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On the Importance of the Artificial Bee Colony Control Parameter 'Limit'

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Artificial Bee Colony (ABC) is a successful meta-heuristic algorithm that has been greatly utilised by researchers. Through our practical experience of ABC, we have noticed that the recommended formula 'limit' = n_e * D may not be the best choice for different problems. In this work, a set of experiments using horizontal and vertical approaches has been designed and executed with the aim of observing the effect of 'limit' on ABC. The results have been statistical analysed using Null Hypothesis Significance Testing (NHST) as well as the Chess Rating System for Evolutionary Algorithms (CRS4EAs), which is a novel approach for comparing meta-heuristic algorithms. It is shown that the recommended formula is not the best setting for different problems and approaches. Hence, the control parameter 'limit' should be tuned or controlled. The other important result of this study is to show that CRS4EAs is comparable but also shows benefits over NHST.

KEYWORDS: ABC, control parameter setting, sensitivity analysis, significance testing, chess rating system for evolutionary algorithms.

1. Introduction

Comparisons between different meta-heuristic algorithms [5] are inevitably necessary within the field of Evolutionary Computation (EC). Although the scientific testing [20] approach, the aim of which is to learn about which kinds of problems and why one algorithm performs better, is preferred over the horse racing approach [12], [23], [45], the aim of which is to outperform other algorithms, the latter approach still prevails during current EC experimental practices. However, even in the scientific testing approach, simply understanding parameter interactions and placing emphasis on the analysis of robustness may not be enough if an algorithm under investigation performs badly. Hence, there is still a need for comparing the performances of the algorithms under investigation using the currently best available algorithms [9].

This paper deals with the Artificial Bee Colony (ABC) algorithm [25], [26], [39], which is a swarm intelligence algorithm that accomplishes optimisation tasks through social cooperation among bees (i.e., individuals) – employed bees exploit food source and share food source information to onlooker bees; onlooker bees probabilistically choose and exploit food source based on the provided information; and scout bees explore new food source when current ones are exhausted. ABC exhibits remarkable balance between exploitation and exploration [8] (raw data for experiments presented in this paper are available in [47]). This balance between exploitation (employed bee phase and onlooker bee phase) and exploration (scout bee phase) is controlled by population size (*SN*) and 'limit', respectively. The formula 'limit' = $n_e * D$ ('limit' is the threshold for determining whether a scout bee should be introduced or not, n_e is the number of employed bees, D is the dimension of a problem) was recommended in a very influential paper [26]. As ABC is a very successful algorithm, it has been used extensively over recent years [28], [29]. The suggested formula for setting the 'limit' control parameter is indeed mostly used (e.g., [2]). We came across only a few studies where the 'limit' was set at a certain fixed number (e.g., 10 in [38], [55], 30 in [53], 40 in [56], 50 in [44], 100 in [22], [52], 200 in [34], [57]), or better where the 'limit' was tuned [35]. When experimenting using ABC we have noticed its sensitiveness to 'limit' control parameter and that its relationship between population size ($SN = 2 * n_e$) and the dimension of a problem (D) is not straightforward. However, this was just our speculation driven by practical experience with ABC. Hence, we decided to perform extensive statistical analysis of ABC and support it by stronger conclusions, using the Null Hypothesis Significance Testing (NHST) [41] and Chess Rating System for Evolutionary Algorithms (CRS4EAs) [48]. For finding the significant differences with NHST, the Wilcoxon's test [51] was a more appropriate test with the post-hoc analysis supported by the Holm's test [19]. Both the Wilcoxon's test and CRS4EAs compare the results pairwisely but whilst the Wilcoxon's comparison concentrates only on $1{\times}k$ comparison, the comparison in CRS4EAs allows $k{\times}k$ comparison and the detections of significant differences amongst all algorithms. Note that when attempting to apply statistical $k{\times}k$ comparison, the more appropriate test would be the Friedman test [13], [14]. However, as the number of problems is really small and the goal was to analyse different 'limit' settings regarding different problems, the Friedman test could not be taken into consideration [50]. Hence, the choice of Wilcoxon's test with post-hoc Holm's test is shown as an appropriate one. Even though CRS4EAs allows $k{\times}k$ comparison and Wilcoxon's test allows only $1{\times}k$ comparison, the analysis of CRS4EAs was applied as $1{\times}k$ comparison, as well as assuring that both methods are applied equally. Our results show that ABC's performance is very sensitive to a control parameter 'limit', which is often independent regarding the population size. Whilst the 'limit' depends on dimension D, it is much more dependent on the problem under investigation. Although, the characteristics of a problem might drastically change when changing dimension D and can become a completely different problem (e.g., an optimisation function becomes multi-modal instead of uni-modal or vice verse, and the fitness-distance correlation is changed from high to low correlation or vice verse [7]). Hence, dimension D can be seen as part of a problem as well.

The main contributions of this paper are:

- Sensitivity analysis is applied for the first time on ABC control parameters using vertical and horizontal approaches showing that control parameter *SN* is much more robust than control parameter 'limit', which must be carefully set for the best results;

- An example of how from sensitivity analysis one might conclude that a suggested formula for setting control parameters is not most appropriate;
- Deep statistical investigations about setting ABC control parameter 'limit' as a full factorial design using NHST and CRS4EAs showing that the recommended formula for setting the control parameter 'limit' regarding population size *SN* and the dimension of the problem *D* is not the best for every problem and approach;
- For the first time, it is shown that even the control parameter 'limit' depends on the available maximum number of fitness evaluations, and that ABC convergence using the suggested formula is not amongst the fastest; and
- First application of CRS4EAs as *1×k* comparison showing its applicability and suitability as a feasible replacement of NHST.

The main conclusion from this study is that ABC does not always perform best when under the setting 'limit' = n_e * D. Hence, the 'limit' control parameter should be tuned or controlled.

However, such a conclusion should not come as a surprise in EC and confirms already established knowledge within the meta-heuristic field. Namely, fixed formulae for setting a control parameter usually lead to poor performances when applying to different problems. However, a systematic mapping study from [39] shows that this formula is indeed very frequently used indicating that still many researchers believe that some fixed formulae can be a robust choice. Our speculation is that this dichotomy between theory and practice exists due to lack of ABC studies showing that such a parameter setting is not the best. In this respect, our work can be seen as remedying this situation for ABC. There should be no excuse not to perform tuning on control parameter 'limit' anymore.

The other important conclusion from this study is that CRS4EAs is comparable with NHST but CRS4EAs also showed many benefits during experimentation where a greater number of experiments needed to be conducted. When executing one tournament in CRS4EAs, all the necessary data for analysis are obtained and calculated, whilst for NHST there are always additional tests required. Having so many different situations and approaches, the results analysed by CRS4EAs are far quicker and easier than with NHST.

The paper is organized as follows. Section 2 describes the conducted experiment in detail. This section is divided into three major parts: in Section 2.1 the sensitivity analysis is conducted for one optimisation problem; in Section 2.2 the results of experiment are analysed with NHST and the results reported regarding the different approaches; in Section 2.3 the results of the experiment are analysed with CRS4EAs and results are again reported regarding the different approaches. Section 3 displays the results of tuning the parameters of ABC on different dimensionalities of one optimisation problem. Section 4 discusses other similar researches as presented in the past. Lastly, Section 5 concludes the paper. All the algorithms, figures and tables are also placed online at https://lpm.feri.um.si/research/abc/.

2. Experiment

The amount of exploration [8] of ABC is controlled by the control parameter 'limit'. ABC is exploring the search space more often when the 'limit' is set at a small number, and vice versa by exploiting the search space when the 'limit' is set to a higher number (Algorithm 1). The amount of exploration and exploitation depends on the problem and even on the evolution stage [8]. Hence, it is difficult to quantify. The formula 'limit' = n_e * D [26] suggests that higher-dimensional problems require less exploration (higher dimension increases 'limit', which in turn decreases exploration), and that bigger population size increases exploitation, which is indeed correct for ABC. However, the relationship between population size and the needed amount of exploration is unclear, as well as the fact that higher-dimensional problems might require more exploration. Overall, the suggested formula was not intuitive for us and we decided to further explore the relationships between population size *SN* (*SN* = 2 * n_e), dimension *D*, and control parameter 'limit'. Our experiment was divided into two parts. In the first part, the importance of *SN*, *D*, and 'limit' to ABC was investigated by performing sensitivity analysis [33], which showed that indeed the most influential one amongst the aforementioned factors is 'limit'. In the second part of the experiment, emphasis was given to the ABC control parameter 'limit', where different settings were statistically analysed by NHST and CRS4EAs.

During the experiment, we used the same benchmark functions as in the original ABC work [26]. Although this benchmark suite contained only five numerical benchmark functions: (1) multi-modal, non-separable Schaffer function f_1, (2) uni-modal, separable Sphere function f_2, (3) multi-modal, non-separable Griewank function f_3, (4) multi-modal, separable Rastrigin function f_4, and (5) uni-modal, non-separable Rosenbrock function f_5, it was enough to arrive at appropriate conclusions. Even this small benchmark suite confirmed our hypothesis and there was no need to perform the experiment on more comprehensive benchmarks. On the other hand, whenever a statistical formula is suggested, it should be tested on comprehensive sets of benchmarks that can really support it on a vast number of different optimisation problems. For example, Piotrowski in [43] suggested that both the problems, minimisation and maximisation should be used on the same benchmark functions since a good performance of a meta-heuristic algorithm on the minimisation of some function does not also guarantee a good performance on the maximisation of the same function, and vice versa.

We extended Karaboga's experiment [26] by performing a full factorial design on this benchmark suite using the following factors and their values: $SN = \{24, 50, 100\}$, $D = \{2, 5, 10, 30, 50\}$, and 'limit' = {0, 100, 250, 500, 750, 1000, 1250, 1500, ∞}. Hence, altogether there were 3 * 5 * 9 = 135 different combinations tested using 100 independent runs, whilst using both vertical and horizontal approaches [18] when performing the experiments.

In the first case, known also as 'the fixed-cost approach', we measured the quality of a solution reached by a pre-defined number of fitness evaluations (100,000 and 250,000 fitness evaluations for each combination). In the second case, also known as 'the fixed-target approach', we measured the number of fitness evaluations needed to find a (sub-)optimal solution (10^{-6} and 10^{-12}). The horizontal approach would have stopped the algorithm if a (sub-)optimal solution could not be found over 1,000,000 fitness evaluations.

2.1. Sensitivity Analysis

In this subsection, the results of the first part of the experiment are presented showing the importance of SN, D, and 'limit' to the performance of ABC. Sensitivity analysis [33] is shown only for f_1 due to its similarity of results on $f_2 - f_5$. The other reason is that the emphasis of this study was given to the second part of the experiment, where different settings of 'limit' were statistically analysed, and in the third part where the results were analysed using a novel method for pairwise comparison, CRS4EAs.

The aim of sensitivity analysis was to show the robustness of a meta-heuristic algorithm against different settings of control parameters. By performing a sensitivity analysis, we could find those control parameters (if any) that are very sensitive, as well of those (if any) which are very robust. In the former case, a proper setting of a control parameter is crucial for obtaining good performance of a meta-heuristic algorithm, whilst in the latter case, similar performance can be achieved regardless of the different settings of such non-sensitive control parameters. An obvious question may arise as to why the dimensionality of problem D was included within our sensitivity analysis as a factor as it is not a control parameter but D should be considered as part of the optimisation problem? As the formula 'limit' = $n_e * D$ [26] suggested a particular correlation between 'limit' and two other variables: population size and dimensionality of a problem, such a correlation should probably be indicated by sensitivity analysis as well. If at least one of these factors is insensitive, then the suggested formula [26] might not capture the relationships amongst the factors too well. As shown in the continuation, this was indeed the case.

In Tables 1(a) and 1(b), the experimental results of f_1 are presented when using the vertical approach with 100,000 and 250,000 maximum number of fitness evaluations (MaxFEs in the tables appeared later), respectively. In Tables 2(a) and 2(b), the experimental results of f_1 are presented when using the horizontal approach in order to find a (sub-)optimal solution at 10^{-6} and at 10^{-12}, respectively. The best results are highlighted by a light grey colour.

The difference between Tables 1(a) and 1(b) shows that 250,000 fitness evaluations were almost always enough for f_1 to find the exact solution; except for high dimensions $D = 30$ and $D = 50$, or when 'limit' = 0 (high exploration) and 'limit' = ∞ (no exploration). The Karaboga's setting of 'limit' L_k was always the better performer regarding the mean value when 250,000 fitness evaluations were available (Table 1(b)), whilst when only 100,000 were available (Table 1(a)), the

Algoritm 1: The pseudo-code of algorithm *ABC*

```
Data: Set the control parameters of the ABC algorithm
SN: Population size
limit: Maximum number of trials for abandoning a source
MFE: Maximum number of fitness evaluations
begin
    //Initialization;
    num_eval ⟵ 0 ;
    for s = 1 to SN do
        X(s) ⟵ random solution by Eq. 1 [26];
        f_s ⟵ f(X(s));
        trial(s) ⟵ 0;
        num_eval + + ;
    end
    repeat
        //Employed Bees Phase;
        for s = 1 to SN do
            x' ⟵ a new solution produced by Eq. 2 [26];
            f(x') ⟵ evaluate new solution;
            num_eval + +;
            if f(x') < f_s then
                X(s) ⟵ x'; f_s ⟵ f(x'); trial(s) ⟵ 0;
            else
                trial(s) ← trial(s) + 1;
            end
            if num_eval == MFE then
                Memorize the best solution achieved so far and exit main repeat;
            end
        end
        Calculate the probability values p_i for the solutions using fitness values by Eqs. 3 and 4 [26];
        //Onlooker bee phase;
        s ⟵ 1; t ⟵ 1 ;
        repeat
            r ⟵ rand(0, 1);
            if r < p(s) then
                t ⟵ t + 1;
                x' ⟵ a new solution produced by Eq. 2 [26];
                f(x') ⟵ evaluate new solution;
                num_eval + +;
                if f(x') < f_s then
                    X(s) ⟵ x'; f_s ⟵ f(x'); trial(s) ⟵ 0;
                else
                    trial(s) ⟵ trial(s) + 1;
                end
                if num_eval == MFE then
                    Memorize the best solution achieved so far and exit main repeat;
                end
            end
            s ⟵ (s mod SN) + 1;
        until t = SN;
        //Scout Bee Phase;
        mi ⟵ {s : trial(s) = max(trial)};
        if trial(mi) >= limit then
            X(mi) ⟵ random solution by Eq. 1 [26];
            f_mi ⟵ f(X(mi));
            num_eval + + ;
            trial(mi) ⟵ 0;
            if num_eval == MFE then
                Memorize the best solution achieved so far and exit main repeat;
            end
        end
        Memorize the best solution achieved so far;
    until num_eval = MFE;
end
```

Table 1

Mean values (Mean) and standard deviation values (SD) for the vertical approach to problem f_1

		L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24	Mean	0.00E+00	2.04E-15	0.00E+00	0.00E+00	5.55E-19	0.00E+00	5.55E-19	1.67E-18	2.22E-18	3.89E-18
D=2	SD	0.00E+00	5.26E-15	0.00E+00	0.00E+00	5.52E-18	0.00E+00	5.52E-18	9.47E-18	1.09E-17	1.42E-17
SN=24	Mean	6.38E-14	7.39E-06	0.00E+00	0.00E+00	5.55E-19	0.00E+00	3.33E-18	1.67E-18	2.22E-18	6.11E-18
D=5	SD	5.19E-13	9.29E-06	0.00E+00	0.00E+00	5.52E-18	0.00E+00	1.53E-17	9.47E-18	1.09E-17	4.50E-17
SN=24	Mean	4.32E-10	5.50E-04	3.03E-07	0.00E+00	3.01E-16	5.00E-14	4.61E-14	2.20E-14	1.19E-13	1.17E-11
D=10	SD	2.09E-09	6.51E-04	1.14E-06	0.00E+00	2.47E-15	4.70E-13	3.10E-13	1.61E-13	1.04E-12	1.16E-10
SN=24	Mean	9.89E-06	1.49E-02	4.84E-03	1.16E-04	2.76E-06	1.99E-06	1.15E-06	1.11E-06	1.43E-06	4.89E-06
D=30	SD	3.19E-05	1.81E-02	5.66E-03	2.45E-04	8.12E-06	1.27E-05	3.50E-06	3.23E-06	5.46E-06	2.42E-05
SN=24	Mean	2.05E-04	3.76E-02	2.44E-02	4.68E-03	4.52E-04	1.25E-04	1.00E-04	9.53E-05	1.25E-04	9.00E-05
D=50	SD	4.85E-04	4.40E-02	2.76E-02	9.23E-03	7.82E-04	3.77E-04	2.69E-04	2.43E-04	3.22E-04	2.11E-04
SN=50	Mean	0.00E+00	2.62E-15	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.55E-19	1.67E-18	0.00E+00	1.67E-18
D=2	SD	0.00E+00	8.06E-15	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.52E-18	9.47E-18	0.00E+00	9.47E-18
SN=50	Mean	0.00E+00	6.20E-06	0.00E+00	0.00E+00	5.55E-19	2.78E-18	1.11E-18	1.11E-18	3.33E-18	3.89E-18
D=5	SD	0.00E+00	6.95E-06	0.00E+00	0.00E+00	5.52E-18	1.21E-17	7.77E-18	7.77E-18	1.32E-17	1.42E-17
SN=50	Mean	1.11E-18	5.01E-04	1.64E-07	1.11E-18	3.85E-15	4.06E-14	4.75E-13	1.74E-10	1.39E-12	8.82E-13
D=10	SD	7.77E-18	5.65E-04	5.67E-07	7.77E-18	3.61E-14	2.47E-13	3.25E-12	1.51E-09	1.36E-11	8.64E-12
SN=50	Mean	1.43E-06	1.59E-02	5.94E-03	9.03E-05	2.32E-06	1.43E-06	1.86E-06	7.94E-07	8.64E-07	2.99E-06
D=30	SD	5.48E-06	1.51E-02	7.19E-03	2.33E-04	8.21E-06	5.48E-06	7.02E-06	1.94E-06	2.39E-06	1.84E-05
SN=50	Mean	1.11E-04	3.05E-02	2.59E-02	4.43E-03	3.19E-04	2.28E-04	1.16E-04	1.11E-04	1.86E-04	1.32E-04
D=50	SD	2.57E-04	3.05E-02	2.91E-02	6.48E-03	7.56E-04	8.67E-04	2.80E-04	2.57E-04	7.94E-04	4.33E-04
SN=100	Mean	0.00E+00	5.90E-15	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.11E-18	2.22E-18	1.67E-18	2.22E-18
D=2	SD	0.00E+00	2.55E-14	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.77E-18	1.09E-17	9.47E-18	1.09E-17
SN=100	Mean	5.55E-19	6.42E-06	0.00E+00	5.55E-19	0.00E+00	1.67E-18	1.11E-18	3.33E-18	1.67E-18	0.00E+00
D=5	SD	5.52E-18	7.15E-06	0.00E+00	5.52E-18	0.00E+00	9.47E-18	7.77E-18	1.32E-17	9.47E-18	0.00E+00
SN=100	Mean	3.83E-17	3.47E-04	2.60E-07	5.55E-18	3.83E-17	4.25E-14	2.02E-13	2.17E-14	5.95E-12	4.13E-15
D=10	SD	1.75E-16	4.83E-04	9.61E-07	2.00E-17	1.75E-16	2.68E-13	2.00E-12	1.46E-13	5.88E-11	2.61E-14
SN=100	Mean	1.08E-06	1.74E-02	5.28E-03	1.35E-04	4.54E-06	1.23E-06	5.61E-06	9.41E-07	1.08E-06	2.62E-06
D=30	SD	3.98E-06	2.37E-02	6.48E-03	2.70E-04	2.83E-05	4.67E-06	4.13E-05	4.78E-06	3.98E-06	1.33E-05
SN=100	Mean	1.20E-04	4.06E-02	3.46E-02	2.75E-03	3.06E-04	1.38E-04	1.36E-04	8.02E-05	1.35E-04	1.39E-04
D=50	SD	2.37E-04	4.63E-02	3.74E-02	3.41E-03	6.53E-04	2.49E-04	5.53E-04	1.84E-04	3.73E-04	4.21E-04

(a) Vertical approach, f_1, MaxFEs = 100,000

		L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24	Mean	0.00E+00	3.01E-16	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.33E-18
D=2	SD	0.00E+00	6.04E-16	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.32E-17
SN=24	Mean	0.00E+00	1.64E-06	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.44E-18
D=5	SD	0.00E+00	2.21E-06	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.51E-17
SN=24	Mean	0.00E+00	1.54E-04	6.95E-09	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.55E-19	0.00E+00	3.33E-18
D=10	SD	0.00E+00	1.59E-04	5.16E-08	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.52E-18	0.00E+00	1.32E-17
SN=24	Mean	0.00E+00	5.63E-03	1.53E-03	4.29E-06	8.73E-11	3.43E-16	1.43E-14	1.14E-14	2.99E-14	1.53E-11
D=30	SD	0.00E+00	6.22E-03	2.06E-03	8.02E-06	4.18E-10	1.85E-15	1.07E-13	5.52E-14	1.72E-13	1.32E-10
SN=24	Mean	2.22E-18	1.36E-02	8.44E-03	7.80E-04	6.74E-06	9.00E-08	5.63E-09	1.69E-08	7.86E-09	1.27E-08
D=50	SD	1.09E-17	1.18E-02	1.02E-02	1.04E-03	2.99E-05	4.49E-07	2.46E-08	1.08E-07	4.87E-08	6.25E-08
SN=50	Mean	0.00E+00	2.04E-16	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.67E-18
D=2	SD	0.00E+00	2.97E-16	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.47E-18
SN=50	Mean	0.00E+00	1.81E-06	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.55E-19
D=5	SD	0.00E+00	2.45E-06	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.52E-18
SN=50	Mean	0.00E+00	1.75E-04	5.71E-09	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.55E-19	0.00E+00	1.11E-18
D=10	SD	0.00E+00	2.10E-04	2.59E-08	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.52E-18	0.00E+00	7.77E-18
SN=50	Mean	0.00E+00	4.34E-03	1.68E-03	5.42E-06	3.98E-10	0.00E+00	7.29E-15	2.37E-13	3.80E-14	4.53E-12
D=30	SD	0.00E+00	4.03E-03	2.78E-03	1.16E-05	3.92E-09	0.00E+00	5.93E-14	2.32E-12	1.79E-13	2.58E-11
SN=50	Mean	0.00E+00	1.26E-02	8.24E-03	7.57E-04	4.65E-06	2.21E-07	1.12E-08	0.00E+00	4.65E-09	3.42E-07
D=50	SD	0.00E+00	1.27E-02	1.11E-02	1.38E-03	1.38E-05	1.37E-06	6.92E-08	0.00E+00	1.87E-08	2.91E-06
SN=100	Mean	0.00E+00	3.42E-16	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.78E-18
D=2	SD	0.00E+00	8.36E-16	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.21E-17
SN=100	Mean	0.00E+00	1.37E-06	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.55E-19	5.55E-19	0.00E+00	2.22E-18
D=5	SD	0.00E+00	1.60E-06	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.52E-18	5.52E-18	0.00E+00	1.09E-17
SN=100	Mean	0.00E+00	1.83E-04	3.00E-09	0.00E+00	0.00E+00	0.00E+00	5.55E-19	5.55E-19	1.11E-18	2.22E-18
D=10	SD	0.00E+00	2.12E-04	7.24E-09	0.00E+00	0.00E+00	0.00E+00	5.52E-18	5.52E-18	7.77E-18	1.09E-17
SN=100	Mean	0.00E+00	5.73E-03	1.92E-03	6.59E-06	8.08E-13	1.28E-14	2.86E-15	5.96E-14	0.00E+00	1.49E-10
D=30	SD	0.00E+00	5.91E-03	2.18E-03	1.30E-05	3.25E-12	7.89E-14	1.88E-14	4.02E-13	0.00E+00	1.48E-09
SN=100	Mean	0.00E+00	1.26E-02	9.33E-03	8.36E-04	6.16E-06	3.18E-08	3.96E-09	1.44E-09	3.38E-09	1.32E-08
D=50	SD	0.00E+00	1.22E-02	1.06E-02	1.09E-03	2.88E-05	1.16E-07	1.66E-08	5.35E-09	2.32E-08	5.21E-08

(b) Vertical approach, f_1, MaxFEs = 250,000

Table 2

Mean values (Mean) and standard deviation values (SD) for the horizontal approach to problem f_1

		L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24	Mean	6.05E+03	6.15E+03	5.72E+03	5.50E+03	5.42E+03	5.65E+03	5.31E+03	5.43E+03	5.95E+03	5.95E+03
D=2	SD	1.89E+03	1.34E+03	1.58E+03	1.29E+03	1.16E+03	1.35E+03	1.20E+03	1.17E+03	1.56E+03	1.45E+03
SN=24	Mean	1.73E+04	3.25E+05	1.39E+04	1.38E+04	1.41E+04	1.47E+04	1.40E+04	1.35E+04	1.39E+04	1.36E+04
D=5	SD	5.94E+03	3.00E+05	3.39E+03	3.39E+03	3.70E+03	3.66E+03	4.62E+03	3.73E+03	3.65E+03	3.26E+03
SN=24	Mean	3.37E+04	9.80E+05	5.01E+04	2.70E+04	2.72E+04	2.70E+04	2.65E+04	2.72E+04	2.75E+04	2.75E+04
D=10	SD	1.41E+04	1.13E+05	3.03E+04	6.36E+03	6.55E+03	8.60E+03	6.16E+03	7.08E+03	6.29E+03	7.11E+03
SN=24	Mean	1.03E+05	1.00E+06	1.00E+06	3.48E+05	8.28E+04	8.22E+04	8.22E+04	8.69E+04	8.16E+04	7.88E+04
D=30	SD	3.55E+04	0.00E+00	0.00E+00	2.58E+05	2.10E+04	2.02E+04	1.82E+04	2.23E+04	1.84E+04	1.88E+04
SN=24	Mean	1.72E+05	1.00E+06	1.00E+06	1.00E+06	2.53E+05	1.47E+05	1.32E+05	1.38E+05	1.35E+05	1.39E+05
D=50	SD	6.34E+04	0.00E+00	0.00E+00	0.00E+00	1.67E+05	3.52E+04	3.29E+04	3.29E+04	3.25E+04	3.11E+04
SN=50	Mean	5.35E+03	6.17E+03	5.64E+03	5.49E+03	5.57E+03	5.87E+03	5.60E+03	5.61E+03	5.86E+03	5.88E+03
D=2	SD	1.18E+03	1.44E+03	1.42E+03	1.39E+03	1.62E+03	1.69E+03	1.44E+03	1.30E+03	1.47E+03	1.42E+03
SN=50	Mean	1.33E+04	3.41E+05	1.35E+04	1.38E+04	1.43E+04	1.31E+04	1.41E+04	1.47E+04	1.35E+04	1.40E+04
D=5	SD	3.14E+03	3.15E+05	3.57E+03	3.10E+03	3.29E+03	3.05E+03	4.11E+03	3.92E+03	3.15E+03	3.53E+03
SN=50	Mean	2.63E+04	9.87E+05	5.21E+04	2.63E+04	2.77E+04	2.82E+04	2.80E+04	2.72E+04	2.80E+04	2.75E+04
D=10	SD	5.82E+03	9.13E+04	2.94E+04	5.82E+03	6.42E+03	6.42E+03	7.06E+03	7.66E+03	6.86E+03	7.00E+03
SN=50	Mean	7.89E+04	1.00E+06	9.90E+05	4.08E+05	8.48E+04	7.89E+04	8.17E+04	8.10E+04	8.26E+04	7.95E+04
D=30	SD	1.49E+04	0.00E+00	9.45E+04	2.84E+05	1.96E+04	1.49E+04	1.94E+04	1.99E+04	2.24E+04	2.34E+04
SN=50	Mean	1.35E+05	1.00E+06	1.00E+06	1.00E+06	2.47E+05	1.47E+05	1.40E+05	1.35E+05	1.42E+05	1.42E+05
D=50	SD	2.95E+04	0.00E+00	0.00E+00	0.00E+00	1.45E+05	4.68E+04	3.94E+04	2.95E+04	3.32E+04	3.50E+04
SN=100	Mean	5.67E+03	5.94E+03	5.67E+03	5.77E+03	5.62E+03	5.52E+03	5.65E+03	5.57E+03	5.57E+03	5.58E+03
D=2	SD	1.21E+03	1.50E+03	1.21E+03	1.38E+03	1.26E+03	1.31E+03	1.55E+03	1.66E+03	1.36E+03	1.54E+03
SN=100	Mean	1.40E+04	2.93E+05	1.38E+04	1.40E+04	1.30E+04	1.40E+04	1.44E+04	1.38E+04	1.42E+04	1.37E+04
D=5	SD	3.80E+03	2.80E+05	3.29E+03	3.80E+03	2.83E+03	3.45E+03	3.43E+03	3.29E+03	3.30E+03	2.74E+03
SN=100	Mean	2.62E+04	9.88E+05	4.80E+04	2.75E+04	2.62E+04	2.74E+04	2.94E+04	2.71E+04	2.75E+04	2.77E+04
D=10	SD	6.18E+03	8.88E+04	2.53E+04	6.19E+03	6.18E+03	6.86E+03	7.11E+03	8.25E+03	6.58E+03	7.14E+03
SN=100	Mean	8.50E+04	1.00E+06	1.00E+06	3.80E+05	8.87E+04	8.22E+04	8.14E+04	8.45E+04	8.50E+04	8.23E+04
D=30	SD	2.12E+04	0.00E+00	0.00E+00	2.66E+05	1.93E+04	1.77E+04	1.82E+04	1.99E+04	2.12E+04	1.99E+04
SN=100	Mean	1.40E+05	1.00E+06	1.00E+06	9.81E+05	2.71E+05	1.42E+05	1.37E+05	1.35E+05	1.39E+05	1.39E+05
D=50	SD	3.12E+04	0.00E+00	0.00E+00	1.07E+05	1.82E+05	3.52E+04	3.45E+04	3.08E+04	3.43E+04	3.33E+04

(a) Horizontal approach, f_1, (sub-)optimal solution 10^{-6}

		L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24	Mean	1.28E+04	1.83E+04	1.23E+04	1.28E+04	1.26E+04	1.23E+04	1.24E+04	1.34E+04	1.24E+04	1.23E+04
D=2	SD	2.91E+03	7.27E+03	2.39E+03	3.04E+03	2.67E+03	2.22E+03	2.59E+03	2.98E+03	2.92E+03	2.27E+03
SN=24	Mean	4.84E+04	1.00E+06	2.82E+04	3.08E+04	3.20E+04	3.08E+04	3.14E+04	3.08E+04	3.17E+04	3.05E+04
D=5	SD	2.47E+04	0.00E+00	4.69E+03	5.56E+03	6.22E+03	6.06E+03	5.90E+03	6.63E+03	8.98E+03	5.53E+03
SN=24	Mean	1.02E+05	1.00E+06	5.74E+05	5.51E+04	6.10E+04	6.20E+04	6.00E+04	6.31E+04	6.40E+04	6.11E+04
D=10	SD	5.69E+04	0.00E+00	3.26E+05	9.39E+03	1.04E+04	1.37E+04	1.14E+04	1.40E+04	1.26E+04	1.30E+04
SN=24	Mean	3.50E+05	1.00E+06	1.00E+06	1.00E+06	1.85E+05	1.69E+05	1.73E+05	1.76E+05	1.79E+05	1.94E+05
D=30	SD	1.89E+05	0.00E+00	0.00E+00	0.00E+00	4.30E+04	2.19E+04	2.91E+04	2.72E+04	2.74E+04	4.12E+04
SN=24	Mean	5.10E+05	1.00E+06	1.00E+06	1.00E+06	9.41E+05	3.30E+05	2.82E+05	2.74E+05	2.91E+05	3.21E+05
D=50	SD	2.27E+05	0.00E+00	0.00E+00	0.00E+00	1.78E+05	8.88E+04	5.17E+04	4.00E+04	4.72E+04	6.06E+04
SN=50	Mean	1.14E+04	1.67E+04	1.17E+04	1.24E+04	1.22E+04	1.27E+04	1.25E+04	1.23E+04	1.15E+04	1.23E+04
D=2	SD	1.66E+03	6.48E+03	1.77E+03	2.49E+03	2.20E+03	2.88E+03	2.53E+03	3.33E+03	2.43E+03	2.65E+03
SN=50	Mean	2.82E+04	1.00E+06	2.81E+04	3.03E+04	3.16E+04	3.16E+04	3.11E+04	3.11E+04	3.14E+04	3.07E+04
D=5	SD	4.41E+03	0.00E+00	5.15E+03	4.50E+03	6.55E+03	8.22E+03	7.29E+03	6.42E+03	7.15E+03	5.96E+03
SN=50	Mean	5.67E+04	1.00E+06	5.19E+05	5.67E+04	6.00E+04	6.20E+04	6.14E+04	5.87E+04	6.21E+04	6.47E+04
D=10	SD	8.71E+03	0.00E+00	3.48E+05	8.71E+03	1.06E+04	1.43E+04	1.30E+04	1.35E+04	1.50E+04	1.58E+04
SN=50	Mean	1.69E+05	1.00E+06	1.00E+06	1.00E+06	1.74E+05	1.69E+05	1.71E+05	1.74E+05	1.78E+05	1.90E+05
D=30	SD	2.86E+04	0.00E+00	0.00E+00	0.00E+00	3.44E+04	2.86E+04	2.87E+04	3.26E+04	2.97E+04	4.69E+04
SN=50	Mean	2.85E+05	1.00E+06	1.00E+06	1.00E+06	9.28E+05	3.21E+05	2.88E+05	2.85E+05	2.82E+05	3.12E+05
D=50	SD	3.97E+04	0.00E+00	0.00E+00	0.00E+00	1.92E+05	8.33E+04	4.99E+04	3.97E+04	4.60E+04	7.02E+04
SN=100	Mean	1.19E+04	1.74E+04	1.19E+04	1.29E+04	1.30E+04	1.27E+04	1.24E+04	1.26E+04	1.24E+04	1.26E+04
D=2	SD	2.37E+03	6.45E+03	2.37E+03	2.68E+03	2.74E+03	4.08E+03	2.37E+03	2.75E+03	3.04E+03	2.74E+03
SN=100	Mean	2.99E+04	1.00E+06	2.82E+04	2.99E+04	3.10E+04	3.09E+04	3.14E+04	3.21E+04	3.02E+04	3.08E+04
D=5	SD	5.50E+03	0.00E+00	3.84E+03	5.50E+03	6.88E+03	5.87E+03	6.25E+03	7.16E+03	6.18E+03	5.56E+03
SN=100	Mean	5.85E+04	1.00E+06	5.73E+05	5.62E+04	5.85E+04	6.16E+04	6.49E+04	6.17E+04	6.19E+04	6.04E+04
D=10	SD	1.05E+04	0.00E+00	3.40E+05	8.70E+03	1.05E+04	1.33E+04	1.39E+04	1.07E+04	1.19E+04	1.47E+04
SN=100	Mean	1.79E+05	1.00E+06	1.00E+06	1.00E+06	1.81E+05	1.68E+05	1.72E+05	1.73E+05	1.79E+05	1.91E+05
D=30	SD	3.56E+04	0.00E+00	0.00E+00	0.00E+00	4.51E+04	2.62E+04	2.61E+04	3.49E+04	3.56E+04	5.18E+04
SN=100	Mean	3.02E+05	1.00E+06	1.00E+06	1.00E+06	9.38E+05	3.08E+05	2.84E+05	2.88E+05	2.86E+05	3.17E+05
D=50	SD	5.45E+04	0.00E+00	0.00E+00	0.00E+00	1.70E+05	6.99E+04	5.00E+04	4.12E+04	5.08E+04	9.54E+04

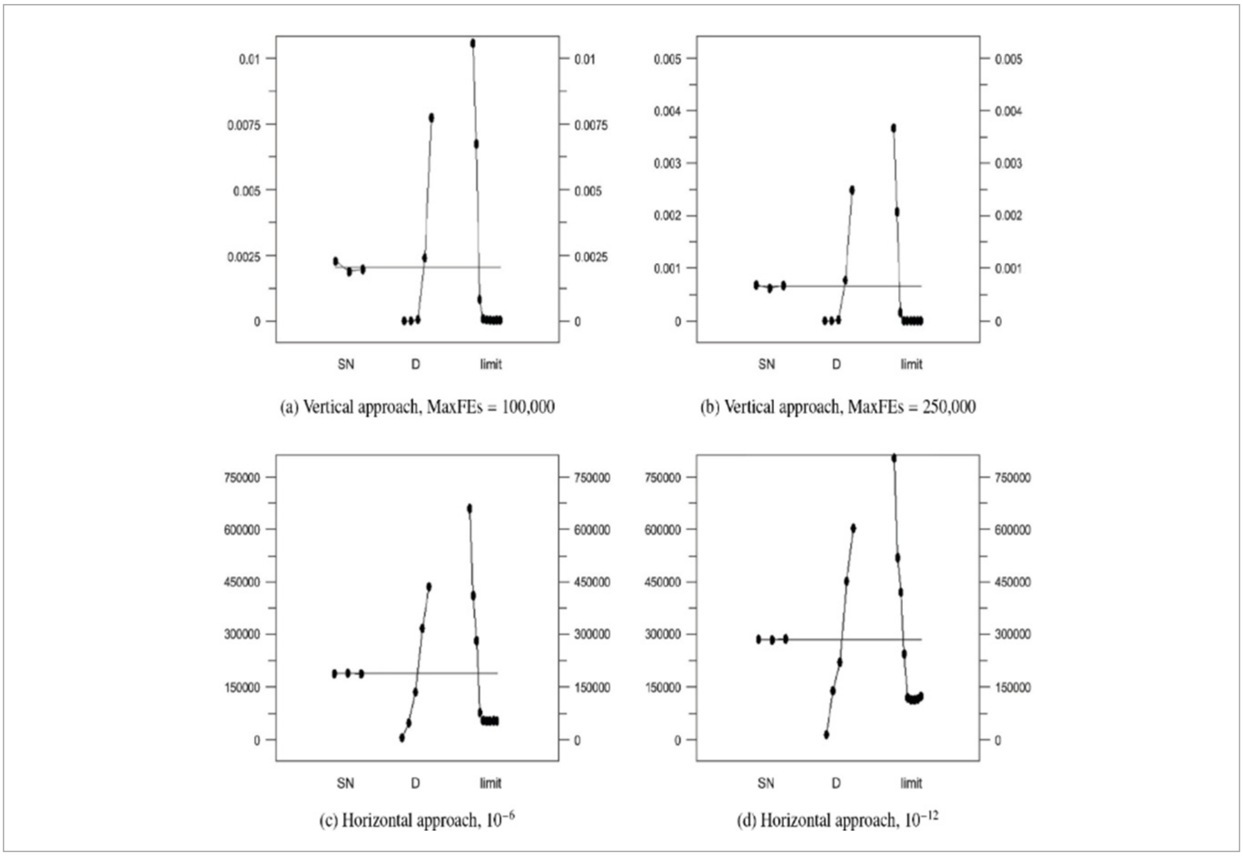
(b) Horizontal approach, f_1, (sub-)optimal solution 10^{-12}

other fixed 'limit' values performed better; in most cases (8 out of 15), the better performing value being $L_{250.}$ A meticulous reader may notice that those problems with higher dimensions always required more fitness evaluations in order to reach a sub-optimal solution (Tables 2(a) and 2(b)), which was an expected property. Less expectedly, the population size did not have a big influence on this property. For example, to reach 10^{-6}, the following average numbers of fitness evaluations were needed at L_{1000}: 5.65E+03 (SN = 100, D = 2), 5.60E+03 (SN = 50, D = 2), and 5.31E+03 (SN = 24, D = 2), whilst 8.14E+04 (SN = 100, D = 30), 8.17E+04 (SN = 50, D = 30), and 8.22E+04 (SN = 24, D = 30). In order to reach 10^{-12}, twice as many fitness evaluations were roughly needed compared to 10^{-6}. Again, an increase in the number of fitness evaluations was expected, although the magnitude of the increase was hard to predict. From these tables, as well as based on the results for f_2 to f_5 (not shown in this paper), we noticed that setting the 'limit' was a difficult task. It can be observed that setting the control parameter 'limit' using the formula from [26] obtained good results only for the vertical approach with 250,000 fitness evaluations. If only one experiment were applied, the wrong conclusions could be drawn. In other cases, a clear winner was hard to discover (if it existed at all). However, we could not define a rule for setting 'limit' based on these results as the statistical significance had not yet been examined.

Figure 1 shows the sensitivity analyses for the (a) vertical approach with a maximum number of 100,000 fitness evaluations; (b) vertical approach with a maximum number of 250,000 fitness evaluations; (c) horizontal approach with (sub-)optimal solution 10^{-6}; (d)

Figure 1

Sensitivity analyses of f_1 using horizontal and vertical approaches



horizontal approach with (sub-)optimal solution 10^{-12}. The X-axis represents the parameter settings of *SN* (3 settings: 100, 50, 24 from left to right), *D* (5 settings: 2, 5, 10, 30, 50 from left to right), and 'limit' (9 settings: 0, 100, 250, 500, 750, 1000, 1250, 1500, ∞ from left to right). The Y-axis represents the sensitivities of three parameters in terms of the average of better solutions found and the average number of fitness evaluations needed to reach (sub-)optimal solution amongst 100 runs for vertical and horizontal approaches, respectively.

As can be observed, *SN* had minimal effect. Changing *SN* amongst 24, 50, and 100 did not make too much difference. Conversely, 'limit' and *D* played important roles when determining the performance of the ABC algorithm. All the figures indicated that 'limit' was more sensitive than *D* because, in terms of the Y-axis, the range of 'limit' was longer than *D*. Conversely, *D* is not the ABC control parameter but the property of the problem. Hence, amongst ABC control parameters the size of the population (*SN*) was much more robust than control parameter 'limit', indicating that much more emphasis should be given to properly setting it.

All four graphs in Figure 1 show remarkable similarities, and although they show that ABC is very sensitive to 'limit', an important question is: "*Are differences in setting 'limit' also statistically significant*?" Hence, we performed NHST and CRS4EAs analyses on the obtained results. Furthermore, all four graphs in Figure 1 clearly indicate that there exists no linear relationship between 'limit', population size *SN*, and dimension *D*, as suggested by formula [26].

2.2. Null Hypothesis Significance Testing

Karaboga's suggestion of 'limit' value $L_k = n_e{*}D = (SN/2){*}D$ [26] was compared to the set of fixed 'limit' values 'limit' = {0, 100, 250, 500, 750, 1000, 1250, 1500, ∞ } for SN = {24, 50, 100} and D = {2, 5, 10, 30, 50}. The whole experiment was divided into four sections (see Table 3). In the first two sections, we measured the quality of a solution reached by a pre-defined number of fitness evaluations (100,000 and 250,000), which is also known as the vertical or 'the fixed-cost' approach. In the other two sections, we measured the number of fitness evaluations needed to find a (sub-)optimal solution (10^{-6} and 10^{-12}), which is also known as the horizontal or 'the fixed-target' approach. The horizontal approach would have stopped the algorithm if a (sub-)optimal solution could not be found over 1,000,000 fitness evaluations. The number of independent runs was in all cases n = 100. By using the vertical approach only the quality of the final solution was taken into consideration but not the convergence. Fast convergence is also a desirable property of meta-heuristic algorithms, which can be captured using the horizontal approach. Convergence can also be analysed by using the vertical approach and additional Page's trend statistics, as shown in [10].

Table 3

Description of all four parts of the experiment

Section	Approach	Termination condition	Measurement
Experiment 1	vertical	number of fitness evaluations is 100,000	quality of solution
Experiment 2	vertical	number of fitness evaluations is 250,000	quality of solution
Experiment 3	horizontal	(sub-)optimal solution 10^{-6} and maximum of fitness evaluations is 1,000,000	number of fitness evaluations
Experiment 4	horizontal	(sub-)optimal solution 10^{-12} and maximum of fitness evaluations is 1,000,000	number of fitness evaluations

The obtained results (readers can find the raw data in [47]) were analysed using Null Hypothesis Significance Testing [41] for multiple comparisons. The non-parametric Wilcoxon's test [51] was used because the distribution of the data was unknown. In Wilcoxon's test, the results L_k obtained over n=100 runs for particular settings *SN*, *D* and problem f_i were pairwisely compared to the results of another fixed 'limit' value obtained over n = 100 runs for the same settings *SN*, *D* and problem f_i. The differences between the corresponding outcomes were ranked and the p value was calculated regarding to the sum of positive ranks (whenever L_k was better) and the sum of negative ranks (whenever L_k was worse). As several multiple Wilcoxon's tests were conducted on the same data and we wished to control the Type-I-Error, the post-hoc procedure known as the Holm test [19] was applied to each such comparison. In Holm's procedure, p values (there is k = 9 of them) obtained using Wilcoxon's test were ordered from the most significant (smallest p value, i.e., p_1) to the least significant (largest p value, i.e., p_k). p_1 was then compared to $\alpha/(k\text{-}1)$, and if it was smaller, the hypothesis (which states that L_k and 'limit' setting linked to p_1 are equal) was rejected. p_2 was compared to $\alpha/(k\ \text{-}2)$, p_3 to $\alpha/(k\ \text{-}3)$ and so on, until the value j for which p_j was not smaller than $\alpha/(k\text{-}j)$ was found. When such a j was found, the procedure stopped and all the remaining hypotheses were retained. All the results from the presented experiments were analysed under a significance level of α = 0.05. The results of these analyses are presented in Tables 5-21. In each table, Karaboga's 'limit' value L_k is compared to other fixed values of 'limit' ($L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$). L_k was either better (>), equal (=), or worse (<) than any fixed value of 'limit'. The decision whether L_k was better or worse depended on the sums of the positive and negative ranks from the Wilcoxon's test. Whenever the difference between the two values was significant under the Holm test, there is a star symbol (*) behind the 'limit' value. Whenever Karaboga's 'limit' value L_k was worse than at least one other fixed 'limit' value, the cell in the table is highlighted in light grey colour. Since L_k was different for different settings of *SN* and *D*, its values are displayed in Table 4.

Table 4

Values of 'limit' $L_k = (SN/2)*D$

	D=2	D=5	D=10	D=30	D=50	D=100	D=200	D=300
SN=24	$L_K = 24$	$L_K = 60$	$L_K = 120$	$L_K = 360$	$L_K = 600$	$L_K = 1200$	$L_K = 2400$	$L_K = 3600$
SN=50	$L_K = 50$	$L_K = 125$	$L_K = 250$	$L_K = 750$	$L_K = 1250$	$L_K = 2500$	$L_K = 5000$	$L_K = 7500$
SN=100	$L_K = 100$	$L_K = 250$	$L_K = 500$	$L_K = 1500$	$L_K = 2500$	$L_K = 5000$	$L_K = 10000$	$L_K = 15000$

2.2.1. Experiment 1: Vertical Approach with *MaxFEs* =100,00

Tables 5-9 show the differences found between L_k and the other 9 fixed 'limit' values on all 5 optimisation problems. While L_k was in most cases better than some fixed 'limit' values, there were some values for which L_k was worse, sometimes even significantly. In particular, for f_1: *SN* = 24 and *D* = 5 where L_k was significantly worse than $L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$; *SN* = 24 and D = 10 where L_k was significantly worse than $L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$; *SN* = 100 and *D* = 10 where L_k was significantly worse than L_{250}; *SN* = 24 and *D* = 30 where L_k was significantly worse than $L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$. For f_5: *SN* = 24 and *D* = 5 where L_k was significantly worse than $L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$; *SN* = 50 and *D* = 5 where L_k was significantly worse than $L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$; *SN* = 100 and *D* = 5 where L_k was significantly worse than L_{1500}; SN = 24 and *D* = 10 where L_k was significantly worse than $L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$. Hence, L_k had significantly better alternatives for problems f_1 and f_5, whilst for f_2, f_3, and f_4 the found differences were not significant. The differences between L_k and some other fixed 'limit' values for f_1 were significant when the population size *SN* equaled 24, and dimension *D* equaled 5, 10, or 30. So for this problem and small population size, L_k would not be a better choice. For f_5, L_k had significantly better alternatives whenever dimension *D* equaled 5, and for dimension *D* = 10 and small population size *SN* = 24. However, for all five problems, L_k had better alternatives (however, these alternatives were not significantly better) when dimension *D* was bigger (10, 30, or 50) and population size *SN* had different values.

Table 5

f_1, vertical approach, MaxFEs = 100,000, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	$L_0*, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{1000}, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$	$L_{100}, L_{250}, L_{750}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}$
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	L_0*	$L_K >$	$L_0*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{750}, L_{1000}, L_{1250}, L_{1500}$
	$L_K =$		$L_K =$	L_{100}, L_{250}	$L_K =$	
	$L_K <$	$L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$		$L_K <$	$L_{100}, L_{250}, L_{500}, L_\infty$
D=10	$L_K >$	$L_0*, L_{100}*$	$L_K >$	$L_0*, L_{100}*, L_{500}, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{750}*, L_{1000}, L_{1500}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$		$L_K <$	$L_{250}*$
D=30	$L_K >$	$L_0*, L_{100}*, L_{250}*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}, L_{1000}$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}, L_{1000}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$	$L_{1250}, L_{1500}, L_\infty$	$L_K <$	L_{750}, L_{1250}
D=50	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{1000}$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{750}, L_{1500}, L_\infty$	$L_K <$	$L_{1000}, L_{1250}, L_{1500}, L_\infty$

Table 6

f_2, vertical approach, MaxFEs = 100,000, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	$L_0*, L_{100}, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}*, L_{1000}, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	L_{250}	$L_K <$		$L_K <$	
D=10	$L_K >$	$L_0*, L_{250}, L_{1000}, L_{1250}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{250}, L_{750}, L_{1000}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{100}, L_{750}, L_\infty$	$L_K <$		$L_K <$	$L_{100}, L_{500}, L_{1500}$
D=30	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{250}, L_{500}, L_\infty$	$L_K >$	$L_0*, L_{250}, L_{750}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	L_{250}, L_{1500}	$L_K <$	$L_{100}, L_{1000}, L_{1250}, L_{1500}$	$L_K <$	L_{500}, L_{1000}
D=50	$L_K >$	L_0*, L_{1000}	$L_K >$	$L_0*, L_{100}, L_{250}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{1000}, L_{1500}$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	L_{500}	$L_K <$	$L_{500}, L_{750}, L_{1250}, L_\infty$

Table 7

f_3, vertical approach, MaxFEs = 100,000, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	$L_0*, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{750}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{750}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{1000}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{1000}$
	$L_K <$		$L_K <$		$L_K <$	
D=10	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	L_0*, L_{500}	$L_K >$	L_0*, L_{750}
	$L_K =$		$L_K =$	$L_{250}, L_{750}, L_{1000}, L_{1500}$	$L_K =$	L_{500}
	$L_K <$		$L_K <$	$L_{100}, L_{1250}, L_\infty$	$L_K <$	$L_{100}, L_{250}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=30	$L_K >$	L_0*, L_{100}, L_{1500}	$L_K >$	$L_0*, L_{250}, L_{500}, L_{1000}, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{750}*, L_{1000}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$	$L_K <$	L_{100}, L_{1500}	$L_K <$	L_{500}
D=50	$L_K >$	$L_0*, L_{100}, L_{750}, L_{1250}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{250}, L_{500}, L_{1000}, L_\infty$	$L_K <$		$L_K <$	L_{750}

Table 8

f_4, vertical approach, MaxFEs = 100,000, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=10	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=30	$L_K >$	$L_0*, L_{100}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	L_0*, L_{1000}, L_{1500}	$L_K >$	$L_0*, L_{100}, L_{250}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	L_{250}, L_{750}	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1250}, L_\infty$	$L_K <$	$L_{500}, L_{750}, L_{1000}, L_{1250}$
D=50	$L_K >$	L_0*	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_\infty$	$L_K >$	$L_0*, L_{250}, L_{500}, L_{750}, L_{1250}, L_{1500}$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	L_{1500}	$L_K <$	$L_{100}, L_{1000}, L_\infty$

Table 9

f_5, vertical approach, MaxFEs = 100,000, NHST

		SN=24		SN=50		SN=100
	$L_K >$	L_0*, L_∞	$L_K >$	L_0*, L_∞	$L_K >$	L_0*, L_∞
D=2	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$	$L_0*, L_\infty*$	$L_K >$	L_0*, L_∞	$L_K >$	$L_0*, L_{1000}, L_{1250}, L_\infty$
D=5	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1500}$
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$	$L_0*, L_{100}*, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=10	$L_K =$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}$	$L_K =$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}$	$L_K =$	$L_{250}, L_{500}, L_{750}$
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_\infty*$
D=30	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
D=50	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$		$L_K <$		$L_K <$	

Table 10

f_1, vertical approach, MaxFEs = 250,000, NHST

		SN=24		SN=50		SN=100
	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
D=2	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
D=5	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
D=10	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_\infty*$
D=30	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
D=50	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$		$L_K <$		$L_K <$	

Table 11

f_2, vertical approach, MaxFEs = 250,000, NHST

		SN=24		SN=50		SN=100
	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
D=2	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$	L_0*, L_∞	$L_K >$	L_0*, L_∞	$L_K >$	L_0*, L_∞
D=5	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
D=10	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_\infty*$
D=30	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
D=50	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$		$L_K <$		$L_K <$	

Table 12

f_3, vertical approach, MaxFEs = 250,000, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=10	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=30	$L_K >$	$L_0*, L_{100}*$	$L_K >$	$L_0*, L_{100}*$	$L_K >$	$L_0*, L_{100}*$
	$L_K =$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=50	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$		$L_K <$		$L_K <$	

Table 13

f_4, vertical approach, MaxFEs = 250,000, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	$L_0*, L_{100}*, L_{250}, L_{500}, L_{1000}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	$L_{750}, L_{1250}, L_{1500}$	$L_K <$		$L_K <$	
D=10	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}, L_{750}, L_{1250}, L_{1000}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	L_{1500}	$L_K <$		$L_K <$	
D=30	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_\infty*$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$		$L_K <$		$L_K <$	
D=50	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$		$L_K <$		$L_K <$	

Table 14

f_5, vertical approach, MaxFEs = 250,000, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty*$	$L_K >$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}*, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_{250}, L_{750}, L_{1000}, L_{1250}, L_{1500}*, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$		$L_K <$		$L_K <$	L_0, L_{500}
D=5	$L_K >$	L_0*	$L_K >$	$L_0*, L_{100}*$	$L_K >$	$L_0*, L_{100}*$
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	$L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$	$L_{250}, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$	$L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}*, L_\infty$
D=10	$L_K >$	L_0*, L_{100}	$L_K >$	$L_0*, L_{100}*, L_{500}$	$L_K >$	$L_0*, L_{100}*, L_{250}, L_{1000}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$	$L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	L_{750}, L_{1250}
D=30	$L_K >$	L_0*, L_{100}, L_{1250}	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{250}\ L_{500}, L_{750}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K <$		$L_K <$	L_{100}, L_{1000}
D=50	$L_K >$	L_0*, L_{1250}	$L_K >$	L_0*, L_{250}, L_{1500}	$L_K >$	L_0*
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{500}, L_{750}, L_{1000}, L_\infty$	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$

On the other hand, for f_1 and f_4, L_k was never the absolute best value, meaning that there was always a better or at least equal 'limit' value. For f_2, f_3, and f_5 that was not the case, as L_k was in some cases better than all 9 fixed 'limit' values. In particular, for f_2: SN = 24 and D = 2, SN = 50 and D = 2, and SN = 50 and D = 5. For f_3: SN = 24 and D = 10. For f_5: SN = 24 and D = 2, and SN = 50 and D = 2. This, however, does not mean that the 'limit' values that could be better than L_k for these problems and settings do not exist; it only means that L_k was better for these problems and settings than these 9 fixed 'limit' values.

2.2.2. Experiment 2: Vertical Approach with *MaxFEs* = 250,000

In this section, there were more fitness evaluations available, and L_k was almost always better than or equal to other settings. This means that when large enough fitness evaluations were available, L_k was an appropriate choice regardless of the population size and dimension of a problem (for the benchmark suite under investigation). Only for problem f_5, which is harder than the other four problems, L_k was in two cases worse than some other settings. Firstly for SN = 24 and D = 5 and secondly for SN = 24 and D = 10. These differences, however, were never significant. All the differences are shown in Tables 10-14. These findings suggest that setting a control parameter 'limit' depends on the available maximum number of fitness evaluations.

2.2.3. Experiment 3: Horizontal Approach – 10^{-6} – *MaxFEs* = 1,000,000

During the horizontal approach where we measured the number of function evaluations needed to reach a (sub-)optimal solution, L_k had the better alternatives in almost all cases. For f_1, these better alternatives were available for the small population size SN = 24 and for the bigger population size SN = 100, whereas for SN = 50, L_k was worse only for D = 5 and better for all other dimension values. For f_2, L_k was worse than all the population sizes and dimension values, except for SN = 50 and D = 10, SN = 100 and D = 10, SN = 100 and D = 50, and SN = 100 and D = 50. For f_3 and f_4, L_k always had a better alternative and was always worse than at least one other 'limit' value, regardless of the population size and dimension of a problem. Lastly, for f_5 and small dimension D =2 (and any population size values), L_k had better alternatives, whilst for other dimensions and population sizes all 'limit' values performed the same. This happened due to the fact that none of these 'limit' values had found the (sub-)optimal solution 10^{-6} after 1,000,000 fitness evaluations. For D = 2, some 'limit' values found (sub-)optimal solutions during some runs, and therefore they performed better than L_k. Whilst there were a lot of differences found between L_k and other 'limit' values, these differences were rarely significant. There were only two problems for which L_k was significantly worse than some other 'limit' values. First was f_1 when L_k was significantly worse for small population size SN =24 for all dimensions. The other was f_4 when L_k was significantly worse than all other 'limit' values except L_0 for small population size SN =24 and small dimension D = 2. These differences are shown in Tables 15-19.

2.2.4. Experiment 4: Horizontal Approach 10^{-12} – *MaxFEs* = 1,000,000

In this section, the (sub-)optimal solution was set at 10^{-12}, which was a harder problem than finding (sub-) optimal solution 10^{-6}. Again, L_k almost always had a better alternative. For f_1 better 'limit' values were found for small population size SN = 24 regardless of dimension D and for the bigger population size SN =100 where the dimension was greater than D = 2, whilst for SN =50, L_k had better alternatives for small dimensions D = 2 and D =5 and bigger dimension D = 50. For f_2, L_k had better alternatives regardless of the population size and dimension values. The same went for f_3, except when the population size was SN =50 and dimension D = 5, where L_k was better than the other fixed 'limit' values. For f_4, L_k was better than the other fixed 'limit' values when dimension D = 30, whilst for other dimensions (regardless of population size value) there were better alternatives. For f_5, all 'limit' values were the same during performances, which was due to the fact that none of them found the (sub-) optimal solution 10^{-12} after 1,000,000 fitness evaluations. These differences are shown in Tables 20-24.

2.2.5. Experiment 5: Large Dimensions

In this section, the horizontal approach with (sub-)optimal solution set at 10^{-6} was repeated for larger dimensions, D = {100, 200, 300}, since we have expected that the recommended formulae might perform even worse for large dimensions (such very large optimisation problems are now common for some benchmark suites [32]). Again, fixed 'limit' values, L = {0, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10000, 11000, 12000, 13000, 14000, 15000, ∞} were compared to Karaboga's setting L_k. Found differences are shown in Tables 25-29. As in previous four experiments, this

Table 15

f_1, horizontal approach, 10^{-6}, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty*$	$L_K >$	L_0, L_{250}
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}*, L_{1250}, L_{1500}, L_\infty$	$L_K <$		$L_K <$	$L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=5	$L_K >$	L_0*	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}*, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{750}, L_{1000}, L_{1500}$
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	$L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$	L_{750}	$L_K <$	$L_{100}, L_{500}, L_{1250}, L_\infty$
D=10	$L_K >$	$L_0*, L_{100}*$	$L_K >$	$L_0*, L_{100}*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{250}, L_{750}, L_{1000}*, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$		$L_K <$	
D=30	$L_K >$	$L_0*, L_{100}*, L_{250}*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$		$L_K <$	$L_{750}, L_{1000}, L_{1250}, L_\infty$
D=50	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$		$L_K <$	$L_{1000}, L_{1250}, L_{1500}, L_\infty$

Table 16

f_2, horizontal approach, 10^{-6}, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	$L_0*, L_{250}, L_{750}, L_{1000}, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	L_0*, L_{250}
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{100}, L_{500}, L_{1500}$	$L_K <$	L_{100}, L_{250}	$L_K <$	$L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=5	$L_K >$	L_0*, L_{750}	$L_K >$	L_0*, L_{1000}	$L_K >$	$L_0*, L_{500}, L_{1000}, L_{1250}$
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{750}, L_{1500}, L_\infty$
D=10	$L_K >$	L_0*, L_{250}, L_{750}	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{100}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$		$L_K <$	
D=30	$L_K >$	L_0*, L_{500}, L_{750}	$L_K >$	$L_0*, L_{100}, L_{500}, L_{1000}, L_{1500}$	$L_K >$	L_0*, L_{750}, L_{1250}
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{100}, L_{250}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{250}, L_{1250}, L_\infty$	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1000}, L_\infty$
D=50	$L_K >$	L_0*, L_{1250}	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K <$	L_{750}	$L_K <$	

Table 17

f_3, horizontal approach, 10^{-6}, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	L_0*, L_{100}	$L_K >$	$L_0*,$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	L_{1250}	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=5	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1000}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K >$	$L_0*, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	$L_{250}, L_{1250}, L_\infty$	$L_K <$	L_{500}, L_∞	$L_K <$	$L_{100}, L_{500}, L_{750}$
D=10	$L_K >$	$L_0*, L_{100}, L_{750}, L_{1000}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1500}, L_\infty$	$L_K >$	L_0*, L_{250}, L_{1500}
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{250}, L_{500}, L_{1250}, L_\infty$	$L_K <$	L_{1000}, L_{1250}	$L_K <$	$L_{100}, L_{750}, L_{1000}, L_{1250}, L_\infty$
D=30	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_\infty$	$L_K >$	L_0*, L_{500}	$L_K >$	L_0*, L_{500}, L_{1250}
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	L_{1250}, L_{1500}	$L_K <$	$L_{100}, L_{250}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{250}, L_{750}, L_{1000}, L_\infty$
D=50	$L_K >$	$L_0*, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	L_0*, L_{100}, L_{500}	$L_K >$	$L_0*, L_{100}, L_{250}, L_{750}, L_{1000}$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	L_{100}	$L_K <$	$L_{250}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K <$	$L_{500}, L_{1250}, L_{1500}, L_\infty$

Table 18

f_4, horizontal approach, 10^{-6}, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*	$L_K >$	L_0*, L_{1000}	$L_K >$	$L_0*, L_{250}, L_{750}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{500}, L_{1000}, L_{1250}$
D=5	$L_K >$	$L_0*, L_{100}, L_{500}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{1000}, L_{1250}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	$L_{250}, L_{750}, L_{1000}, L_{1250}$	$L_K <$	$L_{500}, L_{750}, L_\infty$	$L_K <$	L_{1500}
D=10	$L_K >$	$L_0*, L_{100}, L_{250}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{250}, L_{750}, L_{1000}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{500}, L_{750}, L_{1250}$	$L_K <$	$L_{100}, L_{500}, L_{750}$	$L_K <$	L_{100}, L_{1250}
D=30	$L_K >$	L_0*, L_{750}	$L_K >$	$L_0*, L_{250}, L_{500}, L_{1000}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1250}$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{1250}, L_\infty$	$L_K <$	$L_{250}, L_{1000}, L_\infty$
D=50	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$	$L_K >$	L_0*, L_∞	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1250}$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	L_{1500}	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500},$	$L_K <$	$L_{750}, L_{1000}, L_{1500}, L_\infty$

Table 19

f_5, horizontal approach, 10^{-6}, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_0, L_{100}, L_{500}, L_{1250}, L_{1500},$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$	L_∞	$L_K <$	$L_{250}, L_{750}, L_{1000}, L_\infty$	$L_K <$	L_0, L_{1000}
D=5	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=10	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=30	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=50	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	

Table 20

f_1, horizontal approach, 10^{-12}, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*, L_{500}, L_{1250}	$L_K >$	$L_0*, L_{100}, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}, L_\infty*$	$L_K >$	$L_0*, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{100}, L_{250}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K <$	L_{1500}	$L_K <$	
D=5	$L_K >$	L_0*	$L_K >$	$L_0*, L_{250}, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	$L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$	L_{100}	$L_K <$	L_{100}
D=10	$L_K >$	$L_0*, L_{100}*$	$L_K >$	$L_0*, L_{100}*, L_{500}, L_{750}*, L_{1000}, L_{1250}, L_{1500}, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{750}, L_{1000}*, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$		$L_K <$	L_{250}
D=30	$L_K >$	$L_0*, L_{100}*, L_{250}*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$		$L_K <$	$L_{500}, L_{750}, L_{1000}, L_{1250}$
D=50	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}, L_\infty$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	L_{1500}	$L_K <$	$L_{1000}, L_{1250}, L_{1500}$

Table 21

f_2, horizontal approach, 10^{-12}, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*	$L_K >$	$L_0*, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$	$L_K >$	L_0*
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}*, L_{1000}*, L_{1250}, L_{1500}, L_\infty*$	$L_K <$	L_{100}, L_{1500}	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=5	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1250}, L_\infty$	$L_K >$	L_0*, L_{500}, L_{750}	$L_K >$	L_0*, L_{1000}, L_{1500}
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	L_{1000}, L_{1500}	$L_K <$	$L_{100}, L_{250}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{500}, L_{750}, L_{1250}, L_\infty$
D=10	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{250}, L_{750}, L_{1000}$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	L_{1000}	$L_K <$	L_{1250}	$L_K <$	$L_{100}, L_{1250}, L_{1500}, L_\infty$
D=30	$L_K >$	$L_0*, L_{250}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	L_0*	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{100}, L_{500}, L_{1250}$	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	L_{750}
D=50	$L_K >$	$L_0*, L_{100}, L_{250}, L_{1000}, L_{1250}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_\infty$	$L_K >$	L_0*, L_{250}
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{500}, L_{750}, L_\infty$	$L_K <$	L_{1500}	$L_K <$	$L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$

Table 22

f_3, horizontal approach, 10^{-12}, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	$L_0*, L_{100}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K >$	L_0*, L_{500}, L_{750}
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1250}$	$L_K <$	L_∞	$L_K <$	$L_{250}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=5	$L_K >$	$L_0*, L_{250}, L_{500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	L_0*, L_{1250}, L_∞
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	$L_{100}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K <$		$L_K <$	$L_{100}, L_{500}, L_{750}, L_{1000}, L_{1500}$
D=10	$L_K >$	$L_0*, L_{100}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K >$	L_0*, L_{500}	$L_K >$	$L_0*, L_{100}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{250}, L_{500}, L_\infty$	$L_K <$	$L_{100}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{250}, L_{750}, L_{1000}, L_{1500}$
D=30	$L_K >$	L_0*, L_{250}, L_{500}	$L_K >$	L_0*, L_{250}, L_{1250}	$L_K >$	$L_0*, L_{250}, L_{500}, L_{750}, L_{1250}$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{100}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{500}, L_{1000}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{1000}, L_\infty$
D=50	$L_K >$	$L_0*, L_{500}, L_{750}, L_\infty$	$L_K >$	L_0*, L_{750}	$L_K >$	$L_0*, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{100}, L_{250}, L_{1000}, L_{1250}, L_{1500}$	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1000}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{250}, L_{500}$

Table 23

f_4, horizontal approach, 10^{-12}, NHST

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$	L_{750}, L_{1500}	$L_K <$	L_{1250}
D=5	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}$	$L_K >$	$L_0*, L_{100}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	L_0*
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	L_{1500}, L_∞	$L_K <$	$L_{250}, L_{500}, L_{1250}$	$L_K <$	$L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=10	$L_K >$	$L_0*, L_{100}, L_{500}, L_{1500}$	$L_K >$	L_0*	$L_K >$	$L_0*, L_{100}, L_{750}, L_{1250}$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{250}, L_{750}, L_{1000}, L_{1250}, L_\infty$	$L_K <$	$L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{250}, L_{1000}, L_{1500}, L_\infty$
D=30	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}*, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$		$L_K <$		$L_K <$	
D=50	$L_K >$	$L_0*, L_{100}, L_{500}, L_{1000}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{500}, L_{1000}, L_{1500}$	$L_K >$	L_0*, L_{100}
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{250}, L_{750}, L_{1250}, L_\infty$	$L_K <$	$L_{250}, L_{750}, L_\infty$	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$

Table 24

f_5, horizontal approach, 10^{-12}, NHST

		SN=24		SN=50		SN=100
	$L_K >$		$L_K >$		$L_K >$	
D=2	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$		$L_K >$		$L_K >$	
D=5	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$		$L_K >$		$L_K >$	
D=10	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$		$L_K >$		$L_K >$	
D=30	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
	$L_K >$		$L_K >$		$L_K >$	
D=50	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	

Table 25

f_1, horizontal approach, 10^{-6}, large dimension, NHST

		SN=24		SN=50		SN=100
	$L_K >$	$L_0*, L_{1000}*$	$L_K >$	$L_0*, L_{1000}*, L_{8000}$	$L_K >$	$L_0*, L_{1000}*, L_{2000}, L_{7000}, L_{13000}, L_{15000}, L_\infty$
D=100	$L_K =$		$L_K =$		$L_K =$	L_{5000}
	$L_K <$	$L_{2000}*, L_{3000}*, L_{4000}*, L_{5000}*, L_{6000}*, L_{7000}*, L_{8000}*, L_{9000}*, L_{10000}*, L_{11000}*, L_{12000}*, L_{13000}*, L_{14000}*, L_{15000}*, L_\infty*$	$L_K <$	$L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K <$	$L_{3000}, L_{4000}, L_{6000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{14000}$
	$L_K >$	$L_0*, L_{1000}*, L_{2000}*$	$L_K >$	$L_0*, L_{1000}*, L_{2000}*, L_{3000}, L_{4000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K >$	$L_0*, L_{1000}*, L_{2000}*, L_{3000}*, L_{12000}, L_{14000}, L_{15000}, L_\infty$
D=200	$L_K =$		$L_K =$	L_{5000}	$L_K =$	L_{10000}
	$L_K <$	$L_{3000}, L_{4000}*, L_{5000}*, L_{6000}*, L_{7000}*, L_{8000}*, L_{9000}*, L_{10000}*, L_{11000}*, L_{12000}*, L_{13000}*, L_{14000}*, L_{15000}*, L_\infty*$	$L_K <$		$L_K <$	$L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{11000}, L_{13000}$
	$L_K >$	$L_0*, L_{1000}*, L_{2000}*, L_{3000}$	$L_K >$	$L_0*, L_{1000}*, L_{2000}*, L_{3000}*, L_{4000}, L_{5000}, L_{8000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{15000}, L_\infty$	$L_K >$	$L_0*, L_{1000}*, L_{2000}*, L_{3000}*, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{11000}, L_{14000}, L_\infty$
D=300	$L_K =$		$L_K =$		$L_K =$	L_{15000}
	$L_K <$	$L_{4000}, L_{5000}*, L_{6000}*, L_{7000}*, L_{8000}*, L_{9000}*, L_{10000}*, L_{11000}*, L_{12000}*, L_{13000}*, L_{14000}*, L_{15000}*, L_\infty*$	$L_K <$	$L_{6000}, L_{7000}, L_{9000}, L_{14000}$	$L_K <$	$L_{10000}, L_{12000}, L_{13000}$

Table 26

f_2, horizontal approach, 10^{-6}, large dimension, NHST

		SN=24		SN=50		SN=100
	$L_K >$	$L_0*, L_{4000}, L_{7000}, L_{8000}, L_{13000}, L_\infty$	$L_K >$	$L_0*, L_{1000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{8000}, L_{12000}, L_\infty$	$L_K >$	$L_0*, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{6000}, L_{7000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
D=100	$L_K =$		$L_K =$		$L_K =$	L_{5000}
	$L_K <$	$L_{1000}, L_{2000}, L_{3000}, L_{5000}, L_{6000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{14000}, L_{15000}$	$L_K <$	$L_{2000}, L_{7000}, L_{9000}, L_{10000}, L_{11000}, L_{13000}, L_{14000}, L_{15000}$	$L_K <$	L_{8000}
	$L_K >$	$L_0*, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{14000}, L_{15000}, L_\infty$	$L_K >$	L_0*, L_{1000}	$L_K >$	$L_0*, L_{1000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
D=200	$L_K =$		$L_K =$	L_{5000}	$L_K =$	L_{10000}
	$L_K <$	L_{12000}, L_{13000}	$L_K <$	$L_{2000}, L_{3000}, L_{4000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K <$	L_{2000}
	$L_K >$	$L_0*, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{8000}, L_{9000}$	$L_K >$	$L_0*, L_{1000}, L_{2000}, L_{3000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{14000}, L_{15000}, L_\infty$	$L_K >$	$L_0*, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_\infty$
D=300	$L_K =$		$L_K =$		$L_K =$	L_{15000}
	$L_K <$	$L_{5000}, L_{6000}, L_{7000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K <$	$L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{12000}, L_{13000}$	$L_K <$	

Table 27

f_3, horizontal approach, 10^{-6}, large dimension, NHST

		SN=24		SN=50		SN=100
D=100	$L_K >$	$L_0*, L_{3000}, L_{8000}, L_{11000}$	$L_K >$	L_0*, L_{2000}, L_{3000}	$L_K >$	$L_0*, L_{1000}, L_{9000}, L_{11000}$
	$L_K =$		$L_K =$		$L_K =$	L_{5000}
	$L_K <$	$L_{1000}, L_{2000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{9000}, L_{10000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K <$	$L_{1000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K <$	$L_{2000}, L_{3000}, L_{4000}, L_{6000}, L_{7000}, L_{8000}, L_{10000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
D=200	$L_K >$	$L_0*, L_{1000}, L_{7000}, L_{9000}, L_{11000}, L_{13000}, L_{14000}$	$L_K >$	L_0*, L_{2000}, L_{3000}	$L_K >$	$L_0*, L_{2000}, L_{7000}, L_{11000}, L_{13000}, L_\infty$
	$L_K =$		$L_K =$	L_{5000}	$L_K =$	L_{10000}
	$L_K <$	$L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{8000}, L_{10000}, L_{12000}, L_{15000}, L_\infty$	$L_K <$	$L_{1000}, L_{4000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K <$	$L_{1000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{8000}, L_{9000}, L_{12000}, L_{14000}, L_{15000}$
D=300	$L_K >$	$L_0*, L_{1000}, L_{3000}, L_{4000}, L_{5000}, L_{9000}, L_{11000}, L_{12000}, L_\infty$	$L_K >$	$L_0*, L_{3000}, L_{5000}, L_{9000}, L_{11000}, L_{14000}, L_\infty$	$L_K >$	$L_0*, L_{6000}, L_{12000}, L_{14000}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{15000}
	$L_K <$	$L_{2000}, L_{6000}, L_{7000}, L_{8000}, L_{10000}, L_{13000}, L_{14000}, L_{15000}$	$L_K <$	$L_{1000}, L_{2000}, L_{4000}, L_{6000}, L_{7000}, L_{8000}, L_{10000}, L_{12000}, L_{13000}, L_{15000}$	$L_K <$	$L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{13000}$

Table 28

f_4, horizontal approach, 10^{-6}, large dimension, NHST

		SN=24		SN=50		SN=100
D=100	$L_K >$	$L_0*, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_\infty$	$L_K >$	L_0*	$L_K >$	$L_0*, L_{1000}, L_{2000}, L_{3000}, L_{7000}, L_{8000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{5000}
	$L_K <$	L_{6000}, L_{15000}	$L_K <$	$L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K <$	$L_{4000}, L_{6000}, L_{9000}, L_{10000}$
D=200	$L_K >$	L_0*	$L_K >$	$L_0*, L_{8000}, L_{12000}, L_{13000}, L_{14000}$	$L_K >$	$L_0*, L_{4000}, L_{7000}, L_{8000}, L_{13000}$
	$L_K =$		$L_K =$	L_{5000}	$L_K =$	L_{10000}
	$L_K <$	$L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K <$	$L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{6000}, L_{7000}, L_{9000}, L_{10000}, L_{11000}, L_{15000}, L_\infty$	$L_K <$	$L_{1000}, L_{2000}, L_{3000}, L_{5000}, L_{6000}, L_{9000}, L_{11000}, L_{12000}, L_{14000}, L_{15000}, L_\infty$
D=300	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_\infty$	$L_K =$	$L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_\infty$	$L_K =$	$L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
	$L_K <$	L_{15000}	$L_K <$	L_{15000}	$L_K <$	

Table 29

f_5, horizontal approach, 10^{-6}, large dimension, NHST

		SN=24		SN=50		SN=100
D=100	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K =$	$L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K =$	$L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=200	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K =$	$L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K =$	$L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=300	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K =$	$L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K =$	$L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	

experiment showed that there are other 'limit' values that perform better than L_k, for certain problems (f_1) even significantly. In almost all D and SN settings and problems, at least one better performing 'limit' value was found (the only exceptions are f_1, SN = 50, and D = 200 and f_2, SN = 100, and D = 300). For f_5, none of the 'limit' values reached optimal solution, since all settings performed equally. By comparing Tables 25-29 with Tables 15-19, it can be observed that with higher dimensions L_k setting becomes less appropriate.

2.2.6. Discussion

The analysis with NHST supported our concerns about setting a fixed 'limit' value regarding the suggested formula $L_k = n_e * D = (SN/2) * D$. When a smaller number of fitness evaluations (e.g., 100,000) were available, L_k was the appropriate choice only for small dimensions (D = 2, rarely for D = 5 or D = 10) amongst all the five presented problems. When dimension got bigger, more appropriate alternatives could be chosen. On the other hand, when sufficiently large enough numbers of fitness evaluations were available (e.g., 250,000), L_k was a significantly better choice than the presented fixed 'limit' values for all the presented problems, dimensions, and values of population size. This does not necessarily mean that a better value than L_k does not exist but it was not defined in our set of fixed 'limit' values.

When it was of interest in finding a (sub-)optimal solution (i.e., 10^{-6}) and a larger number of fitness evaluations were available (i.e., 1,000,000), L_k has better alternatives for the all presented problems, dimensions, and values of population size. The only time L_k seemed to be like an appropriate choice was for problem f_1 (multi-modal, non-separable problem) when population size equaled 50 and for problem f_5 (uni-modal, non-separable problem) for which ABC did not reach the given (sub-)optimal solution over 1,000,000 fitness evaluations regardless of the 'limit' value. When the value of this (sub-)optimal solution was even more precise (i.e., 10^{-12}) there were better alternatives than L_k even for problem f_1. In summary, it was shown that even within this small benchmark suite used in our study setting 'limit' is very problem dependent (e.g., see Tables 5-9 for results on f_1 - f_5).

In many cases, better settings existed (even significantly better) than setting 'limit' according to the suggested formula. The results also heavily depended on the number of available fitness evaluations, indicating that ABC convergence with L_k is not amongst the fastest. The results from the horizontal approach further supported this claim.

2.3. Chess Rating System for Evolutionary Algorithms (CRS4EAs)

The Chess Rating System for Evolutionary Algorithms (CRS4EAs) [48] is a novel method for the comparing and ranking of evolutionary algorithms. In this method, each participating algorithm plays the role of a chess player. The comparison between two players is treated as one game that can have only one out of three outcomes: win, lose, or draw. Two algorithms play a draw whenever the difference in their solutions is smaller than predefined ε. Otherwise, the algorithm with the solution closer to the optimum of an optimisation problem wins and the other loses. A pairwise comparison between the solutions of all participating algorithms on all optimisation problems over all independent runs is treated as one tournament. After the tournament has been conducted, the rating R, rating deviation RD, and rating interval RI for each of the players are calculated regarding the formula from the Glicko-2 rating system [16], [17]. Rating is an absolute power of a player that is supported by rating deviation. The higher the rating deviation, the less reliable the player's rating. Rating interval is formed from rating and rating deviation. It can be said with 95% probability that a player's rating R belongs to an interval [R-2RD, R+2RD]. Regarding these rating intervals, the algorithms can then be compared and if their intervals do not overlap, the algorithms are considered significantly different. The result of one such tournament is a leaderboard from which all these data can be read and interpreted. When players enter a tournament their rating power equals 1500, and their rating deviation equals 350, which is the maximum available rating deviation value. The more games the algorithms play, the smaller become the rating deviation values, and the minimum value usually used in CRS4EAs comparisons equals 50.

In this analysis, players were presented as ABC algorithms with different 'limit' value settings. A tournament was executed for each combination of SN and D values for each optimisation problem separately to allow fair comparison with NHST's Wilcoxon's test. The results of both analyses (NHST's and CRS4EAs') were very similar, however, there were some differences. Even though, both the Wilcoxon's test and CRS4EAs compared all runs pairwisely, the results of the Wilcoxon's test were more relative and the results of CRS4EAs' more absolute. The Wilcoxon's test took into consideration only wins and losses against L_k, which were reflected in the p value. CRS4EAs, on the other hand, conducted a tournament between 10 players (L_k, L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_∞) where runs were pairwisely compared. In regard to these wins, losses, and draws, a rating was calculated and not only were the games against L_k taken into consideration but games against all opponents. This is the main reason behind the differences between the results of both methods. However, to point out once again: in both approaches, the results were compared as *1×k* comparison as CRS4EAs being appropriate for both types of comparisons – $1{\times}k$ and $k{\times}k$. There was also a difference in effort put into executing both methods. In CRS4EAs when the ratings of pairwise comparison were obtained, there was no need for further calculations and testing, whilst when p values are calculated with a statistical test, a post-hoc test, such as the Holm test, is always necessary due to the repetitive comparisons of L_k with other settings.

The experiment was again divided into 4 parts for CRS4EAs analysis. Each part of the experiment took a different approach just as the ones shown in Table 3. For a more straightforward comparison, the reports of rating deviations and rating intervals were omitted in the tables with results, even though they were calculated and used in detecting significant differences. The ε for determining the draw was set to 10^{-20} as results were compared up to 20 decimals places in the Wilcoxon's test as well. A less precise ε would affect the detected differences and there would be greater differences in NHST and CRS4EAs analyses. The minimum rating deviation value was set at 50 and the maximum rating deviation value at 350. Glicko-2 also calculates some other measurements we omitted during this analysis, as they were unimportant in this analysis. The other CRS4EAs' parameters used in formulae for calculating rating and rating deviation were determined regarding the Glicko-2 rating system. Readers can find more on this topic and definitions of these parameters in [26].

2.3.1. Experiment 1: Vertical Approach with *MaxFEs* = 100,000

Tables 30(a) – 30(e) showed the ratings obtained for every setting of *SN* and *D* for all 5 minimisation problems. All the players reached the minimum rating deviation value of 50 rating points. The best player of each setting (shown in one row) is marked in light grey background colour. For example, from Table 30 (*SN* = 24, *D* = 10), it can be observed that the highest rating of 1768 points was obtained using L_{250} followed by L_{500} (1693 points), L_{1000} (1631 points), L_{750} (1628 points), L_{1500} (1603 points), L_{1250} (1597 points), L_∞ (1588 points), L_k (1394 points), L_{100} (1116 points), and L_0 (982 points). The difference in rating between the winner L_{250} (1768 points) and L_k (1394 points) was more than 200 points (4*RD*) and hence statistically significant. Overall, these tables show that L_k was not always the more appropriate value for 'limit' – especially for f_5. However, observing the ratings and when calculating the rating intervals as [R-100, R+100] where 100 is $2{*}RD_{min}$ = 2*50, the differences were rarely significant.

Tables 31-35 show more clearly the differences found between L_k and the other 9 fixed 'limit' values on all 5 optimisation problems. Whenever the difference was significant, the star symbol (*) has been placed after

Table 30

Vertical approach, MaxFEs = 100,000

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1562	1029	1562	1562	1557	1562	1557	1546	1540	1525
SN=24 D=5	1478	981	1578	1578	1573	1578	1553	1563	1558	1559
SN=24 D=10	1394	982	1116	1768	1693	1628	1631	1597	1603	1588
SN=24 D=30	1518	1012	1072	1265	1650	1717	1705	1690	1680	1690
SN=24 D=50	1619	1033	1074	1247	1508	1693	1726	1713	1694	1693
SN=50 D=2	1558	1018	1558	1558	1558	1558	1553	1542	1558	1541
SN=50 D=5	1571	981	1571	1571	1566	1545	1561	1561	1540	1535
SN=50 D=10	1745	981	1122	1745	1697	1614	1582	1591	1547	1622
SN=50 D=30	1682	1021	1074	1301	1635	1682	1691	1696	1675	1725
SN=50 D=50	1688	1073	1094	1249	1577	1705	1684	1688	1710	1719
SN=100 D=2	1565	1051	1565	1565	1565	1565	1554	1543	1549	1543
SN=100 D=5	1569	981	1575	1569	1575	1559	1564	1543	1559	1575
SN=100 D=10	1637	981	1113	1711	1637	1579	1643	1611	1601	1624
SN=100 D=30	1705	1021	1083	1278	1666	1710	1679	1721	1705	1636
SN=100 D=50	1650	1060	1060	1247	1530	1645	1711	1718	1660	1717

(a) Vertical approach, f_1, MaxFEs = 100,000

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1827	981	1795	1648	1556	1495	1455	1447	1451	1344
SN=24 D=5	1689	981	1654	1689	1599	1601	1620	1422	1362	1383
SN=24 D=10	1566	981	1576	1538	1579	1574	1555	1542	1521	1569
SN=24 D=30	1574	981	1577	1588	1495	1583	1545	1547	1582	1528
SN=24 D=50	1525	981	1611	1542	1537	1570	1527	1540	1567	1600
SN=50 D=2	1779	981	1767	1627	1596	1532	1456	1475	1429	1358
SN=50 D=5	1725	981	1648	1640	1574	1603	1600	1420	1370	1441
SN=50 D=10	1596	981	1583	1596	1573	1542	1566	1543	1544	1573
SN=50 D=30	1544	981	1584	1561	1562	1544	1542	1570	1618	1538
SN=50 D=50	1600	981	1560	1591	1578	1555	1578	1516	1600	1543
SN=100 D=2	1793	981	1793	1677	1610	1497	1553	1511	1529	1349
SN=100 D=5	1741	981	1679	1741	1602	1616	1525	1492	1462	1401
SN=100 D=10	1542	981	1617	1529	1542	1560	1571	1540	1578	1583
SN=100 D=30	1573	981	1574	1556	1608	1534	1590	1574	1573	1512
SN=100 D=50	1544	981	1544	1539	1555	1570	1515	1599	1552	1600

(b) Vertical approach, f_2, MaxFEs = 100,000

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1557	991	1557	1557	1557	1557	1557	1557	1557	1557
SN=24 D=5	1564	981	1564	1564	1564	1564	1564	1543	1533	1559
SN=24 D=10	1580	981	1554	1570	1554	1554	1544	1570	1544	1549
SN=24 D=30	1517	981	1514	1604	1569	1578	1546	1616	1547	1528
SN=24 D=50	1558	981	1539	1577	1576	1554	1610	1532	1505	1568
SN=50 D=2	1557	986	1557	1557	1557	1557	1557	1557	1557	1557
SN=50 D=5	1566	981	1566	1566	1566	1561	1566	1540	1540	1546
SN=50 D=10	1560	981	1596	1560	1549	1560	1560	1570	1560	1565
SN=50 D=30	1569	981	1603	1551	1571	1569	1544	1566	1599	1516
SN=50 D=50	1568	981	1545	1536	1608	1536	1561	1568	1569	1596
SN=100 D=2	1563	996	1563	1563	1563	1563	1563	1563	1563	1563
SN=100 D=5	1572	981	1572	1572	1572	1567	1572	1551	1562	1551
SN=100 D=10	1555	981	1575	1565	1555	1544	1575	1575	1560	1570
SN=100 D=30	1635	981	1558	1593	1621	1508	1540	1532	1635	1532
SN=100 D=50	1592	981	1540	1547	1570	1623	1552	1510	1548	1535

(c) Vertical approach, f_3, MaxFEs = 100,000

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1558	981	1558	1558	1558	1558	1558	1558	1558	1558
SN=24 D=5	1558	981	1558	1558	1558	1558	1558	1558	1558	1558
SN=24 D=10	1558	981	1558	1558	1558	1558	1558	1558	1558	1558
SN=24 D=30	1569	981	1544	1580	1502	1596	1555	1570	1573	1530
SN=24 D=50	1527	981	1576	1574	1573	1572	1518	1532	1539	1608
SN=50 D=2	1558	981	1558	1558	1558	1558	1558	1558	1558	1558
SN=50 D=5	1558	981	1558	1558	1558	1558	1558	1558	1558	1558
SN=50 D=10	1565	981	1565	1565	1565	1565	1565	1565	1565	1565
SN=50 D=30	1566	981	1593	1534	1610	1566	1529	1564	1516	1608
SN=50 D=50	1595	981	1553	1538	1549	1547	1599	1595	1586	1553
SN=100 D=2	1564	986	1564	1564	1564	1564	1564	1564	1564	1564
SN=100 D=5	1565	981	1565	1565	1565	1565	1565	1565	1565	1565
SN=100 D=10	1565	981	1565	1565	1565	1565	1565	1565	1565	1565
SN=100 D=30	1570	981	1543	1561	1574	1587	1575	1566	1570	1543
SN=100 D=50	1576	981	1597	1551	1550	1516	1597	1521	1512	1600

(d) Vertical approach, f_4, MaxFEs = 100,000

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1563	1502	1512	1521	1509	1510	1471	1503	1532	1375
SN=24 D=5	1261	982	1371	1498	1598	1612	1637	1650	1672	1719
SN=24 D=10	1423	981	1359	1538	1518	1702	1612	1620	1619	1628
SN=24 D=30	1558	981	1521	1602	1593	1570	1583	1528	1551	1514
SN=24 D=50	1536	981	1570	1567	1574	1537	1553	1521	1551	1611
SN=50 D=2	1509	1505	1495	1537	1509	1516	1409	1525	1490	1505
SN=50 D=5	1419	982	1322	1475	1589	1615	1673	1621	1644	1659
SN=50 D=10	1551	981	1340	1551	1557	1582	1667	1641	1565	1616
SN=50 D=30	1606	981	1571	1551	1597	1606	1513	1579	1532	1569
SN=50 D=50	1532	981	1631	1484	1609	1606	1553	1532	1512	1591
SN=100 D=2	1512	1535	1512	1514	1560	1534	1503	1482	1422	1439
SN=100 D=5	1487	982	1327	1487	1580	1613	1613	1630	1636	1631
SN=100 D=10	1616	981	1349	1534	1616	1614	1547	1644	1597	1618
SN=100 D=30	1593	981	1593	1534	1588	1538	1614	1527	1593	1531
SN=100 D=50	1487	981	1589	1597	1544	1560	1530	1576	1583	1553

(e) Vertical approach, f_5, MaxFEs = 100,000

'limit' value, and whenever the L_k had better alternative(s) the background of table cell has been highlighted in light grey colour. Similar to the NHST approach, CRS4EAs also found significant difference only for f_1 and f_5. In particular, for f_1: SN = 24 and D = 10 where L_k was significantly worse than L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}. For f_5: SN = 24 and D = 5 where L_k was significantly worse than L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_∞; SN = 50 and D = 5 where L_k was significantly worse than L_{1000}, L_{1250}, L_{1500}, L_∞; SN = 24 and D = 10 where L_k was significantly worse than L_{750}, L_∞. However, when comparing the detected significant differences between NHST and CRS4EAs (compare Tables 5-9 with Tables 31-35), CRS4EAs appears more conservative than NHST. Whilst the differences were presented for the same settings, in CRS4EAs these differences were hardly ever significant.

For all five problems, L_k had almost always better alternatives (but not significant) when dimension D was greater (10, 30, or 50).

2.3.2. Experiment 2: Vertical Approach with *MaxFEs* = 250,000

Tables 36(a)-36(e) show the ratings obtained for every setting of SN and D on all 5 minimisation problems. All players reached the minimum rating deviation value of 50 rating points. The best player of each setting (shown in one row) is marked in light grey background colour. These tables show that L_k was almost always the more appropriate value for 'limit'. f_5 was the only problem for which better alternatives were found for some SN and D settings. Moreover, L_k was just as in NHST analysis – the significantly better choice in most cases.

Table 31

f_1, vertical approach, MaxFEs = 100,000, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_∞	$L_K >$	L_0*, L_{1000}, L_{1250}, L_∞	$L_K >$	L_0*, L_{1000}, L_{1250}, L_{1500}, L_∞
	$L_K =$	L_{100}, L_{250}, L_{750}	$L_K =$	L_{100}, L_{250}, L_{500}, L_{750}, L_{1500}	$L_K =$	L_{100}, L_{250}, L_{500}, L_{750}
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	L_0*	$L_K >$	L_0*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_∞	$L_K >$	L_0*, L_{750}, L_{1000}, L_{1250}, L_{1500}
	$L_K =$		$L_K =$	L_{100}, L_{250}	$L_K =$	L_{250}
	$L_K <$	L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_∞	$L_K <$		$L_K <$	L_{100}, L_{500}, L_∞
D=10	$L_K >$	L_0*, $L_{100}*$	$L_K >$	L_0*, $L_{100}*$, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_∞	$L_K >$	L_0*, $L_{100}*$, L_{750}, L_{1250}, L_{1500}, L_∞
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{250}*$, $L_{500}*$, $L_{750}*$, $L_{1000}*$, $L_{1250}*$, $L_{1500}*$, L_∞	$L_K <$		$L_K <$	L_{250}, L_{1000}
D=30	$L_K >$	L_0*, $L_{100}*$, $L_{250}*$	$L_K >$	L_0*, $L_{100}*$, $L_{250}*$, L_{500}, L_{1500}	$L_K >$	L_0*, $L_{100}*$, $L_{250}*$, L_{500}, L_{1000}, L_{1500}, L_∞
	$L_K =$		$L_K =$	L_{750}	$L_K =$	
	$L_K <$	L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_∞	$L_K <$	L_{1000}, L_{1250}, L_∞	$L_K <$	L_{750}, L_{1250}
D=50	$L_K >$	L_0*, $L_{100}*$, $L_{250}*$, L_{500}	$L_K >$	L_0*, $L_{100}*$, $L_{250}*$, L_{500}, L_{1000}	$L_K >$	L_0*, $L_{100}*$, $L_{250}*$, L_{500}, L_{750}
	$L_K =$		$L_K =$	L_{1250},	$L_K =$	
	$L_K <$	L_{750}, L_{1000}, L_{1250}, L_{1500}, L_∞	$L_K <$	L_{750}, L_{1500}, L_∞	$L_K <$	L_{1000}, L_{1250}, L_{1500}, L_∞

Table 32

f_2, vertical approach, MaxFEs = 100,000, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{250}, L_{500}, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$		$L_K <$		$L_K <$	
D=10	$L_K >$	$L_0*, L_{250}, L_{1000}, L_{1250}, L_{1500},$	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	L_0*, L_{250}, L_{1250}
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{100}, L_{500}, L_{750}, L_\infty$	$L_K <$		$L_K <$	$L_{100}, L_{750}, L_{1000}, L_{1500}, L_\infty$
D=30	$L_K >$	$L_0*, L_{500}, L_{1000}, L_{1250}, L_\infty$	$L_K >$	L_0*, L_{1000}, L_∞	$L_K >$	$L_0*, L_{250}, L_{750}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1500}$	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1250}, L_{1500}$	$L_K <$	$L_{100}, L_{500}, L_{1000}, L_{1250}$
D=50	$L_K >$	L_0*	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500},$	$L_K >$	L_0*, L_{250}, L_{1000}
	$L_K =$		$L_K =$	$L_{1250},$	$L_K =$	L_{100}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$		$L_K <$	$L_{500}, L_{750}, L_{1250}, L_{1500}, L_\infty$

Table 33

f_3, vertical approach, MaxFEs = 100,000, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	$L_0*, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{750}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{750}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{1000}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{1000}$
	$L_K <$		$L_K <$		$L_K <$	
D=10	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	L_0*, L_{500}	$L_K >$	L_0*, L_{750}
	$L_K =$		$L_K =$	$L_{250}, L_{750}, L_{1000}, L_{1500}$	$L_K =$	L_{500}
	$L_K <$		$L_K <$	$L_{100}, L_{1250}, L_\infty$	$L_K <$	$L_{100}, L_{250}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=30	$L_K >$	L_0*, L_{100}	$L_K >$	$L_0*, L_{250}, L_{1000}, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{500}, L_{1500}$	$L_K <$	
D=50	$L_K >$	$L_0*, L_{100}, L_{750}, L_{1250}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{750}, L_{1000}$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{250}, L_{500}, L_{1000}, L_\infty$	$L_K <$	$L_{500}, L_{1500}, L_\infty$	$L_K <$	L_{750}

Table 34

f_4, vertical approach, MaxFEs = 100,000, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=10	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=30	$L_K >$	$L_0*, L_{100}, L_{500}, L_{1000}, L_\infty$	$L_K >$	$L_0*, L_{250}, L_{1000}, L_{1250}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{250}, L_{750}, L_{1250}, L_{1500}$	$L_K <$	$L_{100}, L_{500}, L_\infty$	$L_K <$	$L_{500}, L_{750}, L_{1000},$
D=50	$L_K >$	L_0*, L_{1000}	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{250}, L_{500}, L_{750}, L_{1250}, L_{1500}$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	L_{1000}	$L_K <$	$L_{100}, L_{1000}, L_\infty$

Table 35

f_5, vertical approach, MaxFEs = 100,000, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0, L_{100}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	$L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{500}	$L_K =$	L_{100}
	$L_K <$		$L_K <$	$L_{250}, L_{750}, L_{1250}$	$L_K <$	$L_0, L_{250}, L_{500}, L_{750}$
D=5	$L_K >$	L_0*	$L_K >$	L_0*, L_{100}	$L_K >$	L_0*, L_{100}
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	$L_{100}, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$	$L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=10	$L_K >$	L_0*, L_{100}	$L_K >$	L_0*, L_{100}	$L_K >$	$L_0*, L_{100}*, L_{250}, L_{750}, L_{1000}, L_{1500}$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{250}, L_{500}, L_{750}*, L_{1000}, L_{1250}, L_{1500}, L_\infty*$	$L_K <$	$L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	L_{1250}, L_∞
D=30	$L_K >$	$L_0*, L_{100}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{250}, L_{500}, L_{750}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{100}, L_{1500}
	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}$	$L_K <$		$L_K <$	L_{1000}
D=50	$L_K >$	L_0*, L_{1250}	$L_K >$	L_0*, L_{250}, L_{1500}	$L_K >$	L_0*
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{500}, L_{750}, L_{1000}, L_\infty$	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$

Table 36

Vertical approach. MaxFEs = 250,000

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1548	1101	1548	1548	1548	1548	1548	1548	1548	1516
SN=24 D=5	1562	981	1562	1562	1562	1562	1562	1562	1562	1521
SN=24 D=10	1615	981	1128	1615	1615	1615	1615	1611	1615	1588
SN=24 D=30	1822	1004	1073	1212	1664	1741	1694	1644	1628	1518
SN=24 D=50	2019	1022	1074	1193	1362	1612	1689	1711	1697	1620
SN=50 D=2	1547	1096	1547	1547	1547	1547	1547	1547	1547	1531
SN=50 D=5	1558	981	1558	1558	1558	1558	1558	1558	1558	1553
SN=50 D=10	1630	981	1124	1630	1630	1630	1630	1625	1630	1621
SN=50 D=30	1831	1009	1082	1240	1678	1831	1685	1720	1653	1601
SN=50 D=50	2019	1033	1076	1225	1407	1649	1730	2019	1704	1658
SN=100 D=2	1553	1106	1553	1553	1553	1553	1553	1553	1553	1526
SN=100 D= 5	1569	981	1569	1569	1569	1569	1564	1564	1569	1548
SN=100 D=10	1634	981	1120	1634	1634	1634	1629	1629	1625	1616
SN=100 D=30	1833	1013	1079	1239	1698	1685	1725	1650	1833	1577
SN=100 D=50	2019	1033	1060	1200	1355	1608	1704	1689	1693	1640

(a) Vertical approach, f_1, MaxFEs = 250,000

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1994	981	1648	1715	1465	1593	1366	1512	1459	1267
SN=24 D=5	1982	981	1695	1637	1598	1517	1502	1434	1420	1233
SN=24 D=10	1967	981	1583	1591	1558	1553	1547	1460	1461	1299
SN=24 D=30	1941	981	1145	1431	1548	1575	1583	1561	1604	1632
SN=24 D=50	1998	981	1119	1539	1530	1614	1547	1557	1543	1572
SN=50 D=2	1987	981	1616	1699	1610	1579	1381	1459	1488	1200
SN=50 D=5	1973	981	1679	1638	1547	1513	1502	1446	1471	1250
SN=50 D=10	1971	981	1570	1971	1550	1557	1566	1506	1495	1304
SN=50 D=30	1980	981	1152	1471	1616	1980	1619	1503	1644	1533
SN=50 D=50	2010	981	1144	1522	1558	1555	1575	2010	1577	1578
SN=100 D=2	1991	981	1991	1697	1610	1562	1529	1473	1441	1216
SN=100 D= 5	1976	981	1678	1976	1599	1531	1530	1467	1445	1292
SN=100 D=10	1973	981	1581	1590	1973	1512	1568	1503	1497	1297
SN=100 D=30	1966	981	1170	1470	1535	1584	1629	1583	1966	1582
SN=100 D=50	2007	981	1124	1475	1567	1557	1588	1548	1547	1607

(b) Vertical approach, f_2, MaxFEs = 250,000

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1550	1048	1550	1550	1550	1550	1550	1550	1550	1550
SN=24 D=5	1561	981	1561	1561	1561	1561	1561	1561	1561	1530
SN=24 D=10	1903	981	1529	1535	1529	1535	1519	1503	1498	1467
SN=24 D=30	1923	981	1263	1460	1520	1559	1599	1538	1589	1569
SN=24 D=50	1937	981	1106	1353	1604	1594	1598	1615	1611	1599
SN=50 D=2	1554	1017	1554	1554	1554	1554	1554	1554	1554	1554
SN=50 D=5	1561	981	1561	1561	1561	1561	1561	1561	1561	1535
SN=50 D=10	1917	981	1523	1917	1543	1533	1528	1502	1507	1466
SN=50 D=30	1934	981	1243	1487	1523	1934	1597	1577	1573	1585
SN=50 D=50	1958	981	1111	1446	1586	1607	1582	1958	1606	1623
SN=100 D=2	1558	1033	1558	1558	1558	1558	1558	1558	1558	1558
SN=100 D= 5	1566	981	1566	1566	1566	1566	1566	1566	1556	1556
SN=100 D=10	1867	981	1550	1555	1867	1550	1540	1488	1508	1462
SN=100 D=30	1940	981	1235	1475	1546	1569	1577	1597	1940	1580
SN=100 D=50	1941	981	1108	1397	1555	1592	1606	1610	1613	1598

(c) Vertical approach, f_3, MaxFEs = 250,000

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1558	981	1558	1558	1558	1558	1558	1558	1558	1558
SN=24 D=5	1558	981	1558	1558	1558	1558	1558	1558	1558	1558
SN=24 D=10	1558	981	1558	1558	1558	1558	1558	1558	1558	1558
SN=24 D=30	1577	981	1405	1577	1577	1577	1577	1577	1577	1577
SN=24 D=50	2016	981	1344	1552	1550	1534	1479	1507	1511	1527
SN=50 D=2	1558	981	1558	1558	1558	1558	1558	1558	1558	1558
SN=50 D=5	1558	981	1558	1558	1558	1558	1558	1558	1558	1558
SN=50 D=10	1565	981	1565	1565	1565	1565	1565	1565	1565	1565
SN=50 D=30	1586	981	1415	1586	1586	1586	1586	1586	1586	1586
SN=50 D=50	1586	981	1415	1586	1586	1586	1586	1586	1586	1586
SN=100 D=2	1565	981	1565	1565	1565	1565	1565	1565	1565	1565
SN=100 D= 5	1565	981	1565	1565	1565	1565	1565	1565	1565	1565
SN=100 D=10	1565	981	1565	1565	1565	1565	1565	1565	1565	1565
SN=100 D=30	1587	981	1410	1587	1587	1587	1587	1587	1587	1587
SN=100 D=50	2011	981	1351	1558	1509	1482	1530	1525	1540	1514

(d) Vertical approach, f_4, MaxFEs = 250,000

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1914	1494	1508	1495	1485	1492	1437	1431	1438	1306
SN=24 D=5	1538	984	1268	1495	1561	1649	1592	1647	1668	1597
SN=24 D=10	1582	981	1272	1476	1547	1610	1608	1621	1649	1655
SN=24 D=30	1838	981	1492	1469	1595	1509	1500	1542	1493	1582
SN=24 D=50	1933	981	1493	1477	1482	1506	1507	1512	1578	1532
SN=50 D=2	1903	1546	1453	1535	1505	1503	1417	1411	1416	1312
SN=50 D=5	1625	982	1230	1439	1562	1627	1620	1625	1653	1637
SN=50 D=10	1595	981	1343	1595	1582	1557	1600	1626	1592	1625
SN=50 D=30	1835	981	1453	1522	1564	1835	1535	1526	1529	1556
SN=50 D=50	1875	981	1474	1556	1523	1531	1506	1875	1540	1513
SN=100 D=2	1749	1551	1749	1491	1495	1503	1500	1465	1443	1304
SN=100 D= 5	1596	981	1269	1596	1573	1617	1609	1622	1614	1619
SN=100 D=10	1704	981	1287	1514	1704	1618	1623	1583	1588	1601
SN=100 D=30	1818	981	1445	1575	1506	1523	1523	1556	1818	1571
SN=100 D=50	1888	981	1529	1551	1482	1506	1506	1523	1509	1527

(e) Vertical approach, f_5, MaxFEs = 250,000

Tables 37-41 show more clearly the differences found between L_k and other 9 fixed 'limit' values on all 5 optimisation problems. As mentioned before, the better alternatives were found only for problem f_5, when the dimensions were either 5 or 10 but the 'limit' values were not significantly better than L_k. As in NHST analysis, the CRS4EAs also showed that whenever sufficiently larger numbers of function evaluations were available, Karaboga's 'limit' setting was an appropriate choice. In this approach, both methods, NHST and CRS4EAs, appeared equally conservative (compare Tables 10-14 with Tables 37-41).

2.3.3. Experiment 3: Horizontal Approach – 10^{-6} - *MaxFEs* = 1,000,000

In the horizontal approach, there were fixed 'limit' values that found (sub-)optimal solutions in fewer fitness evaluations than L_k for all optimisation problems. Tables 42(a)-42(e) show the ratings for every optimisation problem and every setting of *SN* and *D*. All 'limit' values reached the minimum rating deviation value of 50 rating points and the better rating values are again highlighted with light grey colour.

For f_1, better alternatives than L_k were available for the smaller population size *SN* = 24 and for the greater population size *SN* = 100, whereas for *SN* = 50, L_k was only worse for *D* = 5 and *D* = 50 and better for all other dimension values. For f_2, L_k was the worst value for all population sizes and dimensions, except for *SN* =50 and *D* =10, *SN* = 100 and *D* = 10, *SN* = 50 and *D* = 30, and *SN* = 50 and *D* = 50. For f_3, L_k was the better value only for *SN* =24 and *D* =2 and *SN* =24 and *D* = 50, but for other settings there were better alternatives. For f_4, L_k always had a better alternative and was always worse than at

Table 37

f_1, vertical approach, MaxFEs = 250,000, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*, L_∞	$L_K >$	L_0*, L_∞	$L_K >$	L_0*, L_∞
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	L_0*, L_∞	$L_K >$	L_0*, L_∞	$L_K >$	$L_0*, L_{1000}, L_{1250}, L_\infty$
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1500}$
	$L_K <$		$L_K <$		$L_K <$	
D=10	$L_K >$	$L_0*, L_{100}*, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}$	$L_K =$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}$	$L_K =$	$L_{250}, L_{500}, L_{750}$
	$L_K <$		$L_K <$		$L_K <$	
D=30	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty*$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$		$L_K <$		$L_K <$	
D=50	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$		$L_K <$		$L_K <$	

Table 38

f_2, vertical approach, MaxFEs = 250,000, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$		$L_K <$		$L_K <$	
D=10	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$		$L_K <$		$L_K <$	
D=30	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_\infty*$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$		$L_K <$		$L_K <$	
D=50	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$		$L_K <$		$L_K <$	

Table 39

f_3, vertical approach, MaxFEs = 250,000, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	L_0*, L_∞	$L_K >$	L_0*, L_∞	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=10	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$		$L_K <$		$L_K <$	
D=30	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_\infty*$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$		$L_K <$		$L_K <$	
D=50	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$		$L_K <$		$L_K <$	

Table 40

f_4, vertical approach, MaxFEs = 250,000, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=10	$L_K >$	L_0*	$L_K >$	L_0*	$L_K >$	L_0*
	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=30	$L_K >$	L_0*, L_{100}	$L_K >$	L_0*, L_{100}	$L_K >$	L_0*, L_{100}
	$L_K =$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=50	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	L_0*, L_{100}	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	
	$L_K <$		$L_K <$		$L_K <$	

Table 41

f_5, vertical approach, MaxFEs = 250,000, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$	$L_0*, L_{100}*, L_{250}$	$L_K >$	$L_0*, L_{100}*, L_{250}, L_{500}, L_{1000}$	$L_K >$	$L_0*, L_{100}*, L_{500}$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	L_{250}
	$L_K <$	$L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{750}, L_{1500}, L_\infty$	$L_K <$	$L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=10	$L_K >$	$L_0*, L_{100}*, L_{250}, L_{500}$	$L_K >$	$L_0*, L_{100}*, L_{500}, L_{750}, L_{1500},$	$L_K >$	$L_0*, L_{100}*, L_{250}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{1000}, L_{1250}, L_\infty$	$L_K <$	
D=30	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_\infty*$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$		$L_K <$		$L_K <$	
D=50	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1500}*, L_\infty*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$		$L_K <$		$L_K <$	

Table 42

Horizontal approach, 10^{-6}

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1435	1386	1515	1562	1551	1505	1569	1560	1467	1452
SN=24 D=5	1396	1004	1585	1573	1576	1518	1592	1597	1562	1596
SN=24 D=10	1441	981	1252	1625	1636	1629	1637	1607	1610	1582
SN=24 D=30	1542	1040	1040	1290	1688	1689	1677	1630	1671	1733
SN=24 D=50	1587	1097	1097	1097	1519	1688	1755	1702	1743	1716
SN=50 D=2	1559	1382	1530	1533	1550	1491	1532	1499	1457	1465
SN=50 D=5	1604	995	1573	1553	1499	1635	1536	1468	1583	1555
SN=50 D=10	1665	981	1300	1665	1590	1583	1604	1626	1588	1564
SN=50 D=30	1725	1051	1063	1252	1652	1725	1680	1695	1674	1708
SN=50 D=50	1747	1111	1111	1111	1509	1714	1751	1747	1728	1719
SN=100 D=2	1453	1438	1453	1481	1496	1536	1505	1549	1525	1517
SN=100 D= 5	1549	1002	1555	1549	1652	1549	1510	1573	1540	1570
SN=100 D=10	1661	981	1281	1586	1661	1617	1543	1609	1605	1618
SN=100 D=30	1671	1050	1050	1283	1634	1696	1718	1690	1671	1708
SN=100 D=50	1678	1096	1096	1101	1473	1697	1712	1728	1719	1700

(a) Horizontal approach, f_1, 10^{-6}

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1553	1082	1566	1521	1578	1559	1537	1544	1565	1495
SN=24 D=5	1553	981	1576	1581	1571	1516	1548	1576	1553	1546
SN=24 D=10	1517	981	1557	1487	1581	1525	1629	1630	1557	1536
SN=24 D=30	1532	981	1558	1597	1505	1525	1591	1533	1620	1558
SN=24 D=50	1513	981	1528	1607	1573	1550	1583	1494	1552	1620
SN=50 D=2	1560	1058	1624	1589	1557	1555	1500	1501	1535	1521
SN=50 D=5	1538	981	1577	1591	1595	1541	1513	1558	1559	1547
SN=50 D=10	1618	981	1579	1618	1532	1574	1583	1547	1519	1566
SN=50 D=30	1603	981	1510	1582	1544	1603	1564	1592	1528	1595
SN=50 D=50	1579	981	1565	1516	1579	1569	1578	1579	1569	1565
SN=100 D=2	1534	1060	1534	1499	1595	1544	1555	1597	1547	1569
SN=100 D= 5	1592	981	1571	1592	1519	1608	1546	1521	1569	1593
SN=100 D=10	1641	981	1560	1570	1641	1566	1503	1597	1556	1526
SN=100 D=30	1539	981	1558	1603	1626	1481	1639	1500	1539	1574
SN=100 D=50	1589	981	1591	1563	1502	1553	1548	1560	1542	1570

(b) Horizontal approach, f_2, 10^{-6}

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1560	1184	1544	1495	1553	1535	1522	1555	1521	1530
SN=24 D=5	1562	981	1539	1589	1545	1545	1529	1580	1547	1583
SN=24 D=10	1576	981	1562	1622	1566	1507	1522	1585	1508	1571
SN=24 D=30	1566	981	1563	1578	1562	1542	1529	1575	1563	1542
SN=24 D=50	1600	981	1578	1552	1486	1565	1562	1576	1558	1542
SN=50 D=2	1500	1190	1462	1528	1591	1577	1538	1543	1538	1535
SN=50 D=5	1586	981	1548	1561	1574	1535	1559	1556	1497	1605
SN=50 D=10	1560	981	1564	1560	1551	1488	1608	1623	1569	1557
SN=50 D=30	1529	981	1560	1592	1514	1529	1555	1553	1603	1614
SN=50 D=50	1553	981	1544	1558	1516	1592	1577	1553	1613	1566
SN=100 D=2	1501	1169	1501	1523	1508	1575	1596	1520	1595	1513
SN=100 D= 5	1578	981	1613	1578	1603	1590	1571	1527	1514	1523
SN=100 D=10	1531	981	1616	1540	1531	1573	1555	1597	1503	1606
SN=100 D=30	1535	981	1577	1622	1542	1575	1581	1539	1535	1549
SN=100 D=50	1553	981	1559	1524	1572	1544	1514	1603	1591	1560

(c) Horizontal approach, f_3, 10^{-6}

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1411	1115	1588	1573	1528	1557	1534	1550	1518	1628
SN=24 D=5	1562	981	1533	1549	1546	1572	1629	1583	1516	1528
SN=24 D=10	1576	981	1536	1521	1578	1590	1531	1618	1518	1551
SN=24 D=30	1527	981	1598	1593	1554	1499	1593	1561	1585	1508
SN=24 D=50	1582	981	1575	1544	1535	1538	1523	1544	1617	1562
SN=50 D=2	1527	1100	1548	1538	1542	1543	1494	1553	1602	1555
SN=50 D=5	1573	981	1540	1502	1597	1583	1544	1554	1560	1566
SN=50 D=10	1552	981	1597	1552	1614	1587	1532	1551	1552	1535
SN=50 D=30	1578	981	1593	1531	1565	1578	1513	1590	1558	1591
SN=50 D=50	1535	981	1605	1622	1565	1565	1561	1560	1535	1506
SN=100 D=2	1575	1074	1575	1529	1597	1523	1578	1556	1553	1516
SN=100 D= 5	1574	981	1551	1574	1551	1559	1535	1544	1612	1593
SN=100 D=10	1590	981	1583	1549	1590	1557	1570	1592	1540	1538
SN=100 D=30	1586	981	1575	1535	1586	1547	1556	1599	1586	1536
SN=100 D=50	1567	981	1566	1510	1521	1597	1567	1554	1561	1576

(d) Horizontal approach, f_4, 10^{-6}

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1499	1499	1499	1499	1499	1499	1499	1499	1499	1510
SN=24 D=5	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=24 D=10	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=24 D=30	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=24 D=50	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=50 D=2	1498	1498	1498	1503	1498	1503	1503	1498	1498	1503
SN=50 D=5	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=50 D=10	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=50 D=30	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=50 D=50	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=100 D=2	1499	1505	1499	1499	1499	1499	1505	1499	1499	1499
SN=100 D= 5	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=100 D=10	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=100 D=30	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=100 D=50	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500

(e) Horizontal approach, f_5, 10^{-6}

least one other 'limit' value, regardless of the population size and dimension of a problem. Lastly, for f_5 and small dimension D =2, L_k had better alternatives, whilst for other dimensions and population sizes values, all 'limit' values performed the same. This happened due to the fact that none of these 'limit' values found the (sub-)optimal solution 10^{-6} in 1,000,000 available fitness evaluations. For $D = 2$, some 'limit' values found (sub-)optimal solution in some runs, and therefore performed better than L_k. Whilst there were a lot of differences found between L_k and other 'limit' values, these differences were hardly ever significant. There were only two problems for which L_k was significantly worse than some other 'limit' values. The first was f_1 where L_k was significantly worse for small population size $SN = 24$ and dimension $D = 5$. The other problem was f_4 where L_k was significantly worse than L_∞ for small population size SN =24 and small dimension $D = 2$. CRS4EAs again appeared as more conservative than NHST (compare Tables 15-19 with Tables 43-47).

2.3.4. Experiment 4: Horizontal Approach – 10^{-12} - *MaxFEs* = 1,000,000

In the horizontal approach with (sub-)optimal solution 10^{-12}, L_k again had better alternatives in almost all cases. In this approach, L_k was the optimal solution 50% fewer times than when the (sub-)optimal solution equaled 10^{-6}. Tables 48(a)-48(e) show the ratings obtained for all five optimisation problems.

For f_1, L_k always had a better alternative, except when $SN = 50$ and $D = 10$, $SN = 50$ and $D = 30$, and $SN = 100$ and $D = 2$. For f_2, L_k always had a better alternative and was always worse than at least one other 'limit' value, regardless of the population size and dimension of a problem. For f_3, L_k always had a better alternative and was always worse than at least one other ‘limit’ value, except when SN = 50 and D = 5. For f_4, L_k always had a better alternative and was always worse than at least one other ‘limit’ value, except when SN = 24 and D = 30 and SN = 100 and D = 30. Lastly, for f_5, all ‘limit’ values performed the same. This happened due to the fact that none of these ‘limit’ values found the (sub-)optimal solution 10^{-12} in 1,000,000 fitness evaluations. Whilst there were a lot of differences found between L_k and other ‘limit’ values, these differences were rarely significant. There were only two problems for which L_k was significantly worse than some other ‘limit’ values. The first was f_1 where L_k was significantly worse for small population size SN = 24 and dimensions D = {5, 10, 30, 50}. The other was problem f_4 where L_k was significantly worse for small population size SN = 24 and small dimension D = 2. CRS4EAs again appeared as more conservative than NHST (compare Tables 20-24 with Tables 49-53).

2.3.5. Experiment 5: Large Dimensions

In this section, the horizontal approach with (sub-)optimal solution set at 10^{-6} was repeated for larger dimensions, D = {100, 200, 300}. Again, fixed ‘limit’ values, L = {0, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10000, 11000, 12000, 13000, 14000, 15000, ∞}, were compared to Karaboga’s setting L_k. Obtained ratings are shown in Table 54 and found differences are shown in Tables 55-59. As in previous four experiments, this experiment showed that there are other ‘limit’ values that perform better than L_k, for certain problems (f_1) even significantly. In majority of D and SN settings and problems, at least one better performing ‘limit’ value was found. For f_5 none of the ‘limit’ values reached optimal solution, since all settings performed equally. By comparing Tables 55-59 with Tables 43-47, it can be observed that with higher dimensions L_k setting becomes less appropriate.

Table 43

f_1, horizontal approach, 10^{-6}, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0	$L_K >$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	L_0
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$		$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=5	$L_K >$	L_0*	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	L_0*, L_{1000}, L_{1500}
	$L_K =$		$L_K =$		$L_K =$	L_{250}, L_{750}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}*, L_{1500}, L_\infty$	$L_K <$	L_{750}	$L_K <$	$L_{100}, L_{500}, L_{1250}, L_\infty$
D=10	$L_K >$	L_0*, L_{100}	$L_K >$	$L_0*, L_{100}*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{250}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$		$L_K <$	
D=30	$L_K >$	$L_0*, L_{100}*, L_{250}*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$		$L_K <$	$L_{750}, L_{1000}, L_{1250}, L_\infty$
D=50	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	L_{1000}	$L_K <$	$L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$

Table 44

f_2, horizontal approach, 10^{-6}, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	$L_0*, L_{250}, L_{1000}, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{250},$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{100}, L_{500}, L_{750}, L_{1500}$	$L_K <$	L_{100}, L_{250}	$L_K <$	$L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=5	$L_K >$	$L_0*, L_{750}, L_{1000}, L_\infty$	$L_K >$	L_0*, L_{1000}	$L_K >$	$L_0, L_{100}, L_{500}, L_{1000}, L_{1250}, L_{1500}$
	$L_K =$	L_{1500}	$L_K =$		$L_K =$	L_{250}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1250}$	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	L_{750}, L_∞
D=10	$L_K >$	$L_0*, L_{250}*$	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0, L_{100}, L_{250}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$		$L_K <$	
D=30	$L_K >$	L_0*, L_{500}, L_{750}	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	L_0*, L_{750}, L_{1250}
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{100}, L_{250}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$		$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1000}, L_\infty$
D=50	$L_K >$	L_0*, L_{1250}	$L_K >$	$L_0*, L_{100}, L_{250}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	$L_0, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{500}, L_{1250}	$L_K =$	
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K <$		$L_K <$	L_{100}

Table 45

f_3, horizontal approach, 10^{-6}, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}*$	$L_K >$	L_0*
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$		$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=5	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1000}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K >$	$L_0*, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	$L_{250}, L_{1250}, L_\infty$	$L_K <$	L_∞	$L_K <$	$L_{100}, L_{500}, L_{750}$
D=10	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{500}, L_{750}, L_\infty$	$L_K >$	L_0*, L_{1500}
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	L_{250}, L_{1250}	$L_K <$	$L_{100}, L_{1000}, L_{1250}, L_{1500}$	$L_K <$	$L_{100}, L_{250}, L_{750}, L_{1000}, L_{1250}, L_\infty$
D=30	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{500},$	$L_K >$	L_0*
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	L_{250}, L_{1250}	$L_K <$	$L_{100}, L_{250}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$
D=50	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	L_0*, L_{100}, L_{500}	$L_K >$	$L_0*, L_{250}, L_{750}, L_{1000}$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$		$L_K <$	$L_{250}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{500}, L_{1250}, L_{1500}, L_\infty$

Table 46

f_4, horizontal approach, 10^{-6}, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*	$L_K >$	L_0*, L_{1000}	$L_K >$	$L_0*, L_{250}, L_{750}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty*$	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	L_{500}, L_{1000}
D=5	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}$
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	$L_{750}, L_{1000}, L_{1250}$	$L_K <$	$L_{500}, L_{750},$	$L_K <$	L_{1500}, L_∞
D=10	$L_K >$	$L_0*, L_{100}, L_{250}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{1000}, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{750}, L_{1000}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{250}, L_{1500}	$L_K =$	L_{500}
	$L_K <$	$L_{500}, L_{750}, L_{1250}$	$L_K <$	$L_{100}, L_{500}, L_{750}$	$L_K <$	L_{1250}
D=30	$L_K >$	L_0*, L_{750}, L_∞	$L_K >$	$L_0*, L_{250}, L_{500}, L_{1000}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{750}, L_{1000}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{500}, L_{1500}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}$	$L_K <$	$L_{100}, L_{1250}, L_\infty$	$L_K <$	L_{1250}
D=50	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$	$L_K >$	L_0*, L_{1500}, L_∞	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1250}, L_{1500}$
	$L_K =$		$L_K =$		$L_K =$	L_{1000}
	$L_K <$	L_{1500}	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}$	$L_K <$	L_{750}, L_∞

Table 47

f_5, horizontal approach, 10^{-6}, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K =$	$L_0, L_{100}, L_{500}, L_{1250}, L_{1500}$	$L_K =$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$	L_∞	$L_K <$	$L_{250}, L_{750}, L_{1000}, L_\infty$	$L_K <$	L_0, L_{1000}
D=5	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=10	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=30	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=50	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	

Table 48

Horizontal approach, 10^{-12}

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1514	1203	1557	1517	1517	1560	1558	1458	1565	1551
SN=24 D=5	1312	981	1715	1591	1525	1585	1551	1588	1565	1588
SN=24 D=10	1367	995	1100	1770	1636	1630	1672	1619	1570	1641
SN=24 D=30	1434	1097	1097	1097	1678	1787	1764	1719	1695	1633
SN=24 D=50	1507	1152	1152	1152	1207	1704	1821	1840	1785	1681
SN=50 D=2	1613	1193	1555	1494	1507	1480	1477	1533	1636	1511
SN=50 D=5	1663	981	1673	1532	1497	1529	1544	1528	1515	1539
SN=50 D=10	1691	996	1126	1691	1626	1598	1619	1674	1610	1558
SN=50 D=30	1748	1111	1111	1111	1708	1748	1726	1695	1666	1626
SN=50 D=50	1782	1165	1165	1165	1236	1706	1782	1782	1797	1701
SN=100 D=2	1603	1191	1603	1475	1486	1570	1544	1540	1566	1525
SN=100 D= 5	1588	981	1675	1588	1574	1542	1513	1508	1565	1555
SN=100 D=10	1677	1000	1100	1703	1677	1609	1551	1616	1599	1648
SN=100 D=30	1679	1111	1111	1111	1678	1743	1714	1723	1679	1631
SN=100 D=50	1719	1144	1144	1144	1195	1700	1761	1740	1758	1695

(a) Horizontal approach, f_1, 10^{-12}

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1458	983	1565	1565	1568	1579	1571	1547	1569	1596
SN=24 D=5	1571	981	1570	1524	1519	1531	1612	1555	1598	1539
SN=24 D=10	1593	981	1508	1565	1576	1587	1604	1537	1555	1494
SN=24 D=30	1569	981	1604	1557	1583	1558	1535	1561	1524	1530
SN=24 D=50	1558	981	1517	1527	1578	1600	1543	1565	1556	1576
SN=50 D=2	1575	981	1617	1582	1490	1550	1525	1536	1607	1538
SN=50 D=5	1559	981	1558	1550	1553	1531	1556	1591	1565	1555
SN=50 D=10	1582	981	1592	1582	1562	1508	1596	1592	1521	1566
SN=50 D=30	1532	981	1592	1556	1593	1532	1540	1561	1575	1569
SN=50 D=50	1593	981	1578	1555	1543	1555	1572	1593	1602	1521
SN=100 D=2	1526	981	1526	1577	1569	1564	1553	1557	1623	1550
SN=100 D= 5	1540	981	1584	1540	1597	1617	1550	1567	1509	1556
SN=100 D=10	1571	981	1600	1500	1571	1533	1538	1573	1618	1586
SN=100 D=30	1586	981	1545	1543	1559	1616	1558	1555	1586	1557
SN=100 D=50	1576	981	1537	1539	1548	1605	1546	1545	1576	1547

(b) Horizontal approach, f_2, 10^{-12}

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1541	990	1562	1583	1592	1560	1547	1599	1529	1498
SN=24 D=5	1576	981	1585	1543	1523	1585	1565	1566	1566	1510
SN=24 D=10	1569	981	1530	1588	1610	1555	1557	1521	1498	1591
SN=24 D=30	1528	981	1582	1527	1527	1569	1542	1561	1583	1602
SN=24 D=50	1543	981	1580	1566	1568	1541	1554	1595	1582	1491
SN=50 D=2	1596	991	1565	1591	1531	1514	1532	1546	1498	1636
SN=50 D=5	1603	981	1578	1548	1576	1515	1521	1536	1566	1576
SN=50 D=10	1558	981	1591	1558	1529	1547	1573	1548	1565	1609
SN=50 D=30	1544	981	1601	1539	1601	1544	1570	1540	1586	1538
SN=50 D=50	1548	981	1556	1580	1554	1529	1571	1548	1587	1595
SN=100 D=2	1568	985	1568	1563	1556	1492	1566	1573	1608	1590
SN=100 D= 5	1556	981	1542	1556	1602	1573	1582	1557	1575	1533
SN=100 D=10	1552	981	1539	1551	1552	1601	1575	1531	1637	1534
SN=100 D=30	1560	981	1613	1549	1556	1561	1591	1504	1560	1586
SN=100 D=50	1582	981	1566	1595	1599	1575	1528	1515	1552	1508

(c) Horizontal approach, f_3, 10^{-12}

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1370	981	1632	1584	1561	1565	1599	1570	1538	1600
SN=24 D=5	1583	981	1517	1551	1542	1543	1544	1544	1625	1570
SN=24 D=10	1562	981	1535	1576	1521	1562	1581	1602	1519	1561
SN=24 D=30	1617	981	1596	1544	1516	1580	1561	1497	1596	1513
SN=24 D=50	1543	981	1521	1566	1552	1595	1550	1629	1493	1572
SN=50 D=2	1553	981	1572	1561	1558	1543	1587	1566	1524	1557
SN=50 D=5	1560	981	1538	1588	1562	1563	1531	1588	1541	1548
SN=50 D=10	1530	981	1551	1530	1558	1636	1577	1545	1580	1542
SN=50 D=30	1587	981	1528	1565	1578	1587	1593	1574	1502	1592
SN=50 D=50	1582	981	1523	1568	1540	1613	1565	1582	1577	1552
SN=100 D=2	1587	981	1587	1527	1552	1579	1538	1603	1571	1562
SN=100 D= 5	1512	981	1590	1512	1539	1555	1584	1586	1603	1551
SN=100 D=10	1571	981	1514	1574	1571	1544	1612	1522	1583	1599
SN=100 D=30	1618	981	1608	1578	1539	1548	1538	1571	1618	1519
SN=100 D=50	1490	981	1501	1533	1591	1555	1553	1577	1605	1613

(d) Horizontal approach, f_4, 10^{-12}

	L_K	L_0	L_{100}	L_{250}	L_{500}	L_{750}	L_{1000}	L_{1250}	L_{1500}	L_∞
SN=24 D=2	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=24 D=5	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=24 D=10	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=24 D=30	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=24 D=50	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=50 D=2	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=50 D=5	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=50 D=10	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=50 D=30	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=50 D=50	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=100 D=2	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=100 D= 5	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=100 D=10	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=100 D=30	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
SN=100 D=50	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500

(e) Horizontal approach, f_5, 10^{-12}

2.3.6. Discussion

The results between CRS4EAs and NHST were comparable. When smaller numbers of fitness evaluations were available, L_k was an appropriate choice only for small dimensions; and when sufficiently large enough numbers of fitness evaluations were available, L_k was a significantly better choice than the presented fixed 'limit' values. When it was of interest to find a (sub-) optimal solution and large numbers of fitness evaluations were available, better alternatives than L_k were available several times. The main difference between NHST and CRS4EAs comparison was that CRS4EAs was more conservative and detected less significant differences than NHST, however, the conservativity/liberality can be easily controlled through rating deviation *RD* [48]. Otherwise, the methods showed the same trends when and for which population size and dimension L_k was an unsuitable choice and when it was a suitable choice. Hence, the main conclusion as presented in Section 2.2.5 is that using NHST was the same as using CRS4EAs. The main differences between both methods showed in the abilities to detect differences amongst fixed 'limit' values. Whilst for NHST only the differences between L_k and other fixed 'limit' values were calculated and detected, CRS4EAs allowed direct comparisons between fixed 'limit' values. In order to find the differences between the fixed 'limit' values in NHST, additional tests would be needed, which would be both time consuming and require special care to avoid Type-I-Error.

3. ABC Parameter Tuning

In the previous section, it was shown that ABC does not always perform best when under the setting 'limit'

$= n_e * D$. Hence, the 'limit' control parameter should be tuned or controlled. Therefore, this section displays the results of ABC tuning in contrast to the suggested 'limit' setting and to the statistical analysis in Section 2.

Tuning is a process of finding those parameter values for which the meta-heuristic algorithm performs the best for selected sets of problems F. A combination of different parameter values is called configuration. One of the more common and easy-to-apply tuning methods is F-Race [4], which empirically evaluates a set of parameter values and discards the bad ones as soon as statistically sufficient evidence – supported by the Friedman test [13], [14] – is gathered against them.

Table 49

f_1, horizontal approach, 10^{-12}, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*, L_{1250}	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}, L_\infty$	$L_K <$	L_{1500}	$L_K <$	
D=5	$L_K >$	L_0*	$L_K >$	$L_0*, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	$L_{100}*, L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$	L_{100}	$L_K <$	L_{100}
D=10	$L_K >$	$L_0*, L_{100}*$	$L_K >$	$L_0*, L_{100}*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{250}*, L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty*$	$L_K <$		$L_K <$	L_{250}
D=30	$L_K >$	$L_0*, L_{100}*, L_{250}*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{500}*, L_{750}*, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty$	$L_K <$		$L_K <$	$L_{750}, L_{1000}, L_{1250}$
D=50	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}, L_\infty$	$L_K >$	$L_0*, L_{100}*, L_{250}*, L_{500}*, L_{750}, L_\infty$
	$L_K =$		$L_K =$	L_{1000}, L_{1250}	$L_K =$	
	$L_K <$	$L_{750}, L_{1000}*, L_{1250}*, L_{1500}*, L_\infty$	$L_K <$	L_{1500}	$L_K <$	$L_{1000}, L_{1250}, L_{1500}$

Table 50

f_2, horizontal approach, 10^{-12}, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*	$L_K >$	$L_0*, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$	$L_K >$	L_0*
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{250}, L_{1500}$	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=5	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_\infty$	$L_K >$	L_0*, L_{1500}
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	L_{1000}, L_{1500}	$L_K <$	L_{1250}, L_{1500}	$L_K <$	$L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$
D=10	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{500}, L_{750}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{250}, L_{750}, L_{1000}$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	L_{1000}	$L_K <$	$L_{100}, L_{1000}, L_{1250}$	$L_K <$	$L_{100}, L_{1250}, L_{1500}, L_\infty$
D=30	$L_K >$	$L_0*, L_{250}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	L_0*	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	L_{100}, L_{500}	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	L_{750}
D=50	$L_K >$	$L_0*, L_{100}, L_{250}, L_{1000}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{500}, L_{750}, L_{1250}, L_\infty$	$L_K <$	L_{1500}	$L_K <$	L_{750}, L_{1500}

Table 51

f_3, horizontal approach, 10^{-12}, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*, L_{1500}, L_∞	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K >$	$L_0*, L_{250}, L_{500}, L_{750}, L_{1000}$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}$	$L_K <$	L_∞	$L_K <$	$L_{1250}, L_{1500}, L_\infty$
D=5	$L_K >$	$L_0, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	L_0, L_{100}, L_∞
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	L_{100}, L_{750}	$L_K <$		$L_K <$	$L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}$
D=10	$L_K >$	$L_0*, L_{100}, L_{750}, L_{1000}, L_{1250}, L_{1500}$	$L_K >$	$L_0, L_{500}, L_{750}, L_{1250}$	$L_K >$	$L_0, L_{100}, L_{250}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{250}, L_{500}, L_\infty$	$L_K <$	$L_{100}, L_{1000}, L_{1500}, L_\infty$	$L_K <$	$L_{750}, L_{1000}, L_{1500}$
D=30	$L_K >$	L_0*, L_{250}, L_{500}	$L_K >$	$L_0*, L_{250}, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{250}, L_{500}, L_{1250}$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$	$L_{100}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{100}, L_{500}, L_{1000}, L_{1500}$	$L_K <$	$L_{100}, L_{750}, L_{1000}, L_\infty$
D=50	$L_K >$	L_0*, L_{750}, L_∞	$L_K >$	L_0*, L_{750}	$L_K >$	$L_0, L_{100}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_{1500}$	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1000}, L_{1500}, L_\infty$	$L_K <$	L_{250}, L_{500}

Table 52

f_4, horizontal approach, 10^{-12}, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$	L_0*	$L_K >$	L_0*, L_{750}, L_{1500}	$L_K >$	$L_0*, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1500}, L_\infty$
	$L_K =$		$L_K =$		$L_K =$	L_{100}
	$L_K <$	$L_{100}*, L_{250}*, L_{500}, L_{750}, L_{1000}*, L_{1250}, L_{1500}, L_\infty*$	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{1000}, L_{1250}, L_\infty$	$L_K <$	L_{1250}
D=5	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	L_0
	$L_K =$		$L_K =$		$L_K =$	L_{250}
	$L_K <$	L_{1500}	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1250}$	$L_K <$	$L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
D=10	$L_K >$	$L_0*, L_{100}, L_{500}, L_{750}, L_{1500}, L_\infty$	$L_K >$	L_0*	$L_K >$	$L_0*, L_{100}, L_{750}, L_{1250}$
	$L_K =$		$L_K =$	L_{250}	$L_K =$	L_{500}
	$L_K <$	$L_{250}, L_{1000}, L_{1250}$	$L_K <$	$L_{100}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K <$	$L_{250}, L_{1000}, L_{1500}, L_\infty$
D=30	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1250}, L_{1500}$	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$
	$L_K =$		$L_K =$	L_{750}	$L_K =$	L_{1500}
	$L_K <$		$L_K <$	L_{1000}, L_∞	$L_K <$	
D=50	$L_K >$	L_0*, L_{100}, L_{1500}	$L_K >$	$L_0*, L_{100}, L_{250}, L_{500}, L_{1000}, L_{1500}, L_\infty$	$L_K >$	L_0*
	$L_K =$		$L_K =$	L_{1250}	$L_K =$	
	$L_K <$	$L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_\infty$	$L_K <$	L_{750}	$L_K <$	$L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$

Table 53

f_5, horizontal approach, 10^{-12}, CRS4EAs

		SN=24		SN=50		SN=100
D=2	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=5	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=10	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=30	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	
D=50	$L_K >$		$L_K >$		$L_K >$	
	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$	$L_K =$	$L_0, L_{100}, L_{250}, L_{500}, L_{750}, L_{1000}, L_{1250}, L_{1500}, L_\infty$
	$L_K <$		$L_K <$		$L_K <$	

Before the tuning procedure starts, the user has to define the initial set P of all configurations that will be tested, number of initial races r, significance level α under which the statistical tests will be applied, and maximum number of executions. In each iteration, all configurations from P will be executed on one random problem from the set F over n_f independent runs. After that, if the number of iterations is greater than r, a Friedman test will be applied to see if there are significant differences amongst all configurations in P. If the Friedman test shows that there are significant differences, a post-hoc test, such as Holm test [19] is applied between the best performing configuration (the one with the smallest Friedman rank) and other configurations. Those configurations that are significantly worse than the best performing configuration under significance level α are removed from set P. This procedure is repeated until the maximum number of executions is reached or only one configuration remains in P (Algorithm 2). As already described, one execution is treated as the execution of one configuration on one problem from F over n_f independent runs.

To test the suggested formula for parameter 'limit' further, we tuned parameters SN and 'limit' for different dimensions D on problem f_1 with vertical approach and maximum number of fitness evaluations 100,000. The number of independent runs n_f = 25, the number of initial races r = 5, significance level α = 0.05, and maximum number of executions equaled 15,000. A goal of this experiment was to find the parameter values *SN* and 'limit' for which ABC will perform the best on f_1 for different dimensions. Hence, some boundaries and precisions of these two parameters needed to be set. The values parameter *SN* could take were {10, 20, 30, ... , 100}, and the values parameter 'limit' could take were {0, 50, 100, 150, ... , 1450, 1500, ∞}. These values are different from those used in the experiment in Section 2, as the goal of this experiment is different as well. In this experiment, we wanted to tune the parameters of ABC and in the experiment from Section 2 the goal was to make a pairwise comparison of pre-selected values. In other words, the values of *SN* and *D* were fixed in Section 2 and the performances of different 'limit' settings compared to Karaboga's 'limit' setting. In this section, on the other hand, only the allowed values of *SN* and 'limit' were defined, and the best settings of *SN* and 'limit' for each fixed value of dimension *D* were selected with a tuning process. The size of the initial population *P* equaled 320 (10*32 combinations, 10 for *SN* and 32 for 'limit'). The conclusions of the tuning process are summarised as follows.

- When the dimensionality of a problem was set to *D* = 2, 62 configurations remained from the initial set *P*. The values of parameter *SN* were from 20 to 100, and the values of parameter 'limit' were from 100 to 500. The best performing configurations (those with the lowest Friedman ranks) were {*SN* = 60, 'limit' = 200}, {*SN* =40, 'limit' = 200}, and {*SN* =80, 'limit' = 200}. Following the Karaboga's formula, the ratio between 'limit' and *SN* when *D* =2 should be 1:1, meaning that 'limit' should have the same value as *SN*. None of the configurations found by the tuning process corresponded to this formula.
- When the dimensionality of a problem was set to *D* = 5, 19 configurations remained from the initial set *P*. The values of parameter *SN* were from 30 to 50, and the values of parameter 'limit' were from 300 to 700. The best performing configurations were {*SN* =30, 'limit' = 300}, {*SN* = 40, 'limit' = 400}, and {*SN* = 40, 'limit' = 550}. Following the Karaboga's formula, the ratio between 'limit' and *SN* when *D* = 5 should be 2.5:1, meaning that 'limit' should be 2.5-times greater than *SN*. None of the configurations found by the tuning process corresponded to this formula.

Table 54

Horizontal approach, 10^{-6}, large dimension

	L_K	L_0	L_{1000}	L_{2000}	L_{3000}	L_{4000}	L_{5000}	L_{6000}	L_{7000}	L_{8000}	L_{9000}	L_{10000}	L_{11000}	L_{12000}	L_{13000}	L_{14000}	L_{15000}	L_{inf}
SN=24 D=100	1427	983	1277	1577	1576	1537	1531	1566	1629	1482	1609	1587	1617	1524	1526	1515	1532	1504
SN=24 D=200	1450	1031	1031	1254	1503	1543	1612	1604	1603	1611	1565	1590	1529	1652	1606	1634	1584	1597
SN=24 D=300	1418	1165	1165	1165	1337	1467	1547	1591	1645	1620	1613	1597	1676	1611	1580	1639	1612	1552
SN=50 D=100	1487	983	1236	1568	1527	1642	1569	1556	1556	1525	1548	1539	1495	1542	1557	1557	1585	1529
SN=50 D=200	1634	1033	1033	1168	1518	1547	1634	1609	1605	1559	1578	1582	1597	1619	1503	1610	1609	1564
SN=50 D=300	1622	1143	1143	1156	1305	1511	1567	1631	1670	1601	1624	1574	1542	1532	1536	1607	1633	1605
SN=100 D=100	1527	984	1278	1472	1556	1585	1527	1535	1574	1611	1561	1520	1592	1572	1514	1563	1529	1501
SN=100 D= 200	1588	1029	1029	1217	1533	1605	1591	1583	1604	1615	1627	1588	1667	1535	1573	1529	1560	1527
SN=100 D=300	1620	1146	1146	1146	1328	1543	1561	1550	1579	1600	1597	1630	1536	1619	1659	1539	1620	1579

(a) Horizontal approach, f_1, 10^{-6}

	L_K	L_0	L_{1000}	L_{2000}	L_{3000}	L_{4000}	L_{5000}	L_{6000}	L_{7000}	L_{8000}	L_{9000}	L_{10000}	L_{11000}	L_{12000}	L_{13000}	L_{14000}	L_{15000}	L_{inf}
SN=24 D=100	1543	981	1535	1571	1529	1504	1476	1546	1499	1451	1497	1576	1604	1540	1474	1567	1600	1507
SN=24 D=200	1621	981	1498	1423	1583	1526	1480	1513	1588	1533	1504	1565	1557	1524	1608	1467	1469	1561
SN=24 D=300	1533	981	1518	1521	1454	1494	1512	1549	1557	1510	1452	1632	1548	1582	1535	1523	1526	1572
SN=50 D=100	1543	981	1490	1513	1490	1520	1542	1499	1641	1490	1554	1540	1498	1558	1542	1515	1553	1532
SN=50 D=200	1521	981	1518	1573	1507	1540	1521	1567	1508	1583	1491	1578	1523	1519	1538	1479	1540	1515
SN=50 D=300	1567	981	1511	1505	1550	1612	1564	1590	1496	1486	1527	1496	1495	1557	1481	1579	1482	1520
SN=100 D=100	1531	981	1597	1454	1469	1491	1531	1498	1548	1594	1568	1527	1505	1554	1526	1556	1498	1571
SN=100 D= 200	1542	981	1568	1544	1579	1486	1546	1558	1480	1481	1462	1542	1623	1510	1489	1531	1474	1604
SN=100 D=300	1587	981	1548	1468	1545	1582	1483	1502	1518	1467	1578	1551	1546	1542	1522	1507	1587	1487

(b) Horizontal approach, f_2, 10^{-6}

	L_K	L_0	L_{1000}	L_{2000}	L_{3000}	L_{4000}	L_{5000}	L_{6000}	L_{7000}	L_{8000}	L_{9000}	L_{10000}	L_{11000}	L_{12000}	L_{13000}	L_{14000}	L_{15000}	L_{inf}
SN=24 D=100	1490	981	1594	1543	1477	1535	1517	1523	1543	1475	1490	1531	1526	1589	1590	1509	1513	1575
SN=24 D=200	1485	981	1503	1573	1584	1526	1562	1571	1505	1586	1401	1556	1538	1562	1502	1487	1567	1512
SN=24 D=300	1544	981	1570	1553	1496	1544	1496	1546	1539	1515	1527	1565	1497	1498	1529	1488	1570	1541
SN=50 D=100	1478	981	1580	1459	1474	1604	1505	1548	1561	1492	1560	1451	1568	1584	1538	1534	1538	1544
SN=50 D=200	1474	981	1565	1484	1540	1547	1474	1497	1498	1575	1616	1541	1507	1523	1556	1589	1511	1523
SN=50 D=300	1507	981	1531	1553	1503	1553	1540	1553	1528	1520	1467	1513	1488	1584	1559	1531	1581	1508
SN=100 D=100	1523	981	1484	1559	1522	1512	1523	1540	1543	1511	1531	1502	1496	1535	1545	1548	1570	1574
SN=100 D= 200	1540	981	1541	1563	1498	1530	1545	1510	1511	1555	1593	1540	1469	1600	1479	1539	1525	1482
SN=100 D=300	1535	981	1513	1489	1474	1570	1509	1482	1551	1582	1565	1612	1531	1489	1608	1480	1535	1491

(c) Horizontal approach, f_3, 10^{-6}

	L_K	L_0	L_{1000}	L_{2000}	L_{3000}	L_{4000}	L_{5000}	L_{6000}	L_{7000}	L_{8000}	L_{9000}	L_{10000}	L_{11000}	L_{12000}	L_{13000}	L_{14000}	L_{15000}	L_{inf}
SN=24 D=100	1538	981	1543	1462	1525	1533	1499	1585	1573	1512	1553	1518	1502	1516	1545	1500	1597	1520
SN=24 D=200	1464	1083	1573	1534	1437	1565	1492	1480	1547	1494	1540	1563	1562	1527	1521	1552	1419	1645
SN=24 D=300	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1505	1500
SN=50 D=100	1454	981	1509	1524	1507	1545	1509	1502	1620	1436	1534	1643	1481	1532	1571	1600	1513	1540
SN=50 D=200	1531	1093	1506	1515	1572	1487	1531	1596	1526	1471	1544	1528	1547	1463	1503	1465	1507	1616
SN=50 D=300	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1505	1500
SN=100 D=100	1558	981	1525	1520	1366	1517	1558	1580	1513	1554	1597	1571	1484	1582	1493	1527	1641	1433
SN=100 D= 200	1475	1082	1518	1578	1556	1445	1603	1592	1501	1560	1530	1475	1525	1551	1389	1560	1492	1568
SN=100 D=300	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1505	1500

(d) Horizontal approach, f_4, 10^{-6}

	L_K	L_0	L_{1000}	L_{2000}	L_{3000}	L_{4000}	L_{5000}	L_{6000}	L_{7000}	L_{8000}	L_{9000}	L_{10000}	L_{11000}	L_{12000}	L_{13000}	L_{14000}	L_{15000}	L_{inf}
SN=24 D=100	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1505	1500
SN=24 D=200	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1505	1500
SN=24 D=300	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1505	1500
SN=50 D=100	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1505	1500
SN=50 D=200	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1505	1500
SN=50 D=300	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1505	1500
SN=100 D=100	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1505	1500
SN=100 D= 200	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1505	1500
SN=100 D=300	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1505	1500

(e) Horizontal approach, f_5, 10^{-6}

Table 55

f_1, horizontal approach, 10^{-6}, large dimension, CRS4EAs

	SN=24	SN=50	SN=100
D=100	$L_K > L_0*, L_{1000}$	$L_K > L_0*, L_{1000}*$	$L_K > L_0*, L_{1000}*, L_{2000}, L_{10000}, L_{13000}, L_\infty$
	$L_K =$	$L_K =$	$L_K = L_{5000}$
	$L_K < L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}*, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K < L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K < L_{3000}, L_{4000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{11000}, L_{12000}, L_{14000}, L_{15000}$
D=200	$L_K > L_0*, L_{1000}*, L_{2000}$	$L_K > L_0*, L_{1000}*, L_{2000}*, L_{3000}, L_{4000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K > L_0*, L_{1000}*, L_{2000}*, L_{3000}, L_{6000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
	$L_K =$	$L_K = L_{5000}$	$L_K = L_{10000}$
	$L_K < L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}*, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K <$	$L_K < L_{4000}, L_{5000}, L_{7000}, L_{8000}, L_{9000}, L_{11000}$
D=300	$L_K > L_0*, L_{1000}*, L_{2000}*, L_{3000}$	$L_K > L_0*, L_{1000}*, L_{2000}*, L_{3000}, L_{4000}, L_{5000}, L_{8000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K > L_0*, L_{1000}*, L_{2000}*, L_{3000}*, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{11000}, L_{12000}, L_{14000}, L_\infty$
	$L_K =$	$L_K =$	$L_K = L_{15000}$
	$L_K < L_{4000}, L_{5000}, L_{6000}, L_{7000}*, L_{8000}*, L_{9000}, L_{10000}, L_{11000}*, L_{12000}, L_{13000}, L_{14000}*, L_{15000}, L_\infty$	$L_K < L_{6000}, L_{7000}, L_{9000}, L_{15000}$	$L_K < L_{10000}, L_{13000}$

Table 56

f_2, horizontal approach, 10^{-6}, large dimension, CRS4EAs

	SN=24	SN=50	SN=100
D=100	$L_K > L_0*, L_{1000}, L_{3000}, L_{4000}, L_{5000}, L_{7000}, L_{8000}, L_{9000}, L_{12000}, L_{13000}, L_\infty$	$L_K > L_0*, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{8000}, L_{10000}, L_{11000}, L_{13000}, L_{14000}, L_\infty$	$L_K > L_0*, L_{2000}, L_{3000}, L_{4000}, L_{6000}, L_{10000}, L_{11000}, L_{13000}, L_{15000}$
	$L_K =$	$L_K =$	$L_K = L_{5000}$
	$L_K < L_{2000}, L_{6000}, L_{10000}, L_{11000}, L_{14000}, L_{15000}$	$L_K < L_{7000}, L_{9000}, L_{12000}, L_{15000}$	$L_K < L_{1000}, L_{7000}, L_{8000}, L_{9000}, L_{12000}, L_{14000}, L_\infty$
D=200	$L_K > L_0*, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K > L_0*, L_{1000}, L_{3000}, L_{7000}, L_{9000}, L_{12000}, L_{14000}, L_\infty$	$L_K > L_0*, L_{4000}, L_{7000}, L_{8000}, L_{9000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}$
	$L_K =$	$L_K = L_{5000}$	$L_K = L_{10000}$
	$L_K <$	$L_K < L_{2000}, L_{4000}, L_{6000}, L_{8000}, L_{10000}, L_{11000}, L_{13000}, L_{15000}$	$L_K < L_{1000}, L_{2000}, L_{3000}, L_{5000}, L_{6000}, L_{11000}, L_\infty$
D=300	$L_K > L_0*, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{8000}, L_{9000}, L_{14000}, L_{15000}$	$L_K > L_0*, L_{1000}, L_{2000}, L_{3000}, L_{5000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{15000}, L_\infty$	$L_K > L_0*, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_\infty$
	$L_K =$	$L_K =$	$L_K = L_{15000}$
	$L_K < L_{6000}, L_{7000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_\infty$	$L_K < L_{4000}, L_{6000}, L_{14000}$	$L_K <$

Table 57

f_3, horizontal approach, 10^{-6}, large dimension, CRS4EAs

	SN=24	SN=50	SN=100
D=100	$L_K > L_0*, L_{3000}, L_{8000}$	$L_K > L_0*, L_{2000}, L_{3000}, L_{10000}$	$L_K > L_0*, L_{1000}, L_{3000}, L_{4000}, L_{8000}, L_{10000}, L_{11000}$
	$L_K =$	$L_K =$	$L_K = L_{5000}$
	$L_K < L_{1000}, L_{2000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K < L_{1000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K < L_{2000}, L_{6000}, L_{7000}, L_{9000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
D=200	$L_K > L_0*, L_{9000}$	$L_K > L_0*$	$L_K > L_0*, L_{3000}, L_{4000}, L_{6000}, L_{7000}, L_{11000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
	$L_K =$	$L_K = L_{5000}$	$L_K = L_{10000}$
	$L_K < L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K < L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K < L_{1000}, L_{2000}, L_{5000}, L_{8000}, L_{9000}, L_{12000}$
D=300	$L_K > L_0*, L_{3000}, L_{5000}, L_{7000}, L_{8000}, L_{9000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_\infty$	$L_K > L_0*, L_{3000}, L_{9000}, L_{11000}$	$L_K > L_0*, L_{1000}, L_{2000}, L_{3000}, L_{5000}, L_{6000}, L_{11000}, L_{12000}, L_{14000}, L_\infty$
	$L_K =$	$L_K =$	$L_K = L_{15000}$
	$L_K < L_{1000}, L_{2000}, L_{4000}, L_{6000}, L_{10000}, L_{15000}$	$L_K < L_{1000}, L_{2000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{10000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K < L_{4000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{13000}$

Table 58

f_4, horizontal approach, 10^{-6}, large dimension, CRS4EAs

	SN=24	SN=50	SN=100
D=100	$L_K > L_0*, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{8000}, L_{10000}, L_{11000}, L_{12000}, L_{14000}, L_\infty$	$L_K > L_0*, L_{8000}$	$L_K > L_0*, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{7000}, L_{8000}, L_{11000}, L_{13000}, L_{14000}, L_\infty$
	$L_K =$	$L_K =$	$L_K = L_{5000}$
	$L_K < L_{1000}, L_{6000}, L_{7000}, L_{9000}, L_{13000}, L_{15000}$	$L_K < L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K < L_{6000}, L_{9000}, L_{10000}, L_{12000}, L_{15000}$
D=200	$L_K > L_0*, L_{3000}, L_{15000}$	$L_K > L_0*, L_{1000}, L_{2000}, L_{4000}, L_{7000}, L_{8000}, L_{10000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}$	$L_K > L_0*, L_{4000}, L_{13000}$
	$L_K =$	$L_K = L_{5000}$	$L_K = L_{10000}$
	$L_K < L_{1000}, L_{2000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_\infty$	$L_K < L_{3000}, L_{6000}, L_{9000}, L_{11000}, L_\infty$	$L_K < L_{1000}, L_{2000}, L_{3000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{11000}, L_{12000}, L_{14000}, L_{15000}, L_\infty$
D=300	$L_K > L_K >$	$L_K >$	
	$L_K = L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_\infty$	$L_K = L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K = L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
	$L_K < L_{15000}$	$L_K <$	$L_K <$

Table 59

f_5, horizontal approach, 10^{-6}, large dimension, CRS4EAs

	SN=24	SN=50	SN=100
D=100	$L_K >$	$L_K >$	$L_K >$
	$L_K = L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K = L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K = L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
	$L_K <$	$L_K <$	$L_K <$
D=200	$L_K >$	$L_K >$	$L_K >$
	$L_K = L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K = L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K = L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
	$L_K <$	$L_K <$	$L_K <$
D=300	$L_K >$	$L_K >$	$L_K >$
	$L_K = L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K = L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$	$L_K = L_0, L_{1000}, L_{2000}, L_{3000}, L_{4000}, L_{5000}, L_{6000}, L_{7000}, L_{8000}, L_{9000}, L_{10000}, L_{11000}, L_{12000}, L_{13000}, L_{14000}, L_{15000}, L_\infty$
	$L_K <$	$L_K <$	$L_K <$

- When the dimensionality of a problem was set to D = 10, four configurations remained from the initial set P. The values of parameter SN were 30, and the values of parameter 'limit' were from 800 to 1000. Those four configurations were {SN =30, 'limit' = 800}, {SN =30, 'limit' = 1000}, {SN = 30, 'limit' = 950}, and {SN =30, 'limit' = 850}. Following the Karaboga's formula, the ratio between 'limit' and SN when D = 10 should be 5:1, meaning that 'limit' should be 5-times greater than SN. None of the configurations found by the tuning process corresponded to this formula.
- When the dimensionality of a problem was set to D = 30, 70 configurations remained from the initial set P. The values of parameter SN were from 10 to 70, and the values of parameter 'limit' were from 700 to ∞. The best performing configurations were {SN = 20, 'limit' = 1450}, {SN = 20, 'limit' = 1250}, and {SN = 20, 'limit' = 1500}. Following the Karaboga's formula, the ratio between 'limit' and SN when D = 30 should be 15:1, meaning that 'limit' should be 15-times greater than SN. None of the configurations found by the tuning process corresponded to this formula.
- When the dimensionality of a problem was set to D = 50, 13 configurations remained from the initial set P. The values of parameter SN were from 20 to 40, and the values of parameter 'limit' were from 1100 to ∞. The best performing configurations were {SN =20, 'limit' = ∞}, {SN = 30, 'limit' = ∞}, and {SN = 40, 'limit' = ∞}. Following the Karaboga's formula, the ratio between 'limit' and SN when D = 50 should be 25:1, meaning that 'limit' should be 25-times greater than SN. None of the configurations found by the tuning process corresponded to this formula.

One would expect that these results can be compared to those in Tables 5 and 31, however as Tables 5 and 31 display the answers to different questions (as already explained above), the results and conclusions of these experiments cannot be compared directly. There are, however, some similarities between the conclusions of both sections. For example, when D = 10 it can be noticed that F-Race found the following best configurations: SN = 30 and 'limit' = {800, 950, 1000}. Whilst, from Tables 5 and 31 it can be noticed that configurations with SN = 24 and 'limit' = {750, 1000, 1250} are significantly better statistically than L_k under NHST and CRS4EAs. Or, when D = 30 it can be noticed that F-Race recommended the following best configurations: SN = 20 and 'limit' = {1250, 1450, 1500}. Whilst, from Tables 5 and 31 it can be noticed that configurations with SN =24 and 'limit' = {1000, 1250, 1500} are significantly better statistically than L_k under NHST and only better, but not statistically significant, under CRS4EAs as CRS4EAs is more conservative than NHST in this experiment.

Overall, the results of ABC parameter tuning showed that the best performing configurations did not correspond to the Karaboga's formula for f_1. Similar conclusion can be derived from recent study [49] where for ABC parameter tuning F-Race, Revac, and CRS-Tuning have been used. From the best performing configurations found by F-Race, Revac, and CRS-Tuning none conform to the Karaboga formula.

4. Related Work

To date there have been no deep investigations about setting ABC control parameter 'limit'. The formula 'limit' = $n_e * D$ was first proposed in the ABC introductory paper [29] and since then used in many papers (e.g., [6], [21], [24], [27], [28], [31], [54]). The effect of 'limit', as investigated by ABC inventors [29], has been studied on the same benchmark suite f_1, ... , f_5 as presented in Section 2 (actually we used the same benchmark suite as in [29]) using the following factors and their values: SN ={20, 40, 100}, D = {2, 5, 50}, and 'limit' ={0.1*n_e*D, 0.5*n_e*D, n_e*D, ∞}. However, full factorial design has not been used since D = 2 was used only for f_1, D = 5 for f_2, and D = 50 for f_3, ... , f_5. Furthermore, the only vertical approaches applied in [26] used 20,000 fitness evaluations for f_1, f_2, and 100,000 fitness evaluations for f_3, ... , f_5. In our work, Karaboga's experiment [8] has been extended by performing a full factorial design on this benchmark suite using the following factors and their values: SN = {24, 50, 100}, D = {2, 5, 10, 30, 50}, and 'limit' = {0, 100, 250, 500, 750, 1000, 1250, 1500, ∞}, whilst using two different horizontal and vertical approaches [18]. The other difference between these two studies is that 30 independent runs were used in [8], whilst 100 in our study in order to enhance its reliability.

The effect of 'limit' on ABC was briefly studied in [1] on functions f_1 ... f_5, Ackley and Weierstrass with a vertical approach (30,000 fitness evaluations, 30 independent runs) using the following factors and their values: SN = {10}, D = {10}, and 'limit' = {10, 200, 500, 1000, 3000, 5000}. It was found that 'limit' = 200 was more appropriate than other values used in this study. Again, our study can be seen as an extension of [1].

A similar study as in [1] on the effect of 'limit' has been recently performed in [30] using a variant of ABC called the quick artificial bee colony (qABC) algorithm. The vertical approach has been applied with 500,000 function evaluations and 30 independent runs on a benchmark suite containing optimisation functions f_2, ... , f_5. The following factors and their values have been used: SN= {50}, D = {30}, and 'limit' = {10, 50, 187, 375, 750, 1500}. It was found that the 'limit' = 750 is the more suitable value, which is equal to the value calculated from the formula 'limit' = $n_e * D$.

The Enhancing artificial bee colony (EABC) algorithm has been proposed in [15] and tested on 48 benchmark functions. The effect of 'limit' on EABC was investigated with vertical approach (150,000 function evaluations, 30 independent runs) on seven functions out of 48. The following factors and their values were used: SN = {100}, D = {30}, and 'limit' = {50, 100, 200, 400, ∞}. It was reported that 'limit' = 200 was the more appropriate than other values used in that study.

All the aforementioned works exhibit partial experimentation and non-full factorial design on investigating the effect of 'limit' on ABC. However, such partial investigations were still better, in our opinion, than using a fixed setting from a study using different optimisation problem. The results from our study show that the tuning or controlling of the control parameter 'limit' is indeed needed. An example of a study where tuning on 'limit' was applied is presented in [35].

It is worth mentioning that in all of the above mentioned experiments, the better settings for 'limit' were chosen by visual inspection of the results and without any statistical testing. Our study was quite different to the aforementioned works due to the applications of NHST and CRS4EAs. In this respect, our work was similar to [42], where full factorial design and ANOVA statistical analysis were used to investigate the sensitivity of reactive tabu search (RTS) to its meta-parameters.

5. Conclusions

As the horse racing approach is still omnipresent, researchers have often compared their algorithms, which are well tuned for (a) particular problem(s), with some standard versions of meta-heuristic algorithms using recommended control parameter settings, which might not be appropriate for some problems used in an experiment. This situation should be avoided. In the recently published guidelines for replication and comparison of experiments in EC [9], we promoted fair comparisons amongst algorithms where all the algorithms used in the comparisons, not only the researchers' preferred, should be using the best control parameter settings. Hence, performing extensive parameter tuning or control [11] for all algorithms involved in an experiment is a prerequisite for a fairer comparison.

This paper has shown that amongst ABC control parameters 'limit' is very sensitive, whilst population size (*SN*) is quite robust (at least for the benchmark suite used in this study). Hence, properly setting control parameter 'limit' should be of particular interest to every ABC user. Furthermore, it was shown in this study that ABC is not always the best performing when 'limit' = $(SN/2)*D$, although it is a very competitive setting. This formula was the best for the vertical approach using *250,000* fitness evaluations for the benchmark suite used in this study. Better settings for 'limit' exist, occasionally statistically significant, for the vertical approach using *100,000* fitness evaluations, as well as for the both horizontal approaches (reaching (sub-)optimal solution at 10^{-6} and 10^{-12}) for benchmark suite used in this study. When 100,000 fitness evaluations were available, L_k was the appropriate choice only for small dimensions (D = 2, rarely for D = 5 or D = 10) amongst all the five presented problems. When the dimension becomes bigger, more appropriate alternatives could be chosen. Hence, proper setting of 'limit' also depends on the available maximum number of fitness evaluations, indicating that ABC convergence with L_k is not amongst the fastest. Furthermore, as results from the horizontal approach indicate a better ABC convergence whilst obtaining the same accuracy can be achieved with 'limit' settings other than using the recommended formulae. Moreover, setting 'limit' = $(SN/2)*D$ has no theoretical explanation in [26] and is based only on partial experimentation on limited number of numerical optimization. Hence, it is too risky to expect that the suggested formula in [26] would be good for other problems. Our recommendation is to perform tuning or control on ABC parameter 'limit'. These findings are valid for ABC only, and no generalisations regarding other meta-heuristic algorithms can be applied. As extensive parameter tuning using full factorial design [33] is often too expensive, researchers should use various already-available tuning approaches (e.g., F-Race [4], Revac [40], SPO [3], CRS-Tuning [49]) for setting control parameter 'limit' or investigate some parameter control approaches (e.g., driven by diversity [46], entropy [36], exploration and exploitation measures [37]), which will be part of our future work. Last but not least, it is shown that CRS4EAs is comparable to NHST, in particular to the multiple pairwise Wilcoxon's test. Both methods pairwisely compare the results of an optimisation problem over all *n* runs. However, in one tournament, the CRS4EAs compared the results obtained by all participants (more absolute approach), whilst the Wilcoxon's test compared only results of the participants that are of the main interest (more relative approach). Thus, several Wilcoxon's tests were applied separately for each and every comparison. Additionally, for a set of Wilcoxon's tests made on the same data, a post-hoc analysis is needed to avoid inflating Type-I-Error. Nevertheless, the results of CRS4EAs can be compared amongst all participants, whilst in NHST even more additional tests would be needed in this respect. Deeper comparison among CRS4EAs and NHST is presented in our recent work [50].

References

1. Akay, B., Karaboga, D. A Modified Artificial Bee Colony Algorithm for Real-Parameter Optimization. Information Sciences, 2012, 192, 120-142. https://doi.org/10.1016/j.ins.2010.07.015
2. Amarjeet, J., Chhabra, K. Harmony Search Based Remodularization for Object-Oriented Software Systems. Computer Languages, Systems & Structures, 2017, 47, 153-169. https://doi.org/10.1016/j.cl.2016.09.003
3. Bartz-Beielstein, T., Lasarczyk, C., Preuss, M. Sequential Parameter Optimization. The 2005 IEEE Congress on Evolutionary Computation, 2015, 1, 773-780. https://doi.org/10.1109/CEC.2005.1554761
4. Birattari, M., Stützle, T., Paquete, L., Varrentrapp, K. A Racing Algorithm for Configuring Metaheuristics. Proceedings of the Genetic and Evolutionary Computation Conference, 2002, 11-18.
5. Blum, C., Roli, A. Metaheuristics in Combinatorial Optimization: Overview and Conceptual Comparison. ACM Computing Surveys, 2003, 35(3), 268-308. https://doi.org/10.1145/937503.937505
6. Civicioglu, P. Transforming Geocentric Cartesian Coordinates to Geodetic Coordinates by Using Differential Search Algorithm. Computers & Geosciences, 2012, 46, 229-247. https://doi.org/10.1016/j.cageo.2011.12.011
7. Črepinšek, M., Liu, S.-H., Mernik, L. A Note on Teaching-Learning-Based Optimization Algorithm. Information Sciences, 2012, 212, 79-93. https://doi.org/10.1016/j.ins.2012.05.009
8. Črepinšek, M., Liu, S.-H., Mernik, M. Exploration and Exploitation in Evolutionary Algorithms: A Survey. ACM Computing Surveys, 2013, 45(3), 1-35. https://doi.org/10.1145/2480741.2480752
9. Črepinšek, M., Liu, S.-H., Mernik M. Replication and Comparison of Computational Experiments in Applied Evolutionary Computing: Common Pitfalls and Guidelines to Avoid them. Applied Soft Computing, 2014, 19, 161-170. https://doi.org/10.1016/j.asoc.2014.02.009
10. Derrac, J., García, S., Hui, S., Suganthan, P., Herrera, F. Analyzing Convergence Performance of Evolutionary Algorithms: A Statistical Approach. Information Sciences, 2014, 289, 41-58. https://doi.org/10.1016/j.ins.2014.06.009
11. Eiben, Á. E., Hinterding, R., Michalewicz, Z. Parameter Control in Evolutionary Algorithms. IEEE Transactions on Evolutionary Computation, 1999, 3(2), 124-141. https://doi.org/10.1109/4235.771166
12. Eiben, Á. E., Jelasity, M. A Critical Note on Experimental Research Methodology in EC. Proceedings of the World on Congress on Computational Intelligence, 2002, 582-587. https://doi.org/10.1109/CEC.2002.1006991
13. Friedman, M. The Use of Ranks to Avoid the Assumption of Normality Implicit in the Analysis of Variance. Journal of the American Statistical Association, 1937, 32(200), 675-701. https://doi.org/10.1080/01621459.1937.10503522
14. Friedman, M. A Comparison of Alternative Tests of Significance for the Problem of M Rankings. The Annals of Mathematical Statistics, 1940, 11(1), 86-92. https://doi.org/10.1214/aoms/1177731944
15. Gao, W. F., Liu, S.-Y., Huang, L.-L. Enhancing Artificial Bee Colony Algorithm Using More Information-Based Search Equations. Information Sciences, 2014, 270, 112-133. https://doi.org/10.1016/j.ins.2014.02.104
16. Glickman, M. Dynamic Paired Comparison Models with Stochastic Variances. Journal of Applied Statistics, 2001, 28(6), 673-689. https://doi.org/10.1080/02664760120059219
17. Glickman, M. Example of the Glicko-2 System. Boston University.
18. Hansen, N., Auger, A., Finck, S., Ros, R. Real-Parameter Black-Box Optimization Benchmarking 2010: Experimental Setup. Research Report RR-7215, INRIA, 2010.
19. Holm S. A Simple Sequentially Rejective Multiple Test Procedure. Scandinavian Journal of Statistics, 1979, 60-75.
20. Hooker, J. Testing Heuristics: We Have It All Wrong. Journal of Heuristics, 1995, 1(1), 33-42. https://doi.org/10.1007/BF02430364
21. Hsieh, T.-J., Hsiao, H.-F., Yeh, W.-C. Forecasting Stock Markets Using Wavelet Transforms and Recurrent Neural Networks: An Integrated System Based on Artificial Bee Colony Algorithm. Applied Soft Computing, 2011, 11(2), 2510-2525. https://doi.org/10.1016/j.asoc.2010.09.007
22. Hsu, C.-C., Chen, C.-H., Huang, K.-K., Huang, Y.-M. A Personalized Auxiliary Material Recommendation System Based on Learning Style on Facebook Applying an Artificial Bee Colony Algorithm. Computers & Mathematics with Applications, 2012, 64(5), 1506-1513. https://doi.org/10.1016/j.camwa.2012.03.098
23. Johnson, D. A Theoretician's Guide to the Experimental Analysis of Algorithms. Data Structures, Near Neighbor Searches, and Methodology: Fifth and Sixth DIMACS Implementation Challenges, 2002, 59, 215-250.

24. Kang, F., Li, J., Ma, Z. Rosenbrock Artificial Bee Colony Algorithm for Accurate Global Optimization of Numerical Functions. Information Sciences, 2011, 181(16), 3508-3531. https://doi.org/10.1016/j.ins.2011.04.024

25. Karaboga, D. An Idea Based on Honey Bee Swarm for Numerical Optimization. Erciyes University, 2005.

26. Karaboga, D., Basturk, B. On the Performance of Artificial Bee Colony (ABC) Algorithm. Applied Soft Computing, 2008, 8(1), 687-697. https://doi.org/10.1016/j.asoc.2007.05.007

27. Karaboga, D., Akay, B. A Comparative Study of Artificial Bee Colony Algorithm. Applied Mathematics and Computation, 2009, 214(1), 108-132. https://doi.org/10.1016/j.amc.2009.03.090

28. Karaboga, D., Akay, B. A Survey: Algorithms Simulating Bee Swarm Intelligence. Artificial Intelligence Review, 2009, 31(1-4), 61-85. https://doi.org/10.1007/s10462-009-9127-4

29. Karaboga, D., Gorkemli, B., Ozturk, C., Karaboga, N. A Comprehensive Survey: Artificial Bee Colony (ABC) Algorithm and Applications. Artificial Intelligence Review, 2014, 42(1), 21-57. https://doi.org/10.1007/s10462-012-9328-0

30. Karaboga, D., Gorkemli, B. A Quick Artificial Bee Colony (qABC) Algorithm and Its Performance on Optimization Problems. Applied Soft Computing, 2014, 23, 227-238. https://doi.org/10.1016/j.asoc.2014.06.035

31. Kiran, M. S., Findik, O. A Directed Artificial Bee Colony Algorithm. Applied Soft Computing, 2015, 26, 454-462. https://doi.org/10.1016/j.asoc.2014.10.020

32. LaTorre, A., Muelas, S., Pena, J.-M. A Comprehensive Comparison of Large Scale Global Optimizers. Information Sciences, 2015, 316, 517-549. https://doi.org/10.1016/j.ins.2014.09.031

33. Lee, Y., Filliben, J., Micheals, R., Phillips, P. J. Sensitivity Analysis for Biometric Systems: A Methodology Based on Orthogonal Experiment Designs. Computer Vision and Image Understanding, 2013, 117(5), 532-550. https://doi.org/10.1016/j.cviu.2013.01.003

34. Li, G., Niu, P., Xiao, X. Development and Investigation of Efficient Artificial Bee Colony Algorithm for Numerical Function Optimization. Applied Soft Computing, 2012, 12(1), 320-332. https://doi.org/10.1016/j.asoc.2011.08.040

35. Liao, T., Aydin, D., Stützle, T. Artificial Bee Colonies for Continuous Optimization: Experimental Analysis and Improvements. Swarm Intelligence, 2013, 7(4), 327-356. https://doi.org/10.1007/s11721-013-0088-5

36. Liu, S.-H., Mernik, M., Bryant, B. R. To Explore or to Exploit: An Entropy-Driven Approach for Evolutionary Algorithms. International Journal of Knowledge-Based and Intelligent Engineering Systems, 2009, 13(3), 185-206. https://doi.org/10.3233/KES-2009-0184

37. Liu, S.-H., Mernik, M., Hrnčič, D., Črepinšek, M. A Parameter Control Method of Evolutionary Algorithms Using Exploration and Exploitation Measures with a Practical Application for Fitting Sovova's Mass Transfer Model. Applied Soft Computing, 2013, 13(9), 3792-3805. https://doi.org/10.1016/j.asoc.2013.05.010

38. Ma, M., Liang, J., Guo, M., Fan, Y., Yin, Y. SAR Image Segmentation Based on Artificial Bee Colony Algorithm. Applied Soft Computing, 2011, 11(8), 5205-5214. https://doi.org/10.1016/j.asoc.2011.05.039

39. Mernik, M., Liu, S.-H., Karaboga, D., Črepinšek, M. On Clarifying Misconceptions When Comparing Variants of the Artificial Bee Colony Algorithm by Offering a New Implementation. Information Sciences, 2015, 291, 115-127. https://doi.org/10.1016/j.ins.2014.08.040

40. Nannen, V., Eiben, Á. E. Relevance Estimation and Value Calibration of Evolutionary Algorithm Parameters. IJCAI, 2007, 7, 975-980.https://doi.org/10.1109/CEC.2007.4424460

41. Neyman, J., Pearson, E. On the Problem of the Most Efficient Tests of Statistical Hypotheses. Springer, 1992. https://doi.org/10.1007/978-1-4612-0919-5_6

42. Pellegrini, P., Mascia, F., Stutzle, T., Birattari, M., Mauro. On the Sensitivity of Reactive Tabu Search to Its Meta-Parameters. Soft Computing, 2011, 18(11), 2177-2190. https://doi.org/10.1007/s00500-013-1192-6

43. Piotrowski, A. Regarding the Rankings of Optimization Heuristics Based on Artificially-Constructed Benchmark Functions. Information Sciences, 2015, 297, 191-201. https://doi.org/10.1016/j.ins.2014.11.023

44. Samanta, S., Chakraborty, S. Parametric Optimization of Some Non-Traditional Machining Processes Using Artificial Bee Colony Algorithm. Engineering Applications of Artificial Intelligence, 2011, 24(6), 946-957. https://doi.org/10.1016/j.engappai.2011.03.009

45. Smit, S. Parameter Tuning and Scientific Testing in Evolutionary Algorithms. Vrije Universiteit, 2012.

46. Ursem, R. K. Diversity-Guided Evolutionary Algorithms. Parallel Problem Solving from Nature PPSN VII, 2002, 462-471. https://doi.org/10.1007/3-540-45712-7_45

47. Veček, N., Liu, S.-H., Črepinšek, M., Mernik, M. Experimental Results. http://lpm.uni-mb.si/crepinsek/Upload.zip

48. Veček, N., Mernik, M., Črepinšek, M. A Chess Rating System for Evolutionary Algorithms: A New Method for the Comparison and Ranking of Evolutionary Algorithms. Information Sciences, 2014, 277, 656-679. https://doi.org/10.1016/j.ins.2014.02.154

49. Veček, N., Mernik, M., Filipič, B., Črepinšek M. Parameter Tuning with Chess Rating System (CRS-Tuning) for Meta-Heuristic Algorithms. Information Sciences, 2016, 372, 446-469. https://doi.org/10.1016/j.ins.2016.08.066

50. Veček, N., Črepinšek, M., Mernik, M. On the Influence of the Number of Algorithms, Problems and Independent Runs in the Comparison of Evolutionary Algorithms. Applied Soft Computing, 2017, 54, 23-45. https://doi.org/10.1016/j.asoc.2017.01.011

51. Wilcoxon, F. Individual Comparisons by Ranking Methods. Biometrics Bulletin, 1945, 1, 80-83. https://doi.org/10.2307/3001968

52. Xiang, W.-L., An, M.-Q. An Efficient and Robust Artificial Bee Colony Algorithm for Numerical Optimization. Computers & Operations Research, 2013, 40(5), 1256-1265. https://doi.org/10.1016/j.cor.2012.12.006

53. Xu, C., Duan, H., Liu, F. Chaotic Artificial Bee Colony Approach to Uninhabited Combat Air Vehicle (UCAV) Path Planning. Aerospace Science and Technology, 2010, 14(8), 535-541. https://doi.org/10.1016/j.ast.2010.04.008

54. Yeh, W.-C., Hsieh, T.-J. Solving Reliability Redundancy Allocation Problems Using an Artificial Bee Colony Algorithm. Computers & Operations Research, 2011, 38(11), 1465-1473. https://doi.org/10.1016/j.cor.2010.10.028

55. Zhang, C., Ouyang, D., Ning, J. An Artificial Bee Colony Approach for Clustering. Expert Systems with Applications, 2010, 37(7), 4761-4767. https://doi.org/10.1016/j.eswa.2009.11.003

56. Zhang, R., Song, S., Wu, C. A Hybrid Artificial Bee Colony Algorithm for the Job Shop Scheduling Problem. International Journal of Production Economics, 2013, 141(1), 167-178. https://doi.org/10.1016/j.ijpe.2012.03.035

57. Zhang, W., Wang, N., Yang, S. Hybrid Artificial Bee Colony Algorithm for Parameter Estimation of Proton Exchange Membrane Fuel Cell. International Journal of Hydrogen Energy, 2013, 38(14), 5796-5806. https://doi.org/10.1016/j.ijhydene.2013.01.058

Summary / Santrauka

Artificial Bee Colony (ABC) is a successful meta-heuristic algorithm that has been greatly utilised by researchers. Through our practical experience of ABC, we have noticed that the recommended formula 'limit' = ne * D may not be the best choice for different problems. In this work, a set of experiments using horizontal and vertical approaches has been designed and executed with the aim of observing the effect of 'limit' on ABC. The results have been statistical analysed using Null Hypothesis Significance Testing (NHST) as well as the Chess Rating System for Evolutionary Algorithms (CRS4EAs), which is a novel approach for comparing meta-heuristic algorithms. It is shown that the recommended formula is not the best setting for different problems and approaches. Hence, the control parameter 'limit' should be tuned or controlled. The other important result of this study is to show that CRS4EAs is comparable but also shows benefits over NHST.

Dirbtinė bičių kolonija (ABC) yra sėkmingas, mokslininkų plačiai naudojamas metaeuristinis algoritmas. Per savo praktinę ABC patirtį straipsnio autoriai pastebėjo, kad rekomenduojama formulė 'limit' = ne * D ne visuomet yra geriausias pasirinkimas tam tikroms problemoms spręsti. Su tikslu įvertinti formulės elemento 'limit' poveikį ABC, straipsnio autoriai sukūrė ir atliko eksperimentus, paremtus horizontaliais ir vertikaliais metodais. Gauti rezultatai statistiškai analizuoti naudojant hipotezės reikšmingumo testavimą (NHST) bei Šachmatų reitingų sistemą Evoliucijos algoritmui (CRS4EAs). Tai yra naujas metodas metaeuristiniams algoritmams palyginti. Straipsnyje įrodoma, kad rekomenduojama formulė išties nėra geriausias skirtingų problemų ir metodų nustatymas. Taigi, kontrolės parametras 'limit' turėtų būti nustatytas arba kontroliuojamas. Kitas svarbus šio tyrimo rezultatas – parodoma, kad CRS4EAs yra palyginamas, tačiau, palyginus su NHST, yra pranašesnis.