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Stochastic Stability and Stabilization of Semi-Markov Jump Linear Systems with Uncertain Transition Rates

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This paper investigates the problems of robust stochastic stability and stabilization for a general class of continuous-time semi-Markovian jump linear systems (S-MJLSs). The main contribution of the research is to eliminate the limitations of the traditional S-MJLSs with precisely available information by introducing a system with uncertain, time-varying transition rates (TRs) of the jump process, in addition to the imperfect information on the system dynamic matrices. The new system is called the general uncertain semi-Markov jump linear system (GUS-MJLS); it does not contain certain values of the transition rates, but includes nominal time-dependent values in addition to bounded deviations. It is suitable to describe a broader class of dynamical systems with estimated information and modeling errors and also covers the concepts of Markov jump linear system (MJLSs) with time-constant and certain TRs. For this system, the stability is firstly analyzed through the multiple stochastic Lyapunov function approach. Then, based on the stability results, a robust state-feedback controller is formulated. To deal with the time-dependent TRs, a sojourn-time fractionizing technique is used and numerically testable conditions are developed. Finally, discussions on reducing the conservativeness of the robust theorems are provided. The theoretical results are successfully tested on an industrial continuous stirred tank reactor (CSTR) subject to stochastically varying environmental conditions. Comparative simulations are also provided to show the superiority of the presented framework and design method to the existing ones.

KEYWORDS: Semi-Markov Jump Linear System, Stochastic Stability, Stabilization, Time-Varying Transition Rates, Uncertainty, Lyapunov Function.

Introduction

In recent years, jump linear systems (JLSs) have received increasing attention [1-3] due to their potentials of modeling systems subject to abrupt changes and variations in their structures or their parameters, induced by random faults, failures, repairs and unexpected configuration conversions [1, 3, 4]. A JLS is characterized by both time-evolving and event-driven mechanisms. The former is described by a set of linear differential (or difference) equations and the latter is governed by a stochastic rule. For a JLS the length of intervals between two consecutive events is called the sojourn-time, which is an identically independently random variable subject to a specific probability distribution.

A major class of the JLSs is the Markov jump linear system where the stochastic rule is specifically a Markov process [1]. Over the past decades, MJLSs have been studied extensively and large volume of theoretical results has been provided for their analysis and synthesis [1, 5, 6, 7-11]. Additionally, great efforts of researchers have been dedicated to solve the problems of practical applications modeled as MJLSs, such as networked control systems (NCSs) [12], fault prone systems [9] and economic systems [13]. Although MJLSs are theoretically and practically interesting but, unfortunately, they have one inevitable constraint which is the inflexible exponential-type probability distribution function (PDF) of the sojourn-time of the Markov process. This specification renders the TRs of the JLS to be time-constant, which limits the applications of MJLSs and provides conservative results in some sense.

From the system modeling point of view, the sojourn-time may follow any probability distribution rather than exponential distribution. In this case, a JLS is termed as a semi-Markov jump linear system [14, 15]. A S-MJLS relaxes the transition rates of the jump process from a constant value to a time-varying variable [16]. It helps to reflect the specifications of more practical systems rather than the traditional MJLS [14, 16-20]. For example, semi-Markovian structure is very popular in modeling biological systems [18, 19], also it is widely used for the reliability engineering, where a typical transition or failure rate function is in a bathtub shape instead of a constant value [21, 22].

Applications and potentials of systems that include semi-Markovian jump processes have interested the researchers to investigate such structures [2]. Stability analysis of S-MJLSs is reported in [12, 14, 15, 23-28], stabilization concepts are established in [25, 29-31], controllers are designed in [19, 25, 32- 35], and filtering problems are investigated in [21, 36].

Almost all of the aforementioned studies [12, 15, 21, 23-28, 30, 32, 34-36], deploy a completely and precisely known S-MJLS. Nevertheless, the assumption of the availability of a certainly modeled semi-Markovian JLS is very restrictive. In reality, no system model could be identified accurately, and the uncertainties are ubiquitous in practical applications. The most dominant factor of S-MJLS that must be identified for control purposes is the TRs of the jump process. Unfortunately, achieving a semi-Markovian model with precise TRs is very complicated or generally expensive, especially when there are a limited number of data samples or when the information is noisy. The imperfect estimation leads to TR modeling errors (also referred to as switching probability uncertainties), that generally cause instability or, at least, degraded performance of a system. Therefore, rather than the large complexity to estimate accurate TRs, it is significant to study more general S-MJLSs with imprecise or uncertain transition rates from the control perspectives. Stability and stabilization are the most important notions for S-MJLSs with incomplete TR information. Although, the problem is almost solved for the traditional MJLSs with time-constant transition rates [5, 7-11, 37], but, to the best of the authors' knowledge, the problem has not been investigated yet for semi-Markovian structures with time-dependent uncertain transition rates and is the main contribution of the present study.

In this study, a GUS-MJLS is addressed. At this description, the system is assumed to include TRs with bounded time-varying uncertainties, which means that the semi-Markov process does not contain certain values of the transition rates, but includes nominal time-varying values in addition to bounded deviations. This bounded deviation is a significant type of uncertainty and represents the imperfection in many physical cases caused by factors like aging of devices and identification errors [1]. To provide a more general and flexible framework, the system under consideration is also assumed to contain time-dependent bounded uncertainties in the system dynamics. By this new GUS-MJLS, lots of system behaviour could be better captured. As an example, the internet-based NCSs can be mentioned [12, 17]. In the analysis of internet-based NCSs, Markov processes are extensively used to model the transmission delays and packet losses [6], however, these factors are generally imprecise and distinct at different periods of time and presenting them with an uncertain semi-Markovian process is a more realistic scenario [12]. For the introduced GUS-MJLS, sufficient criteria are developed for the stability and stabilizability based on the stochastic multiple Lyapunov function approach. The stability condition is used to design a stabilizing feedback. To overcome the difficulty of analysing S-MJLSs that is encountered due to continuously time-varying TRs, a fractionizing technique is applied to the transition rates. Based on this technique, the stability and controller design conditions are reformulated to provide finite dimensional, time-independent and computationally effective criteria. All the results are in the form of linear matrix inequalities and equalities that can be easily solved by the existing optimization techniques. It is worth mentioning that, the proposed theorems cover the stability and stabilization concepts of the MJLSs with time constant jump probabilities [1]. The obtained theorems are tested on a continuous stirred tank reactor subject to external environmental changes. The CSTR is modeled as a JLS including uncertain time-varying TRs with Weibull distribution function of sojourn-times, which is considered as a better representation than its models by traditional MJLS. Comparing the results with a situation that does not consider the TR uncertainty in the controller design procedure shows the superiority and the effectiveness of the results to the previously developed ones [25].

Problem statement

Consider the following dynamical system defined in a probability space (Ω, F, ρ) where Ω, F and ρ represent, the sample space, the algebra of events and the probability measure on F, respectively:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \hat{\boldsymbol{A}}(r_t,t)\boldsymbol{x}(t) + \hat{\boldsymbol{B}}(r_t,t)\boldsymbol{u}(t) \\ \boldsymbol{x}(t_0) = \boldsymbol{x}_0, r_{t_0} = r_0 \end{cases} \tag{1}$$

$\{r_t, t\geq 0\}$ is a continuous-time discrete-value semi-Markov process on the probability space, which takes values in a finite set $\underline{\mathrm{N}} = \{1, 2, \dots, N\}$. It governs the choice of smooth dynamics for the continuous state and its initial mode at $t = 0$ is r_0. $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state vector and $\boldsymbol{x}_0 \in \mathbb{R}^n$ is the initial state vector with the dimension n, and $\boldsymbol{u}(t) \in \mathbf{R}^m$ is the controlled input vector with the dimension m. $\hat{\boldsymbol{A}}(r_t,t)$ and $\hat{\boldsymbol{B}}(r_t,t)$ specify system matrices with appropriate dimensions. They are supposed to be uncertain with the following form:

$$\begin{aligned} \hat{\boldsymbol{A}}(r_t,t) &= \boldsymbol{A}(r_t) + \Delta\boldsymbol{A}(r_t,t), \\ \hat{\boldsymbol{B}}(r_t,t) &= \boldsymbol{B}(r_t) + \Delta\boldsymbol{B}(r_t,t) \end{aligned} \tag{2}$$

where $\boldsymbol{A}(r_t)$ and $\boldsymbol{B}(r_t)$ are known, mode-dependent system matrices, and $\Delta\boldsymbol{A}(r_t,t)$, $\Delta\boldsymbol{B}(r_t,t)$ are unknown, mode-dependent matrices representing time-varying, norm-bounded parameter uncertainty, that are supposed to have the following form:

$$\begin{aligned} \Delta\boldsymbol{A}(r_t,t) &= \boldsymbol{D}_A(r_t)\boldsymbol{F}_A(r_t,t)\boldsymbol{E}_A(r_t), \\ \Delta\boldsymbol{B}(r_t,t) &= \boldsymbol{D}_B(r_t)\boldsymbol{F}_B(r_t,t)\boldsymbol{E}_B(r_t) \end{aligned} \tag{3}$$

in which $\boldsymbol{D}_A(r_t)$, $\boldsymbol{D}_B(r_t)$, $\boldsymbol{E}_A(r_t)$ and $\boldsymbol{E}_B(r_t)$ are known matrices; and $\boldsymbol{F}_A(r_t, t)$, $\boldsymbol{F}_B(r_t, t)$ are time-varying, unknown, Lebesgue measurable matrices satisfying $\boldsymbol{F}_A^T(r_t,t)\boldsymbol{F}_A(r_t,t) \leq \boldsymbol{I}$ and $\boldsymbol{F}_B^T(r_t,t)\boldsymbol{F}_B(r_t,t) \leq \boldsymbol{I}$, respectively.

The evolution of the semi-Markov process is governed by the transition rate matrix:

$$\hat{\boldsymbol{\Gamma}}(h) = \begin{bmatrix} \hat{\lambda}_{11}(h) & \hat{\lambda}_{12}(h) & \dots & \hat{\lambda}_{1N}(h) \\ \hat{\lambda}_{21}(h) & \hat{\lambda}_{22}(h) & \dots & \hat{\lambda}_{2N}(h) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\lambda}_{N1}(h) & \hat{\lambda}_{N2}(h) & \dots & \hat{\lambda}_{NN}(h) \end{bmatrix} \tag{4}$$

where

$$\Pr\left\{r_{t+h} = j \mid r_t = i\right\} = \begin{cases} \hat{\lambda}_{ij}(h)h + o(h) & i \neq j \\ 1 + \hat{\lambda}_{ii}(h)h + o(h) & i = j \end{cases} \tag{5}$$

$h > 0, \lim_{h\to 0}(o(h)/h) = 0$, is the sojourn-time or the time between two consecutive jumps. $\hat{\lambda}_{ij}(h) \geq 0, i, j \in \underline{\mathrm{N}}$ denotes the uncertain transition rate from mode i at time t to mode j at time $t+h$ with the condition $\hat{\lambda}_{ii}(h) = -\sum_{j=1, j\neq i}^{N} \hat{\lambda}_{ij}(h)$. The sojourn-time h is a random variable following a continuous probability distribution in continuous-time jump linear systems. Clearly, here $\hat{\lambda}_{ij}(h)$ is a time-varying stochastic variable, which represents the time-dependent property of the TRs of the S-MJLS.

It is assumed that the transition rate matrix has parametric uncertainty. The uncertain transition rate matrix is expressed as $\hat{\boldsymbol{\Gamma}}(h) = \boldsymbol{\Gamma}(h) + \Delta\boldsymbol{\Gamma}(h)$, where $\Delta\boldsymbol{\Gamma}(h) = \left[\Delta\lambda_{ij}(h)\right]$ is an unknown matrix such that $\Delta\lambda_{ii}(h) = -\sum_{j=1, j\neq i}^{N} \Delta\lambda_{ij}(h)$. Elements of $\Delta\tilde{\boldsymbol{A}}(h)$ are supposed to be bounded by $\left|\Delta\lambda_{ij}(h)\right| \leq \pi_{ij}$ for all modes. This description means that the TRs have norm-bounded uncertainty with a maximum value $\pi_{ij} > 0$.

***Remark* 1.** The uncertainty bound π_{ij} could be determined empirically by experimental tests or by the historical data of the system. In this paper, it is assumed that the uncertainty bound is given a priori by an uncertain transition rate matrix; how to conduct the TR modeling procedure and how to design tests and mechanisms to obtain the chain TRs are not discussed here since their foundations have been pretty well established by now [22, 38-40].

***Remark* 2.** Hereafter in the whole paper, for simplicity, $r_t = i$ will be used for notation and the matrices will be labeled as $\boldsymbol{A}(i)$, $\boldsymbol{B}(i)$, $\Delta\boldsymbol{A}(i)$, $\Delta\boldsymbol{B}(i)$, $\boldsymbol{F}_A(i)$, $\boldsymbol{F}_B(i)$, $\boldsymbol{D}_A(i)$, $\boldsymbol{D}_B(i)$, $\boldsymbol{E}_A(i)$, $\boldsymbol{E}_B(i)$. Additionally, without loss of generality, the initial time is set to be zero, $t_0 = 0$. The initial state vector, x_0 and the initial mode, r_0 are also supposed to be known.

The main objective of this paper is to derive sufficient conditions for robust stochastic stability and stabilizability of the system (1) and then design the controller. Before proceed, the following definition and lemma are given which will be used in the rest of the paper:

Definition. [4] **For any initial mode r_0, and any given initial state vector x_0, the uncertain system (1) ($u(t) = 0$) is said to achieve stochastic stability in second mean if (6) holds for all admissible uncertainties. $E\{.|.\}$ is the expectation conditioning on the initial values of x_0 and r_0.**

$$\int_0^\infty E\left\{\left\|\boldsymbol{x}(t)\right\|^2 dt \mid \boldsymbol{x}_0, r_0\right\} < \infty \tag{6}$$

Lemma. [4] *Let $\boldsymbol{Y}$ satisfy $\boldsymbol{Y}^T\boldsymbol{Y} \leq I$, and $\boldsymbol{H}$, $\boldsymbol{E}$ be given matrices with the appropriate dimensions, then there exists a scalar $\varepsilon > 0$ such that $\boldsymbol{HYE} + \boldsymbol{E}^T\boldsymbol{Y}^T\boldsymbol{H}^T \leq \varepsilon\boldsymbol{HH}^T + \varepsilon^{-1}\boldsymbol{E}^T\boldsymbol{E}$ holds.*

Main results

In this section, the main results are provided. In the first upcoming subsection the robust stochastic stability of a semi-Markovian jump linear systems subject to dynamic and TR matrix uncertainties is analyzed. In the second subsection, the stabilizability is investigated and the robust controller gains are designed. Finally, numerically testable results are achieved through TR approximations.

Stability

Theorem 1. *The uncertain S-MJLS of* (1) ($\boldsymbol{u}(t)$=0) *is robustly stochastically stable if there are symmetric, positive definite mode-dependent matrices $P(i)$ and a set of positive mode dependent scalars $\varepsilon_A(i)$, $\varepsilon_B(i)$, and $\varepsilon_\lambda(i,j)$, such that the following set of LMIs hold for all possible modes of i*

$$\begin{bmatrix} \boldsymbol{J}(i) & \boldsymbol{P}(i)\boldsymbol{D}_A(i) & \boldsymbol{S}(i) \\ \boldsymbol{D}_A^T(i)\boldsymbol{P}(i) & -\varepsilon_A^{-1}(i)\boldsymbol{I} & 0 \\ \boldsymbol{S}^T(i) & 0 & -\boldsymbol{R}(i) \end{bmatrix} < 0\,, \tag{7}$$

$$\boldsymbol{J}(i) = \boldsymbol{A}^T(i)\boldsymbol{P}(i) + \boldsymbol{P}(i)\boldsymbol{A}(i) + \varepsilon_A^{-1}(i)\boldsymbol{E}_A^T(i)\boldsymbol{E}_A(i) + \sum_{j=1}^{N}\lambda_{ij}(h)\boldsymbol{P}(j) + \frac{1}{4}\sum_{j=1}^{N}\varepsilon_\lambda(i,j)\pi_{ij}^2\boldsymbol{I} \tag{8}$$

$$\boldsymbol{S}(i) = \left[\boldsymbol{P}(i) - \boldsymbol{P}(1), \ldots, \boldsymbol{P}(i) - \boldsymbol{P}(i-1), \boldsymbol{P}(i) - \boldsymbol{P}(i+1) \ldots, \boldsymbol{P}(i) - \boldsymbol{P}(N)\right] \tag{9}$$

$$\boldsymbol{R}(i) = diag\left[\varepsilon_\lambda(i,1)\boldsymbol{I}, \ldots, \varepsilon_\lambda(i,i-1)\boldsymbol{I}, \varepsilon_\lambda(i,i+1)\boldsymbol{I}, \ldots, \varepsilon_\lambda(i,N)\boldsymbol{I}\right] \tag{10}$$

Proof. Consider the following stochastic Lyapunov function with semi-Markovian parameters

$$V(\boldsymbol{x}(t), r_t) = \boldsymbol{x}^T(t)\boldsymbol{P}(r_t)\boldsymbol{x}(t)\ . \tag{11}$$

Eq. (11) is a multiple Lyapunov function [4] where $P(i)$ denounce symmetric and positive definite matrices.

The infinitesimal generator [4] for the semi-Markovian jump linear system is defined as

$$LV(\boldsymbol{x}(t), i) = \lim_{\Delta\to 0}\Big(E[\boldsymbol{V}(\boldsymbol{x}(t+\Delta), r_{t+\Delta}) \mid \boldsymbol{x}(t) = \boldsymbol{x}(t), r_t = i] - \boldsymbol{V}(\boldsymbol{x}(t), i)\Big) / \Delta \tag{12}$$

where $\lim_{\Delta\to 0}(\Delta^2/\Delta) = 0.$ By applying the law of total probability and using the property of the conditional expectation, the infinitesimal generator is written as

$$LV(\boldsymbol{x}(t), i) = \lim_{\Delta\to 0}\Big(E[\boldsymbol{V}(\boldsymbol{x}(t+\Delta), r_{t+\Delta}) \mid \boldsymbol{x}(t) = \boldsymbol{x}(t), r_t = i] - \boldsymbol{V}(\boldsymbol{x}(t), i)\Big) / \Delta \tag{13}$$

Using the probability distribution function related to the semi-Markov chain, the infinitesimal generator becomes as the following:

$$\begin{aligned} LV(\boldsymbol{x}(t), i) = \lim_{\Delta\to 0}\Big[\Big\{\sum_{\substack{j=1\\ j\neq i}}^{N} \frac{q_{ij}(F_i(h+\Delta) - F_i(h))}{1 - F_i(h)} \\ \boldsymbol{x}^T(t+\Delta)\boldsymbol{P}(j)\boldsymbol{x}(t+\Delta) + \frac{1 - F_i(h+\Delta)}{1 - F_i(h)} \\ \boldsymbol{x}^T(t+\Delta)\boldsymbol{P}(i)\boldsymbol{x}(t+\Delta)\Big\} - \boldsymbol{x}(t)\boldsymbol{P}(i)\boldsymbol{x}(t)\Big] / \Delta \end{aligned} \tag{14}$$

Using $\boldsymbol{x}(t+\Delta) = (\hat{\boldsymbol{A}}(i)\Delta + \boldsymbol{I})\boldsymbol{x}(t) = \boldsymbol{x}(t) + \hat{\boldsymbol{A}}(i)\Delta\,\boldsymbol{x}(t)$, (14) turns to (15):

$$\begin{aligned} LV(\boldsymbol{x}(t), i) = \lim_{\Delta\to 0}\frac{1}{\Delta}\Big[\Big\{\sum_{\substack{j=1,\\ j\neq i}}^{N} \frac{q_{ij}(F_i(h+\Delta) - F_i(h))}{1 - F_i(h)} \\ \boldsymbol{x}(t)(\hat{\boldsymbol{A}}^T(i,t)\boldsymbol{P}(j)\Delta + \boldsymbol{P}(j)\hat{\boldsymbol{A}}(i,t)\Delta + \boldsymbol{P}(j))\boldsymbol{x}(t) + \\ \frac{1 - F_i(h+\Delta)}{1 - F_i(h)}\boldsymbol{x}^T(t)(\hat{\boldsymbol{A}}^T(i,t)\boldsymbol{P}(i)\Delta + \boldsymbol{P}(i)\hat{\boldsymbol{A}}(i,t)\Delta \\ +\boldsymbol{P}(i))\boldsymbol{x}(t)\Big\} - \boldsymbol{x}(t)\boldsymbol{P}(i)\boldsymbol{x}(t)\Big] \end{aligned} \tag{15}$$

Equation (15) can be written in the form of $LV(\boldsymbol{x}(t), i) = \boldsymbol{x}^T(t)\boldsymbol{N}(i)\boldsymbol{x}(t)$ with $\boldsymbol{N}(i)$ defined as (16):

$$\begin{aligned} \boldsymbol{N}(i) = \sum_{j=1,\, j\neq i}^{N} q_{ij}\boldsymbol{P}(j)\lim_{\Delta\to 0}\frac{F_i(h+\Delta) - F_i(h)}{(1 - F_i(h))\Delta} + \\ \sum_{j=1,\, j\neq i}^{N} q_{ij}\lim_{\Delta\to 0}\frac{F_i(h+\Delta) - F_i(h)}{(1 - F_i(h))\Delta}(\hat{\boldsymbol{A}}^T(i,t)\boldsymbol{P}(j) + \\ \boldsymbol{P}(j)\hat{\boldsymbol{A}}(i,t))\Delta + \lim_{\Delta\to 0}\boldsymbol{P}(i)\frac{1 - F_i(h+\Delta)}{(1 - F_i(h))\Delta} + \\ \lim_{\Delta\to 0}\frac{1 - F_i(h+\Delta)}{(1 - F_i(h))\Delta}\big(\hat{\boldsymbol{A}}^T(i,t)\boldsymbol{P}(i) + \boldsymbol{P}(i)\hat{\boldsymbol{A}}(i,t)\big)\Delta - \\ \lim_{\Delta\to 0}\boldsymbol{P}(j)/\Delta \end{aligned} \tag{16}$$

Here q_{ij} is the probability intensity from mode i to mode j and F_i is the cumulative distribution function (CDF) of the sojourn-time when system remains at mode i.

Consider the properties of,

$$\lim_{\Delta\to 0}\frac{F_i(h+\Delta) - F_i(h)}{(1 - F_i(h))\Delta} = \hat{\lambda}_i(h), \qquad \lim_{\Delta\to 0}\frac{1 - F_i(h+\Delta)}{1 - F_i(h)} = 1$$

and $\lim_{\Delta\to 0}\frac{F_i(h+\Delta) - F_i(h)}{1 - F_i(h)} = 0$

where $\hat{\lambda}_i(h)$ is the uncertain TR of the system jumping from mode i. Then the TR function is $\frac{f_{ij}(h)}{1 - F_{ij}(h)} = \lambda_{ij}(h)$ and satisfies $q_{ij}\hat{\lambda}_i(h) = \hat{\lambda}_{ij}(h)$ with $f_{ij}(h)$ and $F_{ij}(h)$ as the PDF and CDF of the sojourn-time when system switches between i and j [41]. By simplifying (16), $\boldsymbol{N}(i)$ will be obtained as

$$\boldsymbol{N}(i) = \hat{\boldsymbol{A}}^T(i,t)\boldsymbol{P}(i) + \boldsymbol{P}(i)\hat{\boldsymbol{A}}(i,t) + \sum_{j=1}^{N}\hat{\lambda}_{ij}(h)\boldsymbol{P}(j)\,. \tag{17}$$

One has $LV(\boldsymbol{x}(t), i) < 0$, if $\boldsymbol{N}(i) < 0$. By considering a similar line in the proof of Theorem 1 of [25], if $\boldsymbol{N}(i) < 0$ holds, then the definition (7) is verified and the closed-loop system is stochastically stable.

In order to achieve the LMIs of the theorem, substitute the uncertain system matrix form (2) and (3) as well as uncertain transition probabilities form (5), then $\boldsymbol{N}(i) < 0$ can be written as (18):

$$(\boldsymbol{A}(i)+\boldsymbol{D}_A(i)\boldsymbol{F}_A(i)\boldsymbol{E}_A(i))^T\boldsymbol{P}(i)+\boldsymbol{P}(i)(\boldsymbol{A}(i)+\boldsymbol{D}_A(i)\boldsymbol{F}_A(i)\boldsymbol{E}_A(i))+\sum_{j=1}^{N}\lambda_{ij}(h)\boldsymbol{P}(j)+\sum_{j=1}^{N}\Delta\lambda_{ij}(h)\boldsymbol{P}(j)<0 \tag{18}$$

Simplifications yield:

$$\begin{aligned}&\boldsymbol{A}^T(i)\boldsymbol{P}(i)+\boldsymbol{E}_A^T(i)\boldsymbol{F}_A^T(i,t)\boldsymbol{D}_A^T(i)\boldsymbol{P}(i)+\boldsymbol{P}(i)\boldsymbol{A}(i)\\&+\boldsymbol{P}(i)\boldsymbol{D}_A(i)\boldsymbol{F}_A(i,t)\boldsymbol{E}_A(i)+\sum_{j=1}^{N}\lambda_{ij}(h)\boldsymbol{P}(j)+\\&\sum_{j=1,j\neq i}^{N}\frac{1}{2}\Delta\lambda_{ij}(h)(\boldsymbol{P}(j)-\boldsymbol{P}(i))+\\&\sum_{j=1,j\neq i}^{N}\frac{1}{2}\Delta\lambda_{ij}(h)(\boldsymbol{P}(j)-\boldsymbol{P}(i))<0\end{aligned} \tag{19}$$

Taking the advantage of Lemma and using the bounds for uncertain TRs, the following inequality is writable for the uncertain terms in (19):

$$\begin{aligned}&\boldsymbol{P}(i)\boldsymbol{D}_A(i)\boldsymbol{F}_A(i,t)\boldsymbol{E}_A(i)+\boldsymbol{E}_A^T(i)\boldsymbol{F}_A^T(i,t)\boldsymbol{D}_A^T(i)\boldsymbol{P}(i)\\&+\sum_{j=1}^{N}\Delta\lambda_{ij}^n\left(\boldsymbol{P}(j)-\boldsymbol{P}(i)\right)\le\varepsilon_A(i)\boldsymbol{P}(i)\boldsymbol{D}_A(i)\boldsymbol{D}_A^T(i)\\&\boldsymbol{P}(i)+\varepsilon_A^{-1}(i)\boldsymbol{E}_A^T(i)\boldsymbol{E}_A(i)+\frac{1}{4}\sum_{j=1}^{N}\varepsilon_\lambda(i,j)\pi_{ij}^2\boldsymbol{I}\\&+\sum_{j=1}^{N}\varepsilon_\lambda^{-1}(i,j)\left(\boldsymbol{P}(j)-\boldsymbol{P}(i)\right)^2\end{aligned} \tag{20}$$

Using (20), the inequality of (19) turns to the form of (21):

$$\begin{aligned}&\boldsymbol{P}(i)\boldsymbol{D}_A(i)\boldsymbol{F}_A(i,t)\boldsymbol{E}_A(i)+\boldsymbol{E}_A^T(i)\boldsymbol{F}_A^T(i,t)\boldsymbol{D}_A^T(i)\boldsymbol{P}(i)\\&+\sum_{j=1}^{N}\Delta\lambda_{ij}^n\left(\boldsymbol{P}(j)-\boldsymbol{P}(i)\right)\le\varepsilon_A(i)\boldsymbol{P}(i)\boldsymbol{D}_A(i)\boldsymbol{D}_A^T(i)\\&\boldsymbol{P}(i)+\varepsilon_A^{-1}(i)\boldsymbol{E}_A^T(i)\boldsymbol{E}_A(i)+\frac{1}{4}\sum_{j=1}^{N}\varepsilon_\lambda(i,j)\pi_{ij}^2\boldsymbol{I}\\&+\sum_{j=1}^{N}\varepsilon_\lambda^{-1}(i,j)\left(\boldsymbol{P}(j)-\boldsymbol{P}(i)\right)^2\end{aligned} \tag{21}$$

On the basis of Schur complement lemma and by defining $\boldsymbol{J}(i)$, $\boldsymbol{S}(i)$ and $\boldsymbol{Z}(i)$ by (8), (9) and (10), respectively, the inequality (21) can be written in the form of the LMI (7) of Theorem 1, and the proof is complete.

Remark 3. According to Lemma, the parameter $\varepsilon > 0$ can take any values since it specifies the degree of robustness. But generally, it should be selected through a compromise between conservativeness and feasibility of the conditions. One common approach to deal with such parameter [4], which is also used in the present paper, is to solve the conditions for a priori given ε value to achieve a prescribed degree of robustness. The main advantage of this approach is providing the conditions of a fair comparison between the multiple results.

Controller design

Derivation of the stability condition gives insight to provide stabilizability condition of the system (1). In this section, the stabilizability criterion is obtained and the design problem of a robust state-feedback controller law in the form of Eq. (22) is discussed:

$$\boldsymbol{u}(t)=\boldsymbol{K}(r_t)\boldsymbol{x}(t). \tag{22}$$

To achieve the goal, consider the closed-loop system as the following;

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\overline{\boldsymbol{A}}(r_t,t)\boldsymbol{x}(t)\\ \boldsymbol{x}(t_0)=\boldsymbol{x}_0, r_{t_0}=r_0\end{cases} \tag{23}$$

where $\overline{\boldsymbol{A}}(r_t,t)=\hat{\boldsymbol{A}}(r_t,t)+\hat{\boldsymbol{B}}(r_t,t)\boldsymbol{K}(r_t)$ is the closed-loop system matrix.

The following theorem provides a sufficient condition for the existence of a state feedback controller.

Theorem 2. *There exist controller gains* (22) *such that the closed-loop uncertain S-MJLS is stochastically stable, if there exists a set of symmetric, positive definite mode-dependent matrices* $\boldsymbol{X}(i)$, $\boldsymbol{P}(i)$ *and a set of mode-dependent matrices* $\boldsymbol{Y}(i)$, $\boldsymbol{Z}(i)$, $\boldsymbol{V}(i)$, *and a set of positive mode dependent scalars* $\varepsilon_A(i)$, $\varepsilon_B(i)$, *and* $\varepsilon_\lambda(i,j)$, *such that the following set of constraints hold for all possible modes of* i:

$$\begin{bmatrix}\boldsymbol{J}(i) & \boldsymbol{X}(i)\boldsymbol{E}_A^T(i) & \boldsymbol{Y}^T(i)\boldsymbol{E}_B^T(i) & \boldsymbol{X}(i)\\ \boldsymbol{E}_A(i)\boldsymbol{X}(i) & -\varepsilon_A(i)\boldsymbol{I} & 0 & 0\\ \boldsymbol{E}_B(i)\boldsymbol{Y}(i) & 0 & -\varepsilon_B(i)\boldsymbol{I} & 0\\ \boldsymbol{X}(i) & 0 & 0 & -\boldsymbol{Z}(i)\end{bmatrix}<0 \tag{24}$$

$$\begin{bmatrix}\boldsymbol{Q}(i) & \boldsymbol{S}(i)\\ \boldsymbol{S}^T(i) & -\boldsymbol{R}(i)\end{bmatrix}<0 \tag{25}$$

$$X(i) = P^{-1}(i),\ \ V(i) = Z^{-1}(i) \tag{26}$$

where

$$\begin{aligned} &J(i) = X(i)A^T(i) + A(i)X(i) + Y^T(i)B^T(i) + \\ &B(i)Y(i) + \varepsilon_A(i)D_A(i)D_A^T(i) + \varepsilon_B(i)D_B(i)D_B^T(i) \end{aligned} \tag{27}$$

$$\begin{aligned} &Q(i) = -V(i) + \sum_{j=1}^{N} \lambda_{ij}(h)P(j) + \\ &\quad \frac{1}{4}\sum_{\substack{j=1,\\ j\neq i}}^{N} \varepsilon_\lambda(i,j)\pi_{ij}^2 I \end{aligned} \tag{28}$$

and $S(i)$ and $R(i)$ are given by (9) and (10), respectively.

Thus, the controller is obtained as $K(i) = Y(i)X^{-1}(i)$.

Proof. Consider the inequality $N(i) < 0$ as the robust stability result with $N(i)$ defined by (17), replace $\hat{A}(i,t)$ by $\overline{A}(i,t) = \hat{A}(r_t,t) + \hat{B}(r_t,t)K(r_t)$ as the state matrix of the closed-loop system, also substitute $\hat{\lambda}_{ij}(h)$ with $\lambda_{ij}(h) + \Delta\lambda_{ij}(h)$ as the uncertain TR. Using the terms of Eq. (2) and Eq. (3) related to uncertainties in the system, $N(i) < 0$ can be written as (29) which is rewritable as Eq. (30) after using Lemma for uncertain parts:

$$\begin{aligned} &A^T(i)P(i) + P(i)A(i) + P(i)K(i)B(i) + K^T(i) \\ &B^T(i)P(i) + E_A^T(i)F_A^T(i,t)D_A^T(i)P(i) + P(i)D_A(i) \\ &F_A(i,t)E_A(i) + K^T(i)E_B^T(i)F_B^T(i,t)D_B^T(i)P(i) \\ &+P(i)D_B(i)E_B(i)F_B(i,t)_B(i)K(i) + \sum_{j=1}^{N}\lambda_{ij}(h) \\ &P(j) + \sum_{j=1,\ j\neq i}^{N} \Delta\lambda_{ij}(h)\big(P(j) - P(i)\big) < 0 \end{aligned} \tag{29}$$

$$\begin{aligned} &A^T(i)P(i) + P(i)A(i) + P(i)K(i)B(i) + K^T(i) \\ &B^T(i)P(i) + \varepsilon_A(i)P(i)D_A^T(i)D_A(i)P(i) + \\ &\varepsilon_A^{-1}(i)E_A^T(i)E_A(i) + \varepsilon_B(i)P(i)D_B^T(i)D_B(i)P(i) \\ &+\varepsilon_B^{-1}(i)K^T(i)E_B^T(i)E_B(i)K(i) + \sum_{j=1}^{N}\lambda_{ij}(h)P(j) \\ &+\frac{1}{4}\sum_{j=1}^{N}\varepsilon_\lambda(i,j)\pi_{ij}^2 I \\ &+\sum_{j=1,j\neq i}^{N}\varepsilon_\lambda(i,j)^{-1}\big(P(j) - P(i)\big)^2 < 0 \end{aligned} \tag{30}$$

Define $V(i) = Z^{-1}(i)$ such that

$$\begin{aligned} &\sum_{j=1}^{N}\lambda_{ij}(h)P(j) + \frac{1}{4}\sum_{j=1}^{N}\varepsilon_\lambda(i,j)\pi_{ij}^2 I + \\ &\sum_{j=1}^{N}\varepsilon_\lambda(i,j)^{-1}\big(P(j) - P(i)\big)^2 < V(i) \end{aligned} \tag{31}$$

By defining $Q(i)$, $S(i)$ and $R(i)$ in the form of Eqs. (29), (9), and (10) and applying the Schur complement lemma to Eq. (31), LMI (25) in Theorem 2 is achieved.

Furthermore Eq. (30) can be rewritten to the form of (32), if Eq. (31) is used:

$$\begin{aligned} &A^T(i)P(i) + P(i)A(i) + P(i)K(i)B(i) + K^T(i) \\ &B^T(i)P(i) + \varepsilon_A(i)P(i)D_A^T(i)D_A(i)P(i) + \varepsilon_A^{-1}(i) \\ &E_A^T(i)E_A(i) + \varepsilon_B(i)P(i)D_B^T(i)D_B(i)P(i) + \\ &\varepsilon_B^{-1}(i)K^T(i)E_B^T(i)E_B(i)K(i) + V(i) < 0 \end{aligned} \tag{32}$$

The condition of Eq. (32) is nonlinear in $P(i)$ and $K(i)$. In order to find controller gains it is desired to transform (32) into an LMI form, so let $X(i) = P^{-1}(i)$. Pre- and post-multiplying Eq. (32) by $X(i)$, gives Eq. (33):

$$\begin{aligned} &X(i)A^T(i) + A(i)X(i) + X(i)K^T(i)B^T(i) + B(i) \\ &K(i)X(i) + \varepsilon_A(i)D_A^T(i)D_A(i) + \varepsilon_A^{-1}(i)X(i)E_A^T(i) \\ &E_A(i)X(i) + \varepsilon_B(i)D_B^T(i)D_B(i) + \varepsilon_B^{-1}(i)X(i)K^T(i) \\ &E_B^T(i)E_B(i)K(i)X(i) + X(i)V(i)X(i) < 0 \end{aligned} \tag{33}$$

The closed-loop system is stabilizable if Eq. (33) holds. Defining $Y(i) = K(i)X(i)$ and $J(i)$ along with the Schur complement lemma makes it possible to arrange Eq. (33) to the form of LMI (24). Finally, the state feedback gains are derived as $K(i) = Y(i)X^{-1}(i)$ which ends the proof.

Numerically testable results

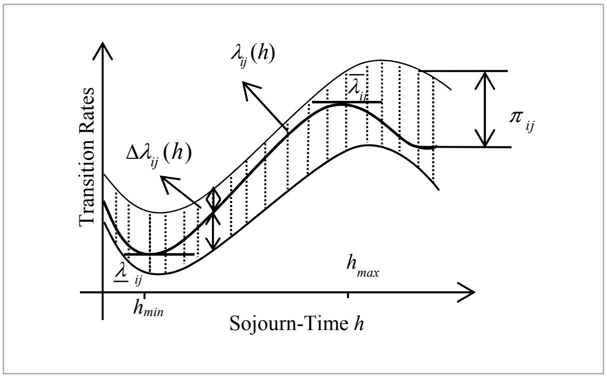
Because of the continuously varying TRs, $\hat{\lambda}_{ij}(h)$ checking the condition of Theorem 1 involves solving a set of infinite number of LMIs (one at each time instant of h), which is numerically impossible. Thus, in order to achieve testable criteria, the approach of approximating the continuously time-varying TR with its bounds is used here. It is assumed that

$\underline{\lambda}_{ij} \leq \lambda_{ij}(h) \leq \overline{\lambda}_{ij}$, where $\underline{\lambda}_{ij}$ and $\overline{\lambda}_{ij}$ specify the probabilistic lower and upper bounds of the nominal TRs. It is worth mentioning that the choice of bounds can guarantee the switching happens between $h_{\min}$ and $h_{\max}$ at 99% confidence level, i.e. ***Pr***{the S-MJLS jumps between $h_{\min}$ and $h_{\max}$} > 0.99. This means that the area of the sojourn-time PDF between $h_{\min}$ and $h_{\max}$ must be larger than 0.99.

The diagram of Figure 1 is depicted to show the uncertain TRs of the S-MJLSs and the related parameters.

Figure 1

Time-varying TRs



The upcoming theorems provide the testable results for stability and stabilizability of uncertain S-MJLSs using TR bounds. Clearly, the results are more conservative than the original theorem, but they are much easier to be solved by the existing tools.

Theorem 3. *The GUS-MJLS of* (1) ($\boldsymbol{u}(t)$=0) *is robustly stochastically stable if there exist symmetric, positive definite mode-dependent matrices* $\boldsymbol{P}(i)$ *and a set of positive mode dependent scalars* $\varepsilon_A(i)$, $\varepsilon_B(i)$, *and* $\varepsilon_\lambda(i,j)$, *such that the following set of constraints hold for all possible modes of* i

$$\begin{bmatrix} \underline{\boldsymbol{J}}(i) & \boldsymbol{P}(i)\boldsymbol{D}_A(i) & \boldsymbol{S}(i) \\ \boldsymbol{D}_A^T(i)\boldsymbol{P}(i) & -\varepsilon_A^{-1}(i)\boldsymbol{I} & 0 \\ \boldsymbol{S}^T(i) & 0 & -\boldsymbol{R}(i) \end{bmatrix} < 0 \tag{34}$$

$$\begin{bmatrix} \overline{\boldsymbol{J}}(i) & \boldsymbol{P}(i)\boldsymbol{D}_A(i) & \boldsymbol{S}(i) \\ \boldsymbol{D}_A^T(i)\boldsymbol{P}(i) & -\varepsilon_A^{-1}(i)\boldsymbol{I} & 0 \\ \boldsymbol{S}^T(i) & 0 & -\boldsymbol{R}(i) \end{bmatrix} < 0 \tag{35}$$

where

$$\begin{aligned} \underline{\boldsymbol{J}}(i) = {} & \boldsymbol{A}^T(i)\boldsymbol{P}(i) + \boldsymbol{P}(i)\boldsymbol{A}(i) + \varepsilon_A^{-1}(i)\boldsymbol{E}_A^T(i)\boldsymbol{E}_A(i) \\ & + \sum_{j=1}^{N} \underline{\lambda}_{ij}\boldsymbol{P}(j) + \frac{1}{4}\sum_{j=1}^{N} \varepsilon_\lambda(i,j)\pi_{ij}^2\boldsymbol{I} \end{aligned} \tag{36}$$

$$\begin{aligned} \overline{\boldsymbol{J}}(i) = {} & \boldsymbol{A}^T(i)\boldsymbol{P}(i) + \boldsymbol{P}(i)\boldsymbol{A}(i) + \varepsilon_A^{-1}(i)\boldsymbol{E}_A^T(i)\boldsymbol{E}_A(i) \\ & + \sum_{j=1}^{N} \overline{\lambda}_{ij}\boldsymbol{P}(j) + \frac{1}{4}\sum_{j=1}^{N} \varepsilon_\lambda(i,j)\pi_{ij}^2\boldsymbol{I} \end{aligned} \tag{37}$$

$\boldsymbol{S}(i)$ and $\boldsymbol{R}(i)$ are defined as (9) and (10), respectively.

Proof. According to Theorem 1, the system is robustly stochastically stable if there exists $\boldsymbol{P}(i)$ such that the inequality (21) holds. On the other hand, $\lambda_{ij}(h)$ can be written as the linear combination $\lambda_{ij}(h) = \varepsilon_1\underline{\lambda}_{ij} + \varepsilon_2\overline{\lambda}_{ij}$ where ε_1 and ε_2 are positive and $\varepsilon_1 + \varepsilon_2 = 1$.

The conditions (34) and (35) of Theorem 3 can be written to the form of the following inequalities

$$\begin{aligned} & \boldsymbol{A}^T(i)\boldsymbol{P}(i) + \boldsymbol{P}(i)\boldsymbol{A}(i) + \varepsilon_A(i)\boldsymbol{P}(i)\boldsymbol{D}_A(i)\boldsymbol{D}_A^T(i) \\ & \boldsymbol{P}(i) + \varepsilon_A^{-1}(i)\boldsymbol{E}_A^T(i)\boldsymbol{E}_A(i) + \sum_{j=1}^{N} \underline{\lambda}_{ij}\boldsymbol{P}(j) + \\ & \frac{1}{4}\sum_{j=1}^{N} \varepsilon_\lambda(i,j)\pi_{ij}^2\boldsymbol{I} + \\ & \sum_{j=1}^{N} \varepsilon_\lambda^{-1}(i,j)\left(\boldsymbol{P}(j) - \boldsymbol{P}(i)\right)^2 < 0 \end{aligned} \tag{38}$$

$$\begin{aligned} & \boldsymbol{A}^T(i)\boldsymbol{P}(i) + \boldsymbol{P}(i)\boldsymbol{A}(i) + \varepsilon_A(i)\boldsymbol{P}(i)\boldsymbol{D}_A(i)\boldsymbol{D}_A^T(i)\boldsymbol{P}(i) + \\ & \varepsilon_A^{-1}(i)\boldsymbol{E}_A^T(i)\boldsymbol{E}_A(i) + \sum_{j=1}^{N} \overline{\lambda}_{ij}\boldsymbol{P}(j) + \\ & \frac{1}{4}\sum_{j=1}^{N}\varepsilon_\lambda(i,j)\pi_{ij}^2\boldsymbol{I} + \frac{1}{4}\sum_{j=1}^{N}\varepsilon_\lambda(i,j)\left(\boldsymbol{P}(j) - \boldsymbol{P}(i)\right)^2 < 0 \end{aligned} \tag{39}$$

Multiplying (38) by ε_1 and (39) by ε_2 and adding up the results leads to the following:

$$\begin{aligned} & \boldsymbol{A}^T(i)\boldsymbol{P}(i) + \boldsymbol{P}(i)\boldsymbol{A}(i) + \varepsilon_A(i)\boldsymbol{P}(i)\boldsymbol{D}_A(i)\boldsymbol{D}_A^T(i)\boldsymbol{P}(i) + \\ & \varepsilon_A^{-1}(i)\boldsymbol{E}_A^T(i)\boldsymbol{E}_A(i) + \sum_{j=1}^{N} (\varepsilon_1\underline{\lambda}_{ij} + \varepsilon_2\overline{\lambda}_{ij})\boldsymbol{P}(j) + \\ & \frac{1}{4}\sum_{j=1}^{N}\varepsilon_\lambda(i,j)\pi_{ij}^2\boldsymbol{I} + \frac{1}{4}\sum_{j=1}^{N}\varepsilon_\lambda(i,j)\left(\boldsymbol{P}(j) - \boldsymbol{P}(i)\right)^2 < 0 \end{aligned} \tag{40}$$

By tuning ε_1 and ε_2 all possible values of $\lambda_{ij}(h)$ can be achieved. Therefore, the condition (21) holds, which means that the system is robustly stochastically stable and the proof is complete.

Similar to Theorem 1, testing the conditions of Theorem 2 involves solving an infinite number of constraints, which is not numerically possible. Applying the same idea of using the lower and upper bounds of the transition rates, $\underline{\lambda}_{ij}, \overline{\lambda}_{ij}$, helps to obtain the following numerically testable theorem.

Theorem 4. *There exist controller gains* (22) *such that the closed-loop uncertain S-MJLS is stochastically stable, if there exists a set of symmetric, positive definite mode-dependent matrices* $\boldsymbol{X}(i)$, $\boldsymbol{P}(i)$, *a set of mode-dependent matrices* $\boldsymbol{Y}(i)$, $\boldsymbol{Z}(i)$, $\boldsymbol{V}(i)$ *and a set of positive mode dependent scalars* $\varepsilon_A(i)$, $\varepsilon_B(i)$, *and* $\varepsilon_\lambda(i,j)$, *such that the following set of constraints hold for all possible modes of* i

$$\begin{bmatrix} \boldsymbol{J}(i) & \boldsymbol{X}(i)\boldsymbol{E}_A^T(i) & \boldsymbol{Y}^T(i)\boldsymbol{E}_B^T(i) & \boldsymbol{X}(i) \\ \boldsymbol{E}_A(i)\boldsymbol{X}(i) & -\varepsilon_A(i)\boldsymbol{I} & 0 & 0 \\ \boldsymbol{E}_B(i)\boldsymbol{Y}(i) & 0 & -\varepsilon_B(i)\boldsymbol{I} & 0 \\ \boldsymbol{X}(i) & 0 & 0 & -\boldsymbol{Z}(i) \end{bmatrix} < 0 \tag{41}$$

$$\begin{bmatrix} \underline{\boldsymbol{Q}}(i) & \boldsymbol{S}(i) \\ \boldsymbol{S}^T(i) & -\boldsymbol{R}(i) \end{bmatrix} < 0 \tag{42}$$

$$\begin{bmatrix} \overline{\boldsymbol{Q}}(i) & \boldsymbol{S}(i) \\ \boldsymbol{S}^T(i) & -\boldsymbol{R}(i) \end{bmatrix} < 0 \tag{43}$$

$$\boldsymbol{X}(i) = \boldsymbol{P}^{-1}(i), \ \ \boldsymbol{V}(i) = \boldsymbol{Z}^{-1}(i) \tag{44}$$

where

$$\boldsymbol{J}(i) = \boldsymbol{X}(i)\boldsymbol{A}^T(i) + \boldsymbol{A}(i)\boldsymbol{X}(i) + \boldsymbol{Y}^T(i)\boldsymbol{B}^T(i) + \boldsymbol{B}(i)\boldsymbol{Y}(i) + \varepsilon_A(i)\boldsymbol{D}_A(i)\boldsymbol{D}_A^T(i) + \varepsilon_B(i)\boldsymbol{D}_B(i)\boldsymbol{D}_B^T(i) \tag{45}$$

$$\underline{\boldsymbol{Q}}(i) = -\boldsymbol{V}(i) + \sum_{j=1}^{N} \underline{\lambda}_{ij}(h)\boldsymbol{P}(j) + \frac{1}{4}\sum_{j=1, j\neq i}^{N} \varepsilon_\lambda(i,j)\pi_{ij}^2 \boldsymbol{I} \tag{46}$$

$$\overline{\boldsymbol{Q}}(i) = -\boldsymbol{V}(i) + \sum_{j=1}^{N} \overline{\lambda}_{ij}(h)\boldsymbol{P}(j) + \frac{1}{4}\sum_{j=1, j\neq i}^{N} \varepsilon_\lambda(i,j)\pi_{ij}^2 \boldsymbol{I} \tag{47}$$

$\boldsymbol{S}(i)$ and $\boldsymbol{R}(i)$ are defined as (9) and (10), respectively. Thus, the controller is obtained as $\boldsymbol{K}(i) = \boldsymbol{Y}(i)\boldsymbol{X}^{-1}(i)$.

Proof. Following the same techniques used in the proof of Theorem 3, Theorem 4 can be readily proved. The details are omitted here due to the similarities.

***Remark* 4.** The condition in Corollaries 1 and 2 are conservative because they use just the upper and the lower bounds on h. To reduce the conservativeness of the condition, the transition rates are fractioned into M sections motivated by Huang and Shi [25]. Based on the fractionizing approach, the upcoming corollaries are obtained.

Corollary 1. *The GUS-MJLS of* (1) ($\boldsymbol{u}(t)$=0) *is robustly stochastically stable if there exist symmetric, positive definite mode-dependent matrices* $\boldsymbol{P}(i, m)$ *and a set of positive mode dependent scalars* $\varepsilon_A(i)$, *and* $\varepsilon_\lambda(i,j)$, *such that the following set of constraints hold for all possible modes of* i *and* m.

$$\begin{bmatrix} \underline{\boldsymbol{J}}(i,m) & \boldsymbol{P}(i,m)\boldsymbol{D}_A(i) & \boldsymbol{S}(i,m) \\ \boldsymbol{D}_A^T(i)\boldsymbol{P}(i,m) & -\varepsilon_A^{-1}(i)\boldsymbol{I} & 0 \\ \boldsymbol{S}^T(i,m) & 0 & -\boldsymbol{R}(i) \end{bmatrix} < 0 \tag{48}$$

$$\begin{bmatrix} \overline{\boldsymbol{J}}(i,m) & \boldsymbol{P}(i,m)\boldsymbol{D}_A(i) & \boldsymbol{S}(i,m) \\ \boldsymbol{D}_A^T(i)\boldsymbol{P}(i,m) & -\varepsilon_A^{-1}(i)\boldsymbol{I} & 0 \\ \boldsymbol{S}^T(i,m) & 0 & -\boldsymbol{R}(i) \end{bmatrix} < 0 \tag{49}$$

where

$$\underline{\boldsymbol{J}}(i,m) = \boldsymbol{A}^T(i)\boldsymbol{P}(i,m) + \boldsymbol{P}(i,m)\boldsymbol{A}(i) + \varepsilon_A^{-1}(i)\boldsymbol{E}_A^T(i)\boldsymbol{E}_A(i) + \sum_{j=1}^{N} \underline{\lambda}_{ij,m}\boldsymbol{P}(j,m) + \frac{1}{4}\sum_{j=1}^{N} \varepsilon_\lambda(i,j)\pi_{ij}^2\boldsymbol{I} \tag{50}$$

$$\overline{\boldsymbol{J}}(i,m) = \boldsymbol{A}^T(i)\boldsymbol{P}(i,m) + \boldsymbol{P}(i,m)\boldsymbol{A}(i) + \varepsilon_A^{-1}(i)\boldsymbol{E}_A^T(i)\boldsymbol{E}_A(i) + \sum_{j=1}^{N} \overline{\lambda}_{ij,m}\boldsymbol{P}(j,m) + \frac{1}{4}\sum_{j=1}^{N} \varepsilon_\lambda(i,j)\pi_{ij}^2\boldsymbol{I} \tag{51}$$

$$\boldsymbol{S}(i,m) = \lfloor \boldsymbol{P}(i,m) - \boldsymbol{P}(1,m), \ldots, \boldsymbol{P}(i,m) - \boldsymbol{P}(i-1,m), \boldsymbol{P}(i,m) - \boldsymbol{P}(i+1,m) \ldots, \boldsymbol{P}(i,m) - \boldsymbol{P}(N,m) \rfloor \tag{52}$$

$\boldsymbol{R}(i)$ is defined as (10) and $\underline{\lambda}_{ij,m}$ and $\overline{\lambda}_{ij,m}$ are the lower and upper bounds for the M^{th} TR section.

Proof. By fractionizing the sojourn time into M sections, the original semi-Markovian jump linear system can be regarded as an individual system with TRs varying in a narrower range. By applying Theorem 3 for the individual S-MJLS in the m-th section and substituting $\underline{\lambda}_{ij}$ and $\overline{\lambda}_{ij}$ by $\underline{\lambda}_{ij,m}$ and $\overline{\lambda}_{ij,m}$ respectively, this corollary can be proved readily.

Based on the given explanations, the following corollary is also obtained for deriving the less conservative robust controller gains.

Corollary 2. *If there exists a set of symmetric, positive definite mode-dependent matrices* $\boldsymbol{X}(i, m)$, $\boldsymbol{P}(i, m)$, *a set of mode-dependent matrices* $\boldsymbol{Y}(i, m)$, $\boldsymbol{Z}(i, m)$, $\boldsymbol{V}(i, m)$ *and a set of positive mode dependent scalars* $\varepsilon_A(i)$, $\varepsilon_B(i)$, *and* $\varepsilon_\lambda(i, j)$, *such that the following set of constraints hold for all possible i and m, then the controller* $\boldsymbol{K}(i,m) = \boldsymbol{Y}(i,m)\boldsymbol{X}^{-1}(i,m)$ *stabilizes the system:*

$$\begin{bmatrix} \boldsymbol{J}(i,m) & \boldsymbol{X}(i,m)\boldsymbol{E}_A^T(i) & \boldsymbol{Y}^T(i,m)\boldsymbol{E}_B^T(i) & \boldsymbol{X}(i,m) \\ \boldsymbol{E}_A(i)\boldsymbol{X}(i,m) & -\varepsilon_A(i)\boldsymbol{I} & 0 & 0 \\ \boldsymbol{E}_B(i)\boldsymbol{Y}(i,m) & 0 & -\varepsilon_B(i)\boldsymbol{I} & 0 \\ \boldsymbol{X}(i,m) & 0 & 0 & -\boldsymbol{Z}(i,m) \end{bmatrix} < 0 \tag{53}$$

$$\begin{bmatrix} \underline{\boldsymbol{Q}}(i,m) & \boldsymbol{S}(i,m) \\ \boldsymbol{S}^T(i,m) & -\boldsymbol{R}(i) \end{bmatrix} < 0 \tag{54}$$

$$\begin{bmatrix} \overline{\boldsymbol{Q}}(i,m) & \boldsymbol{S}(i,m) \\ \boldsymbol{S}^T(i,m) & -\boldsymbol{R}(i) \end{bmatrix} < 0 \tag{55}$$

$$\boldsymbol{X}(i,m) = \boldsymbol{P}^{-1}(i,m), \quad \boldsymbol{V}(i,m) = \boldsymbol{Z}^{-1}(i,m) \tag{56}$$

where

$$\begin{aligned} \boldsymbol{J}(i,m) &= \boldsymbol{X}(i,m)\boldsymbol{A}^T(i) + \boldsymbol{A}(i)\boldsymbol{X}(i,m) + \boldsymbol{Y}^T(i,m) \\ &\boldsymbol{B}^T(i) + \boldsymbol{B}(i)\boldsymbol{Y}(i,m) + \varepsilon_A(i)\boldsymbol{D}_A(i)\boldsymbol{D}_A^T(i) + \\ &\varepsilon_B(i)\boldsymbol{D}_B(i)\boldsymbol{D}_B^T(i) \end{aligned} \tag{57}$$

$$\begin{aligned} \underline{\boldsymbol{Q}}(i,m) &= -\boldsymbol{V}(i,m) + \sum_{j=1}^{N} \underline{\lambda}_{ij,m}\boldsymbol{P}(j,m) + \\ &\frac{1}{4}\sum_{j=1, j\neq i}^{N} \varepsilon_\lambda(i,j,m)\pi_{ij}^2\boldsymbol{I} \end{aligned} \tag{58}$$

$$\begin{aligned} \overline{\boldsymbol{Q}}(i,m) &= -\boldsymbol{V}(i,m) + \sum_{j=1}^{N} \overline{\lambda}_{ij,m}\boldsymbol{P}(j,m) + \\ &\frac{1}{4}\sum_{j=1, j\neq i}^{N} \varepsilon_\lambda(i,j,m)\pi_{ij}^2\boldsymbol{I} \end{aligned} \tag{59}$$

$\boldsymbol{S}(i,m)$ and $\boldsymbol{R}(i,m)$ are defined as (52) and (10), respectively.

Proof. Following the same techniques used in the proof of Corollary 1, Corollary 2 can be readily proved. The details are omitted here due to the similarities.

***Remark* 5.** Evidently the sojourn-time fractionizing technique and the number of sections play an important role in the stochastic stability analysis and the corresponding robust controller design for the GUS-MJLS. However, it is evident that larger M is more likely to yield feasible controllers and less conservative results, but, the effect of using different sojourn-time fractionizing strategies on the stability analysis and controller design has not been fully investigated yet, and deserves further exploration.

***Remark* 6.** The stability conditions of Theorems 1and 3 and Corollary 1 are in the form of linear matrix inequalities and are easily solvable by convex optimization techniques. Unlike the stability criterion, the stabilizability conditions and the controller design procedures of Theorems 2 and 4 and Corollary 2 are no longer LMIs due to the matrix equations. These conditions can be effectively solved by the cone complementarity linearization (CCL) algorithm proposed in [42].

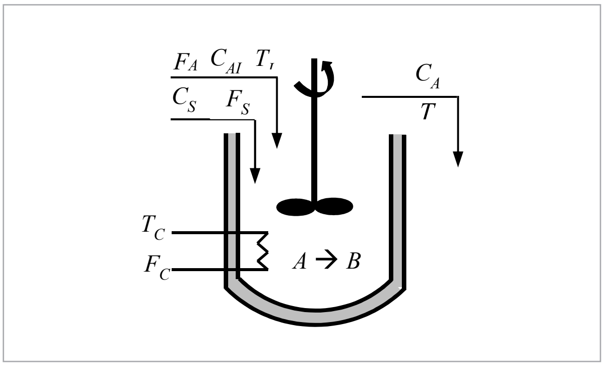
Illustrative example

In this section the effectiveness and flexibility of the proposed technique are tested by solving the stabilization problem for an industrial continuous stirred tank reactor (CSTR). The system is modeled as an uncertain semi-Markov jump linear system. First, a robust state feedback controller is designed for the system, and then its performance is compared to the non-robust controller inspired by [25]. The results show the superiority and potentials of the proposed method.

Consider a CSTR with a single nonreversible reaction $A \rightarrow B$, where the chemical species A reacts to form the species B. The physical structure of the CSTR is depicted in Figure 2.

Figure 2

Diagram of the non-isothermal CSTR



C_{AI} is the input concentration of the reactant A, F_A is the flow rate of the reactant A, T_I is the inlet temperature, C_S is the input concentration of the solvent, F_S is the flow rate of the solvent, C_A is the output concentration of the reactant A, T is the reaction temperature, T_C is the cooling medium temperature and F_C is the cooling medium flow rate.

It is assumed that the tank is non-isothermal, the tank is well mixed, and the shaft work is negligible. It is also assumed that the system is subject to abrupt external environment changes, such as faults, repairs and unexpected configuration conversions. Thus, the system parameters will vary stochastically. The effect of the unprecedented environment change is modeled as a semi-Markov process and the whole system is considered as a semi-Markovian jump linear system.

By selecting the state-variables as $\boldsymbol{x}(t) = \left[C_A(t)^T \quad T(t)^T \right]^T$, $\boldsymbol{u}(t) = \left[C_{AI}(t)^T \quad T_c(t)^T \right]^T$, and by using the data provided in [43, 44] the two-mode semi-Markovian CSTR is presented by the following nominal matrices:

$$\boldsymbol{A}(1) = \begin{bmatrix} 17.49 & 0 \\ 19.49 & -7.84 \end{bmatrix}, \boldsymbol{B}(1) = \begin{bmatrix} 2.52 & 0 \\ 0 & 5.34 \end{bmatrix},$$
$$\boldsymbol{A}(2) = \begin{bmatrix} 16.29 & 0 \\ 19.03 & -6.64 \end{bmatrix}, \boldsymbol{B}(2) = \begin{bmatrix} 2.67 & 0 \\ 0 & 5.21 \end{bmatrix} \tag{60}$$

It is assumed that, due to the imperfect identification of the system parameters, each mode contains uncertainties in dynamics. Switched systems' uncertainty matrices are as the following

$$\boldsymbol{D}_A(1) = \begin{bmatrix} 0 & 0.7 \\ 0.02 & 2.02 \end{bmatrix}, \boldsymbol{E}_A(1) = \begin{bmatrix} 0.01 & 0.5 \\ 0.01 & 1.03 \end{bmatrix},$$
$$\boldsymbol{D}_B(1) = \begin{bmatrix} 0 & 0.7 \\ 0.01 & 2.02 \end{bmatrix}, \boldsymbol{E}_B(1) = \begin{bmatrix} 0.03 & 0.02 \\ 1.01 & 0.02 \end{bmatrix}, \tag{61}$$
$$\boldsymbol{D}_A(2) = \boldsymbol{D}_A(1), \quad \boldsymbol{E}_A(2) = \boldsymbol{E}_A(1),$$
$$\boldsymbol{D}_B(2) = \boldsymbol{D}_B(1), \quad \boldsymbol{E}_B(2) = \boldsymbol{E}_B(1)$$

The governing semi-Markov process of the system is described by the nominal transition rates of $\lambda_{ij}(h) = 2h$. The TRs are a result of sojourn-times with Weibull distribution function $f_{ij}(h) = 2h\exp[-h^2],\ h \geq 0$ with the scale parameter 1 and shape parameter 2.

Weibull distribution is chosen here because it is a natural generalization to the exponential distribution function of sojourn-time [45]. Additionally, it is very popular in mechanical theories [46] and fault-tolerant control systems [47]. This model is a more appropriate description for the fault prone CSTR than the traditional Markovin-type model presented in [43, 44], due to the time-dependent nature of external factors affecting the system.

It is assumed that through inaccurate identification of the distribution function parameters, a bounded uncertainty $\Delta\lambda_{ij}(h)$, that is supposed to satisfy $\left|\Delta\lambda_{ij}(h)\right| \leq 0.7$ must be considered.

A possible realization of the semi-Markov chain, r_t, is depicted in Figure 3. By the initial conditions $x_1(0) = 0.8$, $x_2(0) = 368.25$, and initial mode, $r_0 = 1$, uncontrolled states of the GUS-MJLS become as shown in Figure 4. The states are clearly unstable.

Figure 3

Changing between two modes of the semi-Markov chain during the simulation of the CSTR

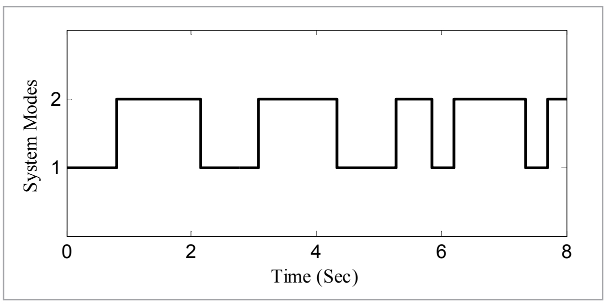


Figure 4

Uncontrolled states of the semi-Markovian CSTR

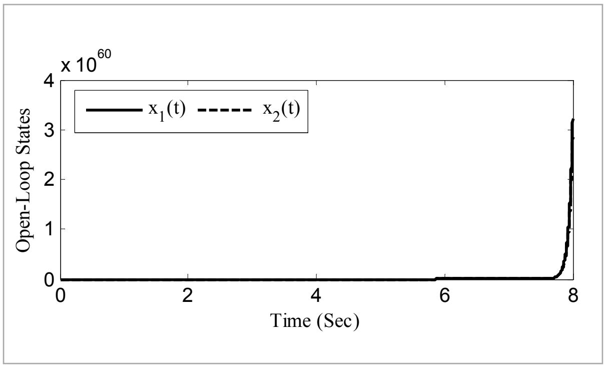
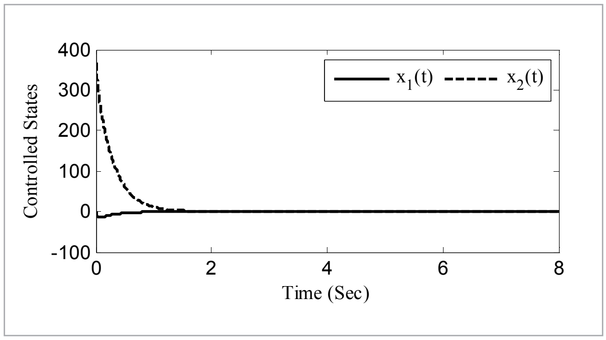


Figure 5

Average controlled states by the designed controller for CSTR for 10000 runs



Considering the control synthesis problem and solving (53) to (56) with prescribed values [4] $\varepsilon_A(1) = \varepsilon_B(1) = 0.5$, $\varepsilon_A(2) = \varepsilon_B(2) = 0.1$, and $\varepsilon_\lambda(i, j) = 0.5$, the state feedback gains become as (62):

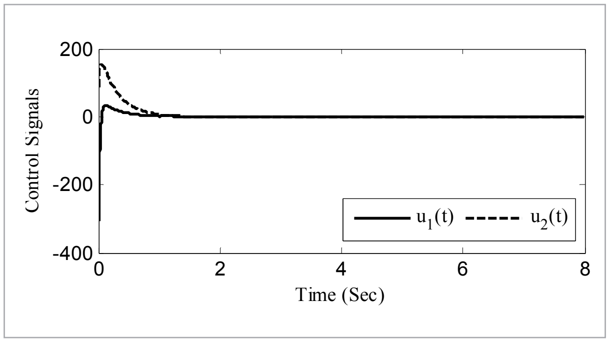
$$\boldsymbol{K}(1,1) = \begin{bmatrix} -15.75 & -0.88 \\ -5.82 & 0.27 \end{bmatrix}, \boldsymbol{K}(2,1) = \begin{bmatrix} -12.93 & -0.93 \\ -5.17 & 0.04 \end{bmatrix},$$
$$\boldsymbol{K}(1,2) = \begin{bmatrix} -15.99 & -0.79 \\ -5.79 & 0.25 \end{bmatrix}, \boldsymbol{K}(2,2) = \begin{bmatrix} -13.48 & -0.85 \\ -5.18 & 0.02 \end{bmatrix} \quad (62)$$

For the sojourn-time following the aforementioned Weibull distribution, $h_{\min}$ and $h_{\max}$ are 0.1 and 4.6, respectively. This means that the jump will occur in the interval $[h_{\min}, h_{\max}]$ with probability greater than 0.99. By setting M to 2 the sojourn-time is partitioned by 0.8326 according to subsection 3.3. Therefore, when $h < 0.8326$, the state feedback control law is $\boldsymbol{K}(1, 1)$ for mode 1 and $\boldsymbol{K}(2, 1)$ for mode 2; when $h > 0.8326$, the state feedback control law is $\boldsymbol{K}(1, 2)$ for mode 1 and $\boldsymbol{K}(2, 2)$ for mode 2. The average state trajectories of the controlled systems resulted from the designed controller are illustrated in Figure 5. Also, the average control signals are shown in Figure 6 for 10000 runs.

The figures clearly demonstrate that in the presence of time-varying transition rate matrix and despite the uncertainties in the TRs and dynamics of each mode, the states tend to the origin. Also, the control signals are enough smooth. In fact, the effect of variations and uncertainties of the TRs is not considerable on either state trajectories or the control signals.

Figure 6

Average control signals for the CSTR for 10000 runs



To further illustrate the effectiveness of the proposed theorems, denote the settling time T_s given by:

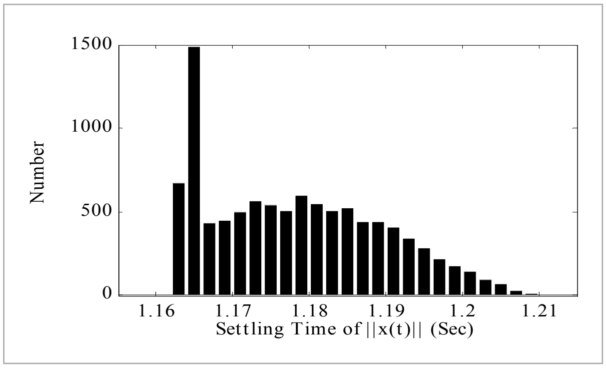
$$\|\boldsymbol{x}(t)\|_2 \le 1.5\% \|\boldsymbol{x}(0)\|_2, \quad t > T_s. \quad (63)$$

Statistics of the settling time are summarized in Figure 7. The average settling time of the controlled states is 1.1792 seconds and its standard deviation is 0.0114.

To clarify the necessity of considering the uncertainties in TR matrices and in the dynamics of semi-Markovian switching system and to show the superiority of the presented robust algorithm, the results are compared to the controller provided by [25]. The controller proposed in Huang and Shi [25] does not consider the effect of TR uncertainties in design. Such a controller is obtained as (64) for each operating mode:

Figure 7

Statistics of the settling time for controlled states for 10000 runs

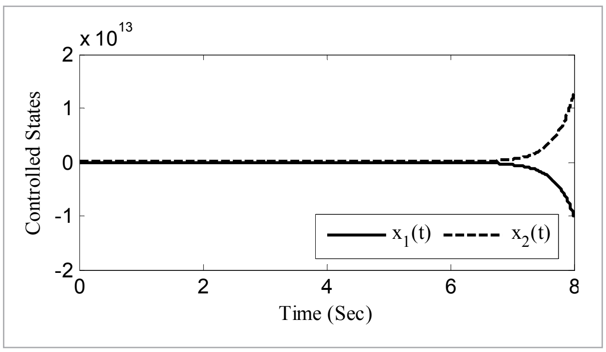


$$\boldsymbol{K}(1,1)=\begin{bmatrix}-7.57 & -2.48\\ -2.48 & 1.19\end{bmatrix},\ \boldsymbol{K}(2,1)=\begin{bmatrix}-6.64 & -2.41\\ -2.41 & 1.00\end{bmatrix},$$
$$\boldsymbol{K}(1,2)=\begin{bmatrix}-7.26 & -2.48\\ -2.48 & 1.34\end{bmatrix},\ \boldsymbol{K}(2,2)=\begin{bmatrix}-6.17 & -2.41\\ -2.41 & 1.24\end{bmatrix} \tag{64}$$

Applying this controller to the system with TR and dynamic uncertainties with the same initial conditions and the same switching times yields a closed-loop response as Figure 8, with control signal depicted in Figure 9.

Figure 8

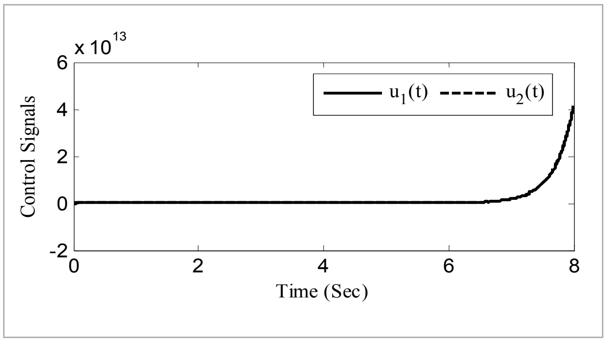
Average controlled states by the controller [25] for CSTR for 10000 runs



For the proposed robust controller, it takes an average 1.1729 seconds for the two states to converge to zero, whereas by the controller designed in [25], the closed-loop state trajectories become unstable. Thus, the proposed control design technique outperforms the existing non-robust methodologies.

Figure 9

Average control signals of the controller by [25] for CSTR for 10000 runs



Remark 7. In the continuous-time semi-Markov process, the probability distribution of the sojourn-time can be any PDF. In the Markovian jump linear systems, the sojourn time h has an exponential distribution function. Therefore, the celebrated MJLSs [1] can be regarded as a special case of the GUS-MJLS discussed in this paper.

Conclusions

In this paper, a general uncertain semi-Markov jump linear system is introduced. The system under consideration finds extensive applications since it eliminates the need to precisely know TRs and perfectly identified dynamics of S-MJLSs. It also covers the concepts of the traditional MJLSs. By combining the multiple stochastic Lyapunov approach with the TR fractionizing technique, the numerically testable sufficient criteria are provided to check the stability and stabilizability of the system.

A state-feedback controller with an acceptable degree of conservativeness is also designed based on the stability results. Some possible directions to extend the proposed scheme are filtering and fault-tolerant control for semi-Markovian systems subject to uncertain TRs because of their widespread applications in the practical systems.

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Summary / Santrauka

This paper investigates the problems of robust stochastic stability and stabilization for a general class of continuous-time semi-Markovian jump linear systems (S-MJLSs). The main contribution of the research is to eliminate the limitations of the traditional S-MJLSs with precisely available information by introducing a system with uncertain, time-varying transition rates (TRs) of the jump process, in addition to the imperfect information on the system dynamic matrices. The new system is called the general uncertain semi-Markov jump linear system (GUS-MJLS); it does not contain certain values of the transition rates, but includes nominal time-dependent values in addition to bounded deviations. It is suitable to describe a broader class of dynamical systems with estimated information and modeling errors and also covers the concepts of Markov jump linear system (MJLSs) with time-constant and certain TRs. For this system, the stability is firstly analyzed through the multiple stochastic Lyapunov function approach. Then, based on the stability results, a robust state-feedback controller is formulated. To deal with the time-dependent TRs, a sojourn-time fractionizing technique is used and numerically testable conditions are developed. Finally, discussions on reducing the conservativeness of the robust theorems are provided. The theoretical results are successfully tested on an industrial continuous stirred tank reactor (CSTR) subject to stochastically varying environmental conditions. Comparative simulations are also provided to show the superiority of the presented framework and design method to the existing ones.

Šis straipsnis tiria stochastinio stabilumo patikimumo ir bendrųjų nepertraukiamo laiko pusiau Markovo šuolio tiesinių sistemų stabilizavimo (S-MJLSs) problemas. Svarbiausias tyrimo tikslas – pašalinti tradicinio S-MJLSs trūkumus su tikslia prieinama informacija, papildomai prie netobulos informacijos dinaminių sistemų matricose įdiegiant sistemą su nekonkrečiais, skirtingo laiko šuolio proceso rodikliais (TRs). Nauja sistema vadinama bendra nekonkrečių pusiau Markovo šuolio tiesine sistema (angl. General Uncertain Semi-Markov Jump Linear System (GUS-MJLS)). Joje nėra konkrečių perėjimo įverčių, tačiau be apribotų deviacijų dar įtraukti nominalūs nuo laiko priklausomi įverčiai. Tokia sistema tinkama apibūdinti platesnę dinamiškų sistemų klasę su apskaičiuota informacija ir kūrimo klaidomis; ji taip pat dengia Markovo šuolio tiesinės sistemos (MJLSs) su laiko konstanta ir tam tikrais TRs, sąvokas. Šios sistemos stabilumas pirmiausia yra analizuojamas per daugialypį stochastinį Liapunovo funkcijos metodą. Tuomet, pasirėmus stabilumo rezultatais, formuluojamas tvirtas grįžtamojo ryšio kontroleris. Tam, kad būtų galima susidoroti su nuo laiko priklausančiais TRs, yra panaudojama gyvavimo laiko frakcionavimo technika ir sukuriamos sąlygos, kurių testavimas gali sugeneruoti skaitinę išraišką. Galiausiai, pateikiama diskusija apie konservatyvaus požiūrio į teoremų patikimumą mažinimą. Teoriniai rezultatai sėkmingai išbandyti industriniame nepertraukiamo ryšio reaktoriuje (CSTR), kuriame vyravo stochastiškai besikeičiančios aplinkos sąlygos. Studija taip pat pateikia lyginamąsias simuliacijas, kurios parodo, kad siūloma sistema ir planavimo metodas yra pranašesni, nei kiti egzistuojantys metodai.