A Novel Iterative Learning Control Design for Linear Discrete Time Systems Based on A 2D Roesser System

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Abstract. This paper is mainly devoted to the iterative learning control (ILC) design for a class of linear discrete time systems. By providing a 2D analysis of the learning process, the ILC design for such systems can be transformed into the problem of state feedback or output feedback control for 2-D systems described by Roesser models. Then, a Lyapunov approach can be used to obtain an ILC law that achieves asymptotical convergence of the tracking error. Sufficient stability conditions are provided in terms of linear matrix inequalities, which can determine learning gains as well. The theoretical results are also verified through simulation tests.

Keywords: iterative learning control; 2D Roesser system; linear matrix inequality.

1. Introduction

Many practical control systems have the repetitive operation tasks, such as batch processes [1, 2]. Iterative learning control (ILC) is an effective technique that attempts to achieve a perfect output tracking for such systems over a finite time interval. Motivated by human learning, ILC uses information from previous iterations to improve the input signal of current iteration, and the control objective is achieved finally in the sense that the improvement of learning proceeds from trial to trial. Since the original work in [3], the general area of ILC has been attracted by more and more researchers.

Until now, there are many approaches available with regard to ILC systems. For example, the early contraction mapping approach for linear ILC and nonlinear ILC systems [1, 4, 5], the Lyapunov-based approach for continuous-time systems [6], the super-vector approach for linear discrete-time ILC systems [7, 8], etc. However, there are some essential limits for the above-mentioned approaches. Contraction mapping approach only focuses on the problem of stability or convergence analysis for ILC systems, and does not present the issue of controller design. Lyapunov-based approach is only considered for a class of special continuous-time systems as described in [6]. For linear discrete-time systems, super-vector approach is an effective technique for ILC. This technique reformulates the 2D ILC system as 1D system that evolves only in the trial domain. Then, the issue of convergence

analysis and controller design can be discussed. For instance, the stability has been analyzed in the frequency domain [9], the time domain [10, 11], and both the frequency and time domains [12]. The design of lifted ILC laws has been studied using H_∞ theory [13, 14], μ analysis [15] or quadratic cost [16], etc. Even though super-vector approach contributes to the ILC theory, it does not address the computational complexity of the lifted ILC design method that might hamper their real-life application [17, 18]. In the trial domain representation, dimensions of the lifted system matrices (system matrices in the trial domain) are proportional to the number of sampling steps in the trial. Besides computational complexity, another difficulty that might hinder the design of ILC for the case of large trial lengths, is an increased demand for memory locations that are needed to store lifted system matrices.

In fact, ILC is a fundamentally 2-D process in essence. Hence, 2-D system theory has been successfully introduced to the ILC approaches in recent years. Contributing by the two independent dynamic processes of the 2-D system, the 2-D model provides an excellent mathematical platform to describe both the dynamics of the control system and the behavior of the learning iteration. Based on the existing 2-D system theory, ILC analysis and design have been addressed for linear time-invariant (LTI) systems [19-21], and the results for LTI system also extended to linear time-varying (LTV) ILC systems in [22] which are based on the time-varying 2-D Roesser models. It should be

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noted that ILC design with linear matrix inequality (LMI) based on 2-D model have been developed to address the control system stability in both the time and batch directions [23-28]. In [23-25], the ILC design for linear systems can be attributed to the rigorous stability theory of linear repetitive process developed in [23]. Moreover, it is also shown that an LMI re-formulation of the stability conditions for discrete linear repetitive processes leads naturally to design algorithms for control laws to ensure stability along the pass under control action. In [26], some new concepts and convergence indices are introduced to describe the dynamical behaviors of the corresponding 2D Roesser system in horizontal and vertical propagation directions. Sufficient conditions for stability of the 2D Roesser system are derived, and then a series of new algorithms for ILC design for discrete-time linear systems based on LMI techniques are presented. Under this framework, the ILC design can also be considered for a class of linear systems with time-delay [27], nonlinear systems[28] and advanced PI control [29].

In this paper, a novel ILC design approach based 2-D system theory for a class of discrete time linear systems is presented. After providing a 2D analysis of the learning process, it has been shown that the design problem of an ILC law for such system can be transformed straightforwardly into feedback control problem of 2-D systems described by Roesser models. Here, due to the state signal of ILC system is measurable or not, the feedback control for 2-D systems can be designed using state feedback or output feedback. Depending on existed 2-D systems results [30-32], a Lyapunov approach can be employed to develop LMI conditions that guarantee asymptotical convergence for tracking error of the ILC system, and the feasibility and effectiveness of the proposed designs are demonstrated by numerical examples. Compared to the existing methods [23-28], the ILC law is designed based on a simple linear matrix inequality in this paper, resulting in less computation than the existing designs. Besides, when only output signal is available, the proposed design can solve the problem of convex optimization by introducing a new slack variable.

The remainder of this paper is organized as follows. Section 2 presents the description and design objectives of a class of discrete systems together with their transformation into equivalent 2D Roesser systems. In Section 3, a Lyapunov approach for 2D system is employed to develop LMI conditions that guarantee asymptotical convergence of the tracking error for ILC system. Based on the condition, a design approach for ILC law is given. In Section 4, the feasibility and effectiveness of the proposed design algorithms are demonstrated through simulation tests. Finally, the conclusions of this paper are given in Section 5.

2. Problem formulations

We consider the following linear discrete time system

$$\begin{cases} x(t+1,k) = Ax(t,k) + Bu(t,k) \\ y(t,k) = Cx(t,k) \end{cases}, \qquad (1)$$

where $x(t,k) \in R^n, u(t,k) \in R^m, y(t,k) \in R^l$ are state, input, and output variables, $A \in R^{n \times n}, B \in R^{n \times m}$, $C \in R^{l \times n}$ are the matrices describing the system in the state space. The letter k denotes iteration and t is discrete time. The system is operated repeatedly in the iteration domain with a desired output $y_d(t), t \in [0, N]$. Basic assumptions of this system are (i) every operation begins at an identical initial condition, that is $x(0,k) = x_0$ for all k; (ii) the desired trajectory $y_d(t)$ is iteration invariant.

The control objective is to find a control input $u(t,k)$ for any $x_0 \in R^n$ and any initial control sequence $u(t,0)$, it generates a output for system such that $y(t,k) \to y_d(t)$ for all $t \in [0, N]$.

A general iterative learning control law can be given as follows:

$$u(t,k+1) = u(t,k) + \Delta u(t,k), \qquad (2)$$

where $\Delta u(t,k)$ denotes modification of the control input.

Remark 1: The ILC updating law (2) means using the error information from previous trials as efficiently as possible in order to achieve a minimal tracking error in as few iterations as possible. The simplest ILC law is using a proportional-integral derivative learning law, which consists of a proportional, integral and derivative gain on the tracking error to update the system input. More advanced learning laws include plant inversion methods [33], norm-optimal ILC methods [34, 35] or adaptive design methods [36, 37], etc. In this paper, we only consider the simple P-type ILC.

The ILC system (1) and (2) is essentially a 2D system with evolution along two independent axes: time t and iteration k. We can use the 2D analysis approach to derive an expression for the tracking error and the state error.

Define $e(t,k) = y_d(t) - y(t,k)$. Using (1) and (2), we can obtain

$$\begin{aligned} &e(t,k+1) - e(t,k) \\ &= y(t,k) - y(t,k+1) \\ &= CAx(t-1,k) + CBu(t-1,k) \\ &\quad - CAx(t-1,k+1) - CBu(t-1,k+1) \\ &= -CA\eta(t,k) - CB\Delta u(t-1,k), \end{aligned} \qquad (3)$$

where $\eta(t,k) = x(t-1,k+1) - x(t-1,k)$.

Next, from (1) and (2), the following can also be obtained

$$\begin{aligned} \eta(t+1,k) &= x(t,k+1) - x(t,k) \\ &= A\eta(t,k) + B\Delta u(t-1,k). \end{aligned} \qquad (4)$$

Equations (3) and (4) can be rewritten as a class of 2D Roesser system as follows:

$$\begin{bmatrix} \eta(t+1,k) \\ e(t,k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -CA & I \end{bmatrix} \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix}$$
$$+ \begin{bmatrix} B & -CB \end{bmatrix}^T \Delta u(t-1,k) \qquad (5)$$
$$= \bar{A} \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix} + \bar{B} \Delta u(t-1,k),$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ -CA & I \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ -CB \end{bmatrix}.$$

Now, we consider the following ILC law

$$u(t,k+1) = u(t,k) + K_1 \begin{pmatrix} x(t,k+1) \\ -x(t,k) \end{pmatrix} \qquad (6)$$
$$+ K_2 e(t+1,k),$$

where K_1, K_2 are designed matrices. (6) can be rewritten as

$$\Delta u(t-1,k) = [K_1 \quad K_2] \begin{bmatrix} \begin{pmatrix} x(t-1,k+1) \\ -x(t-1,k) \end{pmatrix} \\ e(t,k) \end{bmatrix} \qquad (7)$$
$$= K \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix},$$

where $K = [K_1 \quad K_2]$.

Substituting (7) into (5), we can present the ILC system as a class of 2D systems as follows

$$\begin{bmatrix} \eta(t+1,k) \\ e(t,k+1) \end{bmatrix} = (\bar{A} + \bar{B}K) \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix}$$
$$= A_c \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix}, \qquad (8)$$

where $A_c = \bar{A} + \bar{B}K$.

In this case, the problem of ILC design to be addressed in this paper can be transformed as follows: Given a 2-D discrete time system described by (5), design a state feedback controller in the form of (7) such that the resulting closed-loop system (8) is asymptotical stable.

3. Main results

Now, we recall some useful related results on the stability of 2-D discrete systems described by the Roesser model. It has been shown that the stability analysis using Lyapunov functions is efficient to derive sufficient conditions guaranteeing the asymptotic stability for 2-D discrete systems. The well known Lyapunov inequality to test the stability of the 2-D discrete system in (8) is given in the following Lemma.

Lemma 1 *[30]. If there exists a positive definite block diagonal matrix*

$$Q = Q^T = Q_h \oplus Q_v = \begin{bmatrix} Q_h & \\ & Q_v \end{bmatrix},$$

satisfying

$$A_c^T Q A_c - Q < 0,$$

then 2-D discrete closed-loop system (8) is asymptotically stable.

The following well-known lemmas are needed in the proof of our main results.

Lemma 2 *[38] (Schur Complement). Assume W, L, U are given matrices with appropriate dimensions, where W and U are positive definite symmetric matrices. Then*

$$L^T U L - W < 0$$

if and only if

$$\begin{bmatrix} -W & L^T \\ L & -V^{-1} \end{bmatrix} < 0,$$

or

$$\begin{bmatrix} -V^{-1} & L \\ L^T & -W \end{bmatrix} < 0.$$

Lemma 3 *[38]. Given a symmetric matrix $\Theta \in R^{n \times n}$ and two matrices M, N of column dimension n, there exists a W such that the following condition holds*

$$\Theta + MWN^T + NW^T M^T < 0,$$

if and only if the following projection inequalities with respect to W are satisfied

$$M_\perp^T \Theta M_\perp < 0, N_\perp^T \Theta N_\perp < 0,$$

where $M_\perp$ and $N_\perp$ denote arbitrary bases of the nullspaces of M and N, respectively.

The following Theorem reveals that the asymptotical stablilty of 2-D discrete closed-loop system (8) can be recast to a matrix inequality feasibility problem.

Theorem 1. *Consider the 2-D discrete-time system (8). If there exist a block-diagonal matrix $P = diag(P_h, P_v)$ with $P_h \in R^{n \times n}$ and $P_v \in R^{l \times l}$ and matrix S such that*

$$\begin{bmatrix} -\frac{1}{2}(S+S^T) & S^T A_c^T & \frac{1}{2}S+S^T-P \\ A_c S & -P & -A_c S \\ \frac{1}{2}S^T+S-P & -S^T A_c^T & -S-S^T \end{bmatrix} < 0, \qquad (9)$$

then the closed-loop system (8) is asymptotical stable.

Proof. Matrix inequality (9) can be rewritten as follows:

$$\begin{bmatrix} 0 & 0 & -P \\ 0 & -P & 0 \\ -P & 0 & 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}I \\ -A_c \\ -I \end{bmatrix} S \begin{bmatrix} -I & 0 & I \end{bmatrix}$$

$$+ \begin{bmatrix} -I \\ 0 \\ I \end{bmatrix} S^T \begin{bmatrix} \frac{1}{2}I & -A_c^T & -I \end{bmatrix} < 0. \tag{10}$$

Let

$$M^T = \begin{bmatrix} \frac{1}{2}I & -A_c^T & -I \end{bmatrix}, N^T = \begin{bmatrix} -I & 0 & I \end{bmatrix},$$

$$\Theta = \begin{bmatrix} 0 & 0 & -P \\ 0 & -P & 0 \\ -P & 0 & 0 \end{bmatrix},$$

and select the orthogonal complements of M and N ($M_\perp$ and $N_\perp$ respectively) as:

$$M_\perp = \begin{bmatrix} I & 0 \\ 0 & I \\ \frac{1}{2}I & -A_c^T \end{bmatrix}, N_\perp = \begin{bmatrix} I & 0 \\ 0 & I \\ I & 0 \end{bmatrix}.$$

Applying Lemma 2 we can obtain

$$M_\perp^T \Theta M_\perp < 0, N_\perp^T \Theta N_\perp < 0, \tag{11}$$

that is

$$M_\perp^T \Theta M_\perp = \begin{bmatrix} I & 0 & \frac{1}{2}I \\ 0 & I & -A_c \end{bmatrix} \begin{bmatrix} 0 & 0 & -P \\ 0 & -P & 0 \\ -P & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ \frac{1}{2}I & -A_c^T \end{bmatrix}$$

$$= \begin{bmatrix} -P & PA_c^T \\ A_c P & -P \end{bmatrix} < 0, \tag{12}$$

and

$$N_\perp^T \Theta N_\perp = \begin{bmatrix} I & 0 & I \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -P \\ 0 & -P & 0 \\ -P & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ I & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2P & 0 \\ 0 & -P \end{bmatrix} < 0, \tag{13}$$

Using Lemma 3, we have

$$PA_c^T P^{-1} A_c P - P < 0. \tag{14}$$

Thus, multiplying the right and the left hand sides of condition (14) by P^{-1} and taking $Q = P^{-1}$ in the resulting inequality, we obtain

$$A_c^T Q A_c - Q < 0.$$

Using Lemma 1, it is obvious that the asymptotic stability of closed-loop system (8) is guaranteed. This completes the proof of Theorem 1.

To this end, our goal is interested in finding state feedback controller such that the closed loop of 2-D discrete-time system is asymptotically stable. Using Theorem 1 and $A_c = \bar{A} + \bar{B}K$, the synthesis of feedback controllers can be described to find (K, P, S) such that the following matrix inequality holds

$$\begin{bmatrix} -\frac{1}{2}(S+S^T) & S^T(\bar{A}+\bar{B}K)^T & \frac{1}{2}S+S^T-P \\ (\bar{A}+\bar{B}K)S & -P & -(\bar{A}+\bar{B}K)S \\ \frac{1}{2}S^T+S-P & -S^T(\bar{A}+\bar{B}K)^T & -S-S^T \end{bmatrix} < 0. \tag{15}$$

It should be noted that in general the problem of solving numerically (15) for (K, P, S) is non-convex. This makes the control problem difficult to solve. However, the introduction of a new variable Y in (15), such that $Y = KS$ leads to a convex sufficient condition in terms of LMI as in the following theorem

Theorem 2. *If there exist positive definite* $P = diag(P_h, P_v)$ *with* $P_h \in R^{n \times n}$, $P_v \in R^{l \times l}$ *and matrices* S, Y *such that*

$$\begin{bmatrix} -\frac{1}{2}(S+S^T) & (\bar{A}S+\bar{B}Y)^T & \frac{1}{2}S+S^T-P \\ \bar{A}S+\bar{B}Y & -P & -(\bar{A}S+\bar{B}Y) \\ \frac{1}{2}S^T+S-P & -(\bar{A}S+\bar{B}Y)^T & -S-S^T \end{bmatrix} < 0, \tag{16}$$

then 2-D discrete closed-loop system (8) is asymptotical stable. In this situation, a suitable control law for (7) can be given as $K = YS^{-1}$.

Remark 2: Theorem 2 provides an LMI condition for the ILC convergence, which is expressed in terms of learning gains and plant model parameters. This falls into the category of ILC where a convergence condition is given with respect to known plant knowledge. But in contrast, this LMI condition provides a direct approach to determine learning gains.

Next, we extend the above analysis to the case of unavailable state .

In ILC updating law (6), both output signal and state signal of the ILC system are used. However, state signals may be measurable or difficult to be measured in many practical systems. In this case, only output signal can be used. The ILC law can be constructed as Arimoto P-type updating law given by

$$u(t, k+1) = u(t, k) + \Gamma e(t+1, k), \tag{17}$$

where Γ is the learning gain matrix.

We add an output equation for 2D system (5) as follows

$$\tilde{y}(t, k) = \bar{C} \begin{bmatrix} \eta(t, k) \\ e(t, k) \end{bmatrix},$$

where $\bar{C} = \begin{bmatrix} \mathbf{0}_{n \times n} \\ & 1_{l \times l} \end{bmatrix}$. It can be easy obtained that

$\tilde{y}(t,k) = \begin{bmatrix} \mathbf{0} \\ e(t,k) \end{bmatrix}$. Then the problem of ILC design can

be transformed as: Given a 2-D discrete time system described by

$$\begin{cases} \begin{bmatrix} \eta(t+1,k) \\ e(t,k+1) \end{bmatrix} = \bar{A} \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix} + \bar{B} \Delta u(t-1,k) \\ \tilde{y}(t,k) = \bar{C} \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix}, \end{cases} \tag{18}$$

design an output feedback controller in the form of

$$\Delta u(t-1,k) = K\tilde{y}(t,k), \tag{19}$$

such that the resulting closed-loop system is asymptotically stable.

Denote $K_2 = [\mathbf{0}_n \quad L]$. From (17), (18) and (19), we have

$$K_2 = K\bar{C}.$$

Substituting (19) into (18), we can present the ILC system as a 2-D discrete closed-loop system given by

$$\begin{bmatrix} \eta(t+1,k) \\ e(t,k+1) \end{bmatrix} = (\bar{A} + \bar{B}K\bar{C}) \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix}$$
$$= A_c \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix}, \tag{20}$$

where $A_c = \bar{A} + \bar{B}K\bar{C}$.

Using Theorem 1, the synthesis of output feedback controller can be described to find (K,P,S) such that the following matrix inequality holds

$$\begin{bmatrix} -\frac{1}{2}(S+S^T) & S^T(\bar{A}+\bar{B}K\bar{C})^T & \frac{1}{2}S+S^T\text{-}P \\ (\bar{A}+\bar{B}K\bar{C})S & -P & -(\bar{A}+\bar{B}K\bar{C})S \\ \frac{1}{2}S^T+S\text{-}P & -S^T(\bar{A}+\bar{B}K\bar{C})^T & -S\text{-}S^T \end{bmatrix} < 0. \tag{21}$$

We note that the problem of solving numerically (21) is non-convex. This makes the matrix inequality difficult to be solved.

We introduce a new slack variable G in (21), such that $K\bar{C}S = KG\bar{C}$. Then, a convex sufficient condition of solving (21) can be given in terms of LMI.

Theorem 3. *If there exist positive definite* $P = diag(P_h, P_v)$ *with* $P_h \in R^{n \times n}$, $P_v \in R^{l \times l}$ *and matrices* S, G, Y *such that*

$$\begin{bmatrix} -\frac{1}{2}(S+S^T) & S^T\bar{A}^T + \bar{C}^TY^T\bar{B}^T & \frac{1}{2}S+S^T\text{-}P \\ \bar{A}S + \bar{B}Y\bar{C} & -P & -(\bar{A}S+\bar{B}Y\bar{C}) \\ \frac{1}{2}S^T+S\text{-}P & -S^T\bar{A}^T - \bar{C}^TY^T\bar{B}^T & -S\text{-}S^T \end{bmatrix} < 0,$$
$$G+G^T > 0,$$
$$CS = GC, \tag{22}$$

then 2-D discrete closed-loop system (20) is asymptotical stable, and a feasible control law for (19) can be given as $K = YG^{-1}$. In this situation, the ILC gain matrix in (17) can be selected as $K_2 = YG^{-1}\bar{C}$.

Remark 3. It is worthy pointing out that the equality constraint $CS = GC$ is satisfied by the slack variable G. The invertibility of matrix G is guaranteed by the condition $G+G^T > 0$.

Remark 4. The proposed approach is systematic for ILC controller design. This result can be extended to more complex systems (systems with time-delays, uncertainties, etc) and more special performance design such as H_∞, H_2.

4. Numerical Examples

In this section, an example is provided to illustrate the validity of our design. Let us consider the following discrete-time system

$$\begin{cases} x(k,t+1) = \begin{bmatrix} 0.25 & 0.6 \\ 0.6 & 0 \end{bmatrix} x(k,t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k,t), \\ y(k,t) = \begin{bmatrix} 1 & -1.3 \end{bmatrix} x(k,t). \end{cases} \tag{23}$$

The desired repetitive reference trajectory is:

$$y_d(t) = \sin(8.0t / 50), t = 0,1,2,\cdots,100.$$

For the initial state, it is assumed that $x_1(0,k) = x_2(0,k) = 0$ for all k. The ILC law is applied by adopting the zero initial control input $u(t,0) = 0$ for all t. Using system parameters in (23), the matrices in (5) can be calculated with the following parameters

$$\bar{A} = \begin{bmatrix} 0.25 & 0.6 & 0 \\ 0.6 & 0 & 0 \\ 0.53 & -0.6 & 1 \end{bmatrix}, \bar{B} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

Then, the state is measurable and immeasurable are separately considered .

Case 1 state is measurable

In this case, the ILC updating law is given as $u(t,k+1) = u(t,k) + K_1\big(x(t,k+1) - x(t,k)\big) + K_2e(t+1,k)$, and the state feedback controller matrix for 2D system is $K = [K_1 \quad K_2]$. Using Theorem 2, we know that once the LMI (16) is feasible, the learning gain matrix can be determined by $K = YS^{-1}$, and the closed-loop system is asymptotically stable. With the help of the Matlab LMI toolbox, a feasible solution for the LMI (16) can be obtained as follows

$$Y = \begin{bmatrix} -5.2144 & -21.5447 & 4.9220 \end{bmatrix},$$
$$S = \begin{bmatrix} 21.0390 & 5.2645 & -15.3224 \\ 5.2645 & 36.8271 & -3.4198 \\ -15.3224 & -3.4198 & 11.5909 \end{bmatrix},$$

$$P = \begin{bmatrix} 28.3967 & 7.9373 & -20.7490 \\ 7.9373 & 57.4566 & -5.2486 \\ -20.7490 & -5.2486 & 15.6582 \end{bmatrix}.$$

Hence, the ILC gain matrix can be selected as

$$K = YS^{-1} = \begin{bmatrix} 2.2247 & -0.6072 & 3.1864 \end{bmatrix}.$$

The results of simulation are shown in Fig. 1. Fig. 1 (a)-(d) give system output at the 2nd, 3rd, 5th, 10th iterations (solid line) and the desired output trajectory (dash line). Fig. 2 gives the max tracking error on the iteration domain. It can be observed that by using the proposed iterative learning control law, we can guarantee the asymptotic convergence of the tracking error for all $t \in [1,100]$.

Case 2 State is immeasurable

In this case, it is assumed that the state signal is immeasurable, and the ILC updating law is given as $u(t,k+1) = u(t,k) + \Gamma e(t+1,k)$. We add an output equation for 2D system as (18), and the matrix $\bar{C}$ is selected as

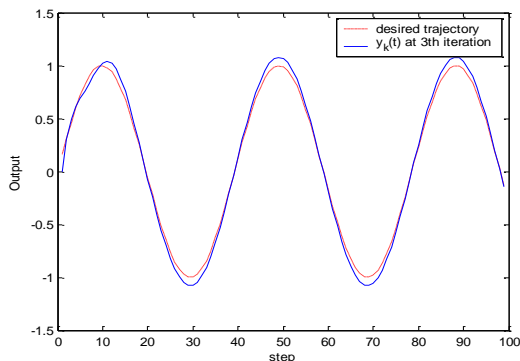$$\bar{C} = \begin{bmatrix} 0 & & \\ & 0 & \\ & & 1 \end{bmatrix}.$$

Using Theorem 3, we know that once the LMI (22) is feasible, the gain matrix for 2D system can be determined by $K = YG^{-1}$, and then the closed-loop
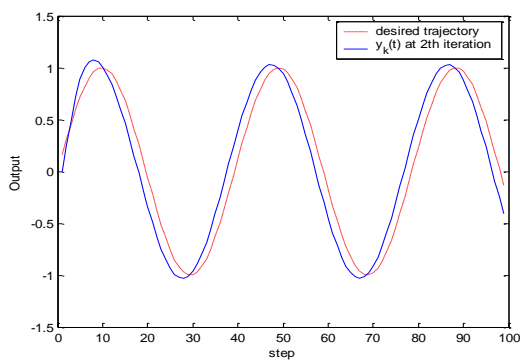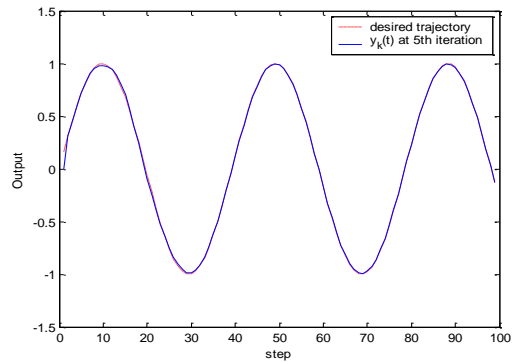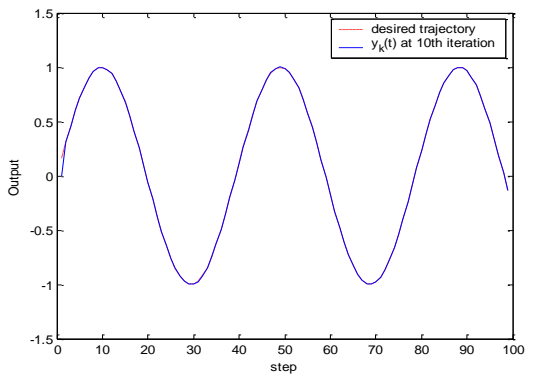

Figure 1. Simulation results for Case 1: (a) output profiles at 2nd iteration; (b) output profiles at 3rd iteration; (c) output profiles at 5th iteration; (d) output profiles at 10th iteration
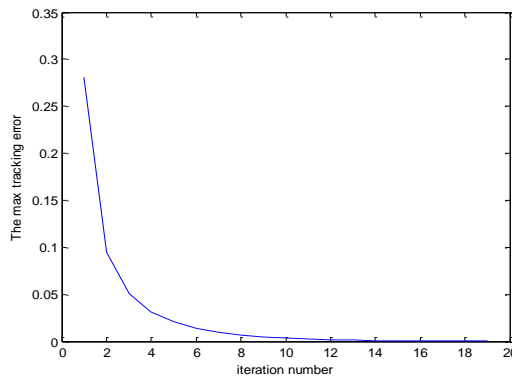

Figure 2. The max tracking error for Case1

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system (20) is asymptotically stable. With the help of the Matlab LMI toolbox, a feasible solution can be obtained as

$$Y = 1.0e + 004 * \begin{bmatrix} 0 & -6.7863 & 0.0091 \end{bmatrix},$$

$$S = 1.0e + 003 * \begin{bmatrix} 2.0199 & -0.0070 & -1.4150 \\ -0.0070 & 8.1148 & 0.0568 \\ -1.4150 & 0.0568 & 1.0347 \end{bmatrix},$$

$$P = 1.0e + 004 * \begin{bmatrix} 0.3183 & 0.0253 & -0.2279 \\ 0.0253 & 1.3932 & -0.0113 \\ -0.2279 & -0.0113 & 0.1687 \end{bmatrix},$$

$$G = 1.0e + 004 * \begin{bmatrix} 5.4964 & -0.0007 & -0.1415 \\ -0.0007 & 0.8115 & 0.0057 \\ -0.1415 & 0.0057 & 0.1035 \end{bmatrix}.$$

Then, the 2D output feedback gain matrix is:

$$K = YG^{-1} = \begin{bmatrix} 0.0135 & -8.3668 & 0.5657 \end{bmatrix},$$

and

$$K_2 = YG^{-1}\bar{C} = \begin{bmatrix} 0 & 0 & 0.5657 \end{bmatrix},$$

hence, ILC gain matrix for (17) is $\Gamma = 0.5657$.

The simulation results are shown in Fig. 3. Fig. 3 (a)-(d) also give system output at the 2nd, 3rd, 5th, 10th iterations (solid line) and the desired output trajectory (dash line), and Fig. 4 gives the max tracking error on the iteration domain. Clearly, the ILC system is stable also in this case. Note that in existing contraction mapping approach, a sufficient stability condition of P-type ILC (17) for linear discrete system (1) is $\|I - \Gamma CB\| < 1$. When $\Gamma = 0.5657$, the condition $\|I - \Gamma CB\| = 0.4343 < 1$ is obvious satisfied. Comparing the simulation results of two cases, we can also observe that when the state signal is available, the fast convergence speed and small transient error can be obtained.

5. Conclusions

In this paper, an ILC design approach for a class of linear discrete time systems has been presented. After providing a 2D analysis of the learning process, it has been shown that the design problem of ILC law for such system can be transformed straightforwardly into state feedback or output feedback control problem of 2-D systems described by Roesser models. Then, a Lyapunov approach has been employed to develop LMI conditions that guarantee asymptotical convergence of the tracking error. It is also shown that the solution can be recast as a convex optimization under LMI form. The theoretical results have been verified through simulation tests.
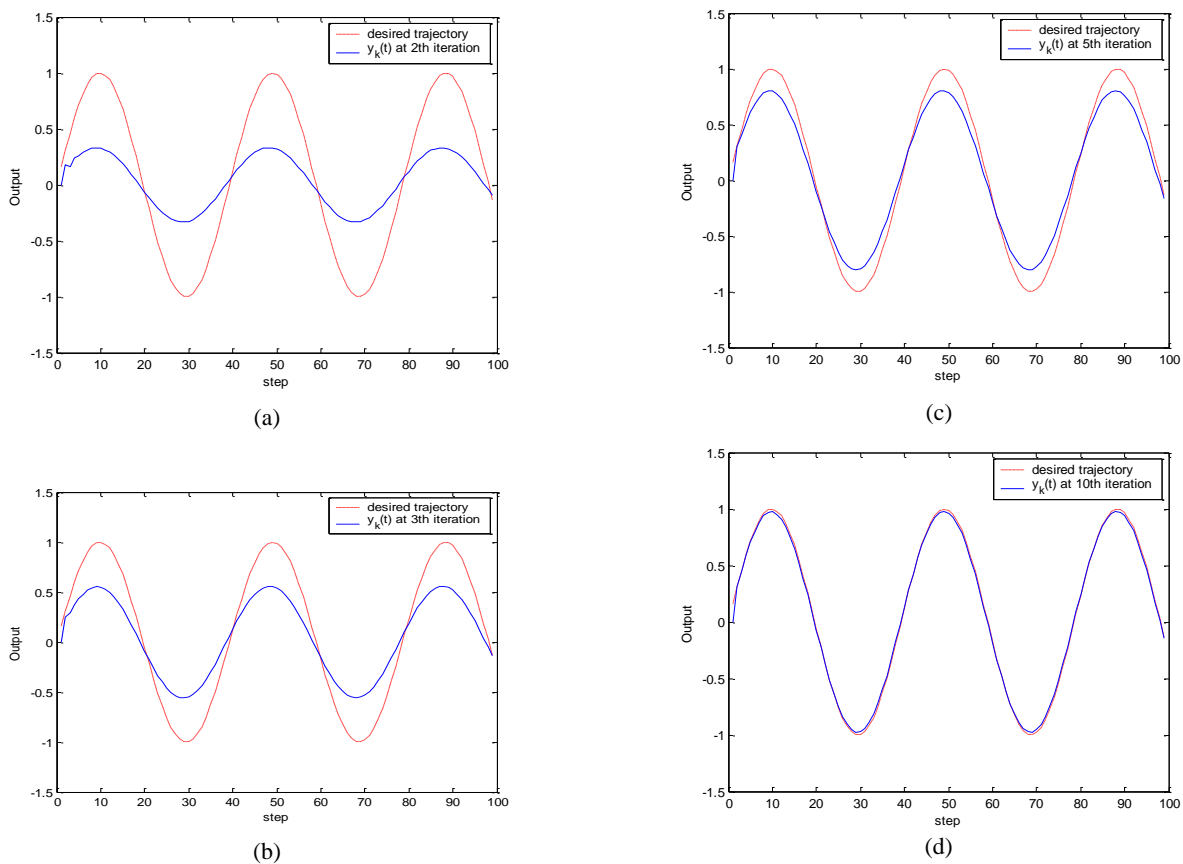

(a)

(b)

(c)

(d)

Figure.3. Simulation results for Case 2: (a) output profiles at 2nd iteration; (b) output profiles at 3rd iteration; (c) output profiles at 5th iteration; (d) output profiles at 10th iteration
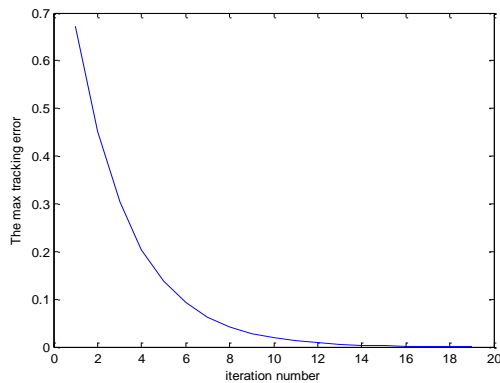
Figure.4. The max tracking error for Case 2

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