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A Two-Phase Optimization Method for Solving the Multi-Type Maximal Covering Location Problem in Emergency Service Networks

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This study introduces the Multi-Type Maximal Covering Location Problem (MTMCLP) that arises from the design of emergency service networks, and represents a generalization of the well-known Maximal Covering Location Problem (MCLP). Differently from the basic MCLP, several types of incidents and emergency units are considered and hierarchy of emergency units of different types is assumed in the MTMCLP. The numbers of available emergency units of each type are limited to some constants. The objective of the MTMCLP is to choose locations for establishing emergency units of each type, such that the total number of covered incidents is maximized. In order to provide a decision maker with optimal solutions in an efficient manner, a two-phase optimization approach to the MTMCLP is designed. In the first phase, a variant of Reduced Variable Neighborhood Search (RVNS) is applied to quickly find a high-quality solution. The obtained RVNS solution is used as a good starting point for the Linear Programming method in the second phase, which returns the optimal solution to the MTMCLP. All constructive elements of the proposed two-phase method, denoted as RVNS-LP, are adapted to the characteristics of the considered problem. The RVNS-LP approach is evaluated on real-life instances obtained from two networks of police units in Montenegro and Serbia, and randomly generated test instances of larger dimensions. Experimental evaluation shows that the proposed RVNS-LP reached all optimal solutions on all real-life test instances in very short CPU time. On generated test instances, the RVNS-LP also returned optimal solutions in all cases, within short running times and significant time savings compared to CPLEX solver. The mathematical model and the proposed two-phase optimization method may be applicable in the design and management of various emergency-service networks.

KEYWORDS: variable neighborhood search, linear programming, emergency service network, maximal covering location problem.

Introduction

Covering models are one the most popular facility location models in the literature, due to their numerous applications in practice, especially for locating services and emergency facilities. Many of real-life problems, such as determining the number and locations of public schools, police stations, fire stations, military bases, medical centers, post offices, bank branches, shopping centers, satellite or radar installations, etc., can be formulated as covering problems [11].

In general, covering problems assume a set of customers and a set of potential locations for establishing facilities. In most of covering problems, it is required that each customer should be served by at least one facility within a given critical distance, denoted as covering radius. However, in many practical applications, located resources are not sufficient to cover all customers with the desired level of coverage. This was a motivation for Church and ReVelle [5] to propose a Maximal Covering Location Problem (MCLP). The MCLP model maximizes the amount of demand covered within the acceptable service distance by locating a fixed number of facilities. The MCLP has showed to be one of the most exploited facility location models from both theoretical and practical points of view. Starting from the work of Church and ReVelle [5], many variants of the MCLP are presented in the literature up to now. White and Case [29] considered the case of MCLP in which demands of all customers are equal, with the goal to find the maximal number of covered customer (demand) nodes. A steepest descent heuristic was proposed in [29] as a solution method to this variant of MCLP. Klastorin [14] showed that MCLP can be formulated as Generalized Assignment Problem (GAP). The variant of MCLP on the plane was considered by Church [6], Drezner [9], and Watson-Gandy [28]. Daskin [8] introduced the maximal coverage location model as one of the variants of set covering model. Probabilistic variant of the MCLP was proposed by ReVelle and Hogan [23], where each potential facility location has assigned a value measuring the probability that a facility will be established on that location. ReVelle et al. [21] proposed a Maximal Conditional Covering Problem, where customer locations need to be covered by facilities at a given coverage radius, while facility locations themselves are supposed to be covered with a different coverage radius by other facilities, in order to provide secondary support. A generalization of MCLP was introduced by Berman and Krass [3], who involved multiple set of coverage levels with the degree of coverage being a non-increasing step function of the distance to the nearest established facility. A general class of covering problems was proposed by Hocbaum and Pathria [13] as the class of problems of maximum k-coverage, and MCLP may be observed as its special case. Another generalization of MCLP is the Multimode Covering Location Problem introduced in [7], which deals with locating a given number of different types of facilities, with a limitation of the number of facilities sharing the same location. A review of papers related to the MCLP and its variants can be found in [10, 15, 24].

In this study, we introduce a generalization of the MCLP that arises from the design of an emergency service network. We consider the set of cities (representing the set of customers) and set of potential locations for emergency units (service providers). An emergency unit can react to an incident in a city if it is located within a given covering radius from this city. In our model, we distinguish several types of incidents, and each city has assigned information on the expected number of incidents of each type, obtained from the statistical data. Different types of emergency units are available, and for each type of emergency unit, it is defined for which types of incident this type of unit is trained to react. A hierarchy among emergency unit types is introduced, meaning that an emergency unit of a certain type can react to incident types handled by emergency units of lower level, but also to some additional types of incidents. The number of available emergency units of each type are limited. The goal of the considered problem is to choose locations for establishing emergency units of each type, such that the number of all covered incidents is maximized. We will refer to the problem as the Multi-Type Maximal Covering Location Problem (MTMCLP). To the best of our knowledge, there are no previously published work on this type of generalization of the MCLP.

The first goal of the study is to formulate the Multi-Type Maximal Covering Location Problem as an Integer Linear Programming (ILP) model. Note that ILP model for the MTMCLP proposed in this study may find its applications in the management and optimization of various emergency systems. In this paper, we have considered two networks of police units in the states of Montenegro and Serbia, but the model may be also applied to smaller administrative units (regions, cities, city districts, etc.). The proposed model may also be used for military purposes, for example, in determining optimal locations for different types of military units involving hierarchical structure. It can also be applied when optimizing the network of health-care providers, i.e., for determining optimal location of medical centers of different types (ambulances, heath-care centers, clinics, etc.). In addition, the model can be used for designing distribution systems, for example, when it is necessary to determine locations of warehouses of different sizes or levels, where a warehouse of a certain level can store not only the products intended for warehouses of lower levels, but some additional product types as well. The proposed ILP model can be also applied for designing a postal delivery system, the network of bank offices, supermarkets, etc.

The second goal of our study is to develop an efficient decision support system for helping the emergency manager to efficiently balance between providing emergency service and the economic aspect of emergency system. Emergency management is a dynamic system and usually a manager is supposed to make the decision within short time. This indicates the importance of developing an efficient optimization algorithm that will provide emergency manager with necessary data (optimal or high-quality solutions) in short time. In this paper, we propose an optimization method for solving the proposed MTMCLP, based on the combination of Reduced Variable Neighborhood Search (RVNS) heuristic and Linear Programming method (LP). The Reduced Variable Neighborhood Search heuristic (RVNS) is applied first in order to quickly find a high-quality solution to the problem. This solution is used as a good initial solution for the Linear Programming method, which is used in the framework of CPLEX commercial solver, returning optimal solution to the MTMCLP.

The proposed RVNS-LP method was first benchmarked on the two sets of real-life instances obtained from statistical data related to the network of police units in Montenegro and Serbia. The presented results on these instances show that the RVNS-LP provides optimal solution in very short CPU times. The obtained experimental results on real-life data sets are also analyzed from the experts' point of view. In order to evaluate the efficiency of the RVNS-LP on larger emergency network, we have generated the set of instances involving lager number of customers and potential locations for emergency unit locations, as well as larger number of incident types and emergency unit types. The RVNS-LP method was additionally benchmarked on the set of generated instances. The obtained results are presented and analyzed, indicating the efficiency of the proposed RVNS-LP method in the case of larger emergency network as well.

The remainder of paper is organized as follows. In Section 2, we present the Integer Linear Programming formulation of the considered MTMCLP. The proposed RVNS-LP method is described in details in Section 3. In Section 4, we present and analyze computational results obtained on smaller-size real-life problem instances and generated test instances of larger dimension. Section 5 provides a summary of results and suggests some possibilities for future work.

Mathematical formulation

Mathematical model of the MTMCLP uses the following notation:

- I denotes the set of cities;
- J represents the set of possible locations for establishing emergency units;
- K stands for the set of types of incidents;
- L is the set of types of emergency units;
- d_{ij} is the distance between a city $i \in I$ and a potential location of an emergency unit $j \in J$;
- a_{ki} represents the number of incidents of type $k \in K$ that occurred in a city $i \in I$;
- b_l denotes the number of available emergency units of type $l \in L$;
- $R > 0$ represents the covering radius, i.e., the maximal distance between an emergency unit at $j \in J$ and a city $i \in I$, such that emergency unit is able to reach the city in a timely manner.

Note that in our model inequality $|L| \leq |K|$ holds meaning that the number of types of emergency units is not greater than the number of incident types. More precisely, it implies that an emergency unit of type $l \in L$ established on location $j \in J$ can react on incident types $1,2, ..., k_l$ in a city i situated within the given covering radius R, i.e., $d_{ij} \leq R$. The hierarchy of emergency units is assumed by $1 \leq k_1 < k_2 < ... < k_{|L|} = |K|$, meaning that an emergency unit of type $l \in L$ can cover all incident types as emergency units of lower types 1,2, ..., l–1, as well as additional incident types up to k_l.

According to security experts, location planning of emergency units is usually performed on a monthly basis, which means that each emergency unit obtains its schedule and location for the following month. Therefore, $a_{ki} \geq 0$, $i \in I$, $k \in K$ denotes the average number of incidents of type $k \in K$ in a city $i \in I$ for a particular month of the year, obtained from statistical data during past years. Naturally, we may have different values of a_{ki} for different months of the year as input data. Note that the values of a_{ki} may represent the average number of incidents of type $k \in K$ in the city $i \in I$ for different planning period.

By taking into account assumptions mentioned above, the goal of the considered Multi-Type Maximal Covering Location Problem (MTMCLP) is to find optimal locations for establishing police units of each type, so that the total number of incidents in covered cities is maximized.

In order to present mathematical model of the MTMCLP, we introduce two sets of binary variables. Variables $x_{ki} \in \{0, 1\}$, $k \in K$, $i \in I$ take the value of 1 if there is an emergency unit that can react on incident of type k in city i, and 0 otherwise. Variables $y_{lj} \in \{0, 1\}$, $l \in L$, $j \in J$ have the value of 1 if there is an established emergency unit of type l on location j, and 0 otherwise.

By using the above notation, the MTMCLP may be formulated as an Integer Linear Program (ILP) as follows:

$$\max \sum_{k \in K} \sum_{i \in I} a_{ki} x_{ki} \tag{1}$$

so that

$$x_{ki} \leq \sum_{l \in L: k_l \geq k} \sum_{j \in J: d_{ij} \leq R} y_{lj} \quad \forall k \in K \quad \forall i \in I, \tag{2}$$

$$\sum_{j \in J} y_{lj} \leq b_l \qquad \forall l \in L, \tag{3}$$

$$x_{ki} \in \{0,1\} \qquad \forall k \in K \qquad \forall i \in I, \tag{4}$$

$$y_{lj} \in \{0,1\} \qquad \forall l \in L \qquad \forall j \in J. \tag{5}$$

The objective function (1) maximizes the total number of covered incidents in the considered emergency system. The constraints (2) ensure that emergency units of type l established at location j may cover the incidents of type k in city i only if the distance between i and j is not greater than R and $k_l \geq k$ holds. The number of available emergency units of type l is equal to b_l, which is indicated by constraints (3). The constraints (4) and (5) denote the binary nature of variables x_{ki} and y_{lj}.

Note that the proposed model represents a generalization of the Maximal Covering Location Problem – MLCP [5]. More precisely, for $|K| = |L| = 1$ and $a_{ki} = 1$, for all $k \in K$, $i \in I$, our model is reduced to the MCLP. Therefore, the proposed model is NP-hard as a generalization of the MCLP, which is proved to be NP-hard in [17].

Proposed RVNS-LP method

The goal of combining different optimization methods is to exploit the complementary characteristics of different search strategies. In the literature, one can find numerous examples of hybridization two or more optimization algorithms [1, 16, 18, 20, 25], etc. Although combination of two or more (meta)heuristic methods is the most exploited type of hybridization, there are also examples of successful combination of the exact algorithms with (meta)heuristics for solving many hard optimization problems [2, 22, 26, 27], etc. The choice of optimization methods to be combined and the way of their hybridization highly depend on the characteristics of the given problem. A detailed survey of state-of-the-art hybrid methods in combinatorial optimization can be found in [4].

In this paper, we develop a combination of a heuristic and exact optimization method for solving the MTMCLP. The proposed method consists of two phases: Reduced Variable Neighborhood Search (RVNS) and Linear Programming method (LP). Reduced Variable Neighborhood Search is a variant of well-known Variable Neighborhood Search heuristic, proposed by Mladenović and Hansen [19, 12]. In the RVNS, the deterministic component (local search part) is excluded, since it is usually time consuming. The RVNS showed to be useful for solving problem instances of large dimension, where local search requires significant amounts of CPU time or when it is necessary to obtain good initial solution for other heuristic method in an efficient manner. RVNS is similar to the Monte-Carlo method, but it is more systematic [19, 12].

The basic idea behind the proposed hybrid method for the MTMCLP is to apply RVNS heuristic in the first phase to quickly find a good initial solution for the second, LP phase, which is implemented within the framework of CPLEX solver. The proposed RVNS-LP method returns optimal solution to the MTMCLP in short CPU time, even in the case of problem instances of larger dimensions. In the next subsections, the structure of the proposed two-phase RVNS-LP method will be explained in details.

Solution representation

Regarding the nature of the considered Multi-Type Maximal Covering Location Problem, the code of potential solution consists of $|L|$ binary segments of length $|J|$, where each segment corresponds to one type of emergency units. Bits in each of the binary segments of length $|J|$ represent potential locations for establishing police units of certain type. More precisely, segment l, $l=1,2,...,|L|$ corresponds to emergency units of type l, and the bits within this segment indicate whether or not an emergency unit of type l is located on a position j, $j=1,2,...,|J|$.

Therefore, the total length of a solution's code is $|L|\cdot|J|$. If a bit on the position $(l-1)\cdot|J|+j$ has the value of 1, it means that an emergency unit of type l is established at location j. In case that this bit has the value of zero, emergency unit of type l is not located at position j.

Neighborhood structures

In our study, we use a neighborhood structure based on the facility swap distance. More precisely, one swap consists of closing one and opening another emergency unit of the same type in the solution. Swapping of emergency units belonging to the same type is performed by inverting two randomly chosen bits belonging to the same segment of the solution code. Swaps are allowed within the same segment in order to preserve the feasibility of solution. Otherwise, the number of emergency units of type l may become greater than b_l for some $l\in L$.

We consider that a solution S' is in the k -th neighborhood of the solution S, if S' can be obtained from S by performing exactly k facility swaps of the same type. We will denote by $N_k(S)$, $k=1,2,...,k_{max}$ a neighborhood of size k of a solution S. Parameter k_{max} denotes the maximal size of the neighborhood used in the RVNS part.

Objective function calculation

Algorithm 1 shows the procedure of calculating the objective function value of a given solution S. Initially, objective value $obj(S)$ is set to zero. From the solution's code S, we obtain the indices of located emergency units and their types. Once the indices of locations with established units are known, for each city $i\in I$ we obtain the set of located units that lie within the given range from this city $N_i=\{j\in J: d_{ij}\leq R\}$, as well as types of these units. For each incident type $k\in K$ and each city $i\in I$, we check if there is at least one emergency unit of type l established at location j for which $k_l\geq k$ and $d_{ij}\leq R$ hold, meaning that incident of type k that occurred in the city i will be covered. If this is the case, $obj(S)$ is increased by the value of a_{ki} representing the average number of incidents of type k in the city i.

In order to speed up objective function calculation, for the considered incident of type k and city i, the procedure checks only emergency units of type k_l for which $k \le k_l \le |K|$ holds and which belong to the set $N_i = \{j \in J : d_{ij} \le R\}$ obtained in the initialization part.

Algorithm 1 Objective function calculation

$obj(S) = 0$
for all $i \in I$ **do**
  Find all pairs $(l, j) \in L \times J$ so that $y_{lj} = 1$ and $j \in N_i$
end for
for all $k \in K$ **do**
  for all $i \in I$ **do**
    $found =$ **false**
    l' is the lowest index for which $k_{l'} \ge k$ holds
    for all $l = l', l' + 1, \ldots, |L|$ **do**
      for all $j \in N_i$ **do**
        if bit on the position $(k_l - 1) \cdot |J| + j$ in solution S has the value of 1 **then**
          $obj(S) = obj(S) + a_{ki}$
          $found =$ **true**
          break
        end if
      end for
      if $found =$ **true then**
        break
      end if
    end for
  end for
end for
return $obj(S)$

Structure of the RVNS-LP

The structure of the proposed RVNS-LP algorithm is presented in Algorithm 2. In the initialization part of the RVNS-LP, the initial set of N feasible solutions is created. Each initial solution is generated in a pseudo-random way such that each segment l, $l = 1,2, \ldots, |L|$ in the solution's code contains up to b_l bits with the value of 1 that are randomly distributed in the segment, while remaining bits are set to 0.

The RVNS heuristic is applied first within the proposed two-phase method. Each solution S_i, $i = 1,2, \ldots, N$ from the generated initial set is taken as the initial solution of the RVNS, and the best solution obtained through all RVNS runs is memorized. Therefore, the RVNS phase can be observed as a variant of multi-start RVNS method. In the main RVNS loop, for each run $i = 1,2, \ldots, N$, we iteratively try to improve the current best solution S_i by searching in its neighborhoods $N_k(S_i)$, $k = 1,2, \ldots, k_{max}$. If a randomly chosen solution $S_{i'} \in N_k(S_i)$ is better than the current best one S_i, we replace S_i with S'_i, and start the search from this new solution. Otherwise, we change the size of neighborhood and continue the search in $N_{k+1}(S_i)$. The maximal neighborhood size is defined by the parameter k_{max}. The RVNS algorithm runs until the maximal number of N_{iter} iterations is reached (stopping criterion).

Algorithm 2 RVNS-LP method

Initialization:
for $i = 1,2, \ldots, N$ **do**
  Generate initial feasible solution S_i
end for
RVNS phase:
for $iter = 1,2, \ldots, N_{iter}$ **do**
  for $i = 1,2, \ldots, N$ **do**
    while there is an improvement **do**
      $k \leftarrow 1$
      while $k \le 3$ **do**
        Randomly choose S'_i from the neighborhood $N_k(S_i)$
        if $obj(S'_i) > obj(S_i)$ **then**
          $S_i \leftarrow S'_i$,
          $k \leftarrow 1$
        else
          $k \leftarrow k + 1$
        end if
      end while
    end while
  end for
end for
$S_{best} \leftarrow$ the best solution obtained in the RVNS phase
LP Phase:
From S_{best} get the indices of locations of established units of each type
for all $l \in L$ **do**
  for all $j \in J$ **do**
    if unit of type l is established at location j **then**

```
        y_lj ← 1
      else
        y_lj ← 0
      end if
  end for
 end for
 for all k ∈ K do
   for all i i ∈ I do
     if there is at least one established unit of type
       l ≥ k at location j so that d_ij ≤ R then
       x_ki ← 1
     else
       x_ki ← 0
     end if
   end for
 end for
Apply CPLEX solver with initial values
  of variables x_ki and y_lj
Return S_opt and obj(S_opt)
```

When calculating the objective function value of the new solution S'_i from the neighborhood $N_k(S_i)$, we apply a strategy that speeds up the evaluation of the new solution S'_i by using previously calculated objective value of S_i. Since solution $S'_i \in N_k(S)$ is obtained by swapping k pairs of bits in the solution S_i, we observe only pairs of bits with changed values. Note that the pair of swapped bits must belong to the same segment of individual's code. Let us consider a pair of bits j_1 and j_2 belonging to the same segment $l \in L$. We will denote them as $(l, j_1), (l, j_2) \in L \times J$. Suppose that the bit (l, j_1) has changed its value from 0 to 1, and bit (l, j_2) has been inverted from 1 to 0.

Let $T(r, s)$ represent the number of established emergency units that are able to react on the incidents of type $r \in K$ in a city $s \in I$. Since bit (l, j_1) has changed from 0 to 1, it means that emergency unit of type l is established at location j_1. Therefore, it is sufficient to identify all cities s that lie within the given range R from location j_2 and all incident types $r \leq l$, and to increase each of the corresponding values $T(r, s)$ by 1. In case the value $T(r, s)$ is increased from 0 to 1, the objective value will be increased by a_{rs}, and therefore $obj(S'_i)$ is updated as $obj(S'_i) = obj(S_i) + a_{rs}$.

Similarly, since bit (l, j_2) has changed from 1 to 0, it means that emergency unit of type l is removed from location j_2. We identify all cities s that lie within the given range from the location j_2 and all incident types $r \leq l$ and decrease each of the corresponding values $T(r, s)$ by 1. In case $T(r, s)$ is changed from 1 to 0, the value a_{rs} is subtracted from objective value of S_i and $obj(S'_i) = obj(S_i) - a_{rs}$ is updated.

The described procedure is repeated for all k pairs of swapped bits, and the objective value of the neighbor solution $obj(S'_i)$ is returned and compared with $obj(S_i)$.

The best solution S_{best} obtained through N runs of RVNS is passed to the LP phase. From the code of S_{best}, the indices of locations of established units of each type are obtained. If a unit of type l is established at location j, decision variable y_{lj} takes the value of 1, and 0 otherwise. For each city $i \in I$ and incident type $k \in K$, we check if there is at least one established unit of type l at location j, such that $d_{ij} \leq R$ and $l \geq k$. If this is the case, the value of 1 is assigned to decision variable x_{ki}. Otherwise x_{ki} is set to 0.

The values of variables y_{lj} and x_{ki} obtained from the solution S_{best}, are used as a starting point for CPLEX solver that is employed in LP part. Starting from these initial values, CPLEX easily solves the linear programming model of the resulting subproblem in short CPU times, i.e., it quickly reaches optimal solution to the MTMCLP and confirms its optimality. As computational results show, the solution S_{best} obtained by multi-start RVNS represents a high-quality initial solution for LP part, which enables CPLEX solver to provide optimal solution in an efficient manner, even in the case of larger problem dimensions.

Computational results

All experiments were carried out on an Intel i5-2430M on 2.4 GHz with 8 GB RAM memory, under Windows 7 operating system. Optimization package CPLEX, version 12.1, was used on the same platform. The implementation of RVNS-LP was coded in C++ programming language. The value of parameter N representing the number of initial solutions is set to 20, while the value of the stopping criterion parameter N_{iter} for the RVNS phase is equal to 5000. The value of k_{max} representing the maximal size of neighborhoods in the RVNS part is set to 3.

In this section, we present results and analysis of extensive computational experiments that were carried out on three sets of instances. Two sets of instances are obtained from the real-life data and they are of smaller and medium size. In order to test the efficiency of the algorithm, we have generated the third set of instances of large dimensions.

Data set 1. The first data set is generated from the data obtained from the network of police units in the state of Montenegro. The instances are generated with the help of security experts in this area and by using statistical data in past several years. Instances from the Data set 1 involve the set of 21 cities in Montenegro, which is at the same time the set of potential locations of police units. Two types of police units are distinguished in this data set: police intervention teams (PIT) and police special forces units (PSFU). The first type of units reacts in the case of criminal act against human life and property, while PSFUs may also react in the case of severe criminal acts and high-risk law enforcement operations. The driving distances between the cities are used as distances between two locations. In Data set 1, the coverage radius R is varied from 15 to 35 km.

Data set 2. As the second data set, we have used data from real-life instances presented in [25]. These instances are related to the network police units in the Republic of Serbia. In Data set 2, we consider the set of 145 cities, which are at the same time potential sites for locating police units. As in the case of Montenegro, two types of police units and two types of criminal acts are considered: police intervention teams (PIT) and police special forces units (PSFU). The average numbers of criminal acts of each type are obtained from the data provided by the Statistical Office of the Republic of Serbia. The driving distances between the cities are calculated by using ViaMichelin Maps and route planner. Having in mind that police units need to react as soon as possible, the shortest driving distances between two cities are chosen. In Data set 2, the coverage radius R is varied from 20 to 40 km.

Data set 3. In order to evaluate the proposed algorithm on larger problem dimensions, we have randomly generated the third data set. In instances belonging to Data set 3, the number of locations of users varies from 200 to 350, while the number of potential locations for establishing emergency units is between 40 and 55. Coordinates of all locations are randomly chosen from the square $[0,300] \times [0,300]$. A different number of incidents and emergency units are considered. Covering radius R varies from 40 to 60.

Results obtained for Data sets 1 and 2

We have first performed computational experiments on instances with real-life data. In Table 1, we present the results of the RVNS-LP method obtained on Data set 1. Column headings in Table 1 represent:

- Number of cities – $|I|$;
- Number of potential locations for emergency units –$|J|$;
- Number of incident types –$|K|$;
- Number of emergency unit types – $|L|$;
- Covering radius – R;
- Gap between the objective value of RVNS solution and the objective value of the optimal one – gap_{RVNS}[%];
- Running time of RVNS phase – t_{RVNS}[s];
- Objective value of the optimal solution obtained by RVNS-LP method – Obj. value;
- Total running time of RVNS-LP method – t[s];
- Number of nodes searched until the optimal solution is found – Nodes. More precisely, it represents the number of nodes of the Branch-and-Bound tree that are visited during the CPLEX run until the optimal solution is reached.

From the results presented in Table 1, it can be seen that for each instance from Data set 1, the solution obtained in RVNS phase coincides with the optimal one. The CPLEX solver that is employed within the LP part quickly proves its optimality, i.e., it is easily confirmed that the solution passed to CPLEX (and taken as a root node of Branch-and-Bound tree) is actually the optimal one. For this reason, in the case of the Data set 1, the number of nodes of the Branch-and-Bound tree generated during the CPLEX run is always equal to 0. The average CPU time of RVNS is 0.070 seconds, while the average CPU time of RVNS-LP is slightly longer – 0.073 seconds. Average gap of the RVNS solution is 0 %, meaning that the RVNS found optimal solution in each of the 20 runs.

Instances from Data set 2 (related to police network in Serbia) are of larger dimensions compared to instances from Data set 1 (the case of police network in Montenegro). The results of the RVNS-LP obtained on Data set 2 are presented in Table 2. For each of the considered instances in Data set 2, the best solution obtained in the first phase by multi-start RVNS

Table 1
Results of the RVNS-LP for Data set 1 – the case of Montenegro

Instance					RVNS		RVNS-LP		
$\|I\|$	$\|J\|$	$\|K\|$	$\|L\|$	R	$gapRVNS[\%]$	$tRVNS[s]$	Obj. value	$t[s]$	Nodes
21	21	2	2	15	0.000	0.070	4998	0.073	0
21	21	2	2	16	0.000	0.066	5055	0.069	0
21	21	2	2	17	0.000	0.066	5191	0.069	0
21	21	2	2	18	0.000	0.067	5191	0.070	0
21	21	2	2	19	0.000	0.066	5191	0.069	0
21	21	2	2	20	0.000	0.066	5191	0.069	0
21	21	2	2	21	0.000	0.068	5389	0.071	0
21	21	2	2	22	0.000	0.068	5389	0.071	0
21	21	2	2	23	0.000	0.068	5389	0.072	0
21	21	2	2	24	0.000	0.069	5389	0.072	0
21	21	2	2	25	0.000	0.069	5491	0.072	0
21	21	2	2	26	0.000	0.069	5626	0.072	0
21	21	2	2	27	0.000	0.069	5676	0.073	0
21	21	2	2	28	0.000	0.070	5676	0.073	0
21	21	2	2	29	0.000	0.071	5780	0.075	0
21	21	2	2	30	0.000	0.075	5780	0.078	0
21	21	2	2	31	0.000	0.072	5780	0.075	0
21	21	2	2	32	0.000	0.074	5788	0.078	0
21	21	2	2	33	0.000	0.075	5838	0.078	0
21	21	2	2	34	0.000	0.075	5838	0.079	0
21	21	2	2	35	0.000	0.074	5838	0.078	0
average					0.000	0.070	5499.238	0.073	0

showed to be the optimal one, since its optimality was quickly confirmed in the second phase by the LP method (the number of nodes is zero). The exception is the last instance with $R = 40$, where CPLEX solver has visited 7 nodes, starting from the best RVNS solution before finding the optimal solution. From the $gap_{RVNS}[\%]$ column, it can be seen that the average gap of RVNS solutions was 0.710 %, which means that some of the best solutions generated during 20 RVNS runs have small gaps from the optimal one. The average time of the RVNS was 0.138 seconds through 20 runs, while RVNS-LP needed 0.157 seconds of running time (in average) to return optimal solution.

In order to analyze the quality of obtained solutions from practical point of view, we compare solutions obtained by the proposed model with the current schedule of police units in Montenegro. The list of 21 municipalities in Montenegro is given in Table 3. The area of each municipality is observed as a location with a number assigned. In the present situation, all police units of type 2 are located in Podgorica, which is the capital of Montenegro, while police units of type 1 are located in municipalities 1, 3, 4, 5, 6, 12, 15, and 19, representing so-called security centers in Montenegro. We have considered different combinations of parameter values b_1, b_2 and coverage radius R. For each parameter combination, we have calculated the objective value of a solution obtained with the present police units schedule and compared it with the objective value of the optimal solution provided by the proposed model. A detailed comparison for different values for parameters b_1 and b_2 and covering radius R

Table 2
Results of the RVNS-LP for Data set 2 – the case of Serbia

Instance					RVNS		RVNS-LP		
$\|I\|$	$\|J\|$	$\|K\|$	$\|L\|$	R	*gapRVNS*[%]	*tRVNS*[*s*]	Obj. value	*t*[*s*]	Nodes
145	145	2	2	20	0.026	0.129	34638	0.138	0
145	145	2	2	21	0.250	0.126	34860	0.135	0
145	145	2	2	22	0.483	0.127	35997	0.136	0
145	145	2	2	23	0.210	0.131	37167	0.140	0
145	145	2	2	24	0.789	0.128	38019	0.137	0
145	145	2	2	25	0.642	0.132	38316	0.142	0
145	145	2	2	26	1.014	0.129	38451	0.138	0
145	145	2	2	27	0.898	0.131	38763	0.141	0
145	145	2	2	28	1.317	0.136	39168	0.147	0
145	145	2	2	29	0.894	0.134	39261	0.144	0
145	145	2	2	30	0.788	0.137	39999	0.147	0
145	145	2	2	31	0.446	0.139	40320	0.150	0
145	145	2	2	32	1.008	0.138	40776	0.149	0
145	145	2	2	33	1.400	0.143	40935	0.154	0
145	145	2	2	34	0.630	0.141	40977	0.153	0
145	145	2	2	35	0.674	0.143	41397	0.190	0
145	145	2	2	36	0.628	0.149	41553	0.172	1
145	145	2	2	37	0.898	0.148	41763	0.176	5
145	145	2	2	38	0.506	0.146	42111	0.171	0
145	145	2	2	39	0.704	0.152	42162	0.201	0
145	145	2	2	40	0.702	0.149	42285	0.226	7
average					0.710	0.138	39472.286	0.157	0.619

Table 3
Municipalities in Montenegro

no.	municip.	no.	municip.	no.	municip.
1	Podgorica	8	Žabljak	15	Plevlja
2	Andrijevica	9	Kolašin	16	Rožaje
3	Bar	10	Kotor	17	Tivat
4	Berane	11	Mojkovac	18	Ulcinj
5	Bijelo Polje	12	Nikšić	19	Herceg Novi
6	Budva	13	Plav	20	Cetinje
7	Danilovgrad	14	Plužine	21	Šavnik

is given in Table 4. The number of police units of type 1 is fixed to 8, since all of them are located in security centers. The number of units of type 2 that are currently all located in Podgorica varies from 1 to 4, while covering radius R varies from 15 to 35 km.

By comparing the objective values of solutions corresponding to the current schedule and the optimal one, it can be seen that in all cases the number of covered criminal acts is larger for the optimal schedule obtained by the MTMCLP model. The last column of Table 4 shows improvements (in percent) of the objective values when using the proposed model. As it was expected, smaller improvements were achieved for the largest considered value of covering radius R. For lower values of R, which imply shorter responding times of police units, the improvements of the objective values are significant. The largest improvement is obtained for b_1=8, b_2=4, R=15 and it is equal

Table 4

Comparisons of current locations and optimal locations of police units in Montenegro

Data			Current solution			Optimal solution			
$b1$	$b2$	R	locations of police units of type 1	locations of police units of type 2	Obj. value	locations of police units of type 1	locations of police units of type 2	Obj. value	Impr. [%]
8	1	15	1, 3, 4, 5, 6, 12 ,15, 19	1	3956	1, 3, 4, 6, 10, 12, 18, 19	13	4528	14.459
8	1	25	1, 3, 4, 5, 6, 12 ,15, 19	1	4727	1, 2, 3, 6, 12, 18, 19, 20	5	5207	10.154
8	1	35	1, 3, 4, 5, 6, 12 ,15, 19	1	5556	2, 3, 4, 6, 7, 9, 15, 19	14	5810	4.572
8	2	15	1, 3, 4, 5, 6, 12 ,15, 19	all in 1	3956	1, 3, 4, 6, 10, 12, 18, 19	5, 13	4638	17.240
8	2	25	1, 3, 4, 5, 6, 12 ,15, 19	all in 1	4727	1, 2, 3, 6, 12, 18, 19, 20	5, 15	5309	12.312
8	2	35	1, 3, 4, 5, 6, 12 ,15, 19	all in 1	5556	3, 4, 6, 7, 11, 13, 15, 19	8, 14	5825	4.842
8	3	15	1, 3, 4, 5, 6, 12 ,15, 19	all in 1	3956	1, 3, 4, 6, 10, 12, 18, 19	5, 13, 15	4740	19.818
8	3	25	1, 3, 4, 5, 6, 12 ,15, 19	all in 1	4727	1, 2, 3, 6, 12, 18, 19, 20	5, 15, 16	5397	14.174
8	3	35	1, 3, 4, 5, 6, 12 ,15, 19	all in 1	5556	2, 3, 4, 6, 7, 9, 15, 19	8, 14, 21	5838	5.076
8	4	15	1, 3, 4, 5, 6, 12 ,15, 19	all in 1	3956	1, 3, 4, 6, 10, 12, 18, 19	5, 13, 15, 16	4828	22.042
8	4	25	1, 3, 4, 5, 6, 12 ,15, 19	all in 1	4727	1, 2, 3, 6, 12, 18, 19, 20	5, 11, 15, 16	5459	15.486
8	4	35	1, 3, 4, 5, 6, 12 ,15, 19	all in 1	5556	2, 3, 4, 6, 7, 9, 15, 19	8, 12, 14, 21	5838	5.076

to 22.042 %. By analyzing the positions of police units in the current and optimal solution, we may notice the difference in locations of units of both types. For example, locations 2 (Andrijevica), 18 (Ulcinj) are often suggested by our model for establishing police units of type 1. For smaller values of covering radius R, location 10 (Kotor) is suggested, while for larger R, the model proposes location 15 (Pljevlja) for establishing police units of type 1. It is interesting that our model suggests relocation of police units of type 2, which are currently all situated at location 1 (Podgorica). For different values of b_1, b_2 and R, different locations are obtained; however, locations 5 (Biljelo Polje) and 15 (Pljevlja) appear the most often. In Figure 1, we present current and optimal locations of police units for $b_1=8$, $b_2=1$, and $R=25$. Current locations of police units of types 1 and 2 are marked with blue digits, while locations in optimal solution are marked with red digits 1 and 2. The objective value of the solution corresponding to present situation is equal to 4727, while the objective value of the optimal solution provided by our model is 5207 (the improvement is around 10 %).

Based on the presented analysis, we may conclude that solutions obtained by using the proposed model and RVNS-LP method may help decision-makers

Figure 1

Current and optimal schedule of PSFUs for $b1=8$, $b2=1$ and $R=25$

and security experts to improve the efficiency of police system by relocating some police units. However, it is important to note that the decision where to locate police units is not only driven by distances and statistical data on the number of criminal acts. In a real-life situation, it is important to take into account some additional conditions, such as the existence of adequate infrastructure, configuration of the terrain, possibility to observe larger geographical areas, etc. The decision making, including some of the mentioned additional conditions, may be the subject of investigation in our future work.

Results on generated instances

We have further benchmarked the RVNS-LP meth-
We have further benchmarked the RVNS-LP method on generated instances from Data set 3, involving $|I|=200, 250$ user nodes and $|J|=40, 45$ potential locations for emergency units. Differently from Data sets 1 and 2, in Data set 3, the number of incident types and the number of emergency unit types may be different, i.e., $|K| \neq |L|$ in general. The results are presented in Table 5 in the same way as in Tables 1–2. In order to confirm that a good-quality RVNS solution that is passed as the initial solution to the LP method may significantly reduce the total running time, we add one more column $\Delta t[\%]$ in Table 5. This column shows time savings (in percents) achieved by using the best RVNS solution from the first phase as the initial solution in the LP phase.

From the results presented in Table 5, it can be seen that for larger problem instances with $|I|=200, 250$ users, the RVNS phase produces high-quality solutions in short CPU times. The average running time of RVNS phase for instances in Table 5 is $t_{RVNS}=0.536$ seconds. The average gap of the objective value of the best solution produced by multi-start RVNS from the optimal one is low – 0.204 %, meaning that the best RVNS solution is close to the optimal one. However, the CPLEX solver applied within the LP part still needs additional effort to reach optimal solution starting from the best RVNS solution from the first phase, and to confirm its optimality. On average, the CPLEX solver visits 2340.104 nodes of the Branch-and-Bound tree until the optimal solution is found. The average running time that RVNS-LP needed to detect optimal solution and to confirm its optimality is quite short (9.641 seconds). Data presented in column $\Delta t[\%]$ shows the advantage of the hybrid RVNS-LP method in respect to running times. Significant time savings (up to 75.597 %) are obtained when using solution from the RVNS phase as the initial solution for the CPLEX solver. In average, time savings are 29.353 % for instances from Table 5.

In Table 6, we present the results obtained on generated instances from Data set 3 with $|I|=300, 350$ user nodes and $|J|=50, 55$ potential locations for emergency units. As in Table 5, different number of incidents and emergency units are considered. The RVNS had similar performance on these instances: the average gap of the RVNS solution was 0.185%, while average running time was 0.852 seconds through 20 runs. However, the LP method needed more effort to find optimal solution, starting from the initial one provided in the RVNS phase. The average number of visited nodes before detecting optimal solution was 18 436.082. In average, total running time of the RVNS-LP method was 118.421 seconds, which is quite short having in mind problem dimensions. From the column $\Delta t[\%]$ of Table 6, it can be seen that CPLEX time savings obtained when using multi-start RVNS to produce initial solution for the LP phase are up to 48.809 %, while average time savings are 18.308 %.

In order to provide a graphical representation of the results, we have divided the set of generated instances into subsets of instances having the same values of three parameters $|I|$, $|J|$ and R, and calculated average values of the obtained results for these subsets. In Figure 2, we present average objective values over the subsets of instances with fixed triples of parameters ($|I|$, $|J|$, R). As it was expected, the average objective values are steadily increasing as the number of nodes increases. It can also be noticed that for fixed values of $|I|$ and $|J|$, the average objective values are generally increasing as coverage radius R increases. Figure 3 shows the comparison of the average CPU times of RVNS and RVNS-LP methods for generated instances with fixed triple of parameters ($|I|$, $|J|$, R). The average CPU time of RVNS method was very short (under 1 second), but the best RVNS solution had a certain gap from the optimal one. Therefore, the LP part needed additional CPU time to detect the optimal solution and confirm its optimality. However, from Figure 3 it can be seen that average CPU time in which RVNS-LP produced optimal solution for the largest considered instances with $|I|$=300, $|J|$=55 and

Table 5

Results of the RVNS-LP for Data set 3 – randomly generated large instances with $|I|=200$ and $|I|=250$ user nodes

Instance					RVNS		RVNS-LP			Time savings
$\|I\|$	$\|J\|$	$\|K\|$	$\|L\|$	R	*gapRVNS*[%]	*tRVNS*[*s*]	Obj. value	*t*[*s*]	Nodes	*t* [%]
200	40	2	2	40	0.760	0.313	15915	1.195	576	37.356
200	40	2	2	50	0.184	0.373	17363	4.232	1694	49.119
200	40	2	2	60	0.408	0.441	18383	0.999	276	31.621
200	40	3	2	40	0.000	0.312	24799	1.052	236	18.513
200	40	3	2	50	0.000	0.312	26937	1.606	654	31.841
200	40	3	2	60	0.280	0.437	28551	8.107	4037	18.954
200	40	3	3	40	0.578	0.375	25592	1.413	243	19.333
200	40	3	3	50	0.288	0.484	27117	1.334	318	58.432
200	40	3	3	60	0.000	0.544	28402	1.728	63	22.098
200	40	4	2	40	0.000	0.368	35504	0.855	49	19.081
200	40	4	2	50	0.000	0.407	37161	1.915	542	2.890
200	40	4	2	60	0.000	0.513	39329	9.307	3446	17.739
200	40	4	3	40	0.639	0.404	37255	2.163	479	59.565
200	40	4	3	50	0.000	0.437	38412	1.501	406	43.091
200	40	4	3	60	0.098	0.564	38827	4.160	1787	17.197
200	40	4	4	40	1.118	0.493	37133	2.240	339	19.339
200	40	4	4	50	0.216	0.591	38448	1.805	274	31.368
200	40	4	4	60	0.000	0.680	39386	1.933	91	13.474
200	40	5	3	40	0.305	0.460	48132	1.879	182	35.668
200	40	5	3	50	0.000	0.540	49380	2.276	412	71.720
200	40	5	3	60	0.000	0.662	50217	5.080	1208	20.126
200	40	5	5	40	0.869	0.502	45594	2.928	523	46.620
200	40	5	5	50	0.140	0.781	47233	3.097	250	55.148
200	40	5	5	60	0.179	0.688	48166	3.627	474	62.516
250	45	2	2	40	0.000	0.358	21151	7.963	3397	0.113
250	45	2	2	50	0.232	0.467	22884	9.482	3846	20.437
250	45	2	2	60	0.421	0.517	23990	1.865	499	54.387
250	45	3	2	40	0.000	0.372	30921	8.105	2196	15.949
250	45	3	2	50	0.000	0.431	33481	19.193	6731	35.620
250	45	3	2	60	0.000	0.482	35406	11.431	3832	75.597
250	45	3	3	40	0.535	0.447	34211	2.171	407	19.680
250	45	3	3	50	0.376	0.496	35332	9.327	1562	24.873
250	45	3	3	60	0.000	0.671	36370	11.098	2058	19.668
250	45	4	2	40	0.000	0.428	39841	2.499	371	3.737
250	45	4	2	50	0.000	0.491	44289	14.738	2580	23.487
250	45	4	2	60	0.000	0.622	47126	34.150	8190	19.915
250	45	4	3	40	0.311	0.509	48534	41.130	13181	22.894
250	45	4	3	50	0.000	0.579	50275	11.326	3882	20.424
250	45	4	3	60	0.000	0.711	50908	2.177	547	68.018
250	45	4	4	40	0.328	0.497	45739	3.162	463	31.283
250	45	4	4	50	0.395	0.635	46887	19.243	3778	18.765
250	45	4	4	60	0.149	0.820	47722	20.708	3915	19.833
250	45	5	3	40	0.043	0.563	58127	38.164	9353	13.853
250	45	5	3	50	0.345	0.661	60580	59.445	12499	30.623
250	45	5	3	60	0.118	0.802	61918	16.684	4183	32.440
250	45	5	5	40	0.112	0.673	57911	23.806	2858	8.354
250	45	5	5	50	0.328	0.865	59754	23.708	3041	2.053
250	45	5	5	60	0.054	0.951	60826	4.765	397	24.153
average					0.204	0.536	39529.563	9.641	2340.104	29.353

Table 6

Results of the RVNS-LP for Data set 3 – randomly generated large instances with $|I|$=300 and $|I|$=350 user nodes

Instance					RVNS		RVNS-LP			Time savings
$\|I\|$	$\|J\|$	$\|K\|$	$\|L\|$	R	$gapRVNS[\%]$	$tRVNS[s]$	Obj. value	$t[s]$	Nodes	$t[\%]$
300	50	2	2	40	0.490	0.458	26961	14.478	4893	0.282
300	50	2	2	50	0.163	0.529	28914	8.585	2422	12.851
300	50	2	2	60	0.292	0.523	29767	10.953	3650	27.684
300	50	3	2	40	0.000	0.421	37739	2.121	289	0.516
300	50	3	2	50	0.000	0.496	40900	111.674	34929	40.209
300	50	3	2	60	0.000	0.744	43609	25.653	6136	26.102
300	50	3	3	40	0.408	0.471	40661	29.041	5540	5.715
300	50	3	3	50	0.099	0.771	42269	26.250	5708	16.313
300	50	3	3	60	0.023	0.933	43251	50.739	11699	13.550
300	50	4	2	40	0.000	0.493	48642	15.006	1508	16.154
300	50	4	2	50	0.000	0.603	52594	61.793	9047	32.108
300	50	4	2	60	0.399	0.844	55935	239.585	38931	7.838
300	50	4	3	40	0.175	0.727	56107	34.225	6070	46.069
300	50	4	3	50	0.187	0.861	57810	19.047	3800	0.422
300	50	4	3	60	0.142	1.036	58345	29.852	8640	11.998
300	50	4	4	40	0.679	0.795	55341	25.591	2892	3.944
300	50	4	4	50	0.021	0.744	57028	5.398	523	40.402
300	50	4	4	60	0.000	1.046	57588	37.257	6226	19.327
300	50	5	3	40	0.540	0.851	68690	53.170	6663	4.750
300	50	5	3	50	0.000	0.968	70902	69.981	8800	24.349
300	50	5	3	60	0.367	1.162	72448	32.570	4653	7.424
300	50	5	5	40	0.442	0.721	70590	27.446	1920	8.638
300	50	5	5	50	0.165	0.973	72114	46.421	4770	9.133
300	50	5	5	60	0.014	1.286	72971	49.777	6120	48.809
350	55	2	2	40	0.057	0.531	31765	24.165	4646	2.130
350	55	2	2	50	0.054	0.573	33055	56.398	12041	16.431
350	55	2	2	60	0.296	0.860	33763	77.698	23187	31.578
350	55	3	2	40	0.024	0.611	42327	41.142	6094	11.614
350	55	3	2	50	0.000	0.712	46551	58.848	8036	37.582
350	55	3	2	60	0.144	0.654	48744	414.896	80479	28.266
350	55	3	3	40	0.202	0.789	49029	53.891	7353	0.562
350	55	3	3	50	0.026	0.815	50270	109.235	16428	33.560
350	55	3	3	60	0.120	1.245	50936	118.387	27022	7.867
350	55	4	2	40	0.000	0.752	54878	108.720	11230	41.265
350	55	4	2	50	0.000	0.888	60857	123.553	12280	38.428
350	55	4	2	60	0.000	1.008	64461	706.457	86305	27.177
350	55	4	3	40	0.382	0.837	65511	538.097	115610	17.304
350	55	4	3	50	0.480	1.006	67693	242.005	45341	16.682
350	55	4	3	60	0.335	1.179	68728	169.871	38834	2.845
350	55	4	4	40	0.435	0.760	66899	80.140	7804	19.391
350	55	4	4	50	0.056	1.030	68203	110.708	12457	22.029
350	55	4	4	60	0.012	1.113	69155	108.952	13562	21.398
350	55	5	3	40	0.700	0.902	80131	266.091	24984	17.478
350	55	5	3	50	0.349	1.140	83288	417.100	49156	6.096
350	55	5	3	60	0.033	1.348	85093	301.165	41495	23.513
350	55	5	5	40	0.249	1.071	85010	100.211	7039	16.238
350	55	5	5	50	0.223	1.184	86229	192.808	17311	4.661
350	55	5	5	60	0.116	1.412	86780	237.053	30409	10.101
average					0.185	0.852	57094.417	118.421	18436.083	18.308

Figure 2
Average objective values

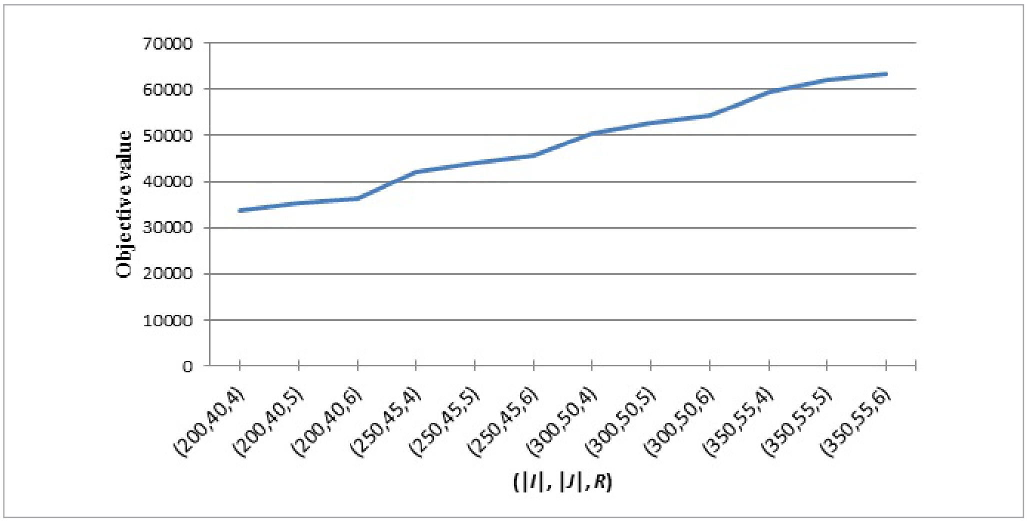


Figure 3
Comparisons of average CPU times of RVNS and RVNS-LP

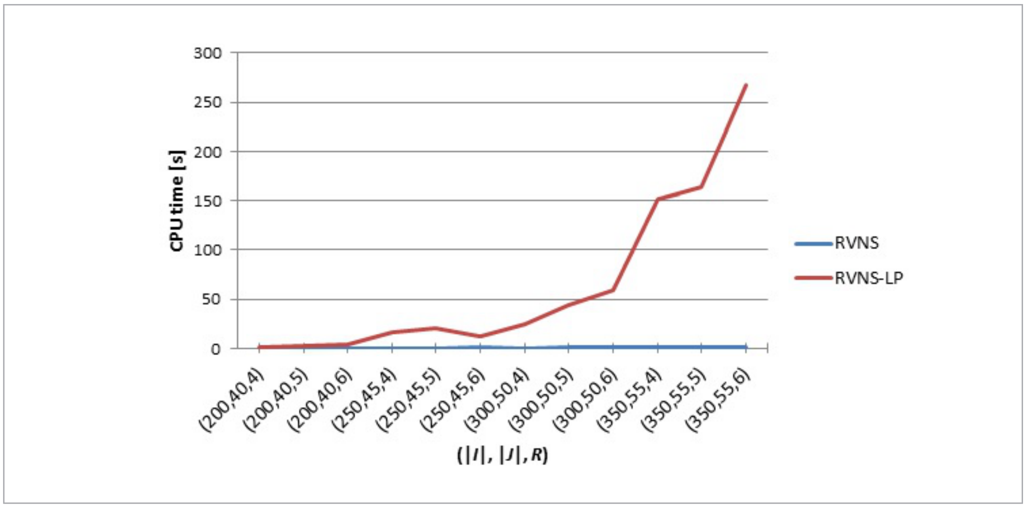
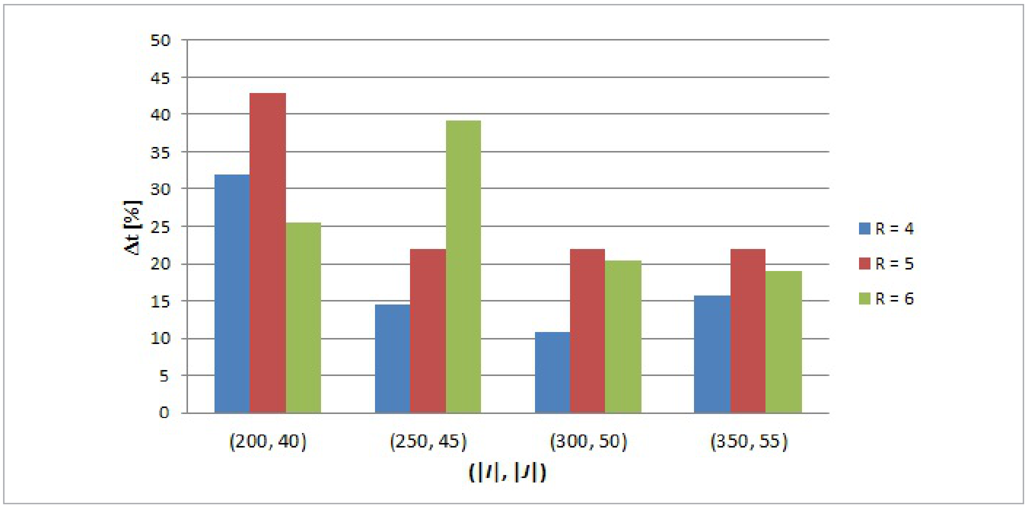


Figure 4
Average time savings obtained when using the best RVNS solution as initial solution for the LP part



R = 60 is around 270 seconds, which is relatively short considering problem dimension and the fact that the optimal solution is provided. Finally, in Figure 4, we present the average time savings (in percents) obtained when using the best RVNS solution as the starting point for the LP part. Note that time-savings depend on the quality of the RVNS solution from the first phase as well as on the nature of the considered instances. As it can be seen in Figure 4, the average time-savings vary between 11 % an 43 % for the considered groups of generated instances with fixed parameters $|I|$, $|J|$ and R.

Conclusions

This paper introduces the Multi-Type Maximal Covering Location Problem (MTMCLP) in emergency service networks, representing a generalization of the well-known Maximal Covering Location Problem (MCLP). In the proposed MTMCLP, different types of incidents and emergency units are considered, and it is assumed that limited number of emergency units of each type is available. A hierarchy among emergency units is introduced, meaning that an emergency unit of a certain type can cover the same incident types as emergency units of lower level, as well as additional incident types. The objective of the MTMCLP is to find optimal locations for establishing emergency units of each type, so that the total sum of covered incidents is maximized. An efficient two-phase optimization algorithm (RVNS-LP) is designed to solve the considered problem. In the first phase of the optimization algorithm, a variant of Reduced Variable Neighborhood Search (RVNS) is applied, producing high-quality solution in very short CPU time. The RVNS uses neighborhood structures that are appropriate for the considered MTMCLP. The neighborhoods of the current solution are explored in an efficient manner by using a time-saving strategy in the procedure for objective function calculation. The RVNS is run on the set of randomly generated initial solutions, and the best solution obtained through multiple RVNS runs is used as the starting point for the Linear Programming method in the second phase. The LP method is used within the framework of commercial CPLEX software, and it was showed that significant savings of CPLEX running times may be obtained when using high-quality solution from the RVNS phase as the initial solution for the LP part.

The proposed RVNS-LP was benchmarked on two sets of real-life instances and on the set of randomly generated instances of larger dimensions. Our experimental evaluation shown that the RVNS-LP solves all real-life instances to optimality in very short CPU times. On generated test instances, the RVNS-LP provided optimal solutions in reasonably short running times, having in mind problem dimensions. From practical point of view, solutions obtained by using the proposed model and RVNS-LP approach show significant improvement compared to current solutions regarding objective values, i.e., the increase of the total number of covered incidents. However, relocation of police units requires additional costs, but on the other side, it may lead to better efficiency of a security system. The solutions proposed in this study have a potential to be considered when creating a long-term security system strategy. Future work may involve a modification of the proposed MTMCLP model in order to include some specific emergency system requirements, as well as adapting the RVNS-LP in order to solve similar covering problems related to emergency networks. The development of some metaheuristic methods for MTMCLP and testing their performances against the RVNS-LP is another future work direction.

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Summary / Santrauka

This study introduces the Multi-Type Maximal Covering Location Problem (MTMCLP) that arises from the design of emergency service networks, and represents a generalization of the well-known Maximal Covering Location Problem (MCLP). Differently from the basic MCLP, several types of incidents and emergency units are considered and hierarchy of emergency units of different types is assumed in the MTMCLP. The numbers of available emergency units of each type are limited to some constants. The objective of the MTMCLP is to choose locations for establishing emergency units of each type, such that the total number of covered incidents is maximized. In order to provide a decision maker with optimal solutions in an efficient manner, a two-phase optimization approach to the MTMCLP is designed. In the first phase, a variant of Reduced Variable Neighborhood Search (RVNS) is applied to quickly find a high-quality solution. The obtained RVNS solution is used as a good starting point for the Linear Programming method in the second phase, which returns the optimal solution to the MTMCLP. All constructive elements of the proposed two-phase method, denoted as RVNS-LP, are adapted to the characteristics of the considered problem. The RVNS-LP approach is evaluated on real-life instances obtained from two networks of police units in Montenegro and Serbia, and randomly generated test instances of larger dimensions. Experimental evaluation shows that the proposed RVNS-LP reached all optimal solutions on all real-life test instances in very short CPU time. On generated test instances, the RVNS-LP also returned optimal solutions in all cases, within short running times and significant time savings compared to CPLEX solver. The mathematical model and the proposed two-phase optimization method may be applicable in the design and management of various emergency-service networks.

Ši studija pristato įvairiatipio maksimalaus zonų padengimo problemą (angl. Multi-Type Maximal Covering Location Problem (MTMCLP)), kuri kyla dėl tam tikro pagalbos tarnybų tinklų išplanavimo. Taip pat apibendrinama ir gerai žinoma Maksimalaus zonų padengimo problema (angl. Maximal Covering Location Problem (MCLP)). Priešingai nei pagrindiniame MCLP, MTMCLP apžvelgiama keletas skirtingų nelaimės ir pagalbos tarnybų tipų, taip pat yra atsižvelgiama į skirtingų pagalbos tarnybų ekipažų hierarchiją. Pasiekiamų kiekvieno tipo pagalbos tarnybų ekipažų skaičius yra apribotas iki tam tikrų konstantų. MTMCLP tikslas – parinkti vietas kiekvienam pagalbos tarnybų ekipažų tipui taip, kad išspręstų incidentų skaičius būtų maksimizuotas. Tam, kad sprendimų priėmėjui būtų galima pasiūlyti efektyvią alternatyvą, sukurtas dvifazis MTMCLP metodas. Pirmojoje fazėje sumažintų kintamųjų paieškos artimoje aplinkoje (angl. Reduced Variable Neighborhood Search (RVNS)) variantas pritaikomas greitam aukštos kokybės sprendimo suradimui. Gautas RVNS sprendimas yra geras pradinis taškas tiesiniam programavimo metodui antrojoje fazėje, kuri grąžina optimalų sprendimą į MTMCLP. Visi konstrukciniai siūlomo dvifazio metodo elementai, pažymėti RVNS-LP, yra pritaikomi pagal konkrečios sprendžiamos problemos charakteristiką. RVNS-LP metodas yra vertinamas atsižvelgiant į realius atvejus, su kuriais susidūrė du policijos ekipažai Montenegro ir Serbijoje, taip pat į atsitiktinumo tvarka sugeneruotas didesnių dimensijų testines situacijas. Bandomasis įvertinimas rodo, kad pasiūlytas RVNS-LP metodas rado visus optimalius sprendimus visuose realių atvejų testavimo atvejuose per labai trumpą centrinio procesoriaus įrenginio (CPU) laiką. Visais sukurtais testinių įvykių variantų atvejais, RVNS-LP grąžino optimalius sprendimus per trumpą veikimo laiką ir sutaupė gerokai daugiau laiko, nei CPLEX. Matematinis modelis ir pasiūlytas dvifazis optimizavimo metodas gali būti pritaikomi įvairių pagalbos paslaugų tinklų projektavime ir valdyme.