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AN ESTIMATE OF FUNDAMENTAL FREQUENCY USING PCC INTERPOLATION – COMPARATIVE ANALYSIS

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Abstract. This paper deals with the algorithm for an estimate of fundamental frequency, which is based on signal processing by window functions in time domain and parametric Cubic interpolation in frequential domain. In the second part of the paper, the results of the simulation of the algorithm for Catmull-Rom's, Greville's and Greville's two-parametric kernel are presented. Taking MSE as a measure of the algorithm quality, optimal parameters of the selected kernel, selected kernel and a suitable window function are defined.

Keywords: Fundamental frequency estimation. Parametric cubic convolution. Window function.

1. Introduction

In many fields of speech signal processing, such as speech coding, speech synthesis, speech and speaker recognition, it is necessary to define a fundamental frequency. One characteristic example is the improvement of speech signal quality by reducing dissonant frequencies [1, 2]. There are a great number of algorithms for defining fundamental frequency, where the analysis is done in time and frequential domain (for example [3, 4]).

Many signals, such as speech, radar or sonar signals, have spectral characteristics variable in time, and, thus, cannot be efficiently described by Fourier's analysis. The problem of the estimate of Instantaneous Frequency (IF) is of special interest. In [5, 6], an algorithm for defining IF and fundamental frequency is presented. The algorithm is based on speech signal filtration by Bandpass filter and making decisions on Discrete IF estimator with time signal processing, using smoothing window functions. In [7,8], the algorithm for IF estimate using Polynomial Wigner-Ville Distribution (PWVD) is described. Polynomial algorithm is based on finding out parameters of peak of PWVD. Some methods of estimate of IF with monocomponential signal are proposed in [9, 10, 11]. In [12], an algorithm for localization of phase monocomponential signal with phase modulation is presented.

PWVD algorithm does not show satisfactory results with multicomponential signals, that is, interpolation of results is hindered [13]. In order to increase time–frequential resolution, various distributions are applied [14]. In [15, 16], the square time-frequential resolution, which gave good results with multicomponential signals, is presented. In [17], two-sided linear prediction (TSP) is presented for IF estimate. In the paper [7], the algorithm for finding out the peak of PWVD is described, where the accuracy of defining the position of the peak is increased by using interpolation. Detailed analysis proved that linear interpolation is suitable for SNR (Signal-to-noise ratios) low values, whereas sinc interpolation is suitable for high values.

Frequently used method for defining fundamental frequency is based on picking peaks of amplitudinal characteristic in specified frequential domain. This method is used to analyse signal values in a spectrum at points where DFT is calculated. More often, the true value of frequency is not at frequencies where DFT is calculated, but between two samples. Error of frequency estimate is therefore caused and it is in the interval between [$-F_s/(2N)Hz$, $F_s/(2N)Hz$], where F_s is the sampling frequency and N is the number of points where DFT is calculated. One way to reduce the error is to increase the sampling frequency. However, it requires hardware-software arrangement of the system for signal processing. The other way is defining interpolation function and estimate of spectrum characteristics in the interval between two sampes. By this procedure, the reconstruction of the spectrum by DFT is done. Parameters of the spectrum are after that defined by analytic procedures (differentiation, integration, extreme values, …) [18].

Calculation of interpolation function using Parametric Cubic Convolution (PCC) is shown in [19, 20]. The special case of PCC interpolation, which is used in computer graphics, is called Catmull-Rom's interpolation [21]. Detailed analysis of fundamental frequency estimate, as well as the advantage of PCC interpolation which is primarily seen in the efficiency of defining parameters of interpolation function, are described in the paper [22]. In [23], there are the results of using PCC interpolation for defining fundamental frequency in case of using certain window functions in discrete speech signal processing. Analyses of the algorithm's efficiency were done by simulation procedures, where Mean Square Error (MSE) is used as a measure of the algorithm quality. The best results were obtained by the algorithm with the implemented Blackman's window function. The analysis of the algorithm's efficiency in the circumstances of variable SNR, with the presence of a great number of relevant harmonics of fundamental frequency, shown in [24], confirmed the algorithm's efficiency with the Blackman's window function.

In the following text, an algorithm of estimate of fundamental frequency using PCC interpolation with Catmull-Rom's, Geville's and two-dimensional kernel is described. The kernel parameters and the implemented window function will be determined so that the minimal error of fundamental estimate can be generated.

The paper is organized as follows. In section 2, the algorithm of fundamental frequency estimate with PCC interpolation is described. The analytic shape of Catmull-Rom's [22], Greville and Greville's two-parametric kernel (G2D), is presented. In section 3, the algorithm for defining optimal values of parameters of interpolation kernel in relation to the implemented window function, is described. In section 4, a tabular presentation of the results of the simulation is given and a comparative analysis is done, in order to define the interpolation kernel with the optimal parameters and an adequate window function.

2. Algorithm of estimate of fundamental frequency

The algorithm of estimate of fundamental frequency is shown in Figure 1. It can be realized throughout a few steps:

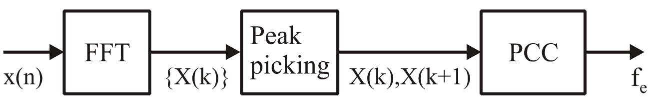


Figure 1. Algorithm of estimate of fundamental frequency

Step 1: The spectrum is calculated by using DFT on the discrete signal $x(n)$ that is achieved by time sampling of a continuous signal $s(t)$:

$$X(k) = DFT(x(n)) . \tag{1}$$

The spectrum is calculated at discrete points $k=0, \ldots N-1$, where N is the length of DFT. The real spectrum of signals $x(n)$ is continuous, whereas DFT defines the values of the spectrum at some discrete points.

Step 2: By using peak picking algorithm [26], the position of the maximum of the real spectrum that is between kth and $(k+1)$th samples is determined, where the values $X(k)$ and $X(k+1)$ are the highest in the specified domain.

Step 3: The position of the maximum of the spectrum is calculated by PCC interpolation. The reconstructed function is:

$$X_r(f) = \sum_{i=k-L}^{k+L+1} p_i \cdot r(f-i), \quad k \le f \le k+1 , \tag{2}$$

where $p_i=X(i)$, $r(f)$ is the kernel of interpolation and L is the number of samples that participate in interpolation.

The quality of the algorithm for the estimate of fundamental frequency can be also expressed by MSE:

$$MSE = \overline{(f - f_e)^2} , \tag{3}$$

where f is true fundamental frequency and f_e is fundamental frequency estimate.

Next, we give definitions of the interpolation kernels which are tested in this paper:

a) **Catmull-Rom's** interpolation kernel:

$$r(f) = \begin{cases} (\alpha+2)|f|^3 - (\alpha+3)|f|^2 + 1, & |f| \le 1, \\ \alpha|f|^3 - 5\alpha|f|^2 + 8\alpha|f| - 4\alpha, & 1 < |f| \le 2 \\ 0, & \text{otherwise.} \end{cases} , \tag{4}$$

The Maximum of the reconstructed function $X_r(f)$ is found by differentiating in spectrum domain and equalizing the first derivative to zero. The position of the maximum is

$$f_{\max} = \begin{cases} k - \dfrac{c}{2b}, & a=0 \\ k + \dfrac{-b \mp \sqrt{b^2 - ac}}{a}, & a \ne 0 \end{cases} , \tag{5}$$

where:

$$\begin{aligned} a &= 2(\alpha p_{k-1} + (\alpha+2)p_k - (\alpha+2)p_{k+1} - \alpha p_{k+2}) \\ b &= -2\alpha p_{k-1} - (\alpha+3)p_k + (2\alpha+3)p_{k+1} + \alpha p_{k+2} , \\ c &= -\alpha p_{k-1} - \alpha p_{k+1} . \end{aligned} \tag{6}$$

b) **Greville**'s interpolation kernel:

$$r(f) = \begin{cases} \left(\alpha+\frac{3}{2}\right)|f|^3 - \left(\alpha+\frac{5}{2}\right)|f|^2 + 1; \\ \quad \text{if } 0 \le |f| \le 1, \\ \frac{1}{2}(\alpha-1)|f|^3 - \left(3\alpha-\frac{5}{2}\right)|f|^2 - \left(\frac{11}{2}\alpha-4\right)|f| - (3\alpha-2); \\ \quad \text{if } 1 \le |f| \le 2, \\ -\frac{1}{2}\alpha|f|^3 + 4\alpha|f|^2 - \frac{21}{2}\alpha|f| + 9\alpha; \\ \quad \text{if } 2 \le |f| \le 3, \\ 0; \quad \text{if } 3 \le |f|. \end{cases} \tag{7}$$

c) **Greville**'s two-parametric cubic convolutional kernel (G2D) [21]:

$$r(f)=\begin{cases}\left(\alpha-\frac{5}{2}\beta+\frac{3}{2}\right)\cdot|f|^3-\left(\alpha-\frac{5}{2}\beta+\frac{5}{2}\right)\cdot|f|^2+1; & 0\le|f|\le 1,\\ \frac{1}{2}(\alpha-\beta-1)\cdot|f|^3-\left(3\alpha-\frac{9}{2}\beta-\frac{5}{2}\right)\cdot|f|^2+\left(\frac{11}{2}\alpha-10\beta-4\right)\cdot|f|-(3\alpha-6\beta-2); & 1\le|f|\le 2,\\ -\frac{1}{2}(\alpha-3\beta)\cdot|f|^3+\left(4\alpha-\frac{25}{2}\beta\right)\cdot|f|^2-\left(\frac{21}{2}\alpha-34\beta\right)\cdot|f|+(9\alpha-30\beta); & 2\le|f|\le 3,\\ -\frac{1}{2}\beta\cdot|f|^3+\frac{11}{2}\beta\cdot|f|^2-20\beta\cdot|f|+24\beta; & 4\le|f|.\end{cases} \quad (8)$$

In Eq. (4-8), there are parameters α and β. The Optimal values of these parameters will be determined by the minimal value of MSE, for Catmull-Rom's, Greville's and G2D kernels. For the first two of them,

$$\alpha_{opt}=\arg\min_{\alpha}(MSE), \quad (9)$$

and for the G2D kernel:

$$(\alpha_{opt},\beta_{opt})=\arg\min_{\alpha,\beta}(\mathrm{MSE}). \quad (10)$$

The Detailed analysis in [22,23,24] showed that the minimal value of MSE depends on the application of window function by which signal processing *x(n)* is carried out in time domain. MSE will be defined for: a) Hamming's, b) Hann's, c) Blackman's, d) rectangular, e) Kaiser's and f) triangular window.

3. Defining the parameters of interpolation kernels

The Parameters of interpolation kernels α and β (Eq. (4), (7), (8)) are defined according to the following algorithm:

Step 1: out of time continual signal *x(t)* by sampling in the time domain a discrete signal *x(n)* is got. The signal *x(n)* is being modified by the window function *w(n)* of length *N*.

Step 2: by using DFT, discrete spectrum *X(k)* is determined;

Step 3: by using PCC interpolation, the reconstruction of continuous function, which stands for the spectrum *X(k)*, is carried out;

Step 4: MSE is calculated for different values of the parameters α and β, depending on the implemented window function;

Step 5: parameters α_{opt} and β_{opt} are defined, for which the minimal value of MSE is achieved.

In the paper [22], the results of the estimate of fundamental frequency, using PCC algorithm and Catmull-Rom's kernel, are presented. The algorithm is applied to the simulation signal:

$$s(t)=\sum_{i=1}^{K}\sum_{g=0}^{M}a_i\,sin\left(2\pi i\left(f_o+g\frac{f_s}{NM}\right)t+\theta_i\right), \quad (11)$$

where f_o is the fundamental frequency, θ_i and a_i are the phase and amplitude of the *i*-th harmonic, respectively, *K* is the number of harmonics, *M* is the number of points between two samples in the spectrum at which PCC interpolation is carried out, f_s is sampling frequency and *N* window length. In the simulation, f_o and θ_i are random variables uniformly distributed in the range [*G2 (97.99Hz), G5 (783.99Hz)*] and [$0,2\pi$], respectively. The sampling frequency is f_s*=16 kHz*, and window length *N=512*, by which the analysis of subsequencies, which last for *32ms*, is provided. The results presented in the rest of the paper relate to f_o*=187.5Hz*, *K=1*, *M=100*.

In the paper [22], optimal values are defined for: a) Hamming's, b) Hann's, c) Blackman's window function. In the following subsections of our paper, optimal values of Catmull-Rom's kernel will be presented, when: a) rectangular, b) Kaiser's and c) triangular window function is applied. In addition, parameters for Greville's and G2D kernels will be defined. The analysis will include rectangular, triangular and Kaiser's window functions.

3.1. Catmull-Rom's kernel

Using the algorithm for defining parameters of interpolation kernel, MSE(α) charts are drawn and α_{opt} values are determined for: a) rectangular (Figure 2, α_{opt}*=-2.61*), b) Kaiser's (Figure 3, α_{opt}*=-1.125*), and c) triangular (Figure 4, α_{opt}*=-1.028*) window functions.

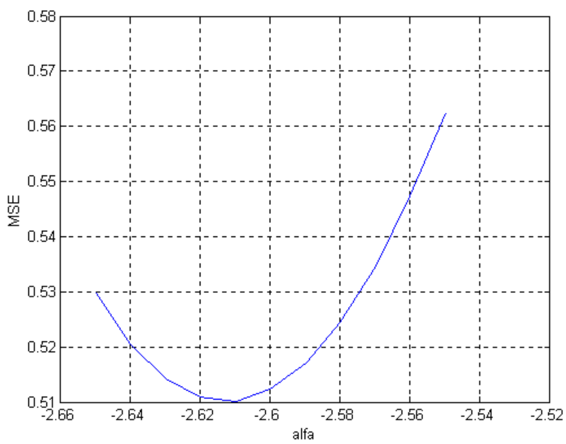


Figure 2. MSE(α) for the application of rectangular window in Catmull-Rom's PCC interpolation

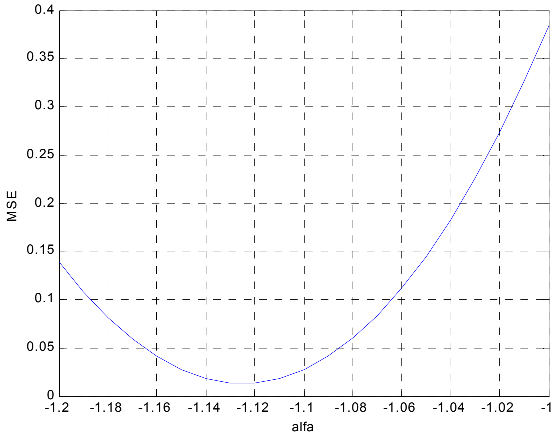


Figure 3. MSE(α) for the application of Kaiser window in Catmull-Rom's PCC interpolation

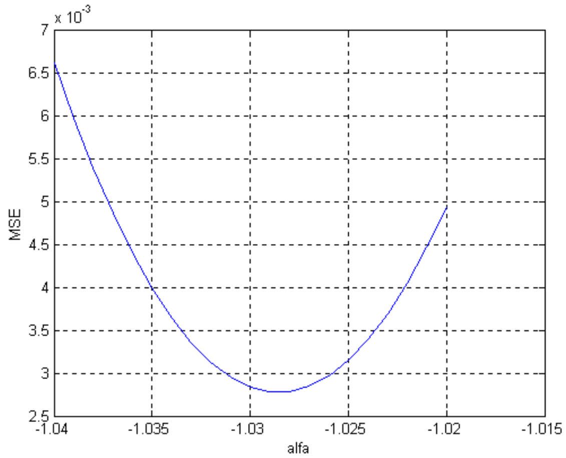


Figure 4. MSE(α) for the application of triangular window in Catmull-Rom's PCC interpolation

3.2. Greville's kernel

MSE(α) charts for Greville's kernel are presented in Figure 5. α_{opt} values are calculated for: a) Hamming's (Figure 5a, α_{opt}=*-0.57*), b) Hann's (Fig.5b, α_{opt}=*-0.449*), c) Blackman's (Figure 5c, α_{opt}=*-0.415*), d) rectangular (Figure 5d, α_{opt}=*-2.25*), e) Kaiser's (Figure 5e, α_{opt}=*-0.6676*) and f) triangular (Figure 5f, α_{opt}=*-0.575*) window function.

3.3 G2D kernel

MSE(α,β) charts for G2D kernel are presented in Figure 6. α_{opt} and β_{opt} values are calculated for: a) Hamming's (Figure 6a, Figure 6b, α_{opt}=*-0.55,* β_{opt}=*0.03*), b) Hann's (Figure 6c, Figure 6d, α_{opt}=*-0.5,* β_{opt}=*0.015*), c) Blackman's (Figure 6e, Figure 6f, α_{opt}=*-0.42,* β_{opt}=*0.002*), d) rectangular (Figure 6g, Figure 6h, α_{opt}=*-2.272,* β_{opt}=*0.005*), e) Kaiser's (Figure 6i, Figure 6h, α_{opt}=*-0.681,* β_{opt}=*0.001*) and f) triangular (Figure 6k, Figure 6l, α_{opt}=*-0.6,* β_{opt}=*-0.001*) window functions. In Figures 6b, 6d, 6f, 6h, 6j and 6l, the positions of MSE(α_{opt},β_{opt}) minimum in (α,β) plane for Gerville's (point **A**) and G2D (point **B**) interpolation kernels, are shown. Vector **AB** shows the position change of **min**(MSE(α_{opt},$\boldsymbol{\beta}_{\mathbf{opt}}$)).

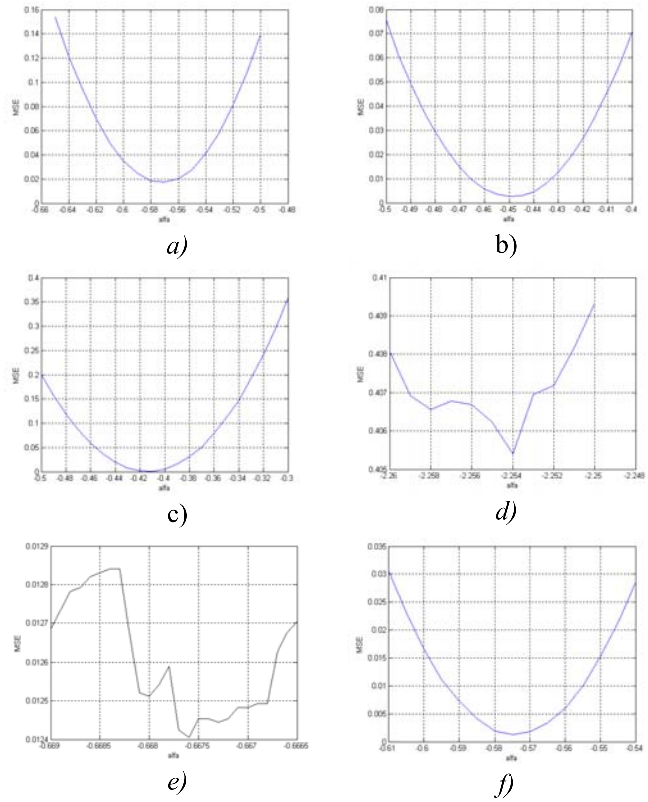
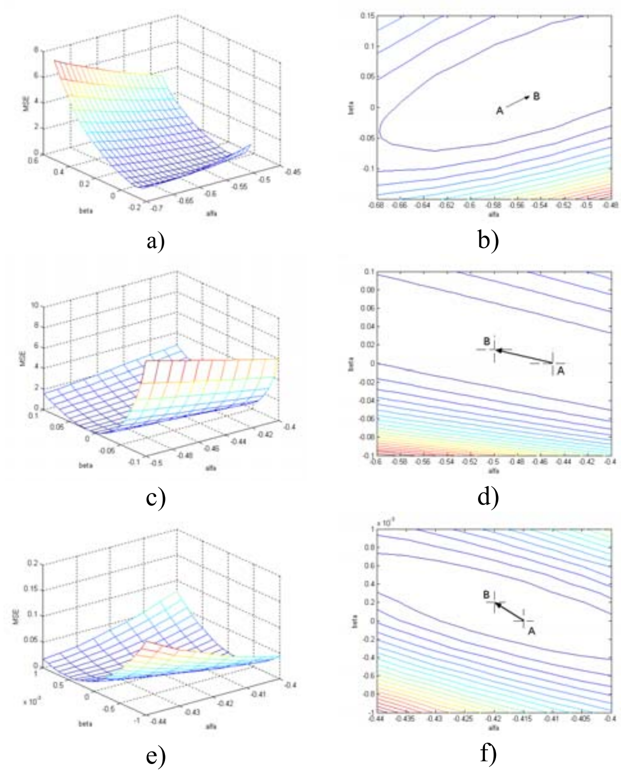


Figure 5. MSE(α) for the application of: a) Hamming, b) Hann, c) Blackman, d) rectangular, e) Kaiser and f) triangular windows in Greville's PCC interpolation



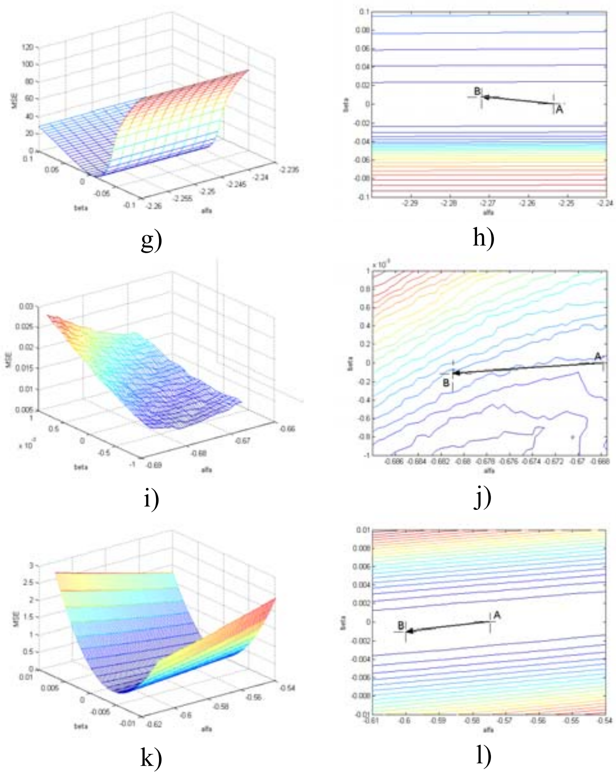


Figure 6. MSE(α,β) for the application of: a) Hamming, c) Hann, e) Blackman, g) rectangular, i) Kaiser and k) triangular windows in G2D PCC interpolation; Positions of min (MSE(α_{opt},β_{opt})) in the plane (α,β) of: b) Hamming, d) Hann, f) Blackman, h) rectangular, j) Kaiser and l) triangular windows in G2D PCC interpolation

4. Comparative analysis

Comparative analysis of the algorithm for the estimate of fundamental frequency is done in relation to MSE. In Table 1, MSE_{min} values for Catmull-Rom's [8] and Greville's interpolation are shown. In Table 2, values α_{opt} and β_{opt} and MSE_{2Dmin} with G2D interpolation are listed.

According to these above reported results, it is clear that:

a) Greville's interpolation estimates the fundamental frequency more accurately than Catmull-Rom's interpolation;

b) G2D interpolation estimates the fundamental frequency more accurately than Greville's interpolation;

c) in all algorithms, **min**(MSE(α,β)) is with the application of Blackman's window function: Catmull-Rom (MSE_{min}=0.001), Greville (MSE_{min}=0.0009) and G2D (MSE_{min}=0.000377).

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Table 1. MSE minimal values and α_{opt} with the application of Catmull-Rom's and Greville's interpolation

	Catmull-Rom		Greville	
	α_{opt}	MSE_{CRmin}	α_{opt}	MSE_{Gmin}
Hamming	-1.005	0.023	-0.57	0.0175
Hann	-0.885	0.004	-0.449	0.0027
Blackman	-1.801	0.001	-0.415	0.0009
Rectangular	-2.61	0.515	-2.254	0.4054
Kaiser	-1.125	0.02	-0.6676	0.0124
Triangular	-1.028	0.0028	-0.575	0.002

Table 2. MSE minimal values, α_{opt} and β_{opt} with the application of G2D interpolation

	α_{opt}	β_{opt}	MSE_{2Dmin}
Hamming	-0.55	0.03	0.0046
Hann	0.5	0.015	0.0018
Blackman	-0.42	0.002	0.000377
Rectangular	-2.272	0.005	0.2244
Kaiser	-0.681	0.001	0.0096
Triangular	0.6	-0.001	0.001

5. Conclusion

The simulation results of the estimate of fundamental frequency by the algorithm with PCC interpolation are presented in this paper. The results of the application of Catmull-Rom's, Greville's and G2D interpolation kernels and several window functions are analysed. The given results clearly point out the advantage of the algorithm with the implemented G2D interpolation kernel and Blackman's window function. Given in percentages, the minimal value of MSE is for 43% and 58,1% lower than MSE with Catmull-Rom's and Geville's interpolation, respectively.

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