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Robust Adaptive Control for Fractional-Order Financial Chaotic Systems with System Uncertainties and External Disturbances

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In this paper, robust adaptive control for uncertain fractional-order financial chaotic systems with bounded unknown external disturbances is studied. By utilizing the fractional-order extension of the classical Lyapunov stability methods, an adaptive controller is presented for controlling the fractional-order financial chaotic system. Quadratic Lyapunov functions are employed in the stability analysis, and fractional-order adaptation laws are designed to update controller parameters online. The proposed controller can ensure that the system states converge to the origin asymptotically and all signals in the closed-loop system remain bounded. Finally, simulation results are presented to confirm our theoretical results.

KEYWORDS: Robust control, fractional-order financial system, fractional-order chaotic system, fractional-order adaptation law.

Introduction

In the last two decades, fractional-order systems have received a significant amount physicists' and engineers' interest due to their attractive properties and potential applications [6, 12, 19, 23, 25, 29, 30, 37, 39]. Compared with classical integer-order systems, the fractional-order derivation has two advantages. First, the traditional integer-order derivative describes a certain attribute or variation at a particular time for a physical process, yet the fractional-order's is concerned throughout the whole time domain. Second, the traditional integer-order derivative indicates the local properties, for example, a certain position, for a physical process, while the fractional-order derivative is related to the whole space. Taking the above facts into account, the fractional calculus plays a very important role in the modeling and tackling of many phenomena and actual systems in various fields, such as quantum mechanics, molecular spectroscopy, stochastic diffusion, control theory, and viscoelastic dynamics [4, 11, 16, 20, 21, 22, 24, 45, 49]. Therefore, research on theory and applications of fractional-order systems is becoming more and more popular.

Recently, research on the complex dynamics of financial systems has also become a very prominent domain in both micro and macroeconomics [43]. Researchers have attempted to elaborate the primary properties of economic data with the help of the dynamical behaviors exhibited in the financial systems. Some continuous nonlinear system models have been established to investigate the complex economic dynamics, for example, the forced van der Pol model [38], the Goodwin's accelerate model [26], the IS-LM model [15]. In fact, it is feasible for nonlinear systems to exhibit chaotic or periodic behaviors. However, if chaotic behaviors exist in economic systems, then the systems will have inherent indefiniteness which makes it hard to provide a reasonable or effective economic prediction. Thus, it is indispensable to investigate the chaotic behaviors in financial and economical systems. Since chaotic behaviors in financial and economical systems were firstly investigated in 1985, great impact had been put on the prominent economics. Many interesting results on integer-order financial chaotic systems were given, for example, sliding mode control and passive control methods were employed to synchronize two chaotic financial systems with different initial conditions in [18]; control of hyperchaotic finance systems was presented in [48] and [42]; chaotic dynamic behavior analysis for a class of financial risk systems was studied in [50]; control of chaotic financial systems with input time-delay by means of H_∞ control was presented in [51]; in [40], a novel 3-D nonlinear financial chaotic system was introduced and its complex dynamic behavior was investigated, etc.

Up to now, controlling and synchronizing chaos in fractional-order financial systems has also been investigated, for example, in [1, 10, 13, 14, 27, 32, 41, 43, 47, 46]. In [14], a sliding mode controller was designed to synchronize fractional-order financial systems in master-slave structure. In [41], a necessary condition was given to show the existence of 1-scroll, 2-scroll even multi-scroll chaotic attractors in fractional-order financial systems. Active control method was used in [27, 47]. An active controller with multiple conflicting objectives was constructed in [32]. It should be highlighted that a key assumption in the above literatures is that the model of the financial systems should be known. However, most of real world systems are subjected to system uncertainties and external disturbances, especially in financial systems [7, 8, 27, 33, 34, 36, 44]. On the other hand, in financial systems, system uncertainties do exist because of limited sizes of weather variables, political events, and other human factors. The existence of systems uncertainties and external disturbances could decrease the control performance, or even lead to instability of the system [35]. It is meaningful to consider the control of financial systems with system uncertainties and external disturbances. Thanks to the works of Li et al. [20], the Lyapunov direct method (also called the Lyapunov second method) has been extended to fractional-order nonlinear systems. In this paper, a robust adaptive controller is proposed to solve the control problem of fractional-order financial chaotic systems with both system uncertainties and bounded external disturbances. The fractional-order Lyapunov approach is used to analyze the stability of the system. Specifically, the main contributions of this study include:

- A robust adaptive controller is derided for fractional-order financial chaotic systems with unknown system dynamics and external disturbances;

- Fractional-order adaptations laws are constructed to eliminate the estimation errors, and a fractional Lyapunov stability criterion as well as quadratic Lyapunov functions are used in the stability analysis.

The remainder of this paper is organized as follows: Section 2 lists mathematical model of the fractional-order financial systems and some basic results on fractional calculus. In Section 3, an adaptive robust controller is designed and stability analysis of the closed-loop system is discussed. Simulation studies are included in Section 4. Finally, Section 5 concludes this work.

Problem Statement and Preliminaries

Preliminaries

The fractional order integrodifferential operator is the extended concept of the integer-order integro-differential operator. The commonly used definitions in literatures are Grunwald-Letnikov, Riemann-Liouville, and Caputo definitions. Because the Caputo derivative takes on the same form as integer-order differential on the initial conditions, which have well-understood physical meanings and have more applications in engineering, we will use this definition. The lower limit of the fractional calculus is set as 0 in this paper. The fractional-order integral with order α can be expressed as

$$ {}_0D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau)d\tau. \tag{1}$$

where $\Gamma(\cdot)$ represents the Euler's function.

The Caputo fractional derivative is defined as follows:

$$ {}_0^C D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)d\tau, \tag{2}$$

where α is the fractional order, and n is an integer satisfying $n-1 \le \alpha < n$.

In nonlinear systems, Lyapunov direct method (also called the Lyapunov second method) provides a way to analyze the stability of a system without explicitly solving the differential equations. Although the Lyapunov stability theory for integer-order systems was proposed in 1892 and it has been studied and modified by lots of expert researchers, the Lyapunov stability theory for fractional order systems has been developed until recently [20].

The following lemmas and definition will be used.

Lemma 1. *[20] Let $x=0$ be an equilibrium of the fractional-order nonlinear system:*

$$ {}_0^C D_t^{\alpha} x(t) = f(x). \tag{3}$$

Suppose there exists a Lyapunov function $V(t,x(t))$ and class- k functions $g_i, i=1,2,3$ such that:

$$ \begin{aligned} &g_1(\|x\|) \le V(t,x(t)) \le g_2(\|x\|),\\ &{}_0^C D_t^{\beta} V(t,x(t)) \le -g_3(\|x\|), \end{aligned} \tag{4}$$

where $0<\beta<1$, then the equilibrium point of system (3) is Mittag-Leffler stable.

Lemma 2. *[20] If the fractional-order nonlinear system (3) is Mittag-Leffler stable, then it will be asymptotically stable, i.e., $\lim_{t\to\infty} x(t)=0$.*

Lemma 3. *[4, 16, 21] Let $x(t) \in \mathcal{R}^n$ be a continuous and derivable function. Then for any $t>0$,*

$$ \frac{1}{2}\,{}_0^C D_t^{\alpha} x^T(t)x(t) \le x^T(t)\,{}_0^C D_t^{\alpha} x(t). \tag{5}$$

As it is known, the exponential function, e^z, is a very important function in the stability analysis of integer-order systems. Its one-parameter generalization function is defined by [31]

$$ E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+1)}. \tag{6}$$

The two-parameter function of Mittag-Leffler type, which plays a very important role in the fractional calculus, was introduced by Agarwal [2]. The Mittag-Leffler function with two parameters can be written as

$$ E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+\beta)}, \tag{7}$$

where $\alpha,\beta>0$ and $z \in C$. The Mittag-leffler function depends on the two parameters α and β. It is a special function and a complex function. If both α and β are real and positive, the series converges for all values of the argument z, so the Mittag-Leffler function is an entire function. The Laplace transform of Mittag-Leffler function is [37]

$$\mathscr{L}\{t^{\beta-1}E_{\alpha,\beta}(-at^{\alpha})\} = \frac{s^{\alpha-\beta}}{s^{\alpha}+a}. \tag{8}$$

Lemma 4. *[37] If $x(t)\in C^1[0,T]$ for some $T>0$, and $0<\alpha\le 1$, then the following equations hold:*

$${}_0^C D_t^{-\alpha}\,{}_0^C D_t^{\alpha} x(t) = x(t) - x(0) \tag{9}$$

and

$${}_0^C D_t^{\alpha}\,{}_0^C D_t^{-\alpha} x(t) = x(t). \tag{10}$$

In this paper, we employ the Caputo version and use an algorithm for fractional order differential equations, which is the generalization of Adams-Bashforth-Moulton one. A brief introduction of the algorithm is given as follows.

Let consider the following fractional order differential equation:

$$\begin{cases} {}_0^C D_t^{\alpha} y(t) = f(t, y(t)), \\ y(0) = y_0. \end{cases} \tag{11}$$

According to Lemma 4, the above equation (11) is equivalent to the Volterra integral equation

$$y(t) = y_0 + \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau, y(\tau))d\tau. \tag{12}$$

Let $h = T/N, N\in Z, t_n = nh, n = 0,1,\cdots,N$. Then (12) can be approximated as [37]

$$\begin{aligned} y_h(t_{n+1}) = \quad & y_0 + \frac{h^{\alpha}}{\Gamma(\alpha+2)} f(t_n+1, y_h^p(t_{n+1})) \\ & + \frac{h^{\alpha}}{\Gamma(\alpha+2)}\sum_{j=0}^{n} a_{j,n+1} f(t_j, y_h(t_j)) \end{aligned} \tag{13}$$

where $a_{j,n+1} = n^{\alpha+1} - (n-\alpha)(n+1)^{\alpha}$ for $j = 0$ and $a_{j,n+1} = (n-j+2)^{\alpha+1} + (n-j)^{\alpha+1} - 2(n-j+1)^{\alpha+1}$ for $1\le j\le n$, $y_0 + \frac{1}{\Gamma(\alpha)}\sum_{j=0}^{n} b_{j,n+1} f(t_j, y_h(t_j))$, $p = \alpha+1$, and $b_{j,n+1} = \frac{h^{\alpha}}{\alpha}\left((n+1-j)^{\alpha} - (n-j)^{\alpha}\right)$.

The estimation error is $\max|y(t_j) - y_h(t_j)| = o(h^p)$ [37].

Description of fractional-order financial chaotic systems

The fractional-order financial chaotic system model to be used in this paper can be seen in [10, 13, 32, 46, 47]. This mathematical model describes a fractional-order financial system by three nonlinear fractional-order differential equations. The model can be expressed as

$$\begin{cases} {}_0^C D_t^{\alpha} x_1(t) = x_3(t) + (x_2(t) - a)x_1(t) \\ {}_0^C D_t^{\alpha} x_2(t) = 1 - bx_2(t) - x_1^2(t) \\ {}_0^C D_t^{\alpha} x_3(t) = -x_1(t) - cx_3(t), \end{cases} \tag{14}$$

where a represents the saving amount, b denotes the cost per investment, c corresponds to the elasticity of demand of commercial market, and $0<\alpha<1$ is the fractional-order derivative. The first state variable $x_1(t)$, which represents the interest rate, can be affected by the surplus between investment and savings as well as structural adjustments of the prices. The second state variable $x_2(t)$ corresponds to the rate of investment, and inversely proportional to the cost of investment and the interest rate. The third state variable $x_3(t)$ depends on the difference between supply and demand in the market, and it can also be affected by the inflation rate.

It is easy to know that system (14) has three equilibrium points:

$$\begin{aligned} E_1 &= \left(0, \frac{1}{b}, 0\right), \\ E_2 &= \left(\sqrt{\frac{c-b-abc}{c}}, \frac{1+ac}{c}, -\frac{1}{c}\sqrt{\frac{c-b-abc}{c}}\right), \\ E_3 &= \left(-\sqrt{\frac{c-b-abc}{c}}, \frac{1+ac}{c}, \frac{1}{c}\sqrt{\frac{c-b-abc}{c}}\right). \end{aligned} \tag{15}$$

The Jacobian matrix, at the equilibrium $E^* = [x_1^*, x_2^*, x_3^*]^T$, can be given as

$$J_E = \begin{bmatrix} -a+x_2^* & x_1^* & 1 \\ -2x_1^* & -b & 0 \\ -1 & 0 & -c \end{bmatrix}. \tag{16}$$

Let $a = 1, b = 0.1$ and $c = 1$. The eigenvalues for the fractional-order financial chaotic system equilibrium $E_1 = (0.000; 10.000; 0.000)$ are $\lambda_1 = 8.8990$,

$\lambda_2 = -0.8990, \lambda_3 = -0.1000$. Clearly, it is a saddle point. For equilibrium points $E_2 = (0.8944; 2.000; -0.8944)$ and $E_3 = (-0.8944; 2.000; 0.8944)$ they are: $\lambda_1 = -0.7609$ and $\lambda_{2,3} = 0.3304 \pm 1.4112i$. It is a saddle-focus point. Since it is an unstable equilibrium, the condition for chaos is satisfied We can easily find that the minimal commensurate order of the system is $\alpha > 0.8537$. Set the initial conditions be $x_1(0) = 1, x_2(0) = 2, x_3(0) = -0.5$, and the fractional order be $\alpha = 0.91$. The chaotic attractor of the fractional-order financial system (14) is shown in Figures 1 and 2 when $\alpha = 0.85$ and $\alpha = 0.97$, respectively.

Figure 1

Chaotic behavior of fractional-order financial system (14) with $\alpha = 0.85$ in (a) $x_1(t) - x_2(t) - x_3(t)$ plane, (b) $x_1(t) - x_2(t)$ plane, (c) $x_1(t) - x_3(t)$ plane and (d) $x_1(t) - x_4(t)$ plane

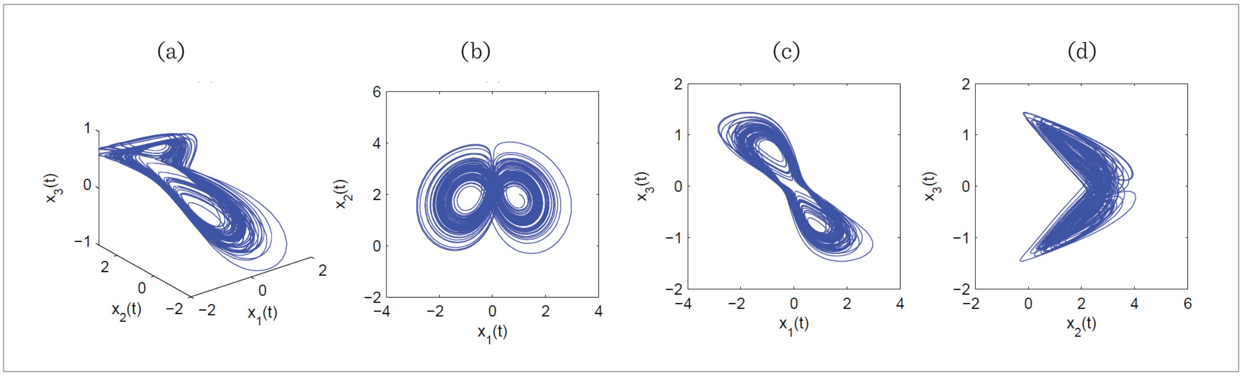
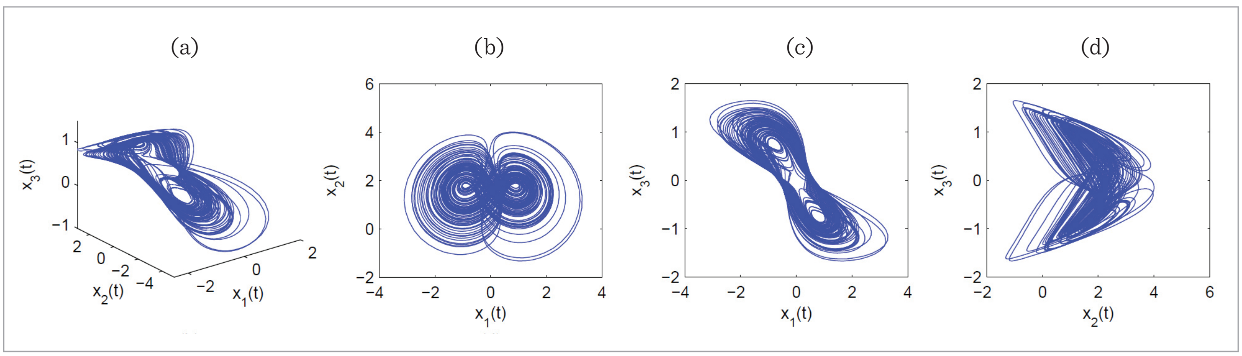


Figure 2

Chaotic behavior of fractional-order financial system (14) with $\alpha = 0.97$ in (a) $x_1(t) - x_2(t) - x_3(t)$ plane, (b) $x_1(t) - x_2(t)$ plane, (c) $x_1(t) - x_3(t)$ plane and (d) $x_1(t) - x_4(t)$ plane



Controller Design and Stability Analysis

According to (14), the controlled model can be expressed as

$$\begin{cases} {}_0^C D_t^\alpha x_1(t) = x_3(t) + (x_2(t) - a)x_1(t) + \Delta f_1(x(t)) + d_1(t) + u_1(t), \\ {}_0^C D_t^\alpha x_2(t) = 1 - bx_2(t) - x_1^2(t) + \Delta f_2(x(t)) + d_2(t) + u_2(t), \\ {}_0^C D_t^\alpha x_3(t) = -x_1(t) - cx_3(t) + \Delta f_3(x(t)) + d_3(t) + u_3(t), \end{cases} \tag{17}$$

where $\Delta f_i(x(t))$ and $d_i(t)$, $i=1,2,3$ are system uncertainties and unknown external disturbances, respectively, and $u_i(t)$ is the control input. Denote $f_1(x(t))=x_3(t)+(x_2(t)-a)x_1(t)$, $f_2(x(t))=1-bx_2(t)-x_1^2(t)$ and $f_3(x(t))=-x_1(t)-cx_3(t)$, then system (17) can be rewritten as

$$ {}_0^C D_t^\alpha x_i(t)=f_i(x(t))+\Delta f_i(x(t))+d_i(t)+u_i(t), i=1,2,3. \quad (18)$$

Let the error state be

$$e_i(t)=x_i(t)-x_i^*. \quad (19)$$

The control objective is to design the control input $u_i(t)$ so that the error variable $e_i(t)$ tends to the origin asymptotically with all signals in the closed-loop system remain bounded. To fulfil this assignment, the following assumptions are needed.

Assumption 1. *The system uncertainty $\Delta f_i(x(t))$ is Lipschitz continuous, and there exist an unknown positive constant γ_i such that*

$$|\Delta f_i(x(t))|\le \gamma_i\|x(t)\|, \quad (20)$$

where $\|\cdot\|$ denotes the Euclid norm.

Assumption 2. *The external disturbance $d_i(t)$ is a bounded continuous function, i.e., $d_i(t)$ satisfies the following inequality*

$$|d_i(t)|\le \bar{d}_i, \quad (21)$$

where d_i is an unknown positive constant.

Remark 1. *Assumptions 1 and 2 are not restrictive, and they are also used in many literatures, such as [22, 27, 28, 47], and so on. It should be pointed out that in this paper we assume that the exact values of $\bar{d}_i$ and γ_i are unknown. In fact, these two assumptions enable us to have a simpler analysis of the system stability.*

Note that the Caputo derivative of a constant is zero, from (18) and (19) we have

$$ {}_0^C D_t^\alpha e_i(t)=f_i(x(t))+\Delta f_i(x(t))+d_i(t)+u_i(t). \quad (22)$$

Multiplying $e_i(t)$ to both sides of (22), and using Assumptions 1 and 2, we have

$$\begin{aligned} e_i(t){}_0^C D_t^\alpha e_i(t) &= e_i(t)f_i(x(t))+e_i(t)\Delta f_i(x(t))+e_i(t)d_i(t)+e_i(t)u_i(t),\\ &\le e_i(t)f_i(x(t))+e_i(t)u_i(t)+\gamma_i\|x(t)\|\,|e_i(t)|+\bar{d}_i\,|e_i(t)|. \end{aligned} \quad (23)$$

Let us design the controller $u_i(t)$ as

$$u_i(t)=-f_i(x(t))+sign(e_i(t))u_{ri}(t), \quad (24)$$

where $u_{ri}(t)$ is a robust controller term which will be constructed later. Substituting (24) into (23) yields

$$e_i(t){}_0^C D_t^\alpha e_i(t)\le |e_i(t)|\Big(u_{ri}(t)+\gamma_i\|x(t)\|+\bar{d}_i\Big). \quad (25)$$

Note that both γ_i and $\bar{d}_i$ are unknown. So their values should be estimated in the controller design. Let $\hat{\gamma}_i(t)$ and $\hat{\bar{d}}_i(t)$ be the estimation of γ_i and $\bar{d}_i$, respectively. Then, the robust term $u_{ri}(t)$ can be defined as

$$u_{ri}(t)=-k_i|e_i(t)|-\hat{\gamma}_i(t)\|x(t)\|-\hat{\bar{d}}_i(t). \quad (26)$$

where k_i is positive design parameter. Substituting (26) into (25), we have

$$e_i(t){}_0^C D_t^\alpha e_i(t)\le -k_i|e_i(t)|^2-|e_i(t)|\tilde{\gamma}_i(t)\|x(t)\|-|e_i(t)|\tilde{\bar{d}}_i, \quad (27)$$

where

$$\tilde{\gamma}_i(t)=\hat{\gamma}_i(t)-\gamma_i \quad (28)$$

and

$$\tilde{\bar{d}}_i(t)=\hat{\bar{d}}_i(t)-\bar{d}_i \quad (29)$$

are the estimation errors of unknown parameters γ_i and $\bar{d}_i$, respectively.

To proceed, let us give the following results on fractional calculus at first.

Lemma 5. *If ${}_0^C D_t^\alpha y(t)\le 0$, then we have $y(t)\le y(0)$ for all $t>0$, and furthermore, the function $y(t)$ is monotone decreasing.*

Proof. There exists some nonnegative function $h(t)$ such that

$$ {}_0^C D_t^\alpha y(t)+h(t)=0. \quad (30)$$

Using the Laplace transform to (30) we have

$$Y(s) = \frac{y(0)}{s} - \frac{H(s)}{s^{\alpha}} \tag{31}$$

where $Y(s)$ and $H(s)$ are the Laplace transform of $y(t)$ and $h(t)$, respectively.

Taking the inverse Laplace transform on (31) yields

$$y(t) = y(0) - {}_0^C D_t^{-\alpha} h(t). \tag{32}$$

Noting that $h(t) \geq 0$, it follows from (1) that ${}_0^C D_t^{-\alpha} h(t) \geq 0$. As a result we know that $y(t) \leq y(0)$ for all $t > 0$, and the function $y(t)$ is monotone decreasing.

Lemma 6. *Let $V_1(t) = \frac{1}{2}x^2(t) + \frac{1}{2}y^2(t)$, where $x(t), y(t) \in \mathcal{R}$ are continuous functions. If*

$${}_0^C D_t^{\alpha} V_1(t) \leq -kx^2(t), \tag{33}$$

where k is a positive constant, then we have

$$x^2(t) \leq 2V_1(0) E_{\alpha}(-2kt^{\alpha}). \tag{34}$$

Proof. Using the fractional integral operator ${}_0^C D_t^{-\alpha}$ to both sides of (33), it follows from Lemma 4 that

$$V_1(t) - V_1(0) \leq -k {}_0^C D_t^{-\alpha} x^2(t). \tag{35}$$

It follows from (35) that

$$x^2(t) \leq 2V_1(0) - 2k {}_0^C D_t^{-\alpha} x^2(t). \tag{36}$$

There exists a nonnegative function $m(t)$ such that

$$x^2(t) + m(t) = 2V_1(0) - 2k {}_0^C D_t^{-\alpha} x^2(t). \tag{37}$$

Taking the Laplace transform ($\mathscr{L}\{\cdot\}$) on (37) gives

$$X_2(s) = 2V_1(0)\frac{s^{\alpha-1}}{s^{\alpha}+2k} - 2\frac{s^{\alpha}}{s^{\alpha}+2k}M(s) \tag{38}$$

where $X_2(s)$ and $M(s)$ are Laplace transform of $x^2(t)$ and $m(t)$, respectively. Using (8), the solution of (38) can be given as

$$x^2(t) = 2V_1(0)E_{\alpha}(-2kt^{\alpha}) - 2m(t) * [t^{-1}E_{\alpha,0}(-2kt^{\alpha})] \tag{39}$$

where * represents the convolution operator. Noting that both $E_{\alpha,0}(-2kt^{\alpha})$ and t^{-1} are nonnegative functions, it follows from (39) that (34) holds. This ends the proof of Lemma 6.

Based on above discussions, now we are ready to give the following results.

Theorem 1. *Consider the fractional-order financial chaotic system (17) or the equivalent form (18). Suppose that Assumptions 1 and 2 are satisfied. Let the control input be (24) and (26). If $\hat{\gamma}_i(t)$ and $\hat{\tilde{d}}_i(t)$ are updated by the following fractional-order differential equations*

$${}_0^C D_t^{\alpha} \hat{\gamma}_i(t) = h_i |e_i(t)| \|x(t)\| \tag{40}$$

and

$${}_0^C D_t^{\alpha} \hat{\tilde{d}}_i(t) = m_i \, | e_i(t) | \tag{41}$$

respectively, where h_i and m_i are positive design parameters, then the system variable $x_i(t)$ will tend to the origin asymptotically, and all signals in the closed-loop system will keep bounded.

Proof. Let us consider the following Lyapunov function candidate:

$$V_i(t) = \frac{1}{2}e_i^2(t) + \frac{1}{2h_i}\tilde{\gamma}_i^2(t) + \frac{1}{2m_i}\tilde{\tilde{d}}_i^2(t). \tag{42}$$

Then by using Lemma 3, we have

$${}_0^C D_t^{\alpha} V_i(t) \leq e_i(t) {}_0^C D_t^{\alpha} e_i(t) + \frac{1}{h_i}\tilde{\gamma}_i(t) {}_0^C D_t^{\alpha} \tilde{\gamma}_i(t) + \frac{1}{m_i}\tilde{\tilde{d}}_i(t) {}_0^C D_t^{\alpha} \tilde{\tilde{d}}_i(t). \tag{43}$$

Noting that the fractional-order derivative of a constant is zero, from (28) and (29) we have

$${}_0^C D_t^{\alpha} \tilde{\gamma}_i(t) = {}_0^C D_t^{\alpha} \hat{\gamma}_i(t) \tag{44}$$

and

$${}_0^C D_t^{\alpha} \tilde{\tilde{d}}_i(t) = {}_0^C D_t^{\alpha} \hat{\tilde{d}}_i(t). \tag{45}$$

Substituting (27), (44) and (45) into (43), we have

$$\begin{aligned}{}_0^C D_t^{\alpha} V_i(t) \leq & -k_i |e_i(t)|^2 - |e_i(t)| \tilde{\gamma}_i(t) \|x(t)\| - |e_i(t)| \tilde{\tilde{d}}_i \\ & + \frac{1}{h_i}\tilde{\gamma}_i(t) {}_0^C D_t^{\alpha} \hat{\gamma}_i(t) + \frac{1}{m_i}\tilde{\tilde{d}}_i(t) {}_0^C D_t^{\alpha} \hat{\tilde{d}}_i(t).\end{aligned} \tag{46}$$

Then substituting (40) and (41) into (46) gives

$$ {}_0^C D_t^\alpha V_i(t) \le -k_i \,|\, x_i(t) \,|^2 . \tag{47}$$

Thus we have ${}_0^C D_t^\alpha V_i(t) \le 0$. According to Lemma 5, we know that all signals in the closed-loop system will keep bounded. From Lemma 6 and (47), we can conclude that $e_i(t)$ will converge to the origin asymptotically. This ends the proof of Theorem 1.

Remark 2. *In the stability analysis of fractional order nonlinear systems, the Lyapunov function candidate $V(t) = 2e^T(t)e(t)$ is often used. The α th-order of $V(t)$ can be given as*

$$ {}_0^C D_t^\alpha V(t) = ({}_0^C D_t^\alpha e(t))^T e(t) + e^T(t) {}_0^C D_t^\alpha e(t) + 2\Lambda \tag{48}$$

where

$$ \Lambda = \sum_{i=1}^{\infty} \frac{\Gamma(1+\alpha)}{\Gamma(1+i)\Gamma(1-i+\alpha)} {}_0^C D_t^i e(t) {}_0^C D_t^{\alpha-i} e(t). \tag{49}$$

We can see that it is very hard to use the above complicated infinite series to analyze the stability of fractional order systems. However, in this paper, by using Lemma 3 and the proposed Lemma 6, we need not to tackle the above complicated infinite series.

Remark 3. *To update $\hat{\gamma}_i(t)$ and $\hat{\bar{d}}_i(t)$, fractional-order adaptation laws (40) and (41) are designed. Compared with classical integer-order adaptation law, the fractional-order adaptation laws enlarge the parameter adaptation performance by heightening one degree of freedom.*

Remark 4. *It is worth to mention that sliding mode control methods are often used to control fractional-order nonlinear systems, for example, in [3, 5, 9, 17] and many others. How to control fractional-order financial chaotic system by using sliding mode control method is one of our research directions.*

Simulation Studies

In the simulation, the system uncertainties are chosen as:

$$\begin{aligned} \Delta f_1(x(t)) &= 0.3\sin(x_1(t)) + 0.1x_2(t) - \sin(x_3(t)), \\ \Delta f_2(x(t)) &= 0.1x_1(t) + 0.2\sin(x_2(t)) + \sin(x_3(t)), \\ \Delta f_3(x(t)) &= 0.1x_1(t) - 0.2x_2(t) + \sin(x_3(t)), \end{aligned} \tag{50}$$

from which we can easily conclude that Assumption 1 is satisfied. Let the external disturbances be $d_1(t) = 0.2\sin(t), d_2(t) = 0.1\cos(t), d_3(t) = 0.1\sin(t) + 0.1\cos(t)$. The controller design parameters are chosen as $k_1 = k_2 = k_3 = 1, h_1 = h_2 = h3 = 0.3, m_1 = m_2 = m_3 = 0.2$. The initial conditions of the fractional order adaptation law are chosen as $\hat{\gamma}_1(0) = 0.2, \hat{\gamma}_2 = 0.5, \hat{\gamma}_3 = 2$, $\hat{\bar{d}}_1(0) = 0.2, \hat{\bar{d}}_2(0) = 0.5, \hat{\bar{d}}_3(0) = 0.7$. To eliminate the chattering phenomenon, the discontinuous term $sign(\cdot)$ is replaced by $\arctan(10\cdot)$.

Let $\alpha = 0.91$. First, let us consider the condition that controlling the fractional-order financial system (17) to the equilibrium point $E_1 = \left(0, \frac{1}{b}, 0\right)$. The simulation results are presented in Figures 3–7. Time responses of system variables $x_1(t)$, $x_2(t)$ and $x_3(t)$ are depicted in Figure 3. Time responses of the control inputs are included in Figure 5. The estimations of the unknown parameters $\hat{\gamma}_i(t)$ and $\hat{\bar{d}}_i(t)$ are depicted in Figures 6 and 7, respectively. From the simulation results, we can see that the good control performance has been obtained, and the system variables converge to the origin rapidly when the controller is activated at $t = 30$.

Figure 3

Controlling the fractional-order financial chaotic system (17) to its equilibrium point $E_1=(0,\frac{1}{b},0)$: time responses of the state variables. The controller is activated at $t = 30$

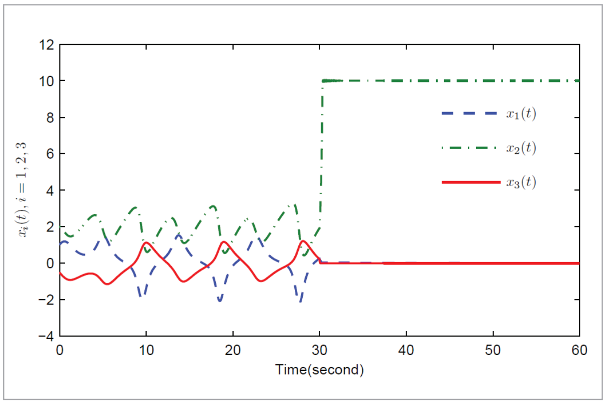


Figure 4

Controlling the fractional-order financial chaotic system (17) to its equilibrium point $E_1=(0,\frac{1}{b},0)$: time responses of the tracking errors $e_i(t)$. The controller is activated at $t = 30$

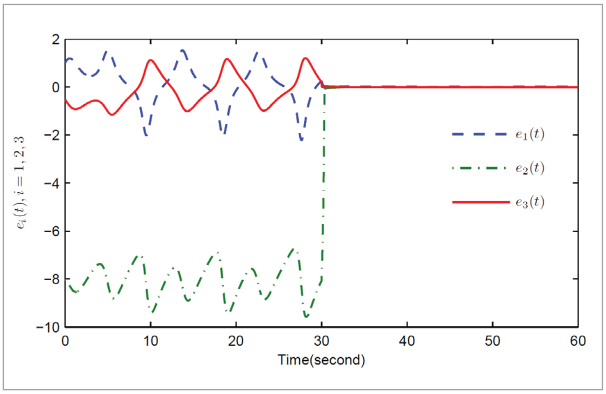


Figure 5

Controlling the fractional-order financial chaotic system (17) to its equilibrium point $E_1=(0,\frac{1}{b},0)$: time responses of the control inputs. The controller is activated at $t = 30$

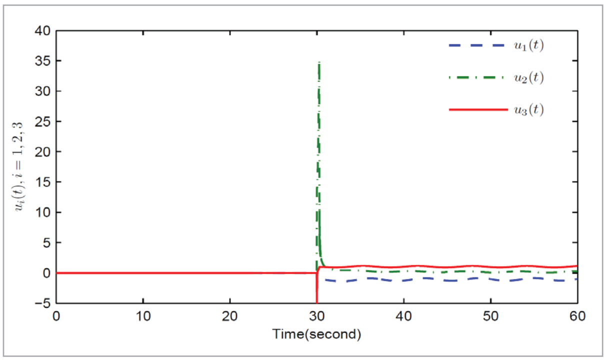


Figure 6

Controlling the fractional-order financial chaotic system (17) to its equilibrium point $E_1=(0,\frac{1}{b},0)$: time responses of $\hat{\gamma}_i(t)$. The controller is activated at $t = 30$

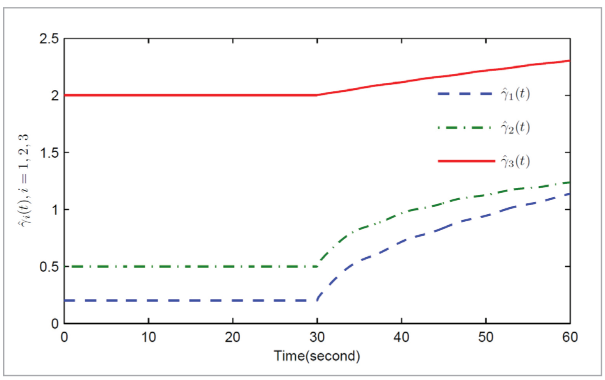


Figure 7

Controlling the fractional-order financial chaotic system (17) to its equilibrium point $E_1=(0,\frac{1}{b},0)$: time responses of $\hat{\bar{d}}_i(t)$. The controller is activated at $t = 30$

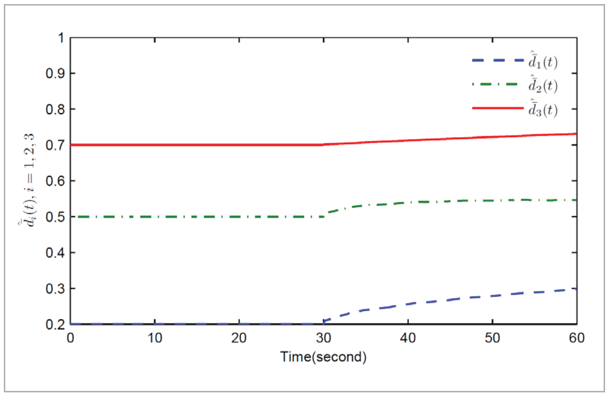


Figure 8

Controlling the fractional-order financial chaotic system (17) to its equilibrium point $E_2=\left(\sqrt{\frac{c-b-abc}{c}},\frac{1+ac}{c},-\frac{1}{c}\sqrt{\frac{c-b-abc}{c}}\right)$. The controller is activated at $t = 30$

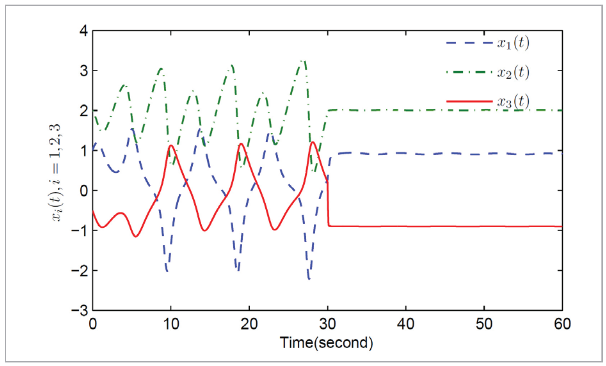
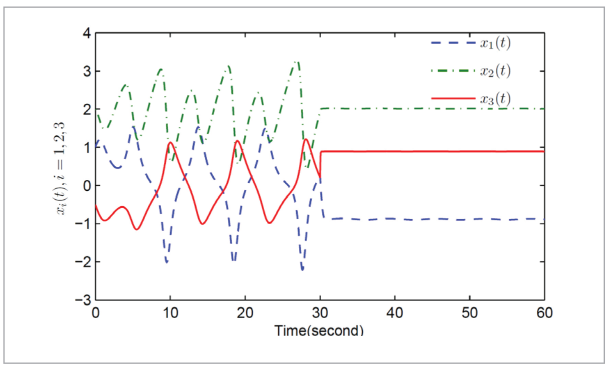


Figure 9

Controlling the fractional-order financial chaotic system (17) to its equilibrium point $E_3=\left(-\sqrt{\frac{c-b-abc}{c}},\frac{1+ac}{c},\frac{1}{c}\sqrt{\frac{c-b-abc}{c}}\right)$. The controller is activated at $t = 30$



Then, the simulation results of controlling the fractional-order financial chaotic system (17) to its equilibrium points

$$E_2=\left(\sqrt{\frac{c-b-abc}{c}},\frac{1+ac}{c},-\frac{1}{c}\sqrt{\frac{c-b-abc}{c}}\right)$$

and $E_3=\left(-\sqrt{\frac{c-b-abc}{c}},\frac{1+ac}{c},\frac{1}{c}\sqrt{\frac{c-b-abc}{c}}\right)$

are presented in Figures 8 and 9, respectively.

It should be stressed that the proposed control method can be used to control a very large scale of fractional-order chaotic systems. Finally, to confirm the effectiveness of the proposed control method, let us consider controlling a novel fractional-order financial chaotic system which can be described by [40]

$$\begin{cases} {}_0^C D_t^\alpha x_1(t)=x_3(t)+(x_2(t)-a)x_1(t)+\Delta f_1(x(t))+d_1(t)+u_1(t),\\ {}_0^C D_t^\alpha x_2(t)=1-bx_2(t)-|x_1(t)|+\Delta f_2(x(t))+d_2(t)+u_2(t),\\ {}_0^C D_t^\alpha x_3(t)=-x_1(t)-cx_3(t)+\Delta f_3(x(t))+d_3(t)+u_3(t), \end{cases} \tag{51}$$

When $\Delta f_i=d_i=u_i=0$, $a=1, b=0.15$, $c=1$ and $x(0)=(1.5,2,-1.5)$, system (51) shows chaotic behavior, which is depicted in Figure 10.

In the simulation, the system uncertainties are chosen as: $\Delta f_1(x(t))=0.3\cos(x_1(t))+0.1x_3(t)$, $\Delta f_2(x(t))=0.1x_2(t)+0.2\sin(x_2(t))$, and $\Delta f_3(x(t))=0.1x_1(t)+\sin(x_3(t))$. Let the external disturbances be $d_1(t)=0.2\sin(t), d_2(t)=0.1\cos(t)$, $d_3(t)=0.1\sin(t)$. The controller design parameters are chosen as $k_1=k_2=k_3=1, h_1=h_2=h3=0.3, m_1=m_2=m_3=0.2$.

The initial conditions of the fractional-order adaptation laws are chosen as

$\hat{\gamma}_1(0)=0.1, \hat{\gamma}_2=0.2, \hat{\gamma}_3=0.3, \hat{\bar{d}}_1(0)=0.1,$

$\bar{d}_2(0)=0.2, \bar{d}_3(0)=0.3.$

The simulation results are presented in Figure 11, from which we can see that good control performance has also been obtained.

Figure 10

Chaotic behavior of fractional-order financial system (51) with $\alpha=0.97$ in (a) $x_1(t)-x_2(t)-x_3(t)$ plane, (b) $x_1(t)-x_2(t)$ plane, (c) $x_1(t)-x_3(t)$ plane and (d) $x_1(t)-x_4(t)$ plane

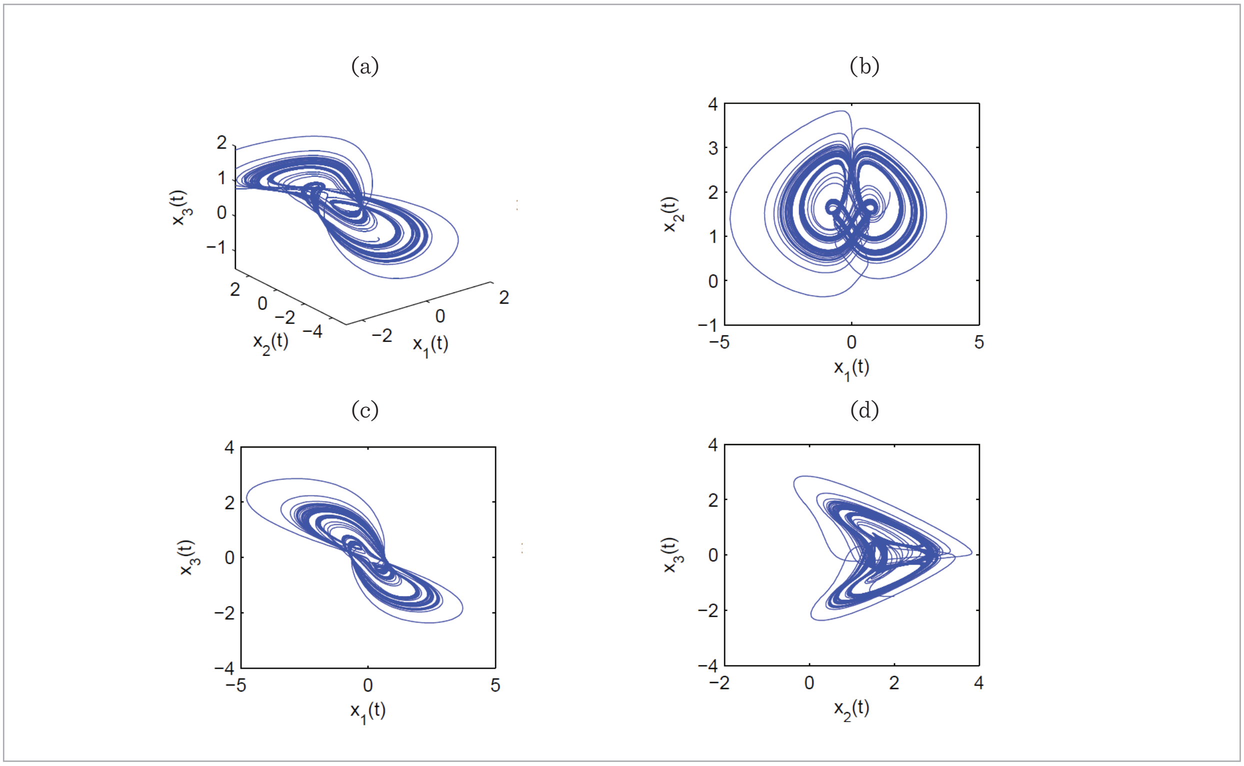
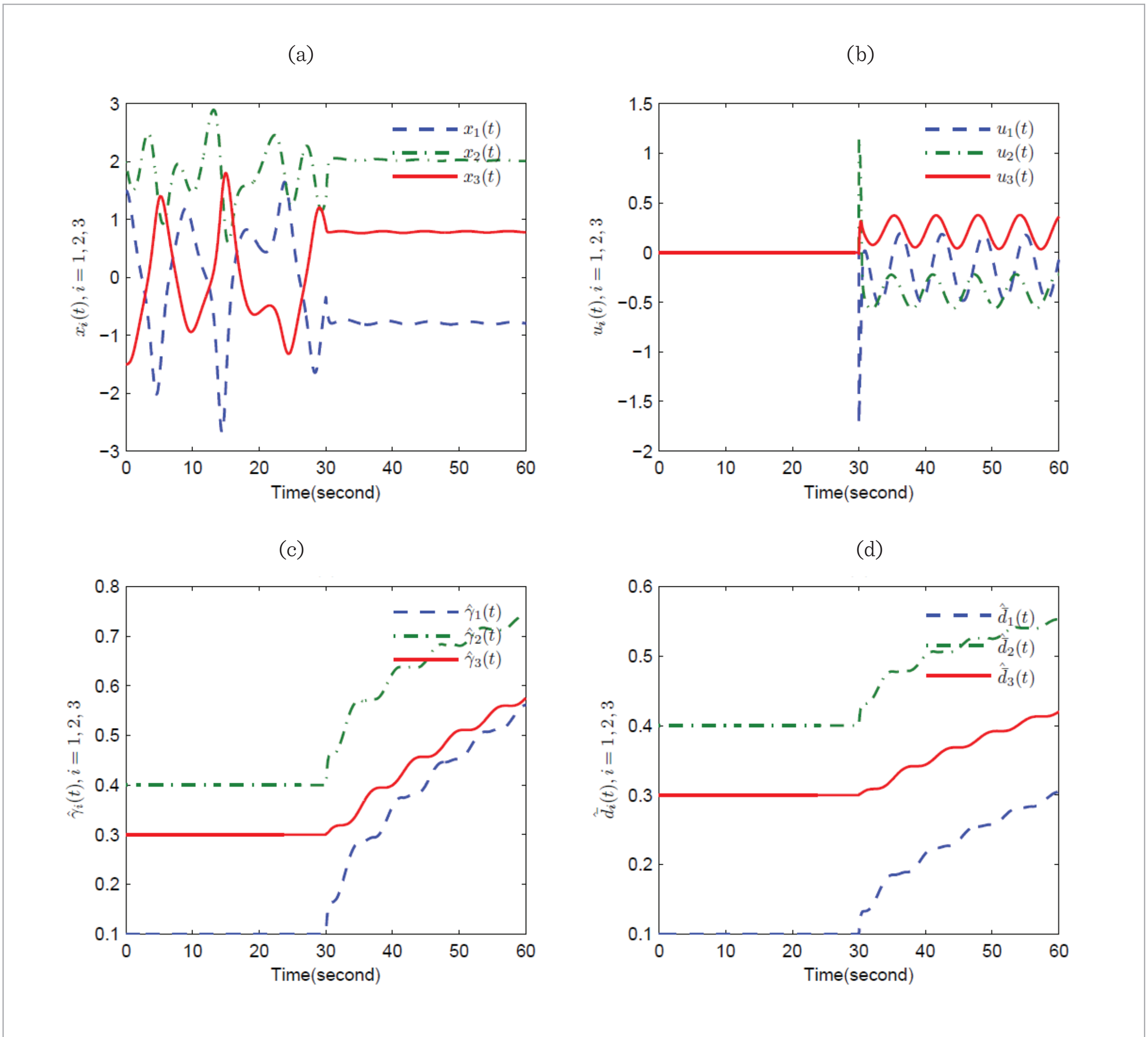


Figure 11

Controlling the fractional-order financial chaotic system (51) to its equilibrium point $E_1 = \left(-\frac{c-b-abc}{c}, \frac{1+ac}{c}, \frac{c-b-abc}{c^2}\right)$ in (a) $x_i(t)$, (b) $u_i(t)$, (c) $\hat{\gamma}_i(t)$ and (d) $\hat{\tilde{d}}_i(t)$. The controller is activated at $t = 30$



Conclusions

The economic systems contain many complex factors which are important to governments. So controlling the fractional-order chaotic financial systems by using effective control method is an interesting yet challenging work. This paper mainly discusses this problem by means of adaptive control. The following three aspects are included: (1) the stability analysis for fractional-order financial chaotic systems based on fractional-order Lyapunov second method; (2) fractional-order adaptation law and its application in the stability analysis for fractional-order nonlinear systems; (3) the usage of quadratic Lyapunov functions in stability analysis of fractional-order systems. The results of our results may enrich the control theorem of fractional-order systems, and the proposed control method can also be extended to other fractional-order systems.

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Summary / Santrauka

In this paper, robust adaptive control for uncertain fractional-order financial chaotic systems with bounded unknown external disturbances is studied. By utilizing the fractional-order extension of the classical Lyapunov stability methods, an adaptive controller is presented for controlling the fractional-order financial chaotic system. Quadratic Lyapunov functions are employed in the stability analysis, and fractional-order adaptation laws are designed to update controller parameters online. The proposed controller can ensure that the system states converge to the origin asymptotically and all signals in the closed-loop system remain bounded. Finally, simulation results are presented to confirm our theoretical results.

Straipsnyje tiriamas stiprus adaptyvus nepastoviųjų frakcinės eilės chaotiškųjų sistemų su apribotais išoriniais trikdžiais valdymas. Panaudojant klasikinių Liapunovo stabilumo metodų frakcinės eilės papildinį, pristatomas adaptyvus valdiklis frakcinės eilės finansinei chaotiškajai sistemai valdyti. Stabilumo analizėje naudojamos kvadratinių Liapunovo lygčių funkcijos, o frakcinės eilės prisitaikymo dėsniai sukurti taip, kad juos taikant būtų atnaujinti valdiklio parametrai internete. Siūlomas valdiklis gali užtikrinti, kad sistemos būsenos konverguotų į kilmės šaltinį asimptomiškai ir visi signalai uždaro rato sistemoje išliktų apriboti. Autorių teoriniai rezultatai patvirtinami simuliacijos rezultatais.