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Trajectory-Linearization Based Robust Model Predictive Control for Constrained Unmanned Surface Vessels

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Abstract. In this paper, a trajectory-linearization based robust model predictive control (MPC) approach is proposed for unmanned surface vessels (USVs) with system constraints and disturbances. The trajectory linearization technique is used to translate a continuous-time nonlinear model of vessels into a linear time-varying predictive model and to decrease the complexity of nonlinear MPC. The control scheme includes a linear feedback control and a MPC term, where the former ensures the real trajectory being contained in a tube centered at the reference trajectory, and the later ensures asymptotic stability of the nominal system. The effectiveness of the designed control is analyzed theoretically and illustrated by simulation results.

Keywords: trajectory linearization; unmanned surface vessel; model predictive control; robustness.

1. Introduction

The trajectory tracking and path-following of unmanned surface vessels (USVs) attracted more and more attention, due to applications in military and civil, including resource detection, environmental surveillance, maritime rescue, reconnaissance, and mine countermeasures [1-5]. However, disturbances from wind, waves, and ocean currents severely affect the stability of USVs and bring difficulties to controller design. Therefore, how to design robust tracking control for surface vessels is of great significance. Up to now, many robust tracking controllers for vessels have been obtained based on sliding-mode control [6-8], H_∞ control [9-11], neural-network control [12-18], fuzzy control [19] and disturbance observer -based control [20]. However, the state and input constraints are seldom considered in these approaches. Factually, the control powers of USVs are limited and the states are constrained due to collision avoidance and limited working space. Thus, it is valuable to design robust control for USVs with state and input constraints.

Model predictive control (MPC) is well known for its advantage of receding horizon optimization, robustness and its ability in actively handling constraints, and has been successfully applied in petro-chemical, robotics, and so on [21, 22]. Nowadays, for constrained uncertain systems, robust model predictive control (RMPC) has been obtained mainly based on min-max MPC [23, 24] and constraint tightening approaches [25, 26]. In these results, the constraints are satisfied in receding horizon optimization, which is online solved for all possible realization of uncertainties. However, this possibly brings infeasibility and conservatism of the online-solved optimal control problem. The tube-based MPC described in [27, 28] mitigates the disadvantages of RMPC in [23-26], since its decision variables include not only the usual control sequences, but also the initial state of the nominal model at each optimization iteration. However, the current results on tube-based MPC are mainly based on discrete-time models, while unmanned surface vessels are usually modeled as continuous-time nonlinear system. Therefore, the continuous-time nonlinear system-based tube RMPC is considered as the approach to design a control law for constrained USVs.

In this paper, a trajectory-linearization based tube MPC is proposed for USVs to track desired trajectories. A linear time-varying predictive model is constructed by trajectory linearization [29, 30] of the vessel's continuous-time nonlinear model. The use of linear time-varying model not only decreases computational complexity of nonlinear MPC, but also maintains the model precision. For simplicity, only kinematic model with additive disturbance is considered in this paper.

The organization of this paper is stated as follows. Section 2 presents the problem considered in this paper. Trajectory linearization of vessel's nonlinear system is constructed and the robust control invariant set for the error system are established in Section 3. Section 4 is devoted to tube-based MPC design. At last, simulations are stated in Section 5 and conclusions are presented in Section 6.

Notation. Denote R^n as the n-dimensional Euclidean space. Define $C_\varepsilon(0)$ as the neighborhood of zero with ε being the radius. The symbols $\oplus$ and $\ominus$ denote Minkowski sum and difference, respectively.

2. Problem statement

The kinematic model of the unmanned surface vessel is described as

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}u\\ v\\ r\end{bmatrix}+\mathrm{d}(t), \tag{1}$$

Where $\eta=[x \quad y \quad \psi]^T\in\mathbb{X}\subset R^3$ denotes the position and heading of vessel in the earth-frame coordinate system; $\upsilon=[u \quad v \quad r]^T\in\mathbb{V}\subset R^3$ represents the surge, sway, and yaw velocities in the vessel-frame coordinate system, respectively; $d(t)$ denotes the system disturbance. The sets $\mathbb{X}$ and $\mathbb{V}$ are two closed sets and both contain zero as their interior point.

The objective of this paper is to design tube-based RMPC for (1) such that the state $[x,y,\psi]^T$ tracks the command signal $[x_{com},y_{com},\psi_{com}]^T$.

3. Main results

3.1. Trajectory linearization

In this section, the trajectory linearization approach is adopted to convert the vessel's nonlinear system into a time-varying linear system.

From (1), the nominal rate for a predetermined trajectory $\begin{bmatrix}\bar{x}(t) & \bar{y}(t) & \bar{\psi}(t)\end{bmatrix}^T$ is

$$\begin{bmatrix}\bar{u}\\ \bar{v}\\ \bar{r}\end{bmatrix}=\begin{bmatrix}\cos\bar{\psi}(t) & \sin\bar{\psi}(t) & 0\\ -\sin\bar{\psi}(t) & \cos\bar{\psi}(t) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{\bar{x}}(t)\\ \dot{\bar{y}}(t)\\ \dot{\bar{\psi}}(t)\end{bmatrix} \tag{2}$$

with

$$\frac{d}{dt}\begin{bmatrix}\bar{x}(t)\\ \dot{\bar{x}}(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ -a_{d1}(t) & -a_{d2}(t)\end{bmatrix}\begin{bmatrix}\bar{x}(t)\\ \dot{\bar{x}}(t)\end{bmatrix}+\begin{bmatrix}0\\ a_{d1}(t)\end{bmatrix}x_{com}, \tag{3}$$

where $\begin{bmatrix}\dot{\bar{x}}(t) & \dot{\bar{y}}(t) & \dot{\bar{\psi}}(t)\end{bmatrix}^T$ and $\begin{bmatrix}\bar{x}(t) & \bar{y}(t) & \bar{\psi}(t)\end{bmatrix}^T$ are calculated in equation (3) by passing command state $\begin{bmatrix}x_{com} & y_{com} & \psi_{com}\end{bmatrix}^T$ into a twice-order, low-pass, command filter. The states $\bar{x}(t)$ and $\dot{\bar{x}}(t)$ in (3) represent the estimations of $x_{com}(t)$ and its derivative, respectively. In (3), $a_{d1}(t)=\omega_{n,diff}^2$, $a_{d2}(t)=2\zeta\omega_{n,diff}$, with ζ being the damping ratio, $\omega_{n,diff}$ being the natural frequency, which determines the bandwidth of the filter. $\dot{\bar{y}}(t)$ and $\dot{\bar{\psi}}(t)$ can be obtained similarly as $\dot{\bar{x}}(t)$.

Define

$$[e_x \quad e_y \quad e_\psi]^T=\begin{bmatrix}x(t) & y(t) & \psi(t)\end{bmatrix}^T-\begin{bmatrix}\bar{x}(t) & \bar{y}(t) & \bar{\psi}(t)\end{bmatrix}^T \tag{4}$$

and

$$\begin{bmatrix}\tilde{u} & \tilde{v} & \tilde{r}\end{bmatrix}^T=\begin{bmatrix}u(t) & v(t) & r(t)\end{bmatrix}^T-\begin{bmatrix}\bar{u}(t) & \bar{v}(t) & \bar{r}(t)\end{bmatrix}^T. \tag{5}$$

Taking linearization of equation (1) along $\begin{bmatrix}\bar{x}(t) & \bar{y}(t) & \bar{\psi}(t)\end{bmatrix}^T$ and $\begin{bmatrix}\bar{u}(t) & \bar{v}(t) & \bar{r}(t)\end{bmatrix}^T$, we can obtain the following linearized error dynamics

$$\begin{bmatrix}\dot{e}_x\\ \dot{e}_y\\ \dot{e}_\psi\end{bmatrix}=A(t)\begin{bmatrix}e_x\\ e_y\\ e_\psi\end{bmatrix}+B(t)\begin{bmatrix}\tilde{u}\\ \tilde{v}\\ \tilde{r}\end{bmatrix}+w(t). \tag{6}$$

Define $e\triangleq[e_x \quad e_y \quad e_\psi]^T$, $\alpha\triangleq\begin{bmatrix}\tilde{u} & \tilde{v} & \tilde{r}\end{bmatrix}^T$. Then, the system (6) can be rewritten in the following form

$$\dot{e}=A(t)e+B(t)\alpha+w(t), \tag{7}$$

where

$$A(t)=\begin{bmatrix}0 & 0 & -\bar{u}(t)\sin\bar{\psi}(t)-\bar{v}(t)\cos\bar{\psi}(t)\\ 0 & 0 & \bar{u}(t)\sin\bar{\psi}(t)-\bar{v}(t)\cos\bar{\psi}(t)\\ 0 & 0 & 0\end{bmatrix},$$

$$B(t)=\begin{bmatrix}\cos\bar{\psi}(t) & -\sin\bar{\psi}(t) & 0\\ \sin\bar{\psi}(t) & \cos\bar{\psi}(t) & 0\\ 0 & 0 & 1\end{bmatrix}, \tag{8}$$

$e(t)\in R^n$ is the state of system (7); $\alpha(t)\in R^m$ is the control of system (7); $w(t)\in R^n$ denotes lumped disturbances containing linearization errors and system disturbances, which satisfies $w(t)\in\mathbb{W}\triangleq\{w\in R^{n_w} \mid \|w\| \le w_{\max}\}$ for all $t\ge 0$.

From the constraints on η and υ in (1), the dynamics (7) is subject to state and control constraints $\mathbb{X}\triangleq\mathbb{E}\oplus\begin{bmatrix}\bar{x} & \bar{y} & \bar{\psi}\end{bmatrix}^T$ and $\mathbb{V}\triangleq\mathbb{U}\oplus\begin{bmatrix}\bar{u} & \bar{v} & \bar{r}\end{bmatrix}^T$ with $\mathbb{E}$ and $\mathbb{U}$ being compact sets containing zero as their interior point.

3.2. Robust control invariant set

The nominal system of (7) can be stated as

$$\dot{\bar{e}}=A(t)\bar{e}+B(t)\bar{\alpha}, \tag{9}$$

where $\bar{\alpha}$ is the nominal control input of (9).

To stabilize the system error of the system (7), a proportional feedback control law is proposed.

Suppose that there exist $K(t)$ such that the matrix $A(t)+B(t)K(t)$ is stable.

Define $\tilde{e}=\begin{bmatrix}\tilde{e}_x & \tilde{e}_y & \tilde{e}_\psi\end{bmatrix}^T \triangleq \begin{bmatrix}e_x & e_y & e_\psi\end{bmatrix}^T - \begin{bmatrix}\bar{e}_x & \bar{e}_y & \bar{e}_\psi\end{bmatrix}^T$.

If the control law for (7) is designed as

$$\alpha=\bar{\alpha}+K(t)(e-\bar{e})=\bar{\alpha}+K(t)\tilde{e}, \tag{10}$$

then, based on (7)-(10), the dynamics of the error system is

$$\begin{bmatrix}\dot{\tilde{e}}_x\\ \dot{\tilde{e}}_y\\ \dot{\tilde{e}}_\psi\end{bmatrix}=A\begin{bmatrix}\tilde{e}_x\\ \tilde{e}_y\\ \tilde{e}_\psi\end{bmatrix}+w, \tag{11}$$

where $A=[A(t)+B(t)K(t)]=diag\begin{bmatrix}a_{11} & a_{22} & a_{33}\end{bmatrix}$, which satisfies that $a_{11}<0,\ a_{22}<0,\ a_{33}<0$.

Lemma 1. *Denote $\beta=\max\{a_{11},a_{22},a_{33}\}<0$, then the set $\Omega=\left\{\tilde{e}:\tilde{e}\,|\,F\le-\frac{w_{\max}^2}{2\xi(2\beta+\xi)}\right\}$ is a robust control invariant set for the controlled uncertain system (11), where $F(\tilde{e})=\tilde{e}^T\tilde{e}/2$ and ξ is a designed positive constant.*

Proof. Taking time derivative of F and substituting (11), yields

$$\dot{F}=\tilde{e}^TA\tilde{e}+\tilde{e}^Tw\le\beta\tilde{e}^T\tilde{e}+\tilde{e}^Tw. \tag{12}$$

Using Young's inequality $\tilde{e}^Tw\le\frac{\xi}{2}\tilde{e}^T\tilde{e}+\frac{1}{2\xi}w^Tw$, then, we can further obtain the following result

$$\begin{aligned}\dot{F}&\le(\beta+\xi/2)\tilde{e}^T\tilde{e}+\frac{1}{2\xi}w^Tw\\ &\le(\beta+\xi/2)\tilde{e}^T\tilde{e}+\frac{1}{2\xi}w_{\max}^2\\ &=(2\beta+\xi)F+\frac{1}{2\xi}w_{\max}^2.\end{aligned} \tag{13}$$

From (13) we can see that the derivative of F is guaranteed to be less than zero, as long as the following expression holds

$$F>-\frac{w_{\max}^2}{2\xi(2\beta+\xi)}. \tag{14}$$

Therefore, the set $\Omega=\left\{\tilde{e}:\tilde{e}\,|\,F\le-\frac{w_{\max}^2}{2\xi(2\beta+\xi)}\right\}$ is a robust control invariant set for the controlled uncertain system (11).

Proposition 1. If $e(0)\in\bar{e}(0)\oplus\Omega$, $w\in\mathbb{W}$ and $\alpha=\bar{\alpha}+K(t)\alpha$, then $e(t)\in\bar{e}(t)\oplus\Omega$ holds for all $t>0$, where $e(t)$ and $\bar{e}(t)$ are the states of system model (10) and (11), respectively.

4. Tube-based MPC

4.1. Construction of Tube-based MPC

Denote $\bar{e}(t,t_k,\bar{e}(t_k),\bar{\alpha}(\cdot))$ as the movement of the nominal system (9) from the initial time t_k and initial state $\bar{e}(t_k)$ for a control signal. Then the cost function $J(\bar{e}(t),\bar{\alpha}(t))$ of the receding horizon optimization problem is formulated as follows:

$$\begin{aligned}J(\bar{e}(t_k),\bar{\alpha}(\cdot))&=\int_{t_k}^{t_k+T_p}l(\bar{e}(\tau),\bar{\alpha}(\tau))d\tau\\ &+G(\bar{e}(t_k+T_p)),\end{aligned} \tag{15}$$

where $l(\bar{e},\bar{\alpha})\triangleq\frac{1}{2}(\bar{e}^TQ\bar{e}+\bar{\alpha}^TR\bar{\alpha})$, $\mathrm{G}(\bar{e})\triangleq=\frac{1}{2}\bar{e}^TP\bar{e}$; Q, R and P are positive definite symmetric matrices; $T_p\ge0$ is defined as the prediction horizon.

Assume there exist a matrix K such that $K(t)\Omega\subset K\Omega$. Then, the system constraints for the nominal system in the MPC can be constructed as follows:

$$\bar{e}(\tau)\in\bar{\mathbb{E}}\triangleq\mathbb{E}\ominus\Omega\,..\ \ \tau\in[t_k,t_k+T_p); \tag{16}$$

$$\bar{\alpha}(\tau)\in\bar{V}\triangleq V\ominus K\Omega\,..\ \ \tau\in[t_k,t_k+T_p); \tag{17}$$

$$\bar{e}(t_k+T_p)\in\mathbb{E}_f\subset\mathbb{E}\ominus\Omega. \tag{18}$$

Hence, the set of feasible control sequences at sampling time t_k can be defined by

$$\begin{aligned}&\mathrm{U}_N(\bar{e}(t_k))\\ &=\{\bar{\alpha}(\tau),\tau\in[t_k,t_k+T_p)\,|\,\bar{\alpha}(\tau)\in\bar{V},\bar{e}(\tau)\in\bar{\mathbb{E}},\\ &\quad\forall\tau\in[t,t+T_p),\bar{e}(t+T_p)\in\mathbb{E}_f\}.\end{aligned} \tag{19}$$

It is assumed that $\mathbb{W}$ is small enough to ensure that $\Omega\subset\mathrm{interior}(\mathbb{E})$ and $K\Omega\subset\mathrm{interior}(\mathbb{U})$, and the terminal constraint set $\mathbb{E}_f$ satisfies:

1) $A\mathbb{E}_f\subset\mathbb{E}_f,\mathbb{E}_f\subset\mathbb{E}\ominus\Omega,K\mathbb{E}_f\subset\mathbb{U}\ominus K\Omega,$
   $\mathbb{E}_f$ is closed and $0\in\mathbb{E}_f$; (20)
2) $\mathbb{E}_f$ is a positively invariant set for
   $\dot{\bar{e}}=A(t)\bar{e}+B(t)K\bar{e}$; (21)
3) $[\dot{G}+l](\bar{e},K\bar{e})\le0,\quad\forall\bar{e}\in\mathbb{E}_f.$ (22)

Remark 1. *If $\bar{e}\in\bar{\mathbb{E}}$ and $e\in\bar{e}\oplus\Omega$, the $e\in\mathbb{E}$ and $\eta\in\mathbb{X}$ can be concluded. If $\bar{\alpha}\in\bar{\mathbb{U}}$ and $e\in\bar{e}\oplus\Omega$ hold, from $\alpha=\bar{\alpha}+K(e-\bar{e})$, we can conclude that $\alpha\in\mathbb{U}$ and $\upsilon\in\mathbb{V}$. Therefore, the defined constraints for the nominal system (9) are reasonable.*

In the conventional continuous-time MPC for the nominal model, the optimal problem at sampling time t_k is defined by

$$\begin{aligned} V^*(\bar{e}(t_k)) &= \min_{\bar{\alpha}}\{J(\bar{e},\bar{\alpha}) \mid \bar{\alpha}(\tau) \in \mathrm{U}_N(\bar{e}(t_k)), \\ &\qquad \tau \in [t_k, t_k+T_p)\}, \\ \bar{\alpha}^*(t_k) &= \arg\ \min_{\bar{\alpha}}\{J(\bar{e},\bar{\alpha}) \mid \bar{\alpha}(\tau) \in \mathrm{U}_N(\bar{e}(t_k)), \\ &\qquad \tau \in [t_k, t_k+T_p)\}. \end{aligned} \tag{23}$$

Compared with the conventional optimal control problem, the new optimization problem in tube MPC involves the initial state. This is permissible because the initial state in the optimal problem is now not equal to the current state $e(t_k)$ of the system, which cannot be instantaneously changed, but a parameter of the control law. The new optimal control problem is defined by

$$\begin{aligned} V_0^*(\bar{e}_0^*(t_k)) &= \min_{\bar{e}_0,\bar{\alpha}}\{J(\bar{e},\bar{\alpha}) \mid \bar{\alpha}(\tau) \in \mathrm{U}_N(\bar{e}(t_k)), \\ &\qquad \tau \in [t_k, t_k+T_p), e(t_k) \in \bar{e}_0(t_k) \oplus \Omega\}, \\ (\bar{e}_0^*(t_k), \bar{\alpha}_0^*(\cdot)) &= \arg\min_{\bar{e}_0,\bar{\alpha}}\{J(\bar{e}_0,\bar{\alpha}) \mid \bar{\alpha}(\tau) \in \mathrm{U}_N(\bar{e}(t_k)), \\ &\qquad \tau \in [t_k, t_k+T_p), e(t_k) \in \bar{e}_0(t_k) \oplus \Omega\}. \end{aligned} \tag{24}$$

If the function $\bar{e}^*(t), t \ge t_0$ is defined as

$$\bar{e}^*(t) = \begin{cases} \bar{e}(t, t_0, \bar{e}_0^*(t_0), \bar{\alpha}_0^*(\cdot)), & t \in [t_0, t_1) \\ \vdots \\ \bar{e}(t, t_k, \bar{e}_0^*(t_k), \bar{\alpha}_0^*(\cdot)), & t \in [t_k, t_{k+1}) \\ \vdots \end{cases}, \tag{25}$$

then according to Proposition 1, we can obtain

$$e(t) \in \bar{e}^*(t) \oplus \Omega, \quad t \ge t_0. \tag{26}$$

Based on the above analysis, the tube MPC for system (7) can be stated as:

$$\alpha(t) = \bar{\alpha}_0^*(t) + K(t)(e(t) - \bar{e}_0^*(t)), \quad t \in [t_k, t_{k+1}), \quad k = 0,1,2,\cdots. \tag{27}$$

4.2. Stability analysis of the proposed control

Definition 1.

1. $X_N = \{e(t_0) \mid \exists \bar{e}(t_0), e(t_0) \in \bar{e}(t_0) \oplus \Omega, U_N(\bar{e})$ is not empty$\}$;
2. The robust control invariant set Ω is robustly asymptotically stable for system (7) controlled through (25) with X_N as the region of attraction if, for all admissible disturbance, a) $dist(e(t),\Omega) \to 0$ as $t \to \infty$ for all $e(t_0) \in X_N$ and b) for all $\varepsilon > 0$, $\exists \delta > 0$ such that, for all $e(t_0) \in C_\delta(0) \oplus \Omega$, then $e(t) \in C_\varepsilon(0) \oplus \Omega$ for all $t \ge t_0$.

Theorem 1. *Suppose the optimization problem (24) is feasible at sampling time t_k. Then,*

1) The optimization problem (24) is feasible for all sampling time t_n with $n>k$;

2) The optimal value function satisfies

$$V_0^*(\bar{e}_0^*(t_{k+1})) \le V_0^*(\bar{e}_0^*(t_k)) - \int_{t_k}^{t_{k+1}} (\|\bar{e}^*(\tau)\|_Q^2 + \|\bar{\alpha}^*(\tau)\|_R^2) d\tau; \tag{28}$$

3) The set Ω is asymptotic stable for the controlled continuous-time nonlinear system $\dot{e} = A(t)e + B(t)\alpha + w(t)$ with $\alpha(t) = \bar{\alpha}^*(t) + K(t)(e(t) - \bar{e}^*(t))$, $t \in [t_k, t_{k+1})$ for a sufficiently small sampling time interval $\delta>0$.

Proof.

1) It is assumed that at sampling time t_k, an optimal solution $(\bar{e}_0^*(t_k),\ \bar{\alpha}^*(\bullet,\ \bar{e}_0^*(t_k),\ t_k,\ t_k+T_p))$ to problem (24) exists and is found. Therefore, the state $\bar{e}(\tau, \bar{e}_0^*(t_k),\ t_k,\ t_k+T_p)$ and the input $\bar{\alpha}^*(\tau;\ \bar{e}_0^*(t_k),\ t_k,\ t_k+T_p)$, $\tau \in [t_k,\ t_k+T_p]$ satisfy the constraints (16)-(18). When applied to the nominal system (9), the state will be driven from $\bar{e}_0^*(t_k)$ to $\bar{e}(t_k+T_p,\ t_k,\ \bar{e}_0^*(t_k),\ \ \bar{\alpha}^*(\cdot)) \in \mathbb{E}_f$.

Since the state of nominal system at time $t_{k+1} = t_k + \delta$ is $\bar{e}(t_{k+1}, t_k, \bar{e}_0^*(t_k), \bar{\alpha}^*(\cdot))$ and $e(t_{k+1}) \in \bar{e}(t_{k+1}, t_k, \bar{e}_0^*(t_k), \bar{\alpha}^*(\cdot)) \oplus \Omega$ holds, $\bar{e}(t_{k+1}, t_k, \bar{e}_0^*(t_k), \bar{\alpha}^*(\cdot))$ is a feasible choice of initial state of the optimization (24) at time t_{k+1}. Since $\zeta \triangleq \bar{e}(t_k+T_p, t_k, \bar{e}_0^*(t_k), \bar{\alpha}(\cdot)) \in \mathbb{E}_f$, $K\mathbb{E}_f \subset \mathbb{E} \ominus K\Omega$ and $\mathbb{E}_f$ is positively invariant for $\dot{\bar{e}} = A(t)\bar{e} + B(t)K\bar{e}$. Then, at sampling time t_{k+1}, the control input $\bar{\alpha}(\bullet)$ on $[t_{k+1}, t_{k+1}+T_p)$ may be chosen as

$$\bar{\alpha}(\tau) = \begin{cases} \bar{\alpha}^*(\tau, t_k, \bar{e}_0^*(t_k)),\ \tau \in [t_{k+1}, t_k+T_p] \\ K\bar{e}(\tau, t_k+T_p, \zeta, \bar{\alpha}(\cdot)),\ \tau \in [t_k+T_p, t_{k+1}+T_p). \end{cases} \tag{29}$$

Therefore, the feasibility of the optimal control problem (24) with constraints (16)-(18) at time t_k implies its feasibility for all sampling time t_n with $n>k$.

2) The optimal value of the objective functional at time t_k can be written as

$$V_0^*(\bar{e}_0^*(t_k)) = \int_{t_k}^{t_k+T_p} (\|\bar{e}^*(\tau)\|_Q^2 + \|\bar{\alpha}^*(\tau)\|_R^2) d\tau + \|\bar{e}(t_k+T_p, t_k, \bar{e}_0^*(t_k), \bar{\alpha}^*(\cdot))\|_P^2. \tag{30}$$

Since at sampling time t_{k+1}, a feasible control input can be chosen as (29), then the value of the objective function at sampling time t_{k+1} is

$$
\begin{aligned}
&V(\bar{e}(t_{k+1}),\bar{\alpha}(\cdot)) \\
&= \int_{t_{k+1}}^{t_{k+1}+T_p} (\|\bar{e}(\tau,t_{k+1},\bar{e}(t_{k+1}),\bar{\alpha}(\cdot))\|_Q^2 + \|\bar{\alpha}(\tau)\|_R^2)d\tau \\
&\quad + \|\bar{e}(t_{k+1}+T_p,t_{k+1},\bar{e}(t_{k+1}),\bar{\alpha}(\cdot))\|_P^2 \\
&= \int_{t_{k+1}}^{t_k+T_p} (\|\bar{e}(\tau,t_k,\bar{e}_0^*(t_k),\bar{\alpha}^*(\cdot,t_k,\bar{e}_0^*(t_k)))\|_Q^2 \\
&\quad + \|\bar{\alpha}^*(\tau,t_k,\bar{e}_0^*(t_k))\|_R^2)d\tau \\
&\quad + \int_{t_k+T_p}^{t_{k+1}+T_p} (\|\bar{e}(\tau,t_{k+1},\zeta,\bar{\alpha}_1(\cdot))\|_Q^2 + \|\bar{\alpha}(\tau)\|_R^2)d\tau \qquad (31)\\
&\quad + \|\bar{e}(t_{k+1}+T_p,t_k+T_p,\zeta,\bar{\alpha}(\cdot))\|_P^2 \\
&= V_0^*(\bar{e}_0^*(t_k)) - \int_{t_k}^{t_{k+1}} (\|\bar{e}(\tau,t_k,\bar{e}_0^*(t_k),\bar{\alpha}^*(\cdot,t_k,\bar{e}_0^*(t_k)))\|_Q^2 \\
&\quad + \|\bar{\alpha}^*(\tau,t_k,\bar{e}_0^*(t_k))\|_R^2)d\tau \\
&\quad + \int_{t_k+T_p}^{t_{k+1}+T_p} (\|\bar{e}(\tau;\bar{e}_0^*(t_{k+1}),t_{k+1},t_{k+1}+T_p)\|_Q^2 + \|\bar{\alpha}(\tau)\|_R^2)d\tau \\
&\quad + \|\bar{e}(t_{k+1}+T_p,t_k+T_p,\zeta,\bar{\alpha}(\cdot))\|_P^2 \\
&\quad - \|\bar{e}(t_k+T_p,t_k,\bar{e}_0^*(t_k),t_k+T_p)\|_P^2 .
\end{aligned}
$$

From $\bar{e}(t_k+T_p,t_k,\bar{e}_0^*(t_k),t_k+T_p) \in \mathbb{E}_f$ and inequality (22), we can obtain the following result

$$
\begin{aligned}
&\|\bar{e}(t_{k+1}+T_p,t_k+T_p,\zeta,\bar{\alpha}(\cdot))\|_P^2 - \\
&\qquad \|\bar{e}(t_k+T_p,t_k,\bar{e}_0^*(t_k),t_k+T_p)\|_P^2 \qquad (32)\\
&\le -\int_{t_k+T}^{t_{k+1}+T_p} \left(\|\bar{e}(\tau;\bar{e}_0^*(t_{k+1}),t_{k+1},t_{k+1}+T_p)\|_Q^2 + \|\bar{\alpha}(\tau)\|_R^2\right)d\tau.
\end{aligned}
$$

Therefore, the following results can be obtained from (31)-(32),

$$
\begin{aligned}
&V^*(\bar{e}(t_{k+1})) \le \\
&\quad V_0^*(\bar{e}_0^*(t_k)) \\
&\qquad - \int_{t_k}^{t_{k+1}} \left(\|\bar{e}(\tau,t_k,\bar{e}_0^*(t_k),\bar{\alpha}^*(\cdot,t_k,\bar{e}_0^*(t_k)))\|_Q^2 \right. \qquad (33)\\
&\qquad \left. + \|\bar{\alpha}^*(\tau,t_k,\bar{e}_0^*(t_k))\|_R^2\right)d\tau.
\end{aligned}
$$

At last, the result (28) can be concluded from the fact that $V_0^*(\bar{e}_0^*(t_{k+1})) \le V^*(\bar{e}(t_{k+1}))$.

3) It can be easily seen from (28) that the sequence $\{V_0^*(\bar{e}_0^*(t_k)), \mathrm{k}=0,1,2,\ldots\}$ is monotonic non-increasing and with 0 being a lower bound. Thus, $\{V_0^*(\bullet)\}$ converges to some non-negative value as k tends to infinity. Then, from (28) and the convergence of $\{V_0^*(\bar{e}_0^*(t_k)), \mathrm{k}=0,1,2,\ldots\}$, the following result can be concluded

$$
\begin{aligned}
&\lim_{k\to+\infty}\sup \int_{t_k}^{t_{k+1}} \left(\|\bar{e}(\tau,t_k,\bar{e}_0^*(t_k),\bar{\alpha}^*(\cdot,t_k,\bar{e}_0^*(t_k)))\|_Q^2 \right. \\
&\qquad \left. + \|\bar{\alpha}^*(\tau,t_k,\bar{e}_0^*(t_k))\|_R^2\right)d\tau \qquad (34)\\
&\le \lim_{k\to+\infty} V_0^*\left(\bar{e}_0^*(t_k)\right) - \lim_{k\to+\infty} V_0^*\left(\bar{e}_0^*(t_{k+1})\right) = 0.
\end{aligned}
$$

Then, we can obtain

$$\lim_{t\to+\infty}\sup\|\bar{e}(t)\| = 0. \qquad (35)$$

Since $0 \le \lim_{t\to+\infty}\inf\|\bar{e}(t)\| \le \lim_{t\to+\infty}\sup\|\bar{e}(t)\|$ and (34), (35) hold, we can get the following result

$$\lim_{t\to+\infty}\|\bar{e}(t)\| = 0. \qquad (36)$$

Since $e(t) \in \bar{e}(t) \oplus \Omega$ and $\bar{e}(t) \to 0$ as $t \to +\infty$, the set Ω is robustly asymptotic stable for the controlled nonlinear uncertain system (7).

5. Simulation results

In this section, the effectiveness of the proposed control law is illustrated by simulation. Based on (3), we set $\zeta = 0.5$, $\omega_{n,diff} = 4$, $x_{com} = 2$, $y_{com} = 2$, $\psi_{com} = \pi/4$ and $(\bar{x}(0),\bar{y}(0),\bar{\psi}(0)) = (0,0,0)$, then we get $[\bar{x}(t),\bar{y}(t),\bar{\psi}(t)]^T$ and $[\bar{u}(t),\bar{v}(t),\bar{r}(t)]^T$.

In the linearization (7) of the system (1), the lumped disturbance is denoted as $w(t) = [0.1\sin(0.1t)$ $0.1\sin(0.1t)$ $0.05\sin(0.1t)]^T$. The constraints to the system (7) are described as: $\|\mathrm{x}\| \le 2.5, \|\mathrm{y}\| \le 2.5,$ $\|\psi\| \le$ 1.2, and $-1 \le u \le 6, -1 \le v \le 3, \text{-}1 \le r \le 1$. In the simulation, we set K=-0.5I, P=0.5I, Q=I and R=0.2I. We set the sampling time as δ=0.1. The terminal state constraint in the MPC is chosen as $\{\bar{e} \mid \bar{e}^T P \bar{e} \le 0.06\}$.

Based on these designed parameters, the control for the system (7) is executed by using MATLAB. The simulation results are showed in Fig. 1-Fig. 3, where Fig. 1 presents the error between the state of

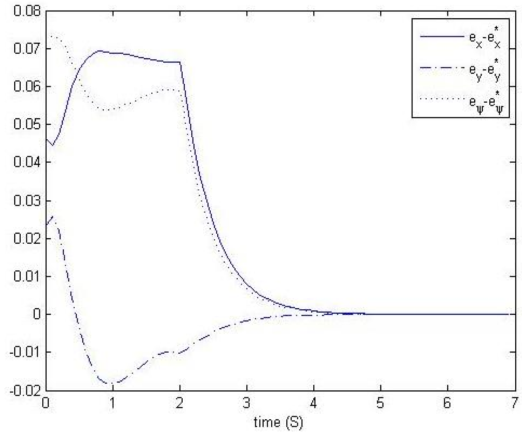

Figure 1. The error between (e_x, e_y, e_ψ) and (e_x^*, e_y^*, e_ψ^*)

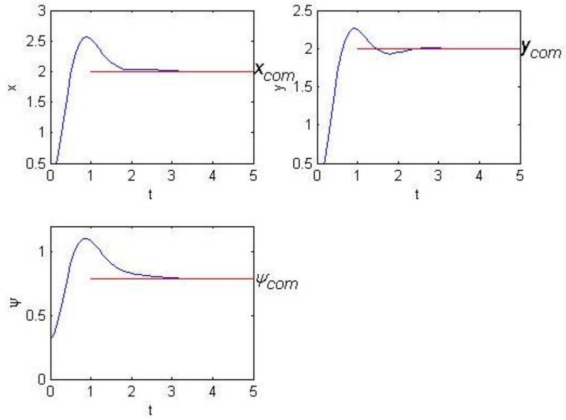

Figure 2. The tracking of command signals for the vessel

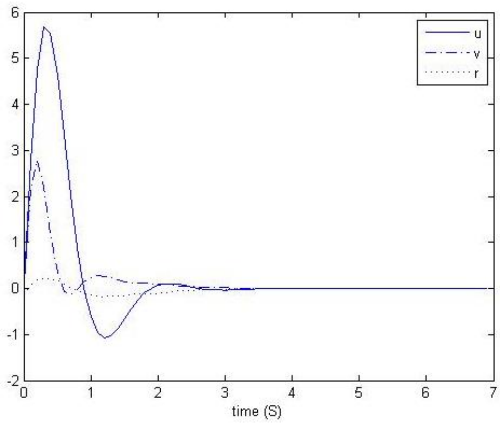

Figure 3. The designed control (u, v, r) for the surface vessel

$(e_x \quad e_y \quad e_\psi)$ and (e_x^*, e_y^*, e_ψ^*), Fig. 2 depicts the tracking of command signals $x_{com} = 2$, $y_{com} = 2$, $\psi_{com} = \pi/4$. From Fig. 1-Fig. 3, we can get the effectiveness of the designed tube-based robust MPC, including input and state constraints satisfaction and robustness of the closed-loop system.

6. Conclusions

In this paper, we have proposed a trajectory-linearization-based robust model predictive control (RMPC) for unmanned surface vessels with system constraints and disturbances. In the proposed RMPC, the linear feedback control ensured the real trajectory contained in a tube of trajectory of a nominal system, while the MPC guaranteed the asymptotical stability of the nominal system. We have also provided theoretical analysis and simulation results to illustrate the effectiveness of the proposed control.

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