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H∞ Control of Network Control System for Singular Plant

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This paper investigates H_∞ control method for a class of Singular Network Control Systems (SNCS) based on singular plant. Considering the network delay, external disturbance, impulse behavior and structural instability of singular plant, the H_∞ control of SNCS with state feedback and dynamic output feedback are investigated respectively by approach of Linear Matrix Inequality (LMI). The existence of the H_∞ control law, the solving of the H_∞ control law and the disturbance degree are discussed in the following sections of the paper. Simulation results illustrate the effectiveness and feasibility of the given approach.

KEYWORDS: Singular Network Controls, H_∞ control, Linear Matrix Inequality, Network delay.

1. Introduction

Network Control System (NCS) is a distributed and a real-time feedback control system where the system node situated at different geographical position exchanges state information and control information with the controller through a communication network [20]. Network bandwidth and restraint of communication mechanism such as network delay and data packet loss exist typically in network communication channel, which makes NCS loses invariability, integrality, causality and certainty [18], and due to this fact the study of NCS is more complicated and challenging.

The traditional control theories and methods are not suitable for NCS, which makes rapid development over the past few years. Since the end of the last century, the research of NCS experiences the process from simple to complex, from single to comprehensive and from special to general. A large number of results have been reported, for instance, system complexity analysis [23], quantized dynamic output feedback control [3], observer-based controller design [25], state estimation and stabilization [6], H_∞ control method [24], fault-tolerant control [4], guaranteed cost control [8], co-design [10].

The results in the existing literature are focused on linear system. However, the study of SNCS based on singular system has not been addressed intensively. The dynamics of singular system is quite different from normal linear system and have many characteristics such as no causality, no solution, no uniqueness and structure instability, etc. [22]. In fact, the research on SNCS is still in the primary stage, and the existing results are limited to system modeling, stability analysis and control method [1-2, 5, 11-12, 14-17].

This paper aims to study the stabilization and H_∞ control method for a class of SNCS subject to the double characteristics of a singular systems and NCS. Network delay, input disturbance of limited energy, and impulse behavior are taken into consideration. The H_∞ control method of SNCS with state feedback and dynamic output feedback are presented respectively by means of LMI. The existence H_∞ control law, H_∞ control law approach and disturbance attenuation degree in different feedback are presented. Finally, a simulation is given to illustrate the effectiveness of the proposed method [7, 9, 13, 19, 21].

Schur Formula and Schur Complement Lemma

Let $A \in R^{r\times r}, B \in R^{r\times(n-r)}, C \in R^{(n-r)\times r}, D \in R^{(n-r)\times(n-r)}$, and A be an invertible matrix. The Schur formula has the following three forms:

(i) $$\begin{bmatrix} A & B \\ 0 & D - CA^{-1}B \end{bmatrix} = \begin{bmatrix} I_{(r)} & 0 \\ -CA^{-1} & I_{(n-r)} \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

(ii) $$\begin{bmatrix} A & 0 \\ C & D - CA^{-1}B \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} I_{(r)} & -A^{-1}B \\ 0 & I_{(n-r)} \end{bmatrix},$$

(iii) $$\begin{bmatrix} A & 0 \\ 0 & D - CA^{-1}B \end{bmatrix} =$$

$$= \begin{bmatrix} I_{(r)} & 0 \\ -CA^{-1} & I_{(n-r)} \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} I_{(r)} & -A^{-1}B \\ 0 & I_{(n-r)} \end{bmatrix},$$

in which, e.g., $I_{(r)}$denotes identity matrix of the size $r \times r$. The other identity matrices are of sizes that fit the Schur lemma.

Schur Complement Lemma:

Let Q$\in R^{r\times r}$, S $\in R^{r\times(n-r)}$, R $\in R^{(n-r)\times(n-r)}$ and $\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}$<0. Then if and only if

(i) R<0, Q-$SR^{-1}S^T$<0

or

(ii) Q<0, R-S^TQ^{-1}S<0.

Proof: Since the contract transform does not change the matrix positive definition, we first prove that $\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}$<0 is equivalent to R<0, Q-$SR^{-1}S^T$.

If Q<0, through the Schur formula, we have the following:

$$\begin{bmatrix} I_{(r)} & -SR^{-1} \\ 0 & I_{(n-r)} \end{bmatrix}\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}\begin{bmatrix} I_{(r)} & 0 \\ -R^{-1}S^T & I_{(n-r)} \end{bmatrix}=$$

$$=\begin{bmatrix} Q-SR^{-1}S^T & 0 \\ 0 & R \end{bmatrix}.$$

Correspondingly, $\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}$<0 is equivalent to R<0, Q-$SR^{-1}S^T$<0.

[End of Proof]

Notes: Q- $SR^{-1}S^T$ is the Schur complement of Q.

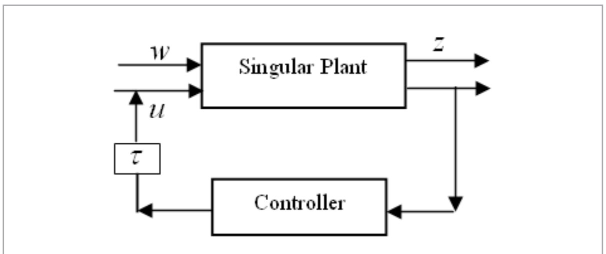
Problem Description

The singular sample of the network control system is described as presented in [1].

In Figure 1, u, w and z are control input, measurement state or measurement output, input external disturbance and expectation output, and τ is network-induced delay. The aim of positioning is to guarantee stable running of the system independently of any external disturbances so that the expected output of the system is not affected.

Figure 1

General Structure of SNCS



For singular plant, its state response contains not only the exponential term as normal systems, but also the pulse term and input derivative item, which will make the whole system has a pulse behavior. The pulse behavior decreases not only the system performance and even leads to the unstable state, which is a fatal destructiveness to the system. For network communication, due to the limited network bandwidth and the restraint of communication mechanism, the network communication obtains uncertainty and complexity.

The presented model shows a singular system state:

$$\left.\begin{cases} E_x \,\&\, (t) = Ax(t) + Bu(t-l+H_0 w(t)) \\ y(t) = C_1 X(t) + H_1 w(t) \\ z(t) = C_2 x(t) + H_2 w(t) \end{cases}\right\}, \tag{1}$$

where x(t) $\in R^n$ u(t) $\in R^m$, y(t) $\in R^l$ and z(t) $\in R^l$ are state vector, control input vector, output vector and expectation output vector, respectively. A $\in R^{n\times n}$, B $\in R^{n\times m}$ and C_1, C_2 $\in R^{n\times l}$ are constant matrices, E $\in R^{n\times n}$ is a singular matrix; w(t) is finite energy external disturbance, and H_0, H_1, H_2 are constant matrices.

When the singular plant is regular and impulse free, the equation (1) can be equivalently transformed as:

$$\begin{cases} x_1 \,\&\, (t) = A_1 x_1(t) + B_1 u(t-l) + W_1 w(t) \\ x_2 \,\&\, (t) = x_2(t) + B_2(t-1) + W_2 w(t) \\ y(t) = C_{11} x_1(t) + C_{12} x_2(t) + H_1 w(t) \\ z(t) = C_{21} x_1(t) + C_{22} x_2(t) + H_2 w(t) \end{cases}.$$

The state feedback control is

$$u(k) = \begin{bmatrix} K_1 & K_2 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}.$$

Let $\hat{x}$=[$x_1^T(k)$ $u^T(k-1)$]^T, the state feedback SNCS close-loop model is

$$\hat{x}(k+1)=\begin{bmatrix} (A_d + B_{10}(l)\ K_1) & (B_{11}(L) - B_{10}(l)\ K_2 B_2 \\ K_1 & -\ K_2 B_2 \end{bmatrix}$$

$$\hat{x}+\begin{bmatrix} W_0 - B_{10}(l)\ K_2\ W_2 \\ -\ K_2 W \end{bmatrix}\text{w(k)}.$$

The dynamic output feedback controller is

$$\begin{cases} x_c(k+1) = A_c x_c(k) + B_c y(k) \\ u(k) = C_c x_c(k) \end{cases}.$$

Let $\bar{x}$=[x_1^T x_c^T u^T]^T, the SNCS close-loop model is as follows:

$$\begin{aligned} \bar{x}(k+1) &= \begin{bmatrix} A_d & B_{10}(l)C_c & b_{11}(l) \\ B_c & A_c & -B_c C_{12} B_2 \\ 0 & C_c & 0 \end{bmatrix} \\ \bar{x}(k) &+ \begin{bmatrix} W_0 \\ B_c H_1 - BcC_{12} W_2 \\ 0 \end{bmatrix} w(k). \end{aligned} \tag{2}$$

Whether the system uses state feedback and output feedback or not, the SNCS close-loop system model is a linear normal system depending on time delay τ.

H_∞ Control

1. State feedback H_∞ control

Theorem 1: If there exist positive definite matrices $\bar{S}$, $\bar{R}$ such that

$$\begin{bmatrix} -\bar{S} & 0 & \bar{S}M_1^T & \bar{S}K_1^T \\ 0 & -\bar{R} & \bar{R}M_2^T & \bar{R}M_3^T \\ M_1\bar{S} & M_2\bar{R} & -\bar{S} & 0 \\ K_1\bar{S} & M_3\bar{R} & 0 & -\bar{R} \end{bmatrix} < 0, \quad (3)$$

where $M_1 = A_d + B_{10}(l)K_1$, $M_2 = B_{11}(l) - B_{10}(l)K_2B_2$ and $M_3 = -K_2B_2$, then the system (2) is asymptotically stable.

Proof: Choose positive definite matrices S and R and define a Lyapunov function as follows:

$V(k) = x_1^T(k)Sx_1(k) + u^T(k-1)Ru(k-1)$.

Then the forward differential of V(k) is $\nabla V(k) = \hat{x}(k)^T \prod \hat{x}(k)$, where

$$\prod = \begin{bmatrix} M_1^TSM_1 + K_1^TRK_1 - S & M_1^TSM_2 + K_1^TRK_3 \\ M_2^TSM_1 + M_3^TRK_1 & M_2^TSM_2 + M_3^TRM_3 - R \end{bmatrix},$$

$M_1 = A_d + B_{10}K_1$, $M_2 = B_{11} - B_{10}K_2B_2$, $M_3 = -K_2B_2$,

$\hat{x} = [\ x_1^T(k) \quad u^T(k-1)]^T$.

By Lyapunov stability theory, if $\Delta V(k) < 0$, then the system (2) is asymptotically stable and asymptotical stability condition is

$$\begin{bmatrix} M_1^TSM_1 + K_1^TRK_1 - S & M_1^TSM_2 + K_1^TRK_3 \\ M_2^TSM_1 + M_3^TRK_1 & M_2^TSM_2 + M_3^TRM_3 - R \end{bmatrix} < 0.$$

By Schur complement lemma, the equation (3) can be transformed to

$$\begin{bmatrix} -S & 0 & M_1^T & K_1^T \\ 0 & -R & M_2^T & M_3^T \\ M_1 & M_2 & -S^{-1} & 0 \\ K_1 & M_3 & 0 & -R^{-1} \end{bmatrix}.$$

Multiplying diag (S^{-1}, R^{-1}, I, I) on the left-hand side and the right-hand side of the equation it is derived that

$$\begin{bmatrix} -S^{-1} & 0 & S^{-1}M_1^T & S^{-1}K_1^T \\ 0 & -R^{-1} & R^{-1}M_2^T & R^{-1}M_3^T \\ M_1S^{-1} & M_2R^{-1} & -S^{-1} & 0 \\ K_1S^{-1} & M_3R^{-1} & 0 & -R^{-1} \end{bmatrix},$$

where we denote $\bar{S} = S^{-1}, \bar{R} = R^{-1}$, then the equation is equivalent to (3).

[End of Proof]

Theorem 2: For the singular plant (1), under state feedback controller, for given $\gamma > 0$, if there exists symmetric positive definite matrices S, R, such that

$$\begin{bmatrix} -S & 0 & 0 & M_1^T & K_1^T & C_{21}^T \\ 0 & -R & 0 & M_2^T & M_3^T & M_4^T \\ 0 & 0 & -\gamma^2 I & W_2^T & W_3^T & W_4^T \\ M_1 & M_2 & W_3 & 0 & 0 & 0 \\ M_1 & M_3 & W_4 & 0 & -R^{-1} & 0 \\ C_{21} & M_4 & W_5 & 0 & 0 & -1 \end{bmatrix} < 0, \quad (4)$$

where $M_1 = A_d + B_{10}K_1$, $M_2 = B_{11} - B_{10}K_2B_2$, $M_3 = -K_2B_2$, $M_4 = -C_{22}B_2$, $W_5 = H_2 - C_{22}W_2$, $W_3 = W_0 - B_{10}(l)K_2B_2$, $W_4 = -K_2W_2$, then the singular plant model (1) will realize second best state feedback H_∞ control.

Proof: The external disturbance is taken into account, in order to make the following equation exist $||z(k)||_2 \le \gamma\ ||w(k)||_2$. Let $J_Z = \sum_{k=0}^{\infty}[z^T(k)z(k) - \gamma^2 w^T(k)w(k)]$, we can take positive definite matrices S, R, and define a Lyapunov function V(k) as follows: $V(k) = x_1^T(k)Sx_1(k) + u^T(k-1)Ru(k-1)$.

For the system (2), when it satisfies Theorem 1, it is asymptotically stable in the zero initial conditions $\forall w(k) \in L_2[0,\infty)$ it is derived that $\sum_{k=0}^{\infty}[z^T(k)z(k) - \gamma^2 w^T(k)w(k) + \Delta V(k)] < 0$.

Let us denote $M_4 = -C_{22}B_2$, $W_3 = W_0 - B_{10}(l)K_2B_2$, $W_4 = -K_2W_2$, $W_5 = H_2 - C_{22}W_2$, so that it is derived that $x^T \Phi x < 0$, where

$x = [\ x_1^T(k) \quad u^T(k-1)$

$w^T(k)\ ]^T$, $\Phi = \begin{bmatrix} A_{11} & * & * \\ A_{21} & A_{22} & * \\ A_{31} & A_{32} & A_{33} \end{bmatrix} < 0.$

$$A_{11} = M_1^TSM_1 + K_1^TRK_1 - S + C_{21}^TC_{21},$$

$A_{21} = M_2^T S M_1 + M_3^T R K_1 + M_4^T C_{21},$

$A_{22} = M_2^T S M_1 + M_3^T R M_3 - R + M_4^T M_4,$

$A_{31} = W_3^T S M_1 + W_4^T R + W_5^T C_{21},$

$A_{32} = W_3^T S M_2 + W_4^T R + W_5^T M_4,$

$A_{33} = W_3^T S W_3 + W_5^T W_5 + W_4^T R M_4 - \gamma^2 I.$

By Schur complement, Equation (4) can be transformed as

$$\begin{bmatrix} -S + C_{21}^T C_{21} & C_{21}^T M_4 & C_{21}^T W_5 & M_1^T & K_1^T \\ C_{21}^T C_{21} & -R + M_4^T M_4 & M_4^T W_5 & M_2^T & M_3^T \\ W_5^T C_{21} & W_5^T M_4 & W_5^T W_5 - \gamma^2 I & W_3^T & W_4^T \\ M_1 & M_2 & M_3 & -S^{-1} & 0 \\ K_1 & M_3 & W_4 & 0 & -R^{-1} \end{bmatrix} < 0. \tag{5}$$

Similarly, by further transforming, we can derive (4).

[End of Proof]

Theorem 3: For the singular plant (1), under the action of state feedback controller, if there exist symmetric positive definite matrices $\widehat{S}, \widehat{R}$, matrices Y_1, Y_2, Y_3, scalars $\varepsilon > 0, \varepsilon_1 > 0, \beta > 0$ and compatible dimension unit matrix I, such that

$$\begin{bmatrix} -\hat{S} & * & * & * & * & * & * & * & * & * \\ 0 & -\hat{R} & * & * & * & * & * & * & * & * \\ 0 & 0 & -\beta I & * & * & * & * & * & * & * \\ A_d\hat{S}+B_{10}Y_1 & B_{11}\hat{R} & W_0 & -\hat{S} & * & * & * & * & * & * \\ Y_1 & 0 & 0 & 0 & -\hat{R} & * & * & * & * & * \\ C_{21}\hat{S} & -C_{22}B_2\hat{R} & H_2 - C_{22}W_2 & 0 & 0 & -I & * & * & * & * \\ 0 & B_2\hat{R} & W_2 & 0 & 0 & 0 & -\varepsilon I & * & * & * \\ 0 & B_2\hat{R} & W_2 & 0 & 0 & 0 & 0 & -\varepsilon_1 I & * & * \\ 0 & 0 & 0 & Y_3 B_{10}^{\mathrm{T}} & 0 & 0 & 0 & 0 & -\varepsilon I & * \\ 0 & 0 & 0 & 0 & Y_2 & 0 & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix} < 0. \tag{6}$$

The $\boldsymbol{H}_\infty$ control law is

$$\mathrm{u(k)} = \left[Y_1 \widehat{S}^{-1} Y_2^T / \varepsilon_1 \right] \begin{bmatrix} x1(k) \\ x2(k) \end{bmatrix}. \tag{7}$$

Proof: For plant (1), if second best state feedback c $\boldsymbol{H}_\infty$ control law exists, then Theorem 2 is established. Spread out $M_1 \sim M_4$, $W_3 \sim W_5$, and then Equation (4) in Theorem 2 can be expressed as

$$\begin{bmatrix} -S & * & * & * & * & * \\ 0 & -R & 0 & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ A_d + B_{10}K_1 & B_{11} & W_0 & -S^{-1} & * & * \\ C & -K_2B_2 & -K_2W_2 & 0 & -R^{-1} & * \\ C_{21} & -C_{22}B_2 & H_2 - C_{22}W_2 & 0 & 0 & -1 \end{bmatrix} < 0. \tag{8}$$

Equation (8) can be rewritten as:

$$\begin{bmatrix} -S & * & * & * & * & * \\ 0 & -R & 0 & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ A_d + B_{10}K_1 & B_{11} & W_0 & -S^{-1} & * & * \\ C & -K_2B_2 & -K_2W_2 & 0 & -R^{-1} & * \\ C_{21} & -C_{22}B_2 & H_2 - C_{22}W_2 & 0 & 0 & -1 \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ -B_{10}K_2 \\ 0 \\ 0 \end{bmatrix} I \begin{bmatrix} 0 & B_2 & W_2 & 0 & 0 & 0 \end{bmatrix} I^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ -B_{10}K_2 \\ 0 \\ 0 \end{bmatrix}^T < 0 \tag{9}$$

From Schur Lemma 1, we can say that the above (9) exists, if and only if there is a scalar $\varepsilon > 0$, such that

$$\begin{bmatrix} -S & * & * & * & * & * \\ 0 & -R & 0 & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ A_d + B_{10}K_1 & B_{11} & W_0 & -S^{-1} & * & * \\ C & -K_2B_2 & -K_2W_2 & 0 & -R^{-1} & * \\ C_{21} & -C_{22}B_2 & H_2 - C_{22}W_2 & 0 & 0 & -1 \end{bmatrix} +$$

$$+ \varepsilon \begin{bmatrix} 0 \\ 0 \\ 0 \\ -B_{10}K_2 \\ 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ -B_{10}K_2 \\ 0 \\ 0 \end{bmatrix}^T +$$

$$+ \varepsilon_{-}{}^{1} \begin{bmatrix} 0 & B_2 & W_2 & 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} 0 & B_2 & W_2 & 0 & 0 & 0 \end{bmatrix} < 0 \tag{10}$$

By Schur complement, Equation (10) can be transformed as

$$\begin{bmatrix} -S & * & * & * & * & * & 0 \\ 0 & -R & * & * & * & * & B_2^{\mathrm{T}} \\ 0 & 0 & -\gamma^2 I & * & * & * & W_2^{\mathrm{T}} \\ A_d+B_{10}K_1 & B_{11} & W_0 & -S^{-1}+\varepsilon B_{10}K_2(B_{10}K_2)^{\mathrm{T}} & * & * & 0 \\ K_1 & -K_2B_2 & -K_2W_2 & 0 & -R^{-1} & * & 0 \\ C_{21} & -C_{22}B_2 & H_2-C_{22}W_2 & 0 & 0 & -I & 0 \\ 0 & B_2 & W_2 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0. \quad (11)$$

Similarly, it is derived that

$$\begin{bmatrix} -S & * & * & * & * & * & * & 0 \\ 0 & -R & * & * & * & * & * & B_2^{\mathrm{T}} \\ 0 & 0 & -\gamma^2 I & * & * & * & * & W_2^{\mathrm{T}} \\ A_d+B_{10}K_1 & B_{11} & W_0 & -S^{-1}+\varepsilon B_{10}K_2(B_{10}K_2)^{\mathrm{T}} & * & * & * & 0 \\ K_1 & 0 & 0 & 0 & -R_1^{-1}+\varepsilon_2 K_2 K^{\mathrm{T}} & * & * & 0 \\ C_{21} & -C_{22}B_2 & H_2-C_{22}W_2 & 0 & 0 & -I & * & 0 \\ 0 & B_2 & W_2 & 0 & 0 & 0 & -\varepsilon I & 0 \\ 0 & B_2 & W_2 & 0 & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix} <0. \quad (12)$$

State feedback $\boldsymbol{H}_\infty$ controller parameter is $K_1 = Y_1\hat{S}^{-1}, K_2 = Y_2^T/\varepsilon_{1=}Y_3^T/\varepsilon$.

The state feedback $\boldsymbol{H}_\infty$ control law (7) is obtained.

[End of Proof]

2. Dynamic output feedback H_∞ control

Theorem 4 when the external disturbance is not taken into account, under the action of dynamic output feedback controller, if there exist positive $\tilde{P}, \tilde{Q}, \tilde{S}$, such that

$$\begin{bmatrix} -\tilde{P} & * & * & * & * & * \\ 0 & -\tilde{Q} & 0 & * & * & * \\ 0 & 0 & -\tilde{S} & * & * & * \\ A_d & M_5\tilde{Q} & M_6\tilde{S} & -\tilde{P} & * & * \\ M_2\tilde{P} & A_c\tilde{Q} & M_8\tilde{S} & 0 & -\tilde{Q} & * \\ 0 & C_c\tilde{Q} & 0 & 0 & 0 & -\tilde{S} \end{bmatrix}, \quad (13)$$

where $M_5 = B_{10}(L)C_C$, $M_6 = B_{11}(L)$, $M_7{=}B_cC_{11}$, $M_8 = -B_cC_{12}B_2$, then the system (2) is asymptotically stable.

Proof: Denote $M_5{=}B_{10}(L)C_C$, $M_6{=}B_{11}(L)$, $M_7{=}B_cC_{11}$, $M_8 = -B_cC_{12}B_2$,$W_6 = B_c(H_1 - C_{12}W_2)$, so that Equation (2) can be written as

$$\bar{x}(k+1) = \begin{bmatrix} A_d & M_5 & M_6 \\ M_7 & A_c & M_8 \\ 0 & M_5 & 0 \end{bmatrix}\bar{x}(k) + \begin{bmatrix} W_0 \\ W_6 \\ 0 \end{bmatrix} w(k).$$

When the external disturbance of the system is not taken into account, choose positive definite matrices P,Q,S and define a Lyapunov function as follows:

$$\mathrm{V(k)}=\mathrm{x}_1^T(k)P(k)+x_c{}^T(k)\mathrm{Q}x_c\ \ (\mathrm{k})+.\ u^T(k-1)\mathrm{Su(k\text{-}1)}$$

Then the forward differential of $V(k)$ along trajectory of close-loop system (6) is as follows:

$$\Delta V(k) =$$
$$(x_1^T(k)A_d{}^T + x_1^T(k)M_5{}^T + u^T(k-1)M_6{}^T PA_d x_1(k) +$$
$$(x_1^T(k)A_d{}^T + x_c{}^T(k)M_5{}^T + u^T(k-1)M_6{}^T PM_5 x_c(k) +$$
$$(x_1^T(k)A_d{}^T + x_c{}^T(k)M_5{}^T + u^T(k-1)M_6{}^T PM_6 u(k-1) +$$
$$(x_1^T(k)M_7{}^T + x_c{}^T(k)A_c{}^T + u^T(k-1)M_8{}^T)QM_7 x_1(k) +$$
$$(x_1^T(k)M_7{}^T + x_c{}^T(k)A_c{}^T + u^T(k-1)M_8{}^T)QA_C x_C(k) +$$
$$(x_1^T(k)M_7{}^T + x_c{}^T(k)A_c{}^T + u^T(k-1)M_8{}^T)\, QM_8 u(k-1) +$$
$$x_c{}^T(k)C_C{}^T SC_C x_C(k) - x_c{}^T(k)Qx_C(k) -$$
$$u^T(k-1)Su(k-1).$$

We define $\tilde{x}(k) = [(x_1^T(k)\ x_c{}^T(k)\ u^T(k-1)\ ]^T$, for which the above equation can be written as $\nabla V(k) = \tilde{x}^T \Psi \tilde{x}$,

$$\varphi = \begin{bmatrix} D_{11} & * & * \\ D_{21} & D_{22} & * \\ D_{31} & D_{32} & D_{33} \end{bmatrix},$$

where $D_{11} = A_d{}^T PA_d + M_7{}^T QM_7 - P$, $D_{21} = M_5{}^T PA_d + A_c{}^T QM_7$, $D_{22} = M_5{}^T PM_5 + A_c{}^T QA_c + C_C{}^T SC_C - Q$ $D_{31} = M_6{}^T PA_d + M_8{}^T QM_7$, $D_{32} = M_6{}^T PM_5 + M_8{}^T QA_c$, $D_{33} = M_6{}^T PM_6 + M_8{}^T QM_8 - S$.

By Schur complement, the above equation can be transformed to

$$\begin{bmatrix} -P & * & * & * & * & * \\ 0 & -Q & 0 & * & * & * \\ 0 & 0 & -S & * & * & * \\ A_d & M_5 & M_6 & -P & * & * \\ M_7 & A_c & M_8 & 0 & -Q^{-1} & * \\ 0 & C_c & 0 & 0 & 0 & -S^{-1} \end{bmatrix}. \tag{14}$$

[End of Proof]

Theorem 5: For the plant in Figure 1, under dynamic output feedback controller, for $\gamma > 0$, if there are symmetric positive definite matrices P,Q,S that

$$\begin{bmatrix} -P & * & * & * & * & * & * & * \\ 0 & -Q & * & * & * & * & * & * \\ 0 & 0 & -S & * & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 & * & * & * & * \\ A_d & M_5 & M_6 & W_0 & -P^{-1} & * & * & * \\ M_7 & A_c & M_8 & W_6 & 0 & -Q^{-1} & * & * \\ 0 & C_c & 0 & 0 & 0 & 0 & -S^{-1} & * \\ C_{21} & 0 & M_4 & W_5 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{15}$$

then the plant in Fig. 1 realizes suboptimal dynamic output feedback H_∞ control.

Proof: The external disturbance is considered in order to make the following equation exist $||z(k)||_2 \le \gamma\ ||w(k)||_2$.

Let $J_Z = \sum_{k=0}^{\infty}[z^T(k)z(k) - \gamma^2 w^T(k)w(k)]$, choose positive definite matrices P,Q,S, and construct a Lyapunov function $V(k) = x_1^T(k)Px_1(k) + x_c{}^T(k)Qx_c(k) +. u^T(k-1)Su(k-1)$.

The dynamic output feedback close-loop system modeled in (2), if satisfies Theorem 4, the system is asymptotically stable in zero initial conditions for $\forall w(k) \in L_2[0,\infty)$. Then we have

$$\sum_{k=0}^{\infty}[z^T(k)z(k) - \gamma^2 w^T(k)w(k)] + \Delta V(k) < 0.$$

Let $W_6 = B_c(H_1 - C_{12}W_2)$, $M_4 = -C_{22}B_2$, $W_5 = H_2 - C_{22}W_2$, $x = [x_1^T\ \ x_c{}^T\ \ u^T\ \ w^T\ ]^T$, we have $z^T(k)z(k) - \gamma^2 w^T(k)w(k)] + \Delta V(k) = x^T \Omega x$,

$$\Omega = \begin{bmatrix} A_{11} & * & * & * \\ A_{21} & A_{22} & * & * \\ A_{31} & A_{32} & A_{33} & * \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \tag{16}$$

$A_{11} = A_d{}^T PA_d + M_7{}^T QM_7 - P + C_{21}{}^T C_{21}$, $A_{21} = M_5{}^T PA_d + A_c{}^T QM_7$, $A_{22} = M_5{}^T PM_5 + A_c{}^T QA_c + C_C{}^T SC_C - Q$, $A_{31} = M_6{}^T PA_d + M_8{}^T QM_7 + M_4{}^T C_{21}$, $A_{32} = M_6{}^T PM_5 + M_8{}^T QA_c$,

$A_{33} = M_6{}^T PM_6 + M_8{}^T QM_8 - S + M_4{}^T M_4$, $A_{41} = W_0{}^T PA_d + W_5{}^T C_{21} + W_6{}^T QM_7$, $A_{42} = W_0{}^T PM_5 + W_6{}^T QA_c$, $A_{43} = W_0{}^T PM_6 + W_5{}^T M_4 + W_6{}^T QM_8$, $A_{44} = W_5{}^T W_5 - \gamma^2 + W_0{}^T PW_0 + W_6{}^T QW_6$.

Now Equation (15) can be transformed to

$$\begin{bmatrix} -P+C_{21}^T C_{21} & 0 & * & * & * & * & * \\ 0 & -Q & * & * & * & * & * \\ M_4{}^T C_{21} & 0 & M_4^T M_4 - S & * & * & * & * \\ W_5{}^T C_{21} & 0 & W_5{}^T M_4 & W_5^T W_5 - \gamma^2 & * & * & * \\ A_d & M_5 & M_6 & W_0 & -P^{-1} & * & * \\ M_7 & A_c & M_8 & W_6 & 0 & -Q^{-1} & * \\ 0 & C_c & 0 & 0 & 0 & 0 & -S^{-1} \end{bmatrix} < 0. \tag{17}$$

[End of Proof]

Theorem 6: For $\gamma > 0$, if there exists a symmetric positive definite matrix $P = P^T > 0$ which satisfies

$$\begin{bmatrix} (A+\Delta A)^T p + p((A+\Delta A)) + I & p(A_d+\Delta A_d) & p(B+\Delta B) & C^T \\ (A_d+\Delta A_d)^T p & -I & 0 & 0 \\ (B+\Delta B)^T p & 0 & -\gamma^2 I & 0 \\ C & 0 & 0 & -I \end{bmatrix} < 0, \tag{18}$$

system (1) is robust stable.

Proof: From Schur complement lemma, (18) is equivalent to

$$\begin{bmatrix} (A+\Delta A)^T p + p((A+\Delta A)) + I & p(A_d+\Delta A_d) & p(B+\Delta B) \\ (A_d+\Delta A_d)^T p & -I & 0 \\ (B+\Delta B)^T p & 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} C^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} C & 0 & 0 \end{bmatrix} < 0. \tag{19}$$

That is,

$$\begin{bmatrix} (A+\Delta A)^T p + p((A+\Delta A)) + I & p(A_d+\Delta A_d) & p(B+\Delta B) \\ (A_d+\Delta A_d)^T p & -I & 0 \\ (B+\Delta B)^T p & 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} C^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} C & 0 & 0 \end{bmatrix} < 0. \tag{20}$$

$$\big[(A+\Delta A)^T \mathrm{p} + p(A+\Delta A) + P(Ad+\Delta Ad)(Ad+\Delta Ad)^T p + I + C^T C + \gamma^{-2} P(B+\Delta B)(B+\Delta B)^T p \big] < 0. \tag{21}$$

We consider Lyapunov functional

$$J(x(t),t) = x^T(t)Px(t) + \int_0^t (y^T y) d\varepsilon + \gamma^{-2} x^T P(B+\Delta B)^T Px d\varepsilon + \int_{t-1}^t x(t+1)^T (t+1) d\varepsilon > 0, \tag{22}$$

differentiating with respect to t at the both sides of J(x(t),t), we have

$$J(x(t),t) = x^T(t)Px(t) + x^T(t)P\dot{x}(t) + x^T(t)C^T Cx(t) + x^T(t)x(t) + \gamma^{-2} x^T P(B+\Delta B)(B+\Delta B)^T Px(t) - x^T(t-l)x(t-l). \tag{23}$$

Then

$$J(x(t),t) = x^T(t)((A+\Delta A)^T P + P(A+\Delta A) + I + C^T C + \gamma^{-2} P(B+\Delta B)(B+\Delta B)^T Px(t) + x^T(t-l)(A_d+\Delta A_d)^T Px(t) + x^T(t)P(A_d+\Delta A_d)^T x(t-l) - x^T(t-l)x(t-l), \tag{24}$$

further,

$$J(x(t),t) = x^T(t)((A+\Delta A)^T P + P(A+\Delta A) + P(A_d+\Delta A_d)(A_d+\Delta A_d)^T P + I + C^T C + \gamma^{-2} P(B+\Delta B)(B+\Delta B)^T Px(t) ((A_d+\Delta A_d)^T Px(t) - x(t-l))^T * (A_d+\Delta A_d)^T Px(t) - x(t-l)). \tag{25}$$

From (21), we have J(x(t),t)<0, so the plant (1) is robust stable.

[End of Proof]

2. Simulation Results

1 Simulation of a Typical Singular Plant

We take state feedback case to illustrate the effectiveness of the proposed method. A typical singular plant model with input external disturbance is as follows:

$$\begin{cases} \begin{bmatrix} x_1(t) \\ 0 \\ x_3(t) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \mathrm{u}(t-l) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.1 \end{bmatrix} \mathrm{w}(t) \\ z(t) = [1 \quad 1 \quad 1 \quad 1]x(t) + 0.1w(t) \end{cases}$$

The sampling period T is 0.1s, the network-induced delay is τ_k= 0.01. The plant model can be transformed as

$$\begin{cases} x_1(t) = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} x_1(t-l) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} -0.1 \\ 0 \end{bmatrix} w(t) \\ 0 = x_2(t) + \begin{bmatrix} -1 \\ 0 \end{bmatrix} u(t-l) + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} w(t) \\ z(t) = [0 \quad 1]x_1(t) + [1 \quad 1]x_2(t) + 0.1w(t) \end{cases}.$$

Its discrete model parameters are

$$A_d = \begin{bmatrix} 0.9 & -0.1 \\ 0.1 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix},\ C_{21} = [1 \quad 0],$$

$C_{22} = [1 \quad 1]$, and $H_2 = 0.1$.

We find the solution of the plant through LMI toolbox using u(t)=[-5 -4 0 0]x(t) for which the system is asymptotically stable. When initial state x(0)=(0,2,1,-1) the system state response external trajectory since disturbance is as solid line shown in Fig. 2.

For H_∞ control, we use Theorem 3. Therefore $\gamma = \sqrt{\beta} = 35.92$ is obtained, and the $\gamma-$suboptimal state feedback H_∞ control law is u(t)=[-0.290 -0.034 0 0]x(t). Under the same conditions, the system state response trajectory is as dotted line shown in Figure 2.

By LMI tool-box, we present solutions for Theorem 4, the obtained corresponding solutions are

$$\hat{S}^* = \begin{bmatrix} 0.0951 & -0.0002 \\ -0.0001 & 0.1051 \end{bmatrix},\ Y_1^* = 1.0^{-6}[-0.211 \quad 0.013],$$

$$Y_2^* = Y_3^* = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \beta^* = 0.009.$$

Therefore, the minimum disturbance attenuation is $\gamma^* = \sqrt{\beta^*} = 0.095$, , the γ–optimal state feedback H_∞ control law is u(t)=$1.0^{-5}[-0.22 \quad 0.01 \quad 0 \quad 0]x(t)$.

After putting optimal H_∞ into effect, the system state response trajectory is as dot dash line shown in Figure 2. Before and after optimization control, the system expectation output is presented as solid line and dotted line shown in Figure 3.

Figure 2

State response simulation

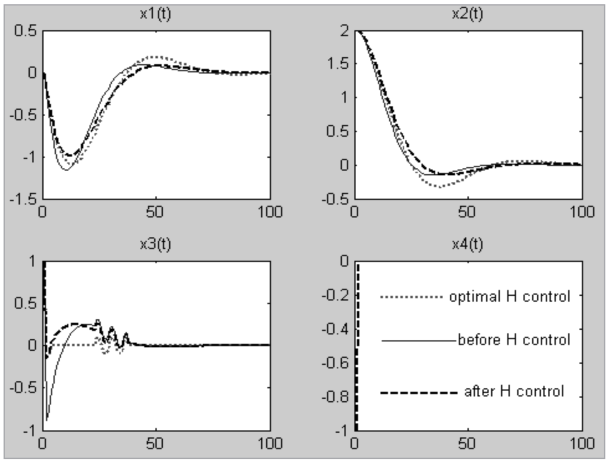
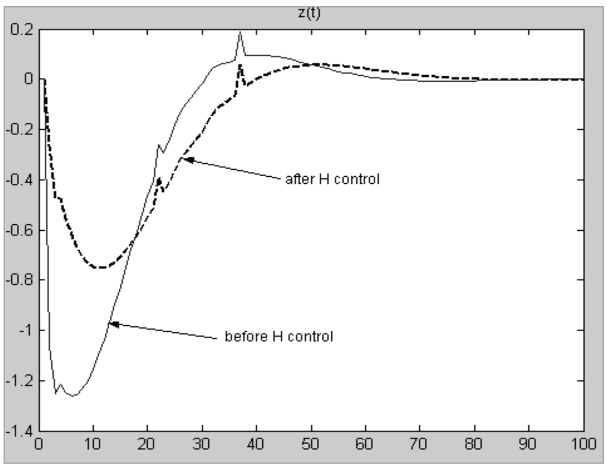


Figure 3

Expectation output simulation



Further,

$$\hat{s} = \begin{bmatrix} 0.097 & -0.015 \\ -0.015 & 0.094 \end{bmatrix}, Y_1 = [-0.028 \quad 0.001],$$

$$Y_2 = Y_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \beta = 1288.6,$$

and the system simulation shows that after implementation H_∞ control and H_∞ optimization control, γ can decrease to 0.06 from the primary 35.9, and the anti-interference performance of the system is enhanced markedly. As a result, the stability performance of system has been improved.

2 Simulation of a Torpedo

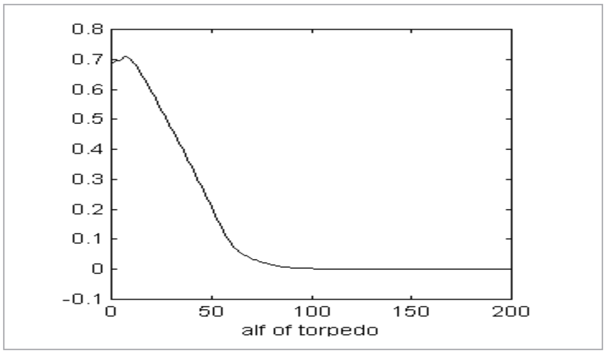
The longitudinal motion of the dynamic equation of a torpedo at the speed $v = 25.7m/s$ can be described by the following state equation:

$$\begin{cases} x\& = \begin{bmatrix} -1.4 & 0.22+0.17\delta_2 \\ 10+0.25\delta_1 & -5.4 \end{bmatrix} x + \\ \qquad \begin{bmatrix} -1.3 & 0.22-0.25\eta_2 \\ 10-0.25\eta_1 & -5 \end{bmatrix} x(t-l) + \\ \qquad \begin{bmatrix} -0.28 & -0.03\delta_2 \\ -0.03\delta_1 & -4.13 \end{bmatrix} \delta_e, \\ y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x \end{cases}$$

where x_1 is the attack angle of the torpedo, x_2 is the angular velocity of the torpedo, δ_e is the rudder angle of the torpedo, and $|\delta_i| \le 1, |\eta_i| \le 1, i = 1,2,$ the simulation result is shown in Figure 4.

Figure 4

Torpedo system simulation



3. Conclusions

In this paper, the H_∞ optimal control problems for a class of SNCS have been addressed with both state feedback case and dynamic output feedback case. The network communications characteristics in the paper are: network-induced delay, input disturbance of limited energy, clock-driven sensors, event-driven controller and actuators. The characteristics for singular system in the paper are: impulse behavior, structural instability, and something like that. When network communication time-delay is less than or equal to a sampling in both cases state the feedback and dynamic output feedback are correct. This paper presents respectively the existence condition of the H_∞ control law, H_∞ optimal control method, and solution of H_∞ control law. The simulation results show that the analytical method and the results are valid and feasible.

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