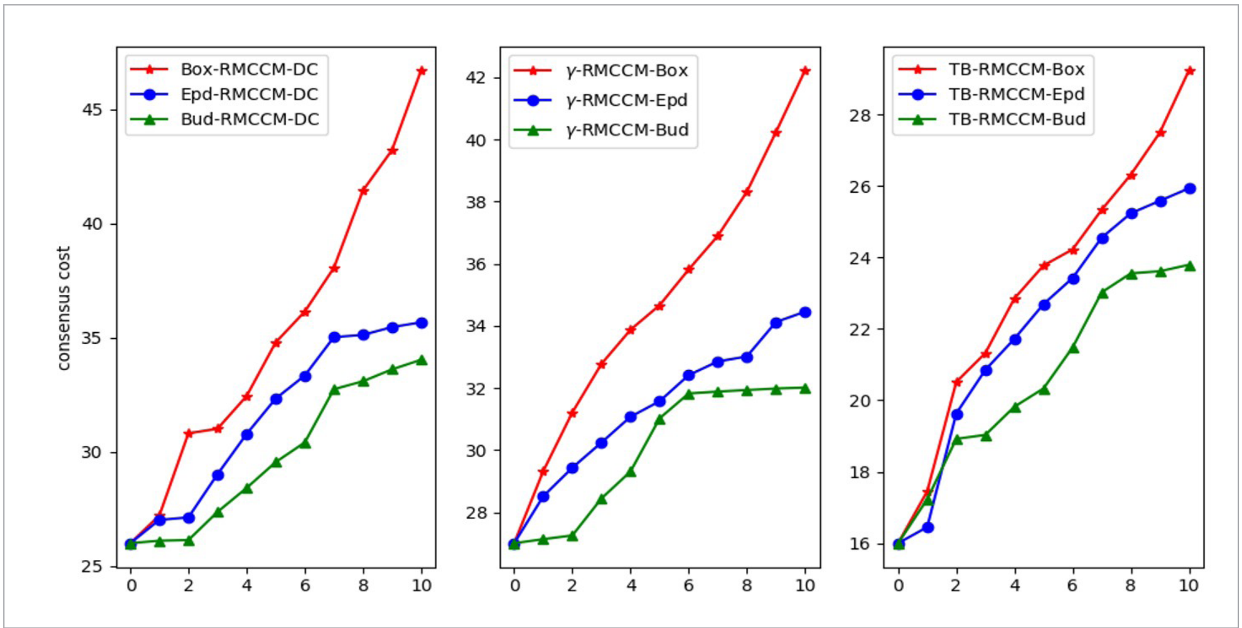


Table 2

Comparison of the total cost of three asymmetric cost models under the same uncertainty set

parameter	0	1	2	3	4	5	6	7	8	9
Box-type										
RMCCM-DC	26	27.21	30.81	31.01	32.44	34.79	36.14	38.05	41.44	43.21
γ-RMCCM	27	29.32	31.21	32.77	33.87	34.66	35.81	36.89	38.31	40.23
TB-RMCCM	16	17.45	20.51	21.32	22.85	23.77	24.22	25.33	26.31	27.49
Ellipsoid-type										
RMCCM-DC	26	27.02	27.13	29.02	30.78	32.33	33.34	35.02	35.12	35.46
γ-RMCCM	27	28.51	29.43	30.24	31.07	31.57	32.41	32.85	33.01	34.12
TB-RMCCM	16	16.45	19.63	20.85	21.72	22.69	23.42	24.55	26.23	28.58
Budget-type										
RMCCM-DC	26	26.11	26.14	27.37	28.42	29.55	30.41	32.74	33.69	35.61
γ-RMCCM	27	27.13	27.25	28.44	29.31	31.02	31.82	31.88	31.93	31.98
TB-RMCCM	16	17.23	18.92	19.03	19.82	20.33	21.48	23.02	23.55	24.11

model from left to right. Set the uncertainty parameter to vary between 0 and 9, with a nominal asymmetric cost consensus model when its value takes 0. The total costs for the three models are 26, 27, and 16, respectively. As the uncertainty parameter increases, the total consensus cost begins to rise. Remarkably, the box model shows the most substantial enhance, whereas the ellipsoid and budget models exhibit a decreasing trend, with their conservativeness falling gradually. The increase of uncertain parameters means that the scope of influence of uncertain factors is expanded, the unit upward and downward adjustment costs are bound to rise, and the total cost is bound to grow, which inevitably leads to difficulties in reaching consensus. Interestingly, when the parameters in the budget model reach about 7, the cost curves of the three models tend to smooth. This means that even if the disturbance of uncertain factors is added, the impact on the consensus result is minimal or even no. Comparing the three models it can be seen that the budgetary consensus model has the minimum total cost and can be considered as the most robust model.

Table 2 shows the results of three asymmetric cost models of same robust-type. The direction constraint model has the weakest anti-interference performance, and subtle disturbances can greatly increase the consensus cost. Under the same uncertain set, the change trend of the three asymmetric cost models is roughly similar.

5.1.2. Effect of Compromise Limit Parameters

Let $\Delta\gamma$ denote changes in the compromise limits of all experts to the same direction, and calculate the optimal consensus opinions and the minimum total cost of the robust asymmetric cost models of interval opinions. Table 3 shows the impact of the compromise limit parameter γ on the total cost and consensus opinions. The compromise limit increases with the magnify of $\Delta\gamma$. The consensus cost and optimal opinions of the robust asymmetric cost models of interval opinions decrease first and then stabilize with the enhance of parameter γ.The consensus cost reaches the equilibrium value of 31.21,29.43,27.25 at about $\Delta\gamma = 1.5$, and the equilibrium point of the consensus opinions is about 19.51. Comparing the three robust asymmetric cost models of interval opinions vertically, γ-RMCCM-Bud consumes the lowest total cost and shows the strongest robustness.

5.1.3. Effect of Cost-free Threshold Parameter φ

Let φ denote the changes of expert cost-free threshold in the same direction, and the minimum total cost and optimal consensus opinions of these models are calculated in Table 4.

Figure 2 elucidates more intuitively that as the no-cost threshold parameter increases, the total consensus cost decreases until it reaches zero. DMs are willing to modify initial opinions within a cost-free

Table 3

The effect of compromise limit parameters

$\Delta\gamma$	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
					γ-RMCCM-Box					
TC	+∞	33.77	33.12	32.83	32.25	31.51	31.21	31.21	31.21	31.21
o'	-	20.66	20.47	20.32	19.51	19.51	19.51	19.51	19.51	19.51
					γ-RMCCM-Epd					
TC	+∞	31.33	30.84	30.27	29.12	29.73	29.43	29.43	29.43	29.43
o'	-	20.88	20.53	20.38	19.51	19.51	19.51	19.51	19.51	19.51
					γ-MCCM-Bud					
MC	+∞	29.11	28.69	28.02	27.77	27.35	27.25	27.25	27.25	27.25
o'	-	20.07	19.92	19.73	19.51	19.51	19.51	19.51	19.51	19.51

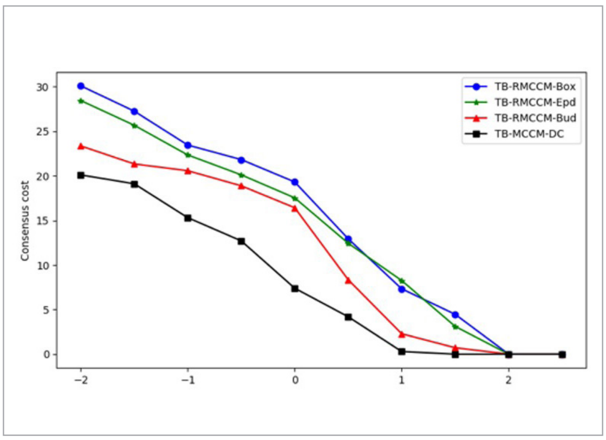
Table 4

The effect of cost-free threshold parameters φ

$\Delta\varphi$	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
					TB-RMCCM-Box					
TC	30.09	27.25	23.45	21.82	19.31	12.94	7.33	4.47	0	0
o'	18.23	18.23	17.91	17.62	17.14	17.25	17.68	18.11	18.11	18.07
					TB-RMCCM-Epd					
TC	28.44	25.67	22.34	20.11	17.53	12.44	8.26	3.12	0	0
o'	18.62	18.51	18.27	17.53	17.14	17.37	17.62	18.14	18.25	18.06
					TB-RMCCM-Bud					
TC	23.35	21.33	20.57	18.88	16.42	8.35	2.29	0.72	0	0
o'	18.92	18.64	18.18	17.74	17.14	17.35	17.44	18.16	18.16	18.03

Figure 2

The effect of cost-free threshold parameter φ



threshold, and moderator is also willing to accept this result, so the total cost is low or even zero. Comparing the experimental analysis of Cheng et al. [3], the total cost of all consensus models with cost-free threshold converges to 0 at $\Delta\varphi = 2$. The consensus opinions of TB-RMCCM-Bud is 18.16. Its total cost curve is closest to TB-RMCCM-DC in terms of graphical structure and can be considered the most robust model.

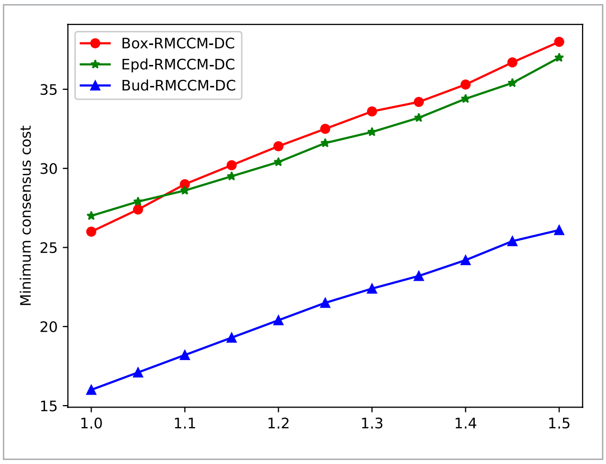
5.1.4. The Influence of Experts ' Initial Opinions on Decision-making Results

o_i denotes the initial unit cultivation compensation cost for the village representative, which is an interval value in this research work. Let it change at a ratio of 0.05, and the experimental results of different DMs' initial opinions are shown in Figure 3.

It can be seen from Figure 3 that the initial opinions of the decision makers become larger, the subsidy received by the villagers becomes more, the government's fiscal expenditure increases, and the pressure on the local government becomes larger. In this case, the local government will suspend the village's "returning farmland to forest" plan. From the perspective of national finance and sustainable development, it is necessary to control a reasonable range of opinions to obtain a win-win result.

Figure 3

The minimum consensus cost for different initial opinions



5.2. Comparison Analysis

In this subsection, the proposed model is compared with existing methods to more clearly demonstrate the superiority of proposed consensus model.

1 Comparison with Cheng et al.'s [3] model from the perspective of determining the consensus cost

Cheng et. al. [3] first proposed the consensus model of asymmetric cost. However, their study was based on the case where the upward/downward unit adjustment costs, initial opinions, compromise limits and no-cost thresholds were deterministic and did not take into account the uncertainties in the consensus negotiation process. The innovation of our research is to introduce robust optimization method to describe the uncertainty of unit consensus cost. Therefore, MCCM-DC, γ-MCCM-DC, and TB-MCCM-DC studied by Cheng et. al. [3] were compared with the robust models in this section, and the results are illuminated in Tables 5-7.

According to the comparison results in Table 5-7, the total cost of the deterministic model is always less than that of the robust model. It is because the deterministic model does not accommodate some uncertainties such as estimation error, and the consensus result is relatively optimistic. In "GTGP" case, if the final decision result is too optimistic, it may cause difficulties

Table 5

Comparison of MCCM-DC and Robust model

	MCCM-DC	Box-RMCCM-DC	Epd-RMCCM-DC	Bud-RMCCM-DC
TC	26	30.81	27.13	26.14
o'	18.05	18.39	18.39	18.39
K	0.000	0.185	0.043	0.004

Table 6

Comparison of γ-MCCM-DC and Robust model

	γ-MCCM-DC	γ-RMCCM-Box	γ-RMCCM-Epd	γ-RMCCM-Bud
TC	27	30.21	29.43	27.25
o'	19.0	19.51	19.51	19.51
K	0.000	0.156	0.09	0.009

Table 7

Comparison of TB-MCCM-DC and Robust model

	TB-MCCM-DC	TB-MCCM-Box	TB-MCCM-Epd	TB-MCCM-Bud
TC	16	19.31	17.53	16.42
o'	16.85	17.14	17.14	17.14
K	0.000	0.207	0.096	0.026

in reaching a consensus, or additional costs need to be paid for secondary operations. While the robust model takes full account of the uncertainty in advance, and appropriately increases the consumption of consensus costs to effectively avoid risks. Therefore, in the complex GDM scenario, the uncertain consensus model is more flexible than the traditional MCCM.

Here, we introduce the conservative degree $K(0 \le K \le 1)$ to more intuitively reflect the robustness of the consensus model. and the larger the value of K, the more pessimistic the decision maker is. When K=0, the consensus opinions are very conservative. When K=1, the decision-making opinions tends to be optimistic. The larger the value of K is, the more conservative DM's opinions are. In order to reach consensus, the moderator has to spend more negotiation costs. Comparing the three robust asymmetric cost consensus models of interval opinions, it is easy to find that TB-MCCM-Bud has the lowest conservative degree and superior performance.

2 Comparison with precision opinions models

For explore the influence of interval opinions on the results of consensus model, this part compares the asymmetric cost model under interval opinions and precise type. Set $o_i = (o_{il} + o_{ir}) / 2$, then $o_1 = 18.5, o_2 = 18.5, o_3 = 23, o_4 = 20.5$, the calculation results are shown in Table 8.

Through the comparison in Table 8, it can be found that interval opinions can significantly reduce consensus consumption. The interval preference opinions can make the decision-making experts have a range of opinion tolerance, and the adjustment of opinions within this range does not need to provide cost compensation. Therefore, the choice of interval opinions preference can greatly reduce the cost of consensus.

6. Conclusion

This study focuses on asymmetric cost consensus problem of interval opinions under uncertain environment. First, we construct an asymmetric cost consensus model for interval-type opinions.

Then, the RO method is integrated to deal with the uncertainty. Three robust-type expressions are given for box, ellipsoid and budget, respectively. Finally, through the case of "The Grains to Greens Program", the model is numerically analyzed and the results are compared.

Compared with precise opinions, interval opinions can greatly reduce the cost of consensus. In the uncertain environment, the increase of robust parameters can significantly enhance the consensus negotiation cost. Compromise limit and threshold-based coefficient are negatively correlated with consensus consumption. By comprehensive comparison, the budget-type model has the best performance.

Future research will further explore the influence of decision makers' psychological factors and the consensus decision-making model of large group participation.

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Table 8

Total cost comparison between interval opinions model and precise opinions model

Uncertainty set	Box-type	Epd-type	Bud-type
RMCCM-DC			
precision	30.81	27.13	26.14
interval	14.71	14.71	14.71
γ-RMCCM			
precision	31.21	29.43	27.25
interval	40.61	34.83	31.25
TB-RMCCM			
precision	19.31	17.53	16.42
interval	24.17	21.63	20.59

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