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ON RECURSIVE PARAMETRIC IDENTIFICATION OF WIENER SYSTEMS

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Abstract. The aim of the given paper is the development of a recursive approach for parametric identification of Wiener systems with non-invertible piecewise linear function in the nonlinear block. It is shown here that the problem of parametric identification of a Wiener system could be reduced to a linear parametric estimation problem by a simple input-output data reordering and by a following data partition. An approach based on sequential reconstruction of the values of intermediate signal by following use of the ordinary recursive least squares (RLS) is proposed here for the estimation of parameters of linear and nonlinear parts of the Wiener system. The unknown threshold of piecewise nonlinearity has been estimated by processing recursively respective particles of current input-output data, too. The results of numerical simulation and parametric estimation of Wiener systems with different piecewise nonlinearities by computer are given.

Keywords: nonlinear Wiener systems, recursive identification, reconstruction.

1. Introduction

It is known [1, 4 – 6, 8 – 10, 12, 13, 15, 16, 20 – 22] that Wiener systems have been used as natural models of many physical plants with respective output nonlinearities. A special class of such systems is piecewise non-invertible Wiener affine (PWA) systems, i.e., when the linear dynamical system is followed by a static nonlinearity, consisting of some subsystems, between which occasional switchings happen at different time moments [6, 7, 13 – 21]. In such a case, one could let the nonlinear part of the Wiener system be represented by some regression functions with unknown parameters. Therefore observations of an output of the Wiener system could be partitioned into distinct data sets according to different descriptions where the boundaries of sets of observations depend on the value of the unknown threshold a—observations are divided into regimes subject to whether the some observed threshold variable is smaller or larger than a. How to partition the available data in order to calculate the estimates of parameters of regression functions, also to get the unknown parameters of slopes, as well as a threshold of nonlinearities using off-line approach is shown in [13, 15]. However, it is emphasized in [21] that recursive identification methods are important, especially, in on-line monitoring and analysis of generally time-varying processes [18, 19]. They can be also combined with on-line control strategies to produce adaptive control algorithms. To this end, we propose in this paper an recursive approach for parametric identification of Wiener systems with non-invertible piecewise-linear nonlinearity. The next section introduces the statement of the problem to be solved. In Section 3, we describe the rearrangement of the data to be used for Wiener systems parametric identification. The approach for reconstruction of intermediate signal by processing reordered observations is given in Section 4. In Section 5 the recursive parametric estimation of linear block of Wiener system is presented. In Section 6, we describe the estimation of parameters of piecewise linear function in nonlinear block of Wiener system's. In Section 7, simulation results are presented. Section 8 contains conclusions.

2. Statement of the Problem

The Wiener system consists of a linear part $G(q^{-1}, \mathbf{\Theta})$ followed by a static non-invertible nonlinearity $f(\cdot, \eta)$ with the vector of parameters η. The linear part of the PWA system is dynamic, time invariant, causal, and stable. It can be represented by a linear time invariant system (LTI) with the rational function of the form

$$G(q^{-1}, \mathbf{\Theta}) = \frac{b_1 q^{-1}+, \ldots, +b_{\mathrm{m}} q^{-\mathrm{m}}}{1 + a_1 q^{-1}+, \ldots, +a_{\mathrm{m}} q^{-\mathrm{m}}} = \frac{B(q^{-1}, \mathbf{b})}{1 + A(q^{-1}, \mathbf{a})} \quad (1)$$

with a finite number of parameters

$$\mathbf{\Theta}^T=(b_1,\ldots,b_{\mathrm{m}},a_1,\ldots,a_{\mathrm{m}}), \quad (2)$$
$$\mathbf{b}^T=(b_1,\ldots,b_{\mathrm{m}}), \quad \mathbf{a}^T=(a_1,\ldots,a_{\mathrm{m}}),$$

that are determined from the set $\mathbf{\Omega}$ of permissible parameter values $\mathbf{\Theta}$. Here q^{-1} is a time-shift operator [10], the set $\mathbf{\Omega}$ is restricted by conditions on the stability of the respective difference equation. The unknown intermediate signal

$$x(k)\equiv x(k;\mathbf{\Theta})=\frac{B(q^{-1},\mathbf{b})}{1+A(q^{-1},\mathbf{a})}u(k)+v(k), \quad (3)$$

generated by the linear part of the PWA system (1) as a response to the input $u(k)$ and corrupted by the additive noise $v(k)$, is acting on the static nonlinear part $f(\cdot,\eta)$ (Fig. 1), i.e.,

$$y(k)=f(x(k;\mathbf{\Theta}),\eta)+e(k). \quad (4)$$

Here the nonlinear part $f(\cdot,\eta)$ of the PWA system is a saturation-like function of the form [6, 7, 13 – 15]

$$f(x(k),\eta)=\begin{cases} c_0-c_1x(k) & \text{if } x(k)\le -a \\ x(k) & \text{if } -a<x(k)\le a, \\ d_0-d_1x(k) & \text{if } x(k)>a \end{cases} \quad (5)$$

that could be partitioned into three functions. These functions are: $f\{x(k;\mathbf{\Theta}),\mathbf{c},a\}=c_0-c_1x(k)$, $f\{x(k;\mathbf{\Theta}),a\}=x(k)$, $f\{x(k;\mathbf{\Theta}),\mathbf{d},a\}=d_0-d_1x(k;\mathbf{\Theta})$.

The function $f\{x(k;\mathbf{\Theta}),\mathbf{c},a\}$ has only negative values, when $x(k)\le -a$, $f\{x(k;\mathbf{\Theta}),a\}$ has arbitrary positive, as well as negative values, when $-a<x(k)\le a$, and $f\{x(k;\mathbf{\Theta}),\mathbf{d},a\}$ has only positive values, when $x(k)>a$. Here $x(k;\mathbf{\Theta})\equiv x(k)$, $\mathbf{c}^T=(c_0,c_1)$, $c_0=-a(1+c_1)$, $0<c_1<<a$, $\mathbf{d}^T=(d_0,d_1)$, $d_0=a(1+d_1)$, $0<d_1<<a$.

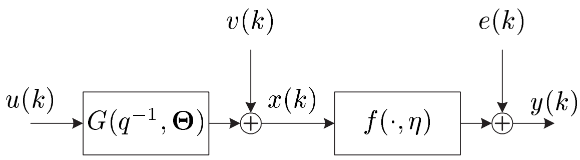

Figure 1. The PWA system.
(The linear dynamic part $G(q^{-1},\mathbf{\Theta})$ of the PWA system is parameterized by $\mathbf{\Theta}$, while the static nonlinear part $f(\cdot,\eta)$—by η. Signals: $u(k)$, $y(k)$, $x(k)$ are an input, an output, and an unmeasurable intermediate signal, respectively)

The process noise $v(k)\equiv\xi(k)$ and the measurement noise $e(k)\equiv\zeta(k)$ are added to an intermediate signal $x(k)$ and the output $y(k)$, respectively, $\xi(k)$, $\zeta(k)$ are mutually non-correlated sequences of independent Gaussian variables with $E\{\xi(k)\}=0$, $E\{\zeta(k)\}=0$, $E\{\xi(k)\xi(k+\tau)\}=\sigma_\xi^2\delta(\tau)$, $E\{\zeta(k)\zeta(k+\tau)\}=\sigma_\zeta^2\delta(\tau)$; $E\{\cdot\}$ is a mean value, $\sigma_\zeta^2,\sigma_\xi^2$ are variances of ζ and ξ, respectively, $\delta(\tau)$ is the Kronecker delta function.

The aim of the given paper is to estimate recursively the parameters (2) of the linear part (1) of the PWA system, parameters $\eta=(c_0,c_1,d_0,d_1)^T$, and the threshold a of non-invertible nonlinearity (5) by processing current pairs of observations $u(k)$ and $y(k)$, as well as current reconstructed values $\hat{x}(k)$ of intermediate signal $x(k)$.

3. The data rearrangement

At first, let us assume that in the memory of computer one has *N* pairs of observations $u(k)$ and $y(k)$ stored in advance. At second, let us rearrange the data $y(k)\ \forall\ k\in\overline{1,N}$ in an ascending order of their values assuming that noises $v(k)\equiv\xi(k)$, $e(k)\equiv\zeta(k)$ are absent, parameters $\eta=(c_0,c_1,d_0,d_1)^T$, and the threshold a of non-invertible nonlinearity (5) are known [15]. Thus, the observations of the reordered output $\tilde{y}(k)$ of the PWA system should be partitioned into three data sets: left-hand side data set (N_1 samples) with values lower than or equal to negative a, middle data set (N_2 samples) with values higher than negative a but lower than or equal to a, and right-hand side data set (N_3 samples) with values higher than a. Here $N=N_1+N_2+N_3$. Hence, the observations with the highest and positive values will be concentrated on the right-hand side set, while the observations with the lowest and negative values on the left-hand side one [15]. It could be noted that on boundaries the small portions of observations of the middle data set of $\tilde{y}(k)$ (approximately by 5 percentage) are mixed together with some portions of observations of the left-hand side and right-hand data sets, respectively, due to negative slopes of the nonlinearity (5). In general case (an noisy environment, unknown parameters and the threshold) it is imperative for the efficient parametric identification of the PWA system that such ambiguities are resolved. On the other hand, one can avoid this problem assuming here that slightly less than 50% unmixed observations are concentrated on the middle-set and approximately by 15% on any side set. Note that the observations of the middle data set of $\tilde{y}(k)$ are coincident with the respective observations of the intermediate signal $x(k)$ in the absence of the measurement noise $e(k)$. Therefore, one could get unmixed observations of $\tilde{y}(k)$ simply by choosing the upper interval bound lower than the 75 percentage and the lower interval bound higher than the 25 percentage of the sampled reordered observations of $\tilde{y}(k)$. Each current observation $\tilde{y}(k)$ ought to be assigned to respective data set. In such a case, the whole number of observations N and numbers of observations N_1, N_2, N_3 will be varying in time. In order to process current observations one can use the corresponding recursive technique that will be given later in this paper.

4. Reconstruction of intermediate signal

Next, let us reconstruct an unmeasurable intermediate signal $x(k)$, using the middle data set $\tilde{y}(k)\ \forall\ k \in \overline{L_1, L_2}$ that is, really, reordered in an ascending order of their values $x(k)$ with small portions of missing observations within it that belong to the left-hand and right-hand side sets of the data. Here $L_1 = N_1 + l_1, L_2 = N_2 - l_2$, where arbitrary integers $l_1, l_2 > 0$. To calculate an auxiliary signal $\hat{x}(k)$ (the reconstructed $x(k)$) $\forall\ k \in \overline{1, N}$, one could approximate the model of the linear part of the PWA system (1) by the finite impulse response (FIR) system of the form [2, 14 – 16]

$$\begin{aligned}\tilde{y}(k) = \beta_0 + \beta_1 \tilde{u}(k) + \beta_2 \tilde{u}(k-1) + \dots \\ +\beta_\nu \tilde{u}(k-\nu+1) + \tilde{e}(k)\end{aligned} \tag{6}$$

$\forall\ k \in \overline{L_1, L_2}$, or the expression in a matrix form

$$\tilde{\mathbf{Y}} = \mathbf{\Lambda}\beta; \tag{7}$$

where $\tilde{\mathbf{Y}} = (\tilde{y}(L_1), \tilde{y}(L_1+1), \dots, \tilde{y}(L_2))^T$ is the $(L_2 - l_1)$ – vector of the middle data set of $\tilde{y}(k)$ without the small portions of observations from left-hand and right-hand side sets of the data,

$$\mathbf{\Lambda} = \begin{bmatrix} 1 & \tilde{u}(\mathrm{L}_1) & \dots \tilde{u}(\mathrm{L}_1-\nu+2) & \tilde{u}(\mathrm{L}_1-\nu+1) \\ 1 & \tilde{u}(\mathrm{L}_1+1) & \dots\ \tilde{u}(\mathrm{L}_1-\nu+3) & \tilde{u}(\mathrm{L}_1-\nu+2) \\ \vdots & \vdots & \vdots \quad\quad \vdots & \vdots \\ 1 & \tilde{u}(\mathrm{L}_2) & \dots\ \tilde{u}(\mathrm{L}_2-\nu+2) & \tilde{u}(\mathrm{L}_2-\nu+1) \end{bmatrix} \tag{8}$$

is the full rank $(L_2 - l_1) \times (\nu + 1)$ regression matrix, consisting only of observations of the non-noisy input $\tilde{u}(k)$;

$$\beta^T = (\beta_0, \beta_1 \dots, \beta_\nu) \tag{9}$$

is a $(\nu+1) \times 1$ vector of unknown parameters, ν is the order of the FIR filter that can be arbitrarily large but fixed, $\tilde{u}(k)$ are observations of $u(k)$ associated with their own $\tilde{y}(k)$, $\tilde{e}(k) = v(k) + e(k)$.

It could be emphasized here that by applying the FIR model one avoids the influence of some missing regressors appearing in the regression matrix $\mathbf{\Lambda}$, if the infinite impulse response (IIR) system is used. In this case, the dependence of some regressors on the process output will be facilitated, and the assumption of the ordinary LS that the regressors depend only on the non-noisy input signal, will be satisfied [3], too. This is the main consequence of replacing the initial transfer function $G(q^{-1}, \mathbf{\Theta})$ of the linear part of the PWA system by the FIR filter (6). Note that all proofs based on the deterministic regression matrix are valid here as well.

The parametric estimation technique, based on ordinary LS, could be applied in the estimation of parameters (9) of the given FIR system (6), using the reordered observations of the middle data-set $\tilde{y}(k)$, because the rearrangement of observations does not influence the accuracy of LS estimates to be calculated. For the recursive estimation of unknown parameters (9), the ordinary prediction error method, based on the RLS of the form

$$\hat{\beta}(k) = \hat{\beta}(k-1) + \frac{\Gamma(k-1)z(k)}{1 + z^T(k)\Gamma(k-1)z(k)}\hat{\varepsilon}(k), \tag{10}$$

$$\Gamma(k) = \Gamma(k-1) - \frac{\Gamma(k-1)z(k)z^T(k)\Gamma(k-1)}{1 + z^T(k)\Gamma(k-1)z(k)}, \tag{11}$$

$\forall\ k = N_4$+1, N_4+2,... could be used with the vector of observations

$$z^T(k) = [1, \tilde{u}(k-1), ..., \tilde{u}(k-\nu+1)],$$

and some initial values, i.e. $\hat{\beta}(N_4)$, and matrix $\Gamma(N_4) = (\mathbf{\Lambda}^T\mathbf{\Lambda})^{-1}$.
Here

$$\hat{\beta}^T(k) = \left(\hat{\beta}_0(k), \hat{\beta}_1(k) \dots, \hat{\beta}_\nu(k)\right) \tag{12}$$

is a current estimate of the parameter vector (9) on the current k-th iteration,

$$\begin{aligned}\hat{\varepsilon}(k) = \tilde{y}(k) - [\hat{\beta}_0(k) + \hat{\beta}_1(k)\tilde{u}(k-1) + \\ \hat{\beta}_2(k)\tilde{u}(k-2) + \dots + \hat{\beta}_\nu(k)\tilde{u}(k-\nu+1)]\end{aligned} \tag{13}$$

is the prediction error on the same k-th iteration,

$$\hat{\beta}(N_4) = (\mathbf{\Lambda}^T\mathbf{\Lambda})^{-1} \mathbf{\Lambda}^T\tilde{\mathbf{Y}}, \tag{14}$$

with

$$\hat{\beta}^T(N_4) = \left(\hat{\beta}_0(N_4), \hat{\beta}_1(N_4) \dots, \hat{\beta}_\nu(N_4)\right) \tag{15}$$

is a $(\nu+1) \times 1$ vector of the estimates of parameters (9) calculated using only $N_4 = L_2 - l_1$ observations of the middle data set.

The next step is to reconstruct the current

k-th value of the intermediate signal $x(k)$,$\forall\ k = N$+1,N+2,... by

$$\hat{x}(k) = \hat{\beta}_0(k) + \hat{\beta}_1(k)\tilde{u}(k-1) + \hat{\beta}_2(k)\tilde{u}(k-2) + \ldots + \hat{\beta}_\nu(k)\tilde{u}(k-\nu+1) \quad (16)$$

that with the previous values of $\hat{x}(k)$ will be used to calculate the estimates of parameters (2) of the transfer function $G(q^{-1}, \mathbf{\Theta})$ at the next followed step.

5. Recursive parametric estimation

Now, let us calculate the current estimates of the parameters (2) of the transfer function $G(q^{-1}, \mathbf{\Theta})$ using $m-1$ values of observations $u(k)$, and $m-1$ values of reconstructed intermediate signal $\hat{x}(k)$. The current estimate of the parameter vector (2) could be determined by the next RLS of the form

$$\hat{\mathbf{\Theta}}(k) = \hat{\mathbf{\Theta}}(k-1) + \frac{\Pi(k-1)\tilde{z}(k)}{1+\tilde{z}^T(k)\Pi(k-1)\tilde{z}(k)}\tilde{\varepsilon}(k), \quad (17)$$

$$\Pi(k) = \Pi(k-1) - \frac{\Pi(k-1)\tilde{z}(k)\tilde{z}^T(k)\Pi(k-1)}{1+\tilde{z}^T(k)\Pi(k-1)\tilde{z}(k)}, \quad (18)$$

$\forall\ k = N$+1, N+2,... with the vector of observations

$$\tilde{z}^T(k) = (u(k-1), ..., u(k-m), -\hat{x}(k-1), ..., -\hat{x}(k-m)) \quad (19)$$

and some initial values of the vector $\hat{\mathbf{\Theta}}(N)$ and matrix $\Pi(N) = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}$. Here

$$\hat{\mathbf{\Theta}}^T(k) = (\hat{\mathbf{b}}^T(k), \hat{\mathbf{a}}^T(k)) = (\hat{b}_1(k), ..., \hat{b}_m(k), \hat{a}_1(k), ..., \hat{a}_m(k)) \quad (20)$$

is the current estimate of the parameter vector (2),

$$\tilde{\varepsilon}(k) = A(q^{-1}, \hat{a}(k-1))\hat{x}(k) - B(q^{-1}, \hat{b}(k-1))u(k)$$

is the prediction error on the current k-th iteration.

$$\hat{\mathbf{\Theta}}(N) = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{U}, \quad (21)$$

$$\hat{\mathbf{\Theta}}^T(N) = \left(\hat{\mathbf{b}}(N), \hat{\mathbf{a}}(N)\right)^T, \quad (22)$$

$$\hat{\mathbf{b}}^T(N) = \left(\hat{b}_1(N), \ldots, \hat{b}_{\mathrm{m}}(N)\right),$$

$$\hat{\mathbf{a}}^T(N) = \left(\hat{a}_1(N), \ldots, \hat{a}_{\mathrm{m}}(N)\right)$$

are $2m \times 1, m \times 1, m \times 1$ vectors of the estimates of parameters, respectively,

$$\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2] \quad (23)$$

is the $(N-m-\nu-1) \times 2m$ matrix, consisting of observations of the input $u(k)$ and the auxiliary signal $\hat{x}(k)$,

$$\mathbf{X}_1 = \begin{bmatrix} u(m) & \ldots & u(1) \\ u(m+1) & \ldots & u(2) \\ \vdots & & \vdots \\ u(N-1) & \ldots & u(N-m) \end{bmatrix}, \quad (24)$$

and

$$\mathbf{X}_2 = \begin{bmatrix} \hat{x}(m) & \ldots & \hat{x}(1) \\ \hat{x}(m+1) & \ldots & \hat{x}(2) \\ \vdots & & \vdots \\ \hat{x}(N-1) & \ldots & \hat{x}(N-m) \end{bmatrix}, \quad (25)$$

$$\mathbf{U} = (\hat{x}(m+1), \hat{x}(m+2), ..., \hat{x}(N))^T \quad (26)$$

is the $(N-m-1)$ – vector, consisting of the observations of $\hat{x}(k)$.

6. Calculation of nonlinear function parameters

Estimates of the parameters c_0, d_0 and c_1, d_1 are calculated by the ordinary LS, too. In such a case, the sums of the form

$$I(c_0, c_1) = \sum_{i=1}^{N_1-l_3} \left[\tilde{y}(i) - c_0 - c_1\tilde{\hat{x}}(i)\right]^2 = min!, \quad (27)$$

$$I(d_0, d_1) = \sum_{j=\{N} {}_2+l_4^N \left[\tilde{y}(j) - d_0 - d_1\tilde{\hat{x}}(j)\right]^2 = min!, \quad (28)$$

are to be minimized in respect of the parameters c_0, c_1 and d_0, d_1, respectively, using side-set data particles of $\tilde{y}(k)$ and associated observations of the auxiliary signal $\hat{x}(k)$. Here $\tilde{\hat{x}}(k)$ are the observations of the signal $\hat{x}(k)$ that are rearranged in accordance with $\tilde{y}(k)$. Note that in respective sums arbitrary integers $l_3, l_4 > 0$.

The estimates of parameters c_1, d_1 and c_0, d_0 are calculated according to

$$\hat{c}_1 = \frac{\sum_{i=1}^{\mathrm{N}_1-l_3} \tilde{y}(i)\tilde{\hat{x}}(i)}{\sum_{i=1}^{\mathrm{N}_1-l_3} \tilde{\hat{x}}^2(i)}, \quad \hat{d}_1 = \frac{\sum_{j=1}^{\mathrm{N}_3-l_4} \tilde{y}(j)\tilde{\hat{x}}(j)}{\sum_{j=1}^{\mathrm{N}_3-l_4} \tilde{\hat{x}}^2(j)}, \quad (29)$$

$$\hat{c}_0 = \frac{\sum_{i=1}^{\mathrm{N}_1 - l_3} [\tilde{y}(i) - \hat{c}_1 \tilde{\hat{x}}(i)]}{\mathrm{N}_1 - l_3}, \tag{30}$$
$$\hat{d}_0 = \frac{\sum_{j=1}^{\mathrm{N}_3 - l_4} [\tilde{y}(j) - \hat{d}_1 \tilde{\hat{x}}(j)]}{\mathrm{N}_3 - l_4},$$

respectively [11], but using side-set data particles of $\tilde{y}(k)$ and associated observations of the auxiliary signal $\hat{x}(k)$, that are reordered in accordance with $\tilde{y}(k)$[13, 15].

It is obvious that one can use expressions (29) and (30) for recursive estimation. In such a case, respective sums in numerators and denominators should be calculated by processing fixed number of observations. Then, in each current iteration k corresponding products ought to be determined. Afterwards, they should be added to previously obtained respective sum values. At last, the ratios of the respective sums are obtained giving us current values of $\hat{c}_1, \hat{d}_1, \hat{c}_0, \hat{d}_0$. The estimates of the threshold a on the right-hand side and left-hand side sets are found according to

$$\hat{a} = \hat{d}_0/(1 + \hat{d}_1), \hat{a} = -\hat{c}_0/(1 + \hat{c}_1), \tag{31}$$

respectively. It could be mentioned that an approach presented in [1] could be applied here if N_1 and N_3 are unknown beforehand.

7. Numerical simulations

The true intermediate signal $x(k)$ $k = \overline{1, \mathrm{N}}$, of the PWA system (Figures 2b, 3b) is given by (3). The true output signal (Figures 2c, 3c) is described by

$$y(k) = \begin{cases} -7.6 - 0.1x(k) & \text{if } x(k) \leq -7.5, \\ x(k) & \text{if } -7.5 < x(k) \leq 7.5, \\ 7.6 - 0.1x(k) & \text{if } x(k) > 7.5 \end{cases} \tag{32}$$

and by

$$y(k) = \begin{cases} -1.1 - 0.1x(k) & \text{if } x(k) \leq -1, \\ x(k) & \text{if } -1 < x(k) \leq 1, \\ 1.1 - 0.1x(k) & \text{if } x(k) > 1 \end{cases} \tag{33}$$

with the sum of sinusoids (Fig. 2a)

$$u(k) = \frac{1}{20} \sum_{i=1}^{20} \sin(i\pi k/10 + \phi_i) \tag{34}$$

and white Gaussian noise with variance 1 (Fig. 3a) as inputs to the linear block

$$G(q^{-1}, \mathbf{\Theta}) = \frac{b_1 q^{-1}}{1 + a_1 q^{-1}}, \tag{35}$$

respectively. Here $b_1 = 1$, $a_1 = -0.7$. In (34), the

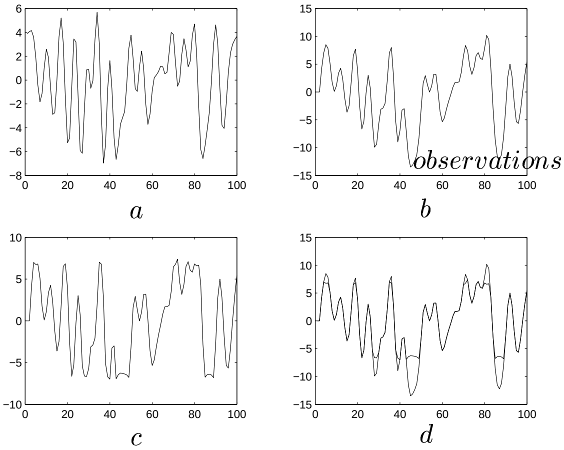

Figure 2. The signals of the simulated PWA system with a piecewise nonlinearity (32).
(Input $u(k)$, calculated by (34)(a), intermediate signal $x(k)$ (b), output $y(k)$ (c), intermediate and output (dotted line) signals (d))

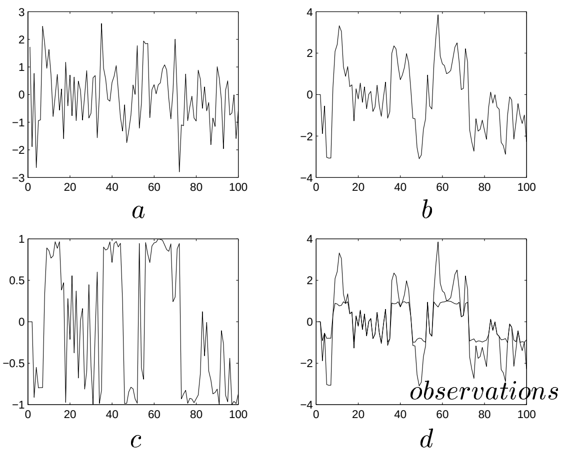

Figure 3. The signals of the simulated PWA system with a piecewise nonlinearity (33).
(Input $u(k)$ is white Gaussian noise (a), intermediate signal $x(k)$ (b), output $y(k)$ (c), intermediate and output signals (d))

stochastic values ϕ_i $i = \overline{1, 20}$ randomly and uniformly drawn from the interval $[0, 2\pi]$ [6]. First of all, $\mathrm{N} = 100$ data points have been generated without additive process and measurement noises (Figures 4, 5). Afterwards, the LS problem (14) was solved, using 38 and 36 rearranged observations of the output, respectively (Figures 6c, 7c), excluding zeros. In (6) the number of the FIR filter parameters $\nu = 14$ has been chosen based on the estimation results, obtained for different ν in the absence of process and measurement noises [15]. The estimate $\hat{x}(k)$

of the intermediate signal $x(k)$ was reconstructed according to (16), replacing unknown true values of parameters by their estimates. The reconstructed versions of the intermediate signal $x(k)$ are shown in Figures 8a, 9a. The estimates $\hat{\mathbf{\Theta}}^T = (b_1, a_1)$ of parameters $\mathbf{\Theta}$ of the function $G(q^{-1}, \mathbf{\Theta})$ were calculated by Eq. (21), using the observations of the auxiliary signal $\hat{x}(k)$. Afterwards, the estimate $\hat{x}_1(k)$ of the intermediate signal $x(k)$ was recalculated by

$$\hat{x}_1(k) = \hat{b}_1\, u(k-1) + \hat{a}_1 \hat{x}_1(k-1), \quad \forall \quad k = \overline{2, 100}, \tag{36}$$

using $\hat{b}_1, \hat{a}_1$ and $\hat{x}_1(1) = 0$. In such a case, the estimates $\hat{b}_1, \hat{a}_1$ were equal to the true parameters: $b_1 = 1, a_1 = 0.7$. The reconstructed versions of the intermediate signal $x(k)$, calculated by eq. (36) are shown in Figures 8b, 9b. The accuracy of estimates of the intermediate signal, calculated by formulas (16) and (36), is more or less similar except for the first 15 observations, when the FIR model (16) was used. If $\hat{x}(k)$ has been obtained, then it is simple to separate different particles of observations that belong to the respective side-sets. The estimates of parameters c_1, d_1 and c_0, d_0 are calculated according to formulas (29) and (30), respectively. In such a case, the rearranged observations of $\hat{x}(k)$ and $y(k)$ were substituted in formulas (29) and the estimates of c_1 and d_1 were determined: $\hat{c}_1 = \hat{d}_1 = 0.1$. Then, the estimates $\hat{c}_0$ and $\hat{d}_0$ were calculated by (30). Their values are also coincidental with the values of true coefficients: $\hat{c}_0$=–7.6, while $\hat{d}_0$=7.6 for the nonlinearity (32) and $\hat{c}_0$=–1.1, while $\hat{d}_0$=1.1 for the nonlinearity (33), respectively. Note that N_1=14, N_3=8 for the periodical signal (34) (Fig. 2a), and N_1=32, N_3=28 for the Gaussian white noise (Fig. 3a) were used to calculate the estimates $\hat{c}_0, \hat{c}_1, \hat{d}_0, \hat{d}_1$, respectively. The estimates of the threshold were established by Eqs. (31). The values of estimates $\hat{a}$ were equal to the true values: a=7.5 and a=1, respectively.

In order to determine how realizations of different process- and measurement noises affect the accuracy of recursive estimation of unknown parameters, we have used the Monte Carlo simulation with 10 data samples, each containing 100 pairs of input-output observations. 10 experiments with the same realization of the process noise $v(k)$ and different realizations of the measurement noise $e(k)$ with different levels of its intensity have been carried out. The intensity of noises was assured by choosing respective signal-to-noise ratios SNR (the square root of the ratio of signal to noise variances). For the process noise, SNR was equal to 10, and for the measurement noise, SNR^e was varying, thus SNR^e=(1, 10, 100). As inputs for all given nonlinearities the

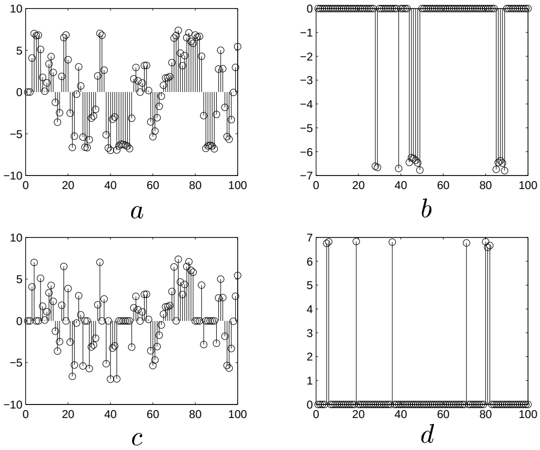

Figure 4. Samples of $y(k)$ (a) (see Fig. 2c). (Data sets: left (b), middle (c), right (d)(here the observations, that belong to the other data set, are equal to zero). Input $u(k)$ is of the form (34) (see Fig. 2a))

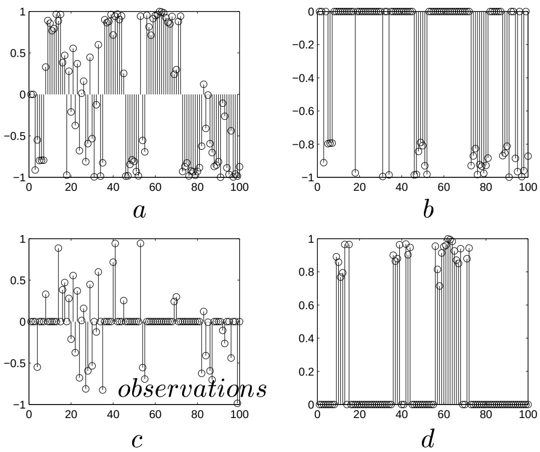

Figure 5. Samples of signal $y(k)$ (a) (see Fig. 3c). (Data sets: left (b), middle (c), right (d). Input $u(k)$ is white Gaussian noise (see Fig. 3a))

periodical signal (34) and white Gaussian noise with variance 1 were chosen. In each ith experiment the estimates of parameters were calculated. During the Monte Carlo simulation averaged values of estimates of the parameters and of the threshold and their confidence intervals were calculated. In Tables 1 and 2, for each input the averaged recursive estimates of parameters and the threshold a of the simulated PWA system (Fig. 1) with the linear part (35) ($b_1 = 1; a_1 = -0.7$) and the piecewise nonlinearities (32), (33) with ($c_0 = -1.1, c_1 = 0.1, d_0 = 1.1, d_1 = 0.1$), and ($c_0 = -7.6, c_1 = 0.1, d_0 = 7.6, d_1 = 0.1$), respectively, with their confidence intervals are presented. Note that in each experiment here the value of SNR was fixed and was the same, while the values of SNR were varying due to different realizations of $e(k)$. The

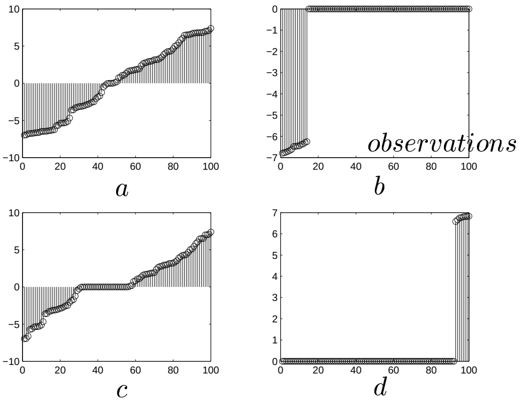

Figure 6. Values of the signal $y(k)$ that are reordered in an ascending order (a) (see also Figures 2c, 4a).
(Rearranged data sets: left (b), middle (c), right (d) (here the observations, that belong to the other data set, are equal to zero). Input $u(k)$ is of the form (34) (see Fig. 2a))

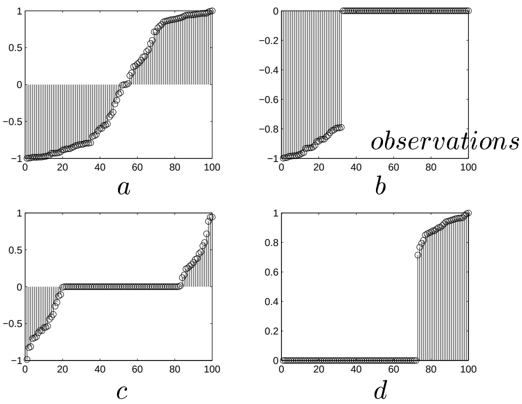

Figure 7. Values of the signal $y(k)$ that are reordered in an ascending order (a) (see also Figures 3c, 5a).
(Rearranged data sets: left (b), middle (c), right (d). Input $u(k)$ is white Gaussian noise (see Fig. 3a))

Monte Carlo simulation implies that the accuracy of recursive parametric identification of the PWA system depends on the intensity of measurement noise.

8. Conclusions

The problem of identification of PWA systems (Fig. 1) could be essentially reduced by a simple data rearrangement. Afterwards, the available data are partitioned into three data sets that correspond to distinct threshold regression models. Thus, the estimates of unknown parameters of linear regression models can be calculated by processing respective sets of the rearranged output and associated input observations. A technique, based on ordinary recursive LS, is proposed here for estimating the parameters of linear

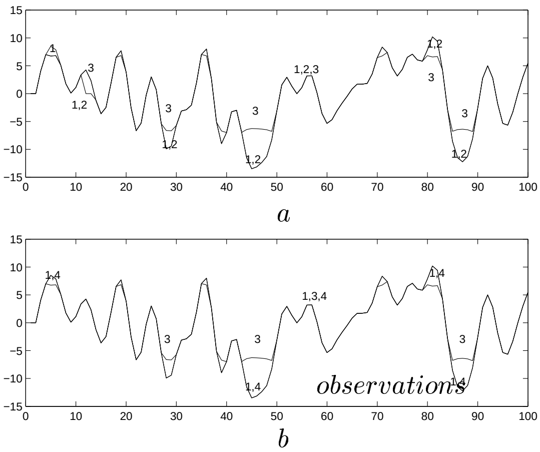

Figure 8. Signals of the PWA system.
(The intermediate signal $x(k)$ (curve 1), the output signal $y(k)$ (curve 3), the reconstructed versions of $x(k)$ (curves 2, 4), calculated using Eq. (16)(a) and Eq. (36)(b), respectively. Input $u(k)$ is of the form (34) (see Fig. 2a))

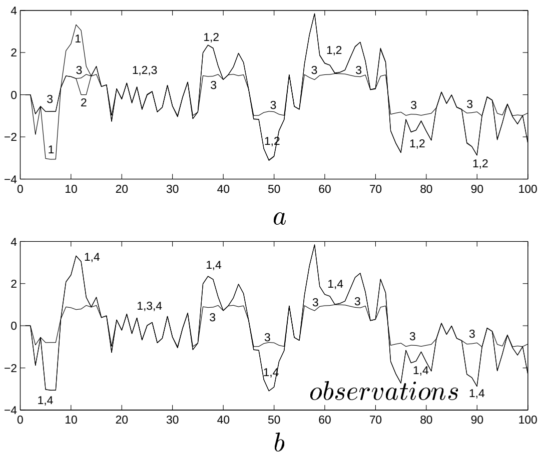

Figure 9. Signals of the PWA system.
(The intermediate signal $x(k)$ (curve 1), the output signal $y(k)$ (curve 3), the reconstructed versions of $x(k)$ (curves 2, 4), calculated using Eq. (16)(a) and Eq. (36)(b), respectively. Input $u(k)$ is white Gaussian noise (see Fig. 3a))

Table 1. Averaged estimates of the parameters $b_1, a_1, c_0, c_1, d_0, d_1$, and thresholds a, $-a$ with their confidence intervals. Input: the periodical signal (34). SNR=10.

Values	$\mathrm{SNR}^e = 1$	$\mathrm{SNR}^e = 10$	$\mathrm{SNR}^e = 100$
$\hat{b}_1$	1.42 ± 0.33	1.15 ± 0.03	1.13 ± 0.00
$\hat{a}_1$	-0.68 ± 0.05	-0.68 ± 0.01	-0.68 ± 0.00
$\hat{c}_0$	-8.47 ± 4.9	-7.86 ± 0.65	-7.71 ± 0.07
$\hat{c}_1$	0.27 ± 0.72	0.01 ± 0.11	0.02 ± 0.01
$\hat{d}_0$	3.11 ± 9.28	6.49 ± 1.14	6.49 ± 0.11
$\hat{d}_1$	-0.03 ± 0.39	0.07 ± 0.05	0.09 ± 0.01
$\hat{a}$	13.71 ± 12.51	6.5 ± 0.45	6.62 ± 0.04
$-\hat{a}$	1.81 ± 24.4	-7.42 ± 1.11	-7.09 ± 0.1

Table 2. Averaged estimates of the parameters $b_1, a_1, c_0, c_1, d_0, d_1$, and thresholds a, $-a$ with their confidence intervals. Input—the Gaussian white noise

Values	$\mathrm{SNR}^e = 1$	$\mathrm{SNR}^e = 10$	$\mathrm{SNR}^e = 100$
$\hat{b}_1$	0.79 ± 0.21	0.95 ± 0.02	0.97 ± 0.00
$\hat{a}_1$	-0.48 ± 0.17	-0.71 ± 0.01	-0.72 ± 0.00
$\hat{c}_0$	-1.17 ± 0.17	-1.04 ± 0.03	-1.03 ± 0.00
$\hat{c}_1$	0.24 ± 0.3	0.12 ± 0.02	0.12 ± 0.00
$\hat{d}_0$	0.89 ± 0.4	1.07 ± 0.05	1.09 ± 0.01
$\hat{d}_1$	-0.02 ± 0.32	-0.09 ± 0.03	-0.1 ± 0.00
$\hat{a}$	1.02 ± 0.28	0.92 ± 0.04	0.91 ± 0.00
$-\hat{a}$	0.81 ± 0.29	-0.98 ± 0.02	-0.99 ± 0.00

and nonlinear parts of the Wiener system, including the unknown threshold of the piecewise nonlinearity, too. During successive steps the unknown intermediate signal is reconstructed and the missing values of observations of respective data particles are replaced by their estimates. Results of numerical simulation (Figures 2—9 and Tables 1, 2) prove the efficiency of the proposed recursive approach.

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