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EFFICIENT COMPUTATION OF THE MULTIVARIATE SYSTEM CHARACTERISTICS

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Abstract. In this paper, we propose a simple and computationally efficient method for computation of the transfer function and other characteristics of the multivariate systems. Multivariate systems are described by autoregressive moving-average (ARMA) equations. The fast Fourier transform algorithm is applied. The method is highly suitable for computer realization. An example of the multivariate system with two inputs and two outputs is provided to demonstrate the efficiency of the proposed method. The method may be extended to multidimensional systems described by the ARMA equations.

Keywords: multivariate systems, determination of characteristics, autoregressive moving-average model, fast Fourier transform, polynomial matrices.

1. Introduction

Multivariate systems with multiple inputs and multiple outputs play an important role in control theory [1], communication problems [2], as well as in information theory [3]. By using the methods of multichannel processing of noisy signals, it is possible to obtain better results than those that are achieved with single channel methods [4]. Technological progress in recording and processing multichannel data has been especially applied in the area of biomedicine [5]. Today data are registered not through one or two channels, but through several dozen channels. Greater amount of information about a particular object may be obtained from an analysis of multichannel data. Besides experimental difficulties in data recording, there is also a problem of efficient and simple computation of the characteristics of multivariate systems.

In computation of the main characteristic of multivariate system, i.e, the transfer function, we must solve the problem of inversion of polynomial matrices [6, 7]. By applying Cramer's rule, it is possible to determine the inverse by calculating determinants [8]. Another approach is a direct computation of the associated matrices [9]. However, polynomial operations involve numerous computational difficulties, hence some methods have been proposed that use the operations with constant matrices [10,11].

In the present paper, a simple and computationally efficient method for determining the transfer function and other characteristics of multivariate system described by autoregressive moving average equations is proposed. The method relies on the results of the previous article [12]. In Section 2, a statement of the problem is presented. In Section 3, the method for computation of characteristics of multivariate system is given. In Section 4, the algorithm's computational complexity is analyzed. Section 5 presents an example. Section 6 contains conclusions.

2. Problem statement

We address the problem of computation of the characteristics of the linear time-invariant multivariate systems. We assume that the multivariate system has p inputs and r outputs, and that N_1 point data records of p input sequences $\{v_m(n)\}, m=1,2,...,p,$

$n = 0, 1, 2, \ldots, N_1$, as well as r output sequences $\{y_k(n)\}, k=1,2,...,r$, are available. We assume also that the data are related according to the multivariate autoregressive-moving average (MARMA) model of the multivariate system

$$y_k(n)=\sum_{i=1}^{M}\sum_{j=1}^{r}\alpha_{kj}(i)y_j(n-i)+ \\ +\sum_{l=0}^{N}\sum_{m=1}^{p}\beta_{km}(l)v_m(n-l), \quad k=1,2,...,r, N\le M \quad ,(1)$$

where $\alpha_{kj}(i)$ and $\beta_{km}(l)$ are the parameters of MARMA system, M is the number of delays of the autoregressive part and N is the number of delays of the moving average part of the MARMA system.

Equation (1) can be rewritten in the matrix-vector form:

$$\mathbf{y}(n) = \sum_{i=1}^{M} \mathbf{A}(i)\mathbf{y}(n-i) + \sum_{l=0}^{N} \mathbf{B}(l)\mathbf{v}(n-l), \qquad (2)$$

where $\mathbf{A}(i) = \{\alpha_{kj}(i)\}$ is the $r \times r$ matrix, $\mathbf{B}(l) = \{\beta_{km}(l)\}$ is the r x p matrix,

$\mathbf{y}(n) = [y_1(n), ..., y_k(n), ..., y_r(n)]^T$,

$\mathbf{v}(n) = [v_1(n), ..., v_m(n), ..., v_p(n)]^T$, T is the sign of matrix transposition.

Performing z transform of (2), we obtain

$$\mathbf{Y}(z) = \mathbf{A}(z)\mathbf{Y}(z) + \mathbf{B}(z)\mathbf{V}(z), \qquad (3)$$

where

$$\mathbf{A}(z) = \{ a_{kj}(z) \} \qquad (4)$$

is the $r \times r$ matrix with entries $a_{kj}(z) = \sum_{i=1}^{M} \alpha_{kj}(i) z^{-i}$,

$$\mathbf{B}(z) = \{ b_{km}(z) \}, \qquad (5)$$

is the r x p matrix with entries $b_{km}(z) = \sum_{l=0}^{N} \beta_{km}(l) z^{-l}$,

$$\mathbf{Y}(z) = [y_1(z), ..., y_k(z), ..., y_r(z)]^T \qquad (6)$$

is the $r \times 1$ matrix with entries $y_k(z) = \sum_{n=0}^{\infty} y_k(n) z^{-n}$,

$$\mathbf{V}(z) = [v_1(z), ..., v_m(z), ..., v_p(z)]^T, \qquad (7)$$

is the $p \times 1$ matrix with entries $v_m(z) = \sum_{n=0}^{\infty} v_m(n) z^{-n}$.

From (3), we obtain the transfer function [14] of the MARMA system

$$\mathbf{H}(z) = \frac{\mathbf{Y}(z)}{\mathbf{V}(z)} = \mathbf{G}^{-1}(z)\mathbf{B}(z), \qquad (8)$$

where

$$\mathbf{G}(z) = \mathbf{I} - \mathbf{A}(z) = \{g_{kj}(z)\}, \; k, j = 1, 2, \dots, r \qquad (9)$$

is the $r \times r$ polynomial matrix with entries

$$g_{kj}(z) = -a_{kj}(z), \quad \text{if } k \neq j \qquad (10)$$

$$g_{kj}(z) = 1 - a_{kj}(z), \text{ if } k = j,$$

$\mathbf{I}$ is the $r \times r$ identity matrix.

From (8) it follows that the transfer function of the MARMA system (1) is equal to the inverse polynomial matrix $\mathbf{G}^{-1}(z)$ multiplied by polynomial matrix $\mathbf{B}(z)$. It is known [13] that the inverse polynomial matrix exists if and only if the determinant det $\mathbf{G}(z)$ is a nonzero number. If det $\mathbf{G}(z)$ is a polynomial, then although the inverse matrix $\mathbf{G}^{-1}(z)$ exists, it will not be a polynomial matrix, since its elements are not polynomials, but the ratios of polynomials.

3. Method for computation of the MARMA system characteristics

Define the (k,j)-th minor of the polynomial matrix $\mathbf{G}(z)$, denoted $M_{kj}(z)$ as the determinant of the $(r-1)$ x $(r-1)$ matrix, that results from deleting row k and column j of $\mathbf{G}(z)$. Define the (k, j)-th cofactor of $\mathbf{G}(z)$ as $C_{kj}(z) = (-1)^{k+j} M_{kj}(z)$, $k, j = 1, 2, \dots, r$. Then, the adjugate matrix $\hat{\mathbf{G}}(z)$ of the polynomial matrix $\mathbf{G}(z)$ is the $r \times r$ matrix whose (k, j)-th entry is the (j, k)-th cofactor of $\mathbf{G}(z)$, i.e., $\hat{\mathbf{G}}(z) = \{C_{jk}(z)\}$, $j, k = 1, 2, \dots, r$.

To determine the inverse of the polynomial matrix $\mathbf{G}(z)$, we calculate the $r \times r$ matrix [6, 13]

$$\mathbf{G}^{-1}(z) = \frac{\hat{\mathbf{G}}(z)}{g(z)}, \qquad (11)$$

where

$$g(z) = \det \mathbf{G}(z) = \sum_{m=0}^{Q} g_m z^{-m} \qquad (12)$$

is the determinant of the polynomial matrix $\mathbf{G}(z)$, where g_m are scalars. Q is the maximal order of the determinant det $\mathbf{G}(z)$, i.e.,

$$Q = \sum_{j=1}^{r} q_j, \qquad (13)$$

where q_j is the maximal order of the j-th column of the polynomial matrix $\mathbf{G}(z)$. In case $q_j = M$ for all j = 1, 2, …, r, the maximal order of the determinant is equal to rM.

It is follows from (11) that the adjugate matrix

$$\hat{\mathbf{G}}(z) = \{G^{kj}(z)\} = \{adj\; g_{kj}(z)\} \quad k, j = 1, 2, \dots, r, \qquad (14)$$

has entries

$$G^{kj}(z) = \sum_{m=0}^{Q_1} G_m^{kj} z^{-m}, \qquad (15)$$

where $Q_1 = (r-1)M$.

If the maximal order of the polynomials of the polynomial matrix $\hat{\mathbf{G}}(z)$ is M, then matrices $\mathbf{G}_m$, m = 0, 1, …, M may be compiled from coefficients of identical degrees z^{-1} of the matrix $\hat{\mathbf{G}}(z)$ [13]. Then, the adjugate matrix $\hat{\mathbf{G}}(z)$ may be written as a polynomial with matrix coefficients $\mathbf{G}_m$, i.e.,

$$\hat{\mathbf{G}}(z) = \sum_{m=0}^{M} \mathbf{G}_m z^{-m}, \qquad (16)$$

where $\mathbf{G}_m = \{G_m^{kj}\}$ are the $r \times r$ matrices with entries G_m^{kj}.

To determine $\mathbf{G}^{-1}(z)$, it is necessary to calculate scalars g_m using (12) and G_m^{kj} using (15).

The following solution approach may be used. First, multiply the corresponding polynomials and then calculate the determinant and all cofactors of the polynomial matrix **G**(*z*). However, this is a rather complicated solution technique.

To calculate $\mathbf{G}^{-1}(z)$ according to (11), we will apply the method used to compute the determinant of a polynomial matrix, which is proposed in [12]. We calculate the values of the determinant of the polynomial matrix **G**(*z*) at *Q*+1 points z_l equally spaced on the unit circle:

$$d_l = \det \mathbf{G}(z_l) = \sum_{m=0}^{Q} g_m z_l^{-m}, \; l = 0, 1, \ldots, Q, \tag{17}$$

where

$$z_l = \exp(i\frac{2\pi l}{Q+1}), \tag{18}$$

in which *i* is a sign of the complex variable.

We find from (17) and (18) that

$$d_l = \sum_{m=0}^{Q} g_m \exp(-i\frac{2\pi lm}{Q+1}), \; l = 0, 1, \ldots, Q. \tag{19}$$

From (19) it follows that d_l and g_m are related according to the discrete Fourier transform, hence the coefficients g_m may be calculated from the inverse discrete Fourier transform

$$g_m = \frac{1}{Q+1}\sum_{l=0}^{Q} d_l \exp(i\frac{2\pi lm}{Q+1}), \; m = 0, 1, \ldots, Q. \tag{20}$$

Similarly, we calculate the entries of the matrices $\mathbf{G}_m = \{G_m^{kj}\}$, $m = 0, 1, \ldots, Q_1$, $k, j = 1, 2, \ldots, r$. We find the values of all adjugates of the polynomial matrix $\mathbf{G}(z) = \{g_{kj}(z)\}$ at Q_1+1 points z_l equally spaced on the unit circle:

$$D_l^{kj} = adj\; g_{kj}(z_l), \; l = 0, 1, \ldots, Q_1, \quad k, j = 1, 2, \ldots, r, \tag{21}$$

where D_l^{kj} is the (*k*, *j*)-th entry of the adjugate matrix $\hat{\mathbf{G}}(z)$ at points z_l, $l = 0,1,\ldots, Q_1$.

Substituting the value D_l^{kj} from (21) and $z_l = \exp(i\frac{2\pi l}{Q_1+1})$ into (15), we obtain

$$D_l^{kj} = \sum_{m=0}^{Q_1} G_m^{kj} \exp(-i\frac{2\pi lm}{Q_1+1}). \tag{22}$$

Consequently, D_l^{kj} and G_m^{kj} are related by the discrete Fourier transform relation (22), hence the coefficients G_m^{kj} may be calculated from the inverse discrete Fourier transform

$$G_m^{kj} = \frac{1}{Q_1+1}\sum_{l=0}^{Q_1} D_l^{kj} \exp(i\frac{2\pi lm}{Q_1+1}), \; m = 0, 1, \ldots, Q_1; \quad k, j = 1, 2, \ldots, r. \tag{23}$$

From (8) and (11), we obtain the z transform of the output of the MARMA system

$$\mathbf{Y}(z) = \mathbf{H}(z)\mathbf{V}(z) = \frac{\hat{\mathbf{G}}(z)}{g(z)}\mathbf{B}(z)\mathbf{V}(z), \tag{24}$$

where **H**(z) is the transfer function of the MARMA system.

From (24), we obtain

$$\mathbf{Y}(z)g(z) = \hat{\mathbf{G}}(z)\mathbf{B}(z)\mathbf{V}(z). \tag{25}$$

Define $\mathbf{F}(z) = \hat{\mathbf{G}}(z)\mathbf{B}(z) = \{F^{km}(z)\}$, $k = 1, 2, \ldots, r$, $m = 1, 2, \ldots, p$,

where $F^{km}(z) = \sum_{j=1}^{r} G^{kj}(z) b_{jm}(z)$.

Then from (25) we find the z transform $y_k(z)$ of the *k*-th output signal $y_k(n)$

$$y_k(z) = \frac{\sum_{m=1}^{p} F^{km}(z) v_m(z)}{g(z)}, \; k = 1, 2, \ldots, r. \tag{26}$$

Using the convolution property of the z transform [14], from (26) we obtain a difference equation that describes the relationship between the *k*-th output signal and all input signals of the MARMA system

$$y_k(n) = -\sum_{i=1}^{Q} g_i y_k(n-i) + \sum_{m=1}^{p}\sum_{l=0}^{R_{km}} F_l^{km} v_m(n-l), \quad k = 1, 2, \ldots, r; \tag{27}$$

where R_{km} is the order of the polynomial $F^{km}(z)$.

We can see that the multivariate system may be considered as a system consisting of *r* mutually independent subsystems each of which has an output $y_k(n)$ and *p* inputs $v_m(n)$, $m = 1, 2, \ldots, p$.

From (26) it follows that the transfer function between the *k*-th output and the *m*-th input of the MARMA system is given by

$$H^{km}(z) = \frac{y_k(z)}{v_m(z)} = \frac{F^{km}(z)}{g(z)}, \; k = 1, 2, \ldots, r; \quad m = 1,2,\ldots, p \tag{28}$$

and impulse response between the *k*-th output and *m*-th input is given as

$$h^{km}(n) = -\sum_{i=1}^{Q} g_i h^{km}(n-i) + \sum_{l=0}^{R_{km}} F_l^{km} \delta_m(n-l), \tag{29}$$

where $\delta_m(n) = 1$, if $n = 0$ and $\delta_m(n) = 0$, if $n \neq 0$, provided that all input signals except the *m*-th signal are equal to zero.

Substituting $z = e^{iw}$ into (28), we obtain the frequency response between the *k*-th output and *m*-th input of the MARMA system:

$$H^{km}(e^{iw}) = \frac{F^{km}(e^{iw})}{g(e^{iw})},\ k = 1, 2, \ldots, r,$$
$$m = 1, 2, \ldots, p, \qquad (30)$$

where i is the sign of the complex variable and w is the frequency.

The amplitude-frequency response between the k-th output and the m-th input is an absolute value of the frequency response $|H^{km}(e^{iw})|$, while the phase-frequency response is the argument of the frequency response $\arg(H^{km}(e^{iw}))$, $k = 1, 2, \ldots, r$, $m = 1, 2, \ldots, p$.

A multivariate system will be stable if and only if all roots of the denominator $g(z)$ of the transfer system $\mathbf{H}(z)$ are inside the unit circle. Thus, it follows from (30) that the stability between the k-th output and m-th input depends on the roots of the polynomial $g(z)$.

Algorithm for computation of the inverse polynomial matrix of the multivariate system:

Input: r – dimension of the polynomial matrix $\mathbf{G}(z)$; M – maximal order of the polynomials of the polynomial matrix $\mathbf{G}(z)$; α_{kj} – entries of $\mathbf{G}(z)$.

Output: G_m^{kj} – entries of the adjugate matrix $\hat{\mathbf{G}}(z)$; g_m – coefficients of the determinant $g(z)$.

1: Compute $Q = rM$ and $Q_1 = (r-1)M$

2 : Compute $W = \exp(i\frac{2\pi}{Q+1})$

3: **for** $l = 0$ **until** Q **do**

3.1: Compute $z_l = W^l$

3.2. Compute numerical values of all entries of $\mathbf{G}(z_l) = \{g_{kj}(z_l)\}$ at point z_l

3.3. Compute $d_l = \det \mathbf{G}(z_l)$

end

4: **for** $m = 0$ **until** Q **do**

Compute $g_m = \frac{1}{Q+1}\sum_{l=0}^{Q} d_l W^{lm}$

end

5: **for** $k = 1$ **until** r **do**

for $j = 1$ **until** r **do**

for $l = 0$ **until** Q_1 **do**

Compute $D_l^{kj} = adj\ g_{kj}(z_l)$

end

for $m = 0$ **until** Q_1 **do**

Compute $G_m^{kj} = \frac{1}{Q_1+1}\sum_{l=0}^{Q_1} D_l^{kj} W^{lm}$

end

end

end

4. Computational Complexity

The complexity of the proposed algorithm for determination of the MARMA transfer function is defined as the number of multiplication operations:

$$n = Q + r^3M^2 + (Q+1)\frac{r^3}{3} + (Q+1)\log_2(Q+1) +$$
$$r^2[((r-1)M+1)\frac{(r-1)^3}{3} + (Q+1)\cdot \qquad (31)$$
$$\cdot\log_2(Q+1)] + r^3 pMN,$$

where $Q = rM$ – number of multiplications for computation of z_l, $l = 0,1,\ldots,Q$;

r^3M^2 – number of multiplications for numerical evaluation of the r^2 polynomial entries of $\mathbf{G}(z) = \{g_{kj}(z_l)\}$ at points z_l;

$(Q+1)\frac{r^3}{3}$ – number of multiplications for computation of the determinant $d_l = \det \mathbf{G}(z_l)$ at points z_l;

$((r-1)M+1)\frac{(r-1)^3}{3}$ – number of multiplications for computation of one adjugate $adj\ g_{kj}(z_l)$ at points z_l;

$(Q+1)\log_2(Q+1)$ – number of multiplications of the inverse fast Fourier transform;

$r^3 pMN$ – number of multiplications for computation of $\mathbf{F}(z) = \hat{\mathbf{G}}(z)\mathbf{B}(z)$. In case of multivariate autoregressive system (MAR), this component is equal to zero.

Table 1. Number of multiplication operations. r – number of outputs; p – number of inputs; M – number of delays of autoregressive part; N – number of delays of moving-average part of MARMA system

r	M	p	N	Proposed Algorithm	Algorithm [15]
2	3	2	2	266	3012
3	4	3	3	2082	37836
4	4	3	3	6397	67840
5	4	3	3	18124	106900
5	3	10	10	47393	55725
5	5	10	10	80136	453125
7	4	3	3	114260	213052
7	6	3	3	174450	2304666
10	5	10	10	1680100	2062500
10	10	10	10	3392100	101000000
15	10	10	10	33072000	228375000
15	15	10	10	49777000	2568000000
20	10	10	10	184440000	408000000

An investigation on existing methods proposed to calculate the coefficients of the determinant of polynomial matrix shows [12] that the method [15] is one of the best methods. The total operation count required by this method is $n = r^2M^6$. Then, the multiplication complexity of computation of the transfer function of MARMA system is obtained:

$$n_L = r^2 M^6 + r^3 pMN. \tag{32}$$

From Table 1, we can see that the computational complexity of the proposed algorithm as compared with algorithm [15] is small and decreases, especially for large M values.

5. Example

In the following example, we choose the MARMA system with two inputs and two outputs:

$$y_k(n) = \sum_{j=1}^{2}\sum_{i=1}^{3}\alpha_{kj}(i)y_j(n-i) + \sum_{m=1}^{2}\sum_{l=0}^{1}\beta_{km}(l)v_m(n-l), \quad k=1,2, \tag{33}$$

where

$\alpha_{11}(1) = 0.314;\quad \alpha_{11}(2) = 0.537;\quad \alpha_{11}(3) = 0.12$
$\alpha_{12}(1) = -0.576;\ \alpha_{12}(2) = 0.364;\quad \alpha_{12}(3) = 0.245$
$\alpha_{21}(1) = -0.46;\quad \alpha_{21}(2) = 0.865;\quad \alpha_{21}(3) = 0.364$
$\alpha_{22}(1) = 0.76;\quad \alpha_{22}(2) = -0.67;\quad \alpha_{22}(3) = 0.465;$

$\beta_{11}(0) = 0.5;\quad \beta_{11}(1) = -0.25$
$\beta_{12}(0) = 0;\quad \beta_{12}(1) = 0$
$\beta_{21}(0) = 0;\quad \beta_{21}(1) = 0$
$\beta_{22}(0) = 0.3;\quad \beta_{22}(1) = -0.28.$

Performing z transform of (33) and writing the result in matrix form, we obtain

$$\mathbf{Y}(z) = \mathbf{A}(z)\mathbf{Y}(z)+\mathbf{B}(z)\mathbf{V}(z), \tag{34}$$

where $\mathbf{A}(z) = \{a_{kj}(z)\}$ is the 2 × 2 matrix with entries

$a_{11}(z) = 0.314z^{-1} + 0.537z^{-2} + 0.12z^{-3}$
$a_{12}(z) = -0.576z^{-1} + 0.364z^{-2} + 0.245z^{-3}$
$a_{21}(z) = -0.46z^{-1} + 0.865z^{-2} + 0.324z^{-3}$
$a_{22}(z) = 0.76z^{-1} - 0.67z^{-2} + 0.465z^{-3},$

$\mathbf{B}(z) = \{b_{km}(z)\}$ is the 2 × 2 matrix with entries

$b_{11}(z) = 0.5 - 0.25z^{-1};\quad b_{12}(z) = 0;$
$b_{21}(z) = 0;\quad b_{22}(z) = 0.3 - 0.28z^{-1},$

$\mathbf{G}(z) = \mathbf{I} - \mathbf{A}(z) = \{g_{kj}(z)\}$ is the 2 × 2 matrix with entries

$g_{11}(z) = 1 - 0.314z^{-1} - 0.537z^{-2} - 0.12z^{-3}$
$g_{12}(z) = 0.576z^{-1} - 0.364z^{-2} - 0.245z^{-3}$
$g_{21}(z) = 0.46z^{-1} - 0.865z^{-2} - 0.324z^{-3}$
$g_{22}(z) = 1 - 0.76z^{-1} + 0.67z^{-2} - 0.465z^{-3},$

$$\mathbf{Y}(z) = [y_1(z) \quad y_2(z)]^T, \quad \mathbf{V}(z) = [v_1(z) \quad v_2(z)]^T.$$

The maximal order Q of the determinant det $\mathbf{G}(z)$ is equal to 4. From (17), (20), and using the proposed algorithm, we have

$\{g_m\} = \{1, -1.074, 0.1067, 0.5184, -0.3205, 0.0002, -0.1352\}.$

From (21), (23), and using the proposed algorithm, we obtain

$\{G_m^{11}\} = \{1, -0.76, 0.67, -0.465\}$
$\{G_m^{12}\} = \{0, -0.576, 0.364, 0.245\}$
$\{G_m^{21}\} = \{0, -0.46, 0.865, 0.324\}$
$\{G_m^{22}\} = \{0, -0.314, -0.537, 0.12\}.$

From (25), we have

$\{F_l^{11}\} = \{0.5, -0.63, 0.525, -0.4, 0.1163\}$
$\{F_l^{12}\} = \{0, -0.1728, 0.2705, -0.0284, -0.0686\}$
$\{F_l^{21}\} = \{0, -0.23, 0.5475, -0.0542, -0.081\}$
$\{F_l^{22}\} = \{0.3, -0.3742, -0.0732, 0.184, -0.0336\}.$

The transfer function of the two-channel MARMA system (33) can be written as

$$H(z) = \frac{\{F^{km}(z)\}}{g(z)}, \quad k,m = 1,2, \tag{35}$$

where

$$g(z) = 1 - 1.074z^{-1} + 0.1067z^{-2} + 0.5184z^{-3} - 0.3205z^{-4} + 0.0002z^{-5} - 0.1352z^{-6};$$

$F^{11}(z) = 0.5 - 0.63z^{-1} + 0.525z^{-2} - 0.4z^{-3} + 0.1163z^{-4}$
$F^{12}(z) = -0.1728z^{-1} + 0.2705z^{-2} - 0.0284z^{-3} - 0.0686z^{-4}$
$F^{21}(z) = -0.23z^{-1} + 0.5475z^{-2} - 0.0542z^{-3} - 0.081z^{-4}$
$F^{22}(z) = 0.3 - 0.3742z^{-1} - 0.0732z^{-2} + 0.184z^{-3} - 0.0336z^{-4}$

The transfer function between, for example, the 1-st output channel and the 2-nd input channel is given by

$$H^{12}(z) = \frac{F^{12}(z)}{g(z)} \tag{36}$$

and frequency response

$$H^{12}(e^{iw}) = \frac{F^{12}(e^{-iw})}{g(e^{-iw})}. \tag{37}$$

The relationship between the 1-st output channel and 2-nd input channel (see Eq. (27)) is described by the difference equation:

$$y_1(n) = 1.074y_1(n-1) - 0.1067y_1(n-2) - 0.5184y_1(n-3) + 0.3205y_1(n-4) - 0.0002y_1(n-5) + 0.1352y_1(n-6) - 0.1728v_2(n-1) + 0.2705v_2(n-2) - 0.0284v_2(n-3) - 0.0686v_2(n-4).$$

The absolute values of the roots of the polynomial $g(z)$ are 0.9419, 0.9000, 0.9000, 0.7807, 0.4764, 0.4764 , i.e., the absolute values of the roots are less than 1, consequently, the two channel system satisfies the stability condition.

In the Figure 1 we show the impulse response h11 between the 1-st output and the 1-st input, impulse response h12 between the 1-st output and the 2-nd input, impulse response h21 between the 2-nd output and the 1-st input, and impulse response h22 between the 2-nd output and the 2-nd input of the system (33). In the Figure 2 we show the amplitude frequency response h11 of the system (33) between the 1-st output and the 1-st input, amplitude frequency response h12 between the 1-st output and the 2-nd input and so on. Figure 3 shows the positions of the poles and zeros in z plane of the system (33). All poles are inside unit circle, so system (33) is stable.

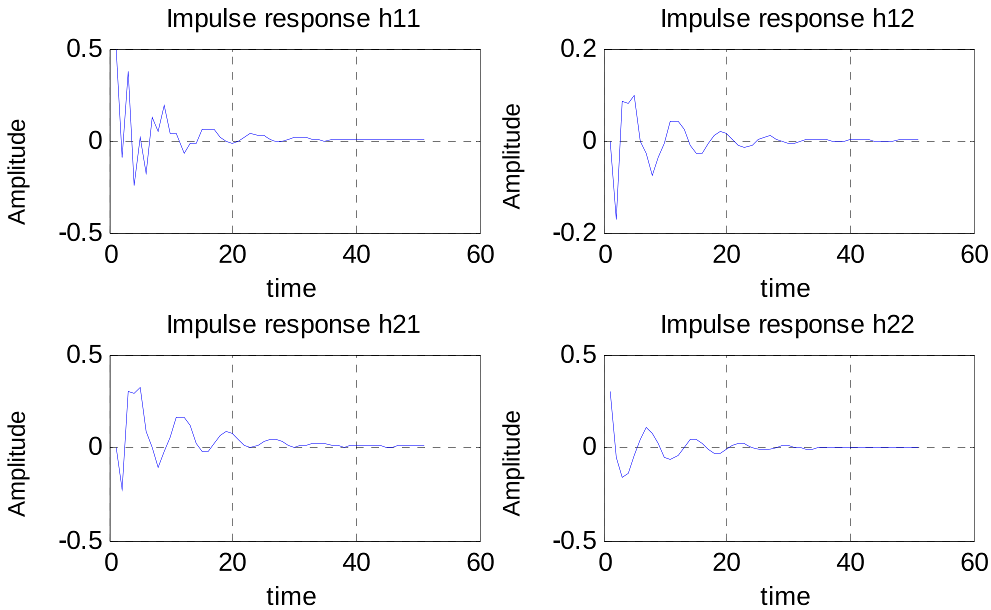

Figure 1. Impulse responses of MARMA system (33)

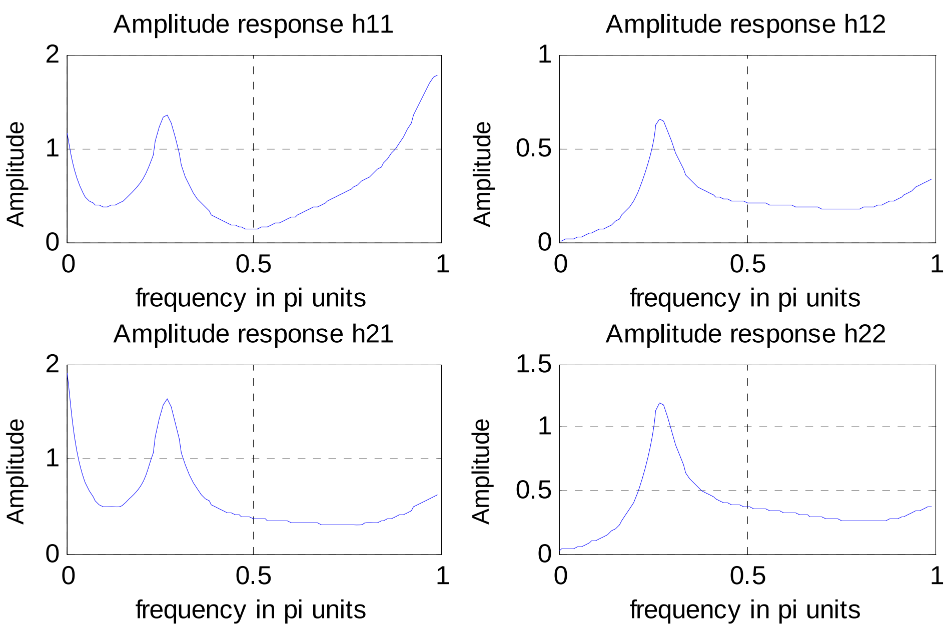

Figure 2. Amplitude-frequency responses of MARMA system (33)

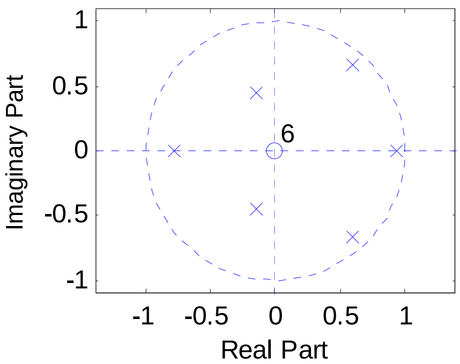

Figure 3. Poles and zeros of the two-channel MARMA system (33)

6. Conclusions

In this paper, a simple and computationally efficient method for calculation the characteristics of the multivariate autoregressive moving-average system is presented. It has been shown that a fast Fourier transform algorithm may be applied to determine the characteristics of multivariate system and that the calculation process may be speeded up. The proposed method uses well-known and efficient algorithms for calculating determinants of scalar matrices and discrete fast Fourier transform algorithm to determine the inverse polynomial matrix. It was showed that a multivariate system may be considered as a system consisting of *r* mutually independent parallel subsystems each of which has one output and the same *p* inputs. An example is used to illustrate practical computation of the coefficients of the transfer functions, difference equations, impulse and amplitude frequency responses of the channels of multivariate system with two inputs and two outputs. The stability condition is determined. The algorithm's computational complexity is analyzed. It was established that the proposed method is more efficient as compared with the algorithm given in [15]. The method may be extended for the evaluation of the multidimensional system characteristics.

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