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MODELLING AND ANALYSIS OF MANUFACTURING SYSTEMS USING AUGMENTED MARKED GRAPHS

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Abstract. A manufacturing system is typically an event-driven system in which the processes are asynchronous in nature and used to compete with each other for common resources. As the sharing of resources may induce erroneous situations such as deadlocks, in manufacturing system design, a major design objective is to achieve a live and reversible system. Augmented marked graphs possess specific structural characteristics and dynamic properties which are especially desirable for modelling and analysing manufacturing systems. This paper begins with a review of augmented marked graphs and their properties. Then, we show the modelling of a manufacturing system using an augmented marked graph, and the analysis of the system liveness and reversibility based on the properties of augmented marked graphs.

Keywords: Manufacturing systems, system design, Petri nets, augmented marked graphs.

1. Introduction

A manufacturing system is an event-driven system that exhibits concurrent, sequential, coordinated or uncoordinated activities among its processes. The processes are asynchronous in nature and used to compete with each other for common resources [1, 2, 3, 4]. As this involves the sharing of resources, erroneous situations such as deadlocks may occur if the system is not carefully analysed and designed. Therefore, in manufacturing system design, a major objective is to achieve a system which is free from deadlocks. On the other hand, it is equally important that the system is capable of reinitialised for fault recovery. These two properties refer to the liveness and reversibility of a system. Thus, in manufacturing system design, verification of the system liveness and reversibility is essentially required. In real practice, without a rigorous method, one need to walk through all possible scenarios for verification. The process is very time-consuming.

A subclass of Petri nets, augmented marked graphs possess a structure which is especially desirable for modelling systems with shared resources. Moreover, there are many desirable properties, pertaining to their liveness and reversibility. For these reasons, augmented marked graphs are often used in manufacturing system design [5, 6, 7, 8, 9, 10].

In the literature, augmented marked graphs are not studied extensively. Only a few publications are found so far. Chu and Xie first investigated the liveness and reversibility of augmented marked graphs, based on siphons [6]. Huang and others investigated the property-preserving composition of augmented marked graphs [7]. Recently, the author further proposed a number of siphon-based and cycle-based characterisations for live and reversible augmented marked graphs [8, 9]. This paper first summarises these known properties of augmented marked graphs. Then, it will be shown how augmented marked graphs can be effectively used for modelling and analysing manufacturing systems. Specific focus will be place on the system liveness and reversibility. Examples drawn from the literature will be used for illustration.

The rest of this paper is structured as follows. Section 2 provides some fundamentals of Petri nets. Section 3 is a review of augmented marked graphs, where the properties of augmented marked graphs are described in details. Section 4 then shows how manufacturing systems can be modelled and analysed using augmented marked graphs. This will be illustrated using examples of manufacturing systems. Section 5 concludes our results. It should be noted that readers of this paper are expected to have basic knowledge of manufacturing systems.

2. Fundamentals of Petri nets

This section provides the preliminaries to be used in this paper for those readers who are not familiar with Petri nets [11, 12, 13, 14].

A place-transition net (PT-net) is a bipartite graph consisting of two sorts of nodes called places and transitions, such that no arcs connect two nodes of the same sort. In graphical notation, a place is represented by a circle, a transition by a box, and an arc by a directed line. A Petri net is a PT-net where tokens are assigned to its places.

Definition 2.1. A place-transition net (PT-net) is a 4-tuple $N = \langle P, T, F, W \rangle$, where P is a set of places, T is a set of transitions, $F \subseteq (P \times T) \cup (T \times P)$ is a flow relation and $W : F \rightarrow \{ 1, 2, ... \}$ is a weight function. N is said to be an ordinary PT-net if and only if $W : F \rightarrow \{ 1 \}$. (Note : An ordinary PT-net can be written as $\langle P, T, F \rangle$. In the rest of this paper, unless specified otherwise, all PT-nets are ordinary.)

Definition 2.2. Let $N = \langle P, T, F, W \rangle$ be a PT-net. For $x \in (P \cup T)$, ${}^\bullet x = \{ y \mid (y, x) \in F \}$ and $x^\bullet = \{ y \mid (x, y) \in F \}$ are called the pre-set and post-set of x, respectively. For $X = \{ x_1, x_2, \ldots, x_n \} \subseteq (P \cup T)$, ${}^\bullet X = {}^\bullet x_1 \cup {}^\bullet x_2 \cup \ldots \cup {}^\bullet x_n$ and $X^\bullet = x_1^\bullet \cup x_2^\bullet \cup \ldots \cup x_n^\bullet$ are called the pre-set and post-set of X, respectively.

Definition 2.3. For a PT-net $N = \langle P, T, F, W \rangle$, a path is a sequence of nodes $\rho = \langle x_1, x_2, ..., x_n \rangle$, where $(x_i, x_{i+1}) \in F$ for i = 1, 2, ..., n-1. ρ is said to be elementary if and only if it does not contain the same node more than once.

Definition 2.4. For a PT-net $N = \langle P, T, F, W \rangle$, a cycle is a sequence of places $\langle p_1, p_2, ..., p_n \rangle$ such that $\exists\, t_1, t_2, ..., t_n \in T : \langle p_1, t_1, p_2, t_2, ..., p_n, t_n \rangle$ forms an elementary path and $(t_n, p_1) \in F$.

Definition 2.5. For a PT-net $N = \langle P, T, F, W \rangle$, a marking is a function $M : P \rightarrow \{ 0, 1, 2, ... \}$, where M(p) is the number of tokens in p. (N, M_0) represents N with an initial marking M_0.

Definition 2.6. For a PT-net $N = \langle P, T, F, W \rangle$, a transition t is said to be enabled at a marking M if and only if $\forall\, p \in {}^\bullet t : M(p) \geq W(p,t)$. On firing t, M is changed to M' such that $\forall\, p \in P : M'(p) = M(p) - W(p,t) + W(t,p)$. In notation, $M\, [N,t\rangle\, M'$ or $M\, [t\rangle\, M'$.

Definition 2.7. For a PT-net (N, M_0), a sequence of transitions $\sigma = \langle t_1, t_2, ..., t_n \rangle$ is called a firing sequence if and only if $M_0\, [t_1\rangle \ldots [t_n\rangle\, M_n$. In notation, $M_0\, [N,\sigma\rangle\, M_n$ or $M_0\, [\sigma\rangle\, M_n$.

Definition 2.8. For a PT-net (N, M_0), a marking M is said to be reachable if and only if there exists a firing sequence σ such that $M_0\, [\sigma\rangle\, M$. In notation, $M_0\, [N,*\rangle\, M$ or $M_0\, [*\rangle\, M$. $[N, M_0\rangle$ or $[M_0\rangle$ represents the set of all reachable markings of (N, M_0).

Definition 2.9. For a PT-net (N, M_0), a transition t is said to be live if and only if $\forall\, M \in [M_0\rangle, \exists\, M' : M\, [*\rangle\, M'\, [t\rangle$. (N, M_0) is said to be live if and only if every transition is live.

Definition 2.10. For a PT-net (N, M_0), a place p is said to be k-bounded or bounded if and only if $\forall\, M \in [M_0\rangle : M(p) \leq k$, where $k > 0$. (N, M_0) is said to be bounded if and only if every place is bounded.

Definition 2.11. A PT-net (N, M_0) is said to be safe if and only if every place is 1-bounded.

Definition 2.12. A PT-net (N, M_0) is said to be reversible if and only if $\forall\, M \in [M_0\rangle : M\, [*\rangle\, M_0$.

Figure 1 shows a PT-net (N, M_0), where every transition is live and every place is 1-bounded. (N, M_0) is live, bounded, safe and reversible.

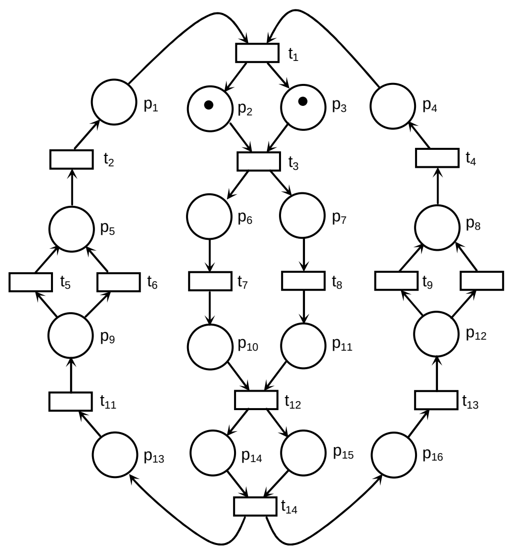


Figure 1. A live, bounded, safe and reversible PT-net

3. Augmented marked graphs

Augmented marked graphs were first introduced by Chu and Xie [6].

Definition 3.1 [6]. An augmented marked graph (N, M_0; R) is a PT-net (N, M_0) with a specific subset of places R, satisfying the following conditions :

(a) Every place in R is marked by M_0.
(b) The net (N', M_0') obtained from (N, M_0; R) by removing the places in R and their associated arcs is a marked graph.
(c) For each place $r \in R$, there exist $k_r > 1$ pairs of transitions $D_r = \{ \langle t_{s1}, t_{h1} \rangle, \langle t_{s2}, t_{h2} \rangle, ..., \langle t_{skr}, t_{hkr} \rangle \}$, where $r^\bullet = \{ t_{s1}, t_{s2}, ..., t_{skr} \}$ and ${}^\bullet r = \{ t_{h1}, t_{h2}, ..., t_{hkr} \}$ and, for each $\langle t_{si}, t_{hi} \rangle \in D_r$, there exists in N' an elementary path ρ_{ri} from t_{si} to t_{hi}.
(d) In (N', M_0'), all cycles are marked and no ρ_{ri} is marked.

Figure 2 shows an augmented marked graph (N, M_0; R), where $R = \{ p_1, p_2 \}$ and both p_1 and p_2 are marked by M_0. For p_1, we have $D_{p1} = \{ \langle t_2, t_8 \rangle, \langle t_1, t_{10} \rangle \}$. For p_2, we have $D_{p2} = \{ \langle t_3, t_9 \rangle, \langle t_1, t_{10} \rangle \}$. If p_1 and p_2 and their associated arcs are removed, the resulting net is a marked graph where every cycle is marked.

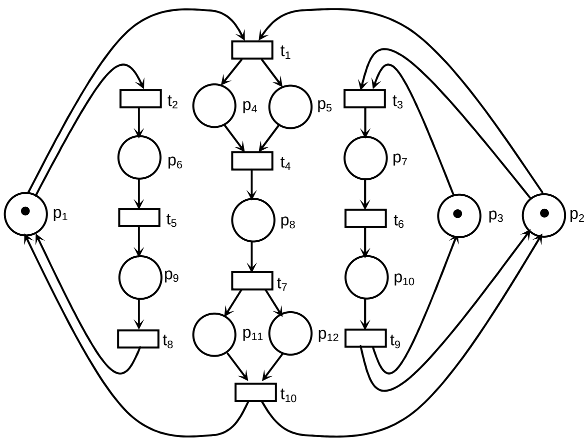


Figure 2. An augmented marked graph

Based on siphons and mathematical programming, Chu and Xie proposed a number of properties for augmented marked graph, pertaining to their liveness and reversibility [6].

Definition 3.2. For a PT-net N, a set of places S is called a siphon if and only if ${}^\bullet S \subseteq S^\bullet$. S is said to be minimal if and only if there does not exist another siphon S' in N, such that $S' \subset S$.

Definition 3.3. For a PT-net N, a set of places Q is called a trap if and only if $Q^\bullet \subseteq {}^\bullet Q$.

Definition 3.4. A PT-net (N, M_0) is said to satisfy the siphon-trap property if and only if every siphon (or minimal siphon) in N contains a marked trap.

Property 3.1 [6]. An augmented marked graph is live if and only if it does not contain any potential deadlock. (Note : A potential deadlock is a siphon which would eventually become empty.)

Property 3.2 [6]. An augmented marked graph is reversible if it is live.

Corollary 3.1. An augmented marked graph is live and reversible if and only if every siphon would never become empty.

Property 3.3 [6]. An augmented marked graph is live and reversible if every minimal siphon, which contains at least one place in R, contains a marked trap.

Corollary 3.2. An augmented marked graph is live and reversible if it satisfies the siphon-trap property.

Based on Chu and Xie's results, the author further proposed a number of siphon-based and cycle-based characterisations for live and reversible augmented marked graphs [8, 9]. Specifically, a new cycle-based property called cycle-inclusion property is proposed. These characterisations are described and illustrated as follows.

For elaborating these characterisations, let us first introduce a number of new notations on cycles, such as the containment of places and transitions in cycles and conflict-free cycles.

Definition 3.5. For a PT-net $N = \langle P, T, F \rangle$ and a place $p \in P$, Ω_N denotes the set of cycles in N, and $\Omega_N[p]$ denotes the set of cycles containing p.

Definition 3.6. Let N be a PT-net. For a set of cycles $Y \subseteq \Omega_N$, $P[Y]$ denotes the set of places in Y, and $T[Y] = {}^\bullet P[Y] \cap P[Y]^\bullet$ denotes the set of transitions generated by Y.

Definition 3.7. For a PT-net N, a set of cycle $Y \subseteq \Omega_N$ is said to be conflict-free if and only if, for any q, $q' \in P[Y]$, there exists in Y a conflict-free path from q to q'. (Note : According to [15], an elementary path $\rho = \langle x_1, x_2, ..., x_n \rangle$ is conflict-free if and only if, for any transition x_i in ρ, $j \neq (i - 1) \Rightarrow x_j \notin {}^\bullet x_i$.)

Figure 3 shows a PT-net N = ⟨ P, T, F ⟩. Consider γ_1, γ_2, $\gamma_3 \in \Omega_N[p_3]$, where γ_1 = ⟨ p_3, p_2, p_7 ⟩, γ_2 = ⟨ p_3, p_4 ⟩ and γ_3 = ⟨ p_3, p_1, p_6, p_{10}, p_8 ⟩. Y_1 = { γ_1, γ_2 } is conflict-free as for any q, q' $\in$ P[Y_1], there exists in Y_1 a conflict-free path from q to q'. Y_2 = { γ_2, γ_3 } is not conflict-free. Consider p_4, $p_8 \in$ P[Y_2]. p_4 is connected to p_8 via only one path ρ = ⟨ p_4, t_5, p_3, t_1, p_1, t_3, p_6, t_6, p_{10}, t_9, p_8 ⟩ in Y_2, where ρ is not conflict-free because p_4, $p_8 \in {}^\bullet t_5$.

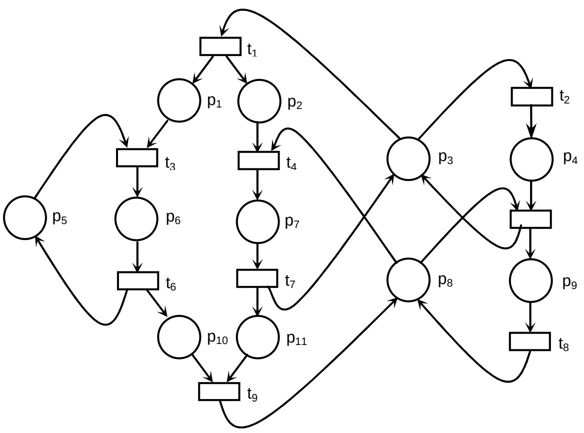


Figure 3. Illustration of conflict-free cycles

Lemma 3.1. Let S be a minimal siphon of an augmented marked graph (N, M_0; R). For every place p $\in$ S, there exists Y $\subseteq \Omega_N[p]$ such that Y is conflict free, P[Y] = S and ${}^\bullet p \subseteq$ T[Y].

Proof. Let S = { p } $\cup$ { p_1, p_2, ..., p_n }. For each p_i, according to [15], p connects to p_i via a conflict-free path ρ_i in S while p_i to p via another ρ_i' in S, forming a cycle $\gamma_i \in \Omega_N[p]$, where p, $p_i \in$ P[γ_i] $\subseteq$ S. Let Y = { γ_1, γ_2, ..., γ_n } $\subseteq \Omega_N[p]$. Then, P[Y] = (P[γ_1] $\cup$ P[γ_2] $\cup$ ... $\cup$ P[γ_n]) = S. Y is conflict-free because there exists a conflict-free path from q to q', for any q, q' $\in$ S = P[Y]. Besides, since S is a siphon, ${}^\bullet p \subseteq ({}^\bullet S \cap S^\bullet)$ = (${}^\bullet$P[Y] $\cap$ P[Y]${}^\bullet$) = T[Y].

Lemma 3.2. Every cycle in an augmented marked graph is marked.

Proof. (by contradiction) Let (N, M_0; R) be an augmented marked graph. Suppose there exists a cycle γ in (N, M_0; R), such that γ is not marked. Then, γ does not contain any place in R. Hence, γ also exists in the net (N', M_0') obtained from (N, M_0; R) after removing the places in R and their associated arcs. However, by definition of augmented marked graphs, every cycle in (N', M_0') is marked.

Property 3.4. An augmented marked graph (N, M_0; R) is live and reversible if and only if no minimal siphons, which contain at least one place of R, eventually become empty.

Proof. (⇐) Consider a siphon S which does not contain any place of R. According to Lemmas 3.1 and 3.2, S is covered by cycles and is marked. As S does not contain any place of R, it follows from the definition of augmented marked graphs that, for any s $\in$ S, | ${}^\bullet s$ | = | $s^\bullet$ | = 1. Then, ${}^\bullet S = S^\bullet$ and S is also a trap. S contains itself as a marked trap and would never become empty. Given that no minimal siphons, which contain at least one place of R, eventually become empty, every minimal siphon never become empty. According to Corollary 3.1, (N, M_0; R) is live and reversible. (⇒) It follows from Corollary 3.1 that no minimal siphons, which contain at least one place of R, eventually become empty.

Definition 3.8. For a PT-net N = ⟨ P, T, F ⟩, a place p $\in$ P is said to satisfy the cycle-inclusion property if and only if, for any set of cycles Y $\subseteq \Omega_N[p]$, such that Y is conflict-free, ${}^\bullet p \subseteq T[Y] \Rightarrow p^\bullet \subseteq T[Y]$.

For the PT-net = ⟨ P, T, F ⟩ shown in Figure 4, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11} and p_{12} satisfy the cycle-inclusion property. For example, for p_8, $\Omega_N[p_8]$ = { γ_{81}, γ_{82}, γ_{83}, γ_{84}, γ_{85} } where γ_{81} = ⟨ p_8, p_1 ⟩, γ_{82} = ⟨ p_8, p_2, p_4 ⟩, γ_{83} = ⟨ p_8, p_2, p_9, p_1 ⟩, γ_{84} = ⟨ p_8, p_1, p_5, p_9, p_2, p_4 ⟩ and γ_{85} = ⟨ p_8, p_1, p_6, p_{10}, p_2, p_4 ⟩. For any $Y_8 \subseteq \Omega_N[p_8]$ such that Y_8 is conflict-free, ${}^\bullet p_8$ = { t_4 } $\subseteq$ T[Y_8] and $p_8^\bullet$ = { t_7 } $\subseteq$ T[Y_8]. Hence, p_8 satisfies the cycle-inclusion property. On the other hand, p_1 and p_2 do not satisfy the cycle-inclusion property. For example, for p_1, let Y_1 = { γ_{11}, γ_{12} } $\subseteq \Omega_N[p_1]$, where γ_{11} = ⟨ p_1, p_8 ⟩ and γ_{12} = ⟨ p_1, p_8, p_2, p_9 ⟩. Y_1 is conflict-free and T[Y_1] = { t_4, t_5, t_7, t_8 }. Since ${}^\bullet p_1$ = { t_7, t_8 } $\subseteq$ T[Y_1] and $p_1^\bullet$ = { t_2, t_4 } $\not\subseteq$ T[Y_1], p_1 does not satisfy the cycle-inclusion property.

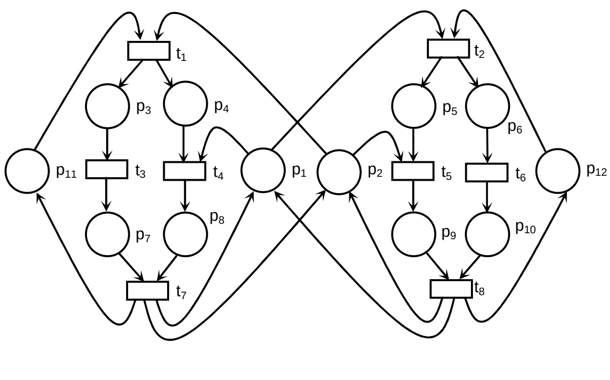


Figure 4. Illustration of the cycle-inclusion property

Lemma 3.3. Let (N, M_0; R) be an augmented marked graph. For every place r $\in$ R, there exists a siphon S which contains only one marked place r.

Proof. Let $D_r = \{ \langle t_{s1}, t_{h1} \rangle, \langle t_{s2}, t_{h2} \rangle, ..., \langle t_{sn}, t_{hn} \rangle \}$, where $r^\bullet = \{ t_{s1}, t_{s2}, ..., t_{sn} \}$ and ${}^\bullet r = \{ t_{h1}, t_{h2}, ..., t_{hn} \}$. For each $\langle t_{si}, t_{hi} \rangle \in D_r$, by definition of augmented marked graphs, there exists an elementary path ρ_i from t_{si} to t_{hi} and ρ_i is not marked. Let $S = P_1 \cup P_2 \cup ... \cup P_n \cup \{ r \}$, where P_i is the set of places in ρ_i for i = 1, 2, ..., n. Consider P_i. We have ${}^\bullet P_i \subseteq (P_i^\bullet \cup r^\bullet)$ because, for every $p \in P_i$, $| {}^\bullet p | = | p^\bullet | = 1$. Then, $({}^\bullet P_1 \cup {}^\bullet P_2 \cup ... \cup {}^\bullet P_n) \subseteq (P_1^\bullet \cup P_2^\bullet \cup ... \cup P_n^\bullet \cup r^\bullet)$. Besides, ${}^\bullet r = \{ t_{h1}, t_{h2}, ..., t_{hn} \} \subseteq (P_1^\bullet \cup P_2^\bullet \cup ... \cup P_n^\bullet)$. Hence, ${}^\bullet S = ({}^\bullet P_1 \cup {}^\bullet P_2 \cup ... \cup {}^\bullet P_n \cup {}^\bullet r) \subseteq (P_1^\bullet \cup P_2^\bullet \cup ... \cup P_n^\bullet \cup r^\bullet) = S^\bullet$, and S is a siphon. Since the places in each P_i are unmarked, r is the only one marked place in S.

Property 3.5. An augmented marked graph (N, M_0; R) satisfies the siphon-trap property if and only if every place of R satisfies the cycle-inclusion property.

Proof. ($\Leftarrow$) Let $S = \{ p_1, p_2, ..., p_n \}$ be a minimal siphon in N. For each $p_i \in S$, according to Lemma 3.1, there exists $Y_i \subseteq \Omega_N[p_i]$, such that Y_i is conflict-free, $P[Y_i] = S$ and ${}^\bullet p_i \subseteq T[Y_i]$. It follows from Lemma 3.2 that S is marked. Any $p_i \notin R$ satisfies the cycle-inclusion property because $| {}^\bullet p_i | = | p_i^\bullet | = 1$ and, for any $Y \subseteq \Omega_N[p_i]$, ${}^\bullet p_i \subseteq T[Y] \Rightarrow p_i^\bullet \subseteq T[Y]$. Given that every place of R satisfies the cycle-inclusion property, every p_i satisfies the cycle-inclusion property. Then, $p_i^\bullet \subseteq T[Y_i] = ({}^\bullet P[Y_i] \cap P[Y_i]^\bullet)$, implying $p_i^\bullet \subseteq {}^\bullet P[Y_i] = {}^\bullet S$. Since $S^\bullet = (p_1^\bullet \cup p_2^\bullet \cup ... \cup p_n^\bullet) \subseteq {}^\bullet S$, S is a trap. S contains itself as a marked trap. The siphon-trap property is satisfied. ($\Rightarrow$ by contradiction) Suppose there exists $r \in R$ which does not satisfy the cycle-inclusion property. According to Lemma 3.3, there exists a siphon S, in which r is the only marked place. Let $S' \subseteq S$ be a minimal siphon in N. It follows from Lemmas 3.1 and 3.2 that S' is covered by cycles and is marked. Since $S' \subseteq S$ and r is the only one marked place in S, r is also the one marked place in S' and $Y \subseteq \Omega_N[r]$. Given that (N, M_0; R) satisfies the siphon-trap property, there exists a marked trap Q in S'. Then, $r \in Q$ and $r^\bullet \subseteq ({}^\bullet Q \cap Q^\bullet)$. Since r does not satisfy the cycle-inclusion property, ${}^\bullet r \subseteq T[Y] \not\Rightarrow r^\bullet \subseteq T[Y]$. Since S' is a siphon, it is always true for ${}^\bullet r \subseteq ({}^\bullet S' \cap S'^\bullet) = ({}^\bullet P[Y] \cap P[Y]^\bullet) = T[Y]$. However, since r does not satisfy the cycle-inclusion property, $r^\bullet \not\subseteq T[Y] = ({}^\bullet P[Y] \cap P[Y]^\bullet) = ({}^\bullet S' \cap S'^\bullet)$ implies $r^\bullet \not\subseteq ({}^\bullet Q \cap Q^\bullet)$.

Corollary 3.3. An augmented marked graph (N, M_0; R) is live and reversible if every place of R satisfies the cycle-inclusion property.

Figure 5 shows an augmented marked graph (N, M_0; R), where R = { p_4, p_7 }. Every siphon would never become empty. According to Corollary 3.1 or Property 3.4, (N, M_0; R) is live and reversible. On the other hand, for (N, M_0; R), both p_4 and p_7 satisfy the cycle-inclusion property. According to Property 3.5, the siphon-trap property is satisfied. It then follows from Corollaries 3.2 or 3.3 that (N, M_0; R) is live and reversible.

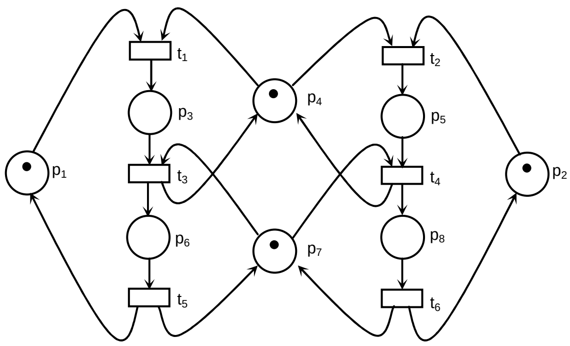


Figure 5. A live and reversible augmented marked graph

Figure 6 shows another augmented marked graph (N, M_0; R), where R = { p_5, p_6 }. There exists a minimal siphon S = { p_5, p_6, p_7, p_8 } which does not contain any marked trap. S will become empty after firing the sequence of transitions $\langle t_1, t_2, ... \rangle$. According to Corollary 3.1 or Property 3.4, (N, M_0; R) is neither live nor reversible. A deadlock would occur after firing $\langle t_1, t_2, ... \rangle$.

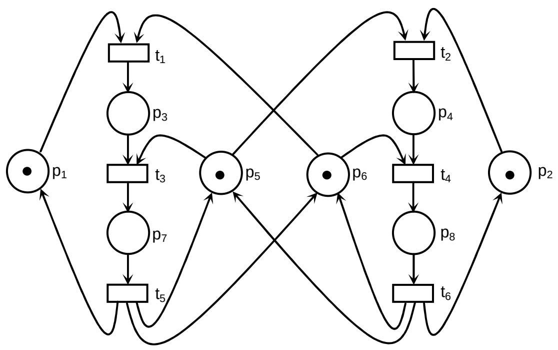


Figure 6. An augmented marked graph which is neither live nor reversible

Properties 3.1, 3.2, 3.3 and 3.4 as well as Corollaries 3.1 and 3.2 give the siphon-based characterisations for live and reversible augmented marked graphs. Property 3.5 and Corollary 3.3, on the other hand, give the cycle-based characterisations, where cycles instead of siphons are considered.

Whereas Corollary 3.1 and Property 3.4 provide the necessary and sufficient conditions for liveness and reversibility of augmented marked graphs, Property 3.3 and Corollaries 3.2 and 3.3 provide only the sufficient conditions.

4. Manufacturing system design

A manufacturing system is typically an event-driven system that exhibits some concurrent, sequential, coordinated or uncoordinated activities among its processes. The processes are asynchronous in nature and used to compete with each other for common resources [1, 2, 3, 4]. Erroneous situation such as deadlocks are easily induced because of the sharing of common resources. Thus, in manufacturing system design, a major design objective is to achieve a live and reversible system - liveness implies deadlock-freeness while reversibility allows system recovery. In real practice, verification of the system liveness and reversibility is essentially required and this verification is very time-consuming.

In the following, we show the modelling of a manufacturing system using an augmented marked graph. Based on the properties of augmented marked graphs, the system liveness and reversibility can be effectively analysed. This is illustrated using examples drawn from the literature.

Example 1. It is a FWS-200 Flexible Workstation System for the production of circuit boards, extracted from [3] (pp. 121-124). The system consists of two robots R_1 and R_2, one feeder area and one PCB area, as shown in Figure 7. There are two asynchronous production processes.

Production process 1 : R_1 picks components from the feeder area, and moves into the PCB area for inserting components. The finished product is then moved out from the PCB area.

Production process 2 : R_2 picks components from the feeder area, and moves into the PCB area for inserting components. The finished product is then moved out from the PCB area.

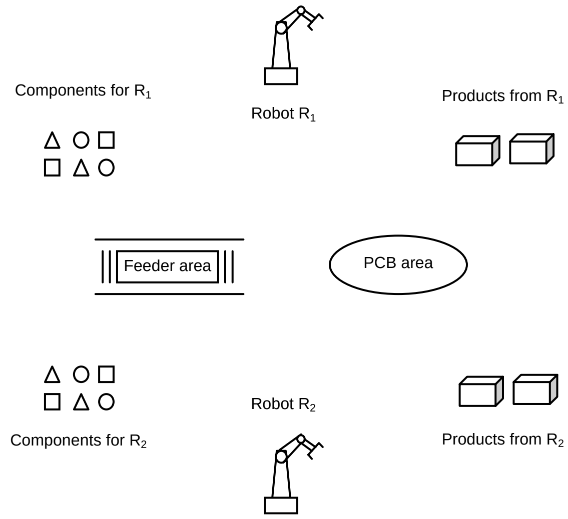
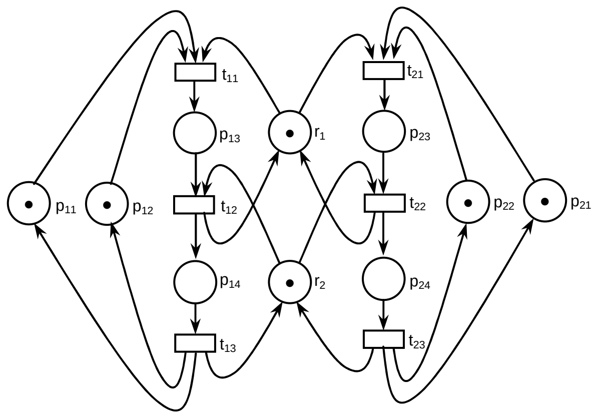


Figure 7. The FWS-200 Flexible Workstation System (Example 1)

Figure 8 shows an augmented marked graph (N, M_0; R), where R = { r_1, r_2 }, representing the FWS-200 system (Example 1).



Semantic meaning for places and transitions

p_{11}	R_1 is ready
p_{12}	Components for R_1 are available
p_{13}	R_1 is picking components from feeder
p_{14}	R_1 is inserting components in PCB area
p_{21}	R_2 is ready
p_{22}	Components for R_2 are available
p_{23}	R_2 is picking components from feeder
p_{24}	R_2 is inserting components in PCB area
r_1	Feeder area is available
r_2	PCB area is available
t_{11}	R_1 starts picking components
t_{12}	R_1 finishes picking components and starts inserting components
t_{13}	R_1 finishes inserting components and starts moving out the product
t_{21}	R_2 starts picking components
t_{22}	R_2 finishes picking components and starts inserting components
t_{23}	R_2 finishes inserting components and starts out the finished product

Figure 8. An augmented marked graph representing the FWS-200 (Example 1)

For the augmented marked graph (N, M_0; R) shown in Figure 8, every siphon would never become empty and, according to Corollary 3.1 or Property 3.4, (N, M_0; R) is live and reversible. Besides, both r_1 and r_2 satisfy the cycle-inclusion property. According to Property 3.5, the siphon-trap property is satisfied. It then follows from Corollaries 3.2 or 3.3 that (N, M_0; R) is live and reversible.

Example 2. It is a flexible assembly system, extracted from [4] (pp.58-61). The system consists of three conveyors C_1, C_2 and C_3 and three robots R_1, R_2 and R_3, as shown in Figure 9. There are three asynchronous assembly processes.

Assembly process 1 : C_1 requests R_1. After acquiring R_1, it requests R_2. After acquiring R_2, it performs assembling and then releases both R_1 and R_2 simultaneously.

Assembly process 2 : C_2 requests R_2. After acquiring R_2, it requests R_3. After acquiring R_3, it perform assembling and then releases both R_2 and R_3 simultaneously.

Assembly process 3 : C_3 requests R_3. After acquiring R_3, it requests R_1. After acquiring R_1, it perform assembling and then releases both R_3 and R_1 simultaneously.

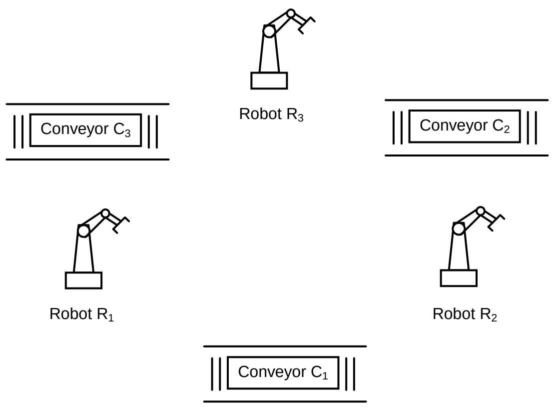
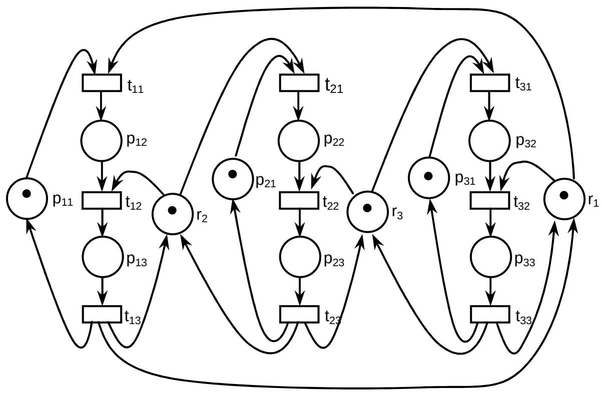


Figure 9. A flexible assembly system

Figure 10 shows an augmented marked graph (N, M_0; R), where R = { r_1, r_2, r_3 }, representing the flexible assembly system (Example 2). There exists a siphon S = { p_{13}, p_{23}, p_{33}, r_1, r_2, r_3 } which become empty after firing the sequence of transitions 〈 t_{11}, t_{21}, t_{31} 〉. According to Corollary 3.1 and Property 3.4, (N, M_0; R) is neither live nor reversible. A deadlock would occur after firing 〈 t_{11}, t_{21}, t_{31} 〉.



Semantic meaning for places and transitions

p_{12}	C_1 is occupying R_1
p_{13}	C_1 is occupying R_1 and R_2
p_{21}	C_2 is ready
p_{22}	C_2 is occupying R_2
p_{23}	C_2 is occupying R_2 and R_3
p_{21}	C_3 is ready
p_{22}	C_3 is occupying R_3
p_{23}	C_3 is occupying R_3 and R_1
r_1	R_1 is available
r_2	R_2 is available
r_3	R_3 is available
t_{11}	C_1 starts acquiring R_1
t_{12}	C_1 starts acquiring R_2
t_{13}	C_1 finishes assembling and release R_1 and R_2 simultaneously
t_{21}	C_2 starts acquiring R_2
t_{22}	C_2 starts acquiring R_3
t_{23}	C_2 finishes assembling and release R_2 and R_3 simultaneously
t_{31}	C_3 starts acquiring R_3
t_{32}	C_3 starts acquiring R_1
t_{33}	C_3 finishes assembling and release R_3 and R_1 simultaneously

Figure 10. An augmented marked graph representing the flexible assembly system (Example 2)

5. Conclusion

Manufacturing systems are typically shared resource systems wherein some common resources used to be shared among different asynchronous processes. Erroneous situations, such as deadlocks, arising from the competition and sharing of resource are easily induced. Therefore, in manufacturing system design, a major design objective is to obtain a live and reversible system. Possessing a specific structure and many desirable behavioural properties, augmented marked graphs are often used for modelling systems involving shared resources, such as manufacturing systems. In this paper, after summarising the properties of augmented marked graphs, we show the modelling of a typical manufacturing system using an augmented marked graph. Then, based on the desirable properties of augmented marked graphs, the system liveness and reversibility can be effectively analysed. These have been illustrated using examples drawn from the literature.

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