

## DYNAMIC MODELS OF LINEAR INDUCTION DRIVE

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**Abstract.** Due to open magnetic circuit and limited length of the linear induction motor inductor phase currents are not equal. The article deals with model of linear induction drive taking into account magnetic asymmetry of linear induction motor at different connection of windings. Developed models are presented and results of simulation are considered.

### 1. Introduction

Linear electric motor (LIM) technology has been broadly used in the aspects of traffic, industry, information, automation, material transportation, elevators and postal mechanical systems [1, 2, 3].

Nevertheless by way of investigation broadening and shrinking areas of application can be defined.

Due to worse efficiency linear electric drives usually are designed for short time duty. Often a linear drive operates just in a dynamic mode and its parameters do not reach steady state values. Electromagnetic transients in the linear motor must be taken into account in order to investigate characteristics of such drives. Due to open magnetic circuit of LIM asymmetry of currents takes place. The paper presents a model of linear induction motor with consideration of electric and magnetic asymmetry due to open magnetic circuit of LIM at different connection of windings. In a model transform of coordinates is not used.

According to traditions rotary or linear motors are modelled by transformation three-phase motor to equivalent two-phase one and deriving equations in the stationary reference frame or revolving (translating) reference frame. Indices  $\alpha, \beta$  denote stationary reference frame and  $u, v$  - reference frame translating with speed  $v_k$ . If  $v_k$  is equal to speed of magnetic field  $v_{0el}$ , such reference frame is called  $x, y$ . Models of linear drives in transformed coordinate systems are analysed in [4, 5, 6]. Modelling of control modes of drives with LIM meets difficulties to transform supply voltage shape. The developed model has no restrictions on supply voltage shape and can be used to model advance control modes. They do not require transform back model variables to real quantities. Models of linear motors are derived on an analogy between rotating and translating motors.

### 2. Mathematical model of a motor with Y connected windings with neutral wire

Three-phase motor with Y connected windings is presented in Figure 1. The figure shows voltages and currents in a circuit. The rotor of the motor is shown turned by angle  $\varphi$ .

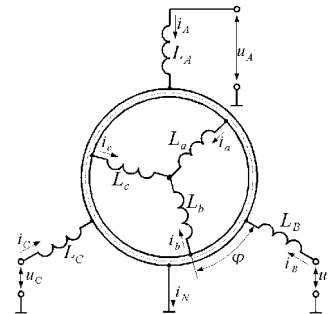


Figure 1. Y connected three-phase motor with neutral wire

The system of equations describing rotating motor and LIM is formed from equations of voltage equilibrium for inductor and secondary, flux linkages and equations to calculate electromagnetic torque and force as well as equations to describe rotational (translational) movement. Equations of voltage equilibrium for one phase of the inductor winding and one phase of secondary winding are:

$$\begin{cases} u_A = \frac{d\Psi_A}{dt} + R_A i_A \\ u_a = \frac{d\Psi_a}{dt} + R_a i_a, \end{cases} \quad (1)$$

where  $u_A, u_a$  – instantaneous values of inductor phase A and phase a of secondary element voltages;

$\Psi_A, \Psi_a$  – flux linkages of inductor phase A and phase a of secondary element;

$R_A (L_A), R_a (L_a)$  – A phase resistances (inductances) of inductor and secondary element;  $i_A, i_a$  – instant-

neous values of inductor phase A winding and phase a secondary element currents.

Phase A flux linkage of a stator and rotor is:

$$\begin{cases} \Psi_A = L_A i_A + M_S (i_B + i_C) + M_{SR} i_a \cos \varphi + \\ + M_{SR} [i_b \cos(\varphi + 120^\circ) + i_c \cos(\varphi - 120^\circ)] \\ \Psi_a = L_a i_a + M_R (i_b + i_c) + \\ + M_{SR} [i_A \cos \varphi + i_B \cos(\varphi - 120^\circ) + i_C \cos(\varphi + 120^\circ)], \end{cases} \quad (2)$$

where  $M_{SR}$  – mutual inductance between inductor and rotor (secondary element) when the axes of coils coincide [H];  $M_S, (M_R)$  – mutual inductance between different coils of inductor (rotor or secondary) in H;  $\varphi$  – angle between rotor and inductor coils in electric degrees.

Electromagnetic torque developed by rotating motor is [6, 7]:

$$\begin{aligned} M_{em} = & -pM_{SR} [(i_A i_a + i_B i_b + i_C i_c) \sin \varphi] - \\ & -pM_{SR} [(i_A i_b + i_B i_c + i_C i_a) \sin(\varphi + 120^\circ)] - \\ & -pM_{SR} [(i_A i_c + i_B i_a + i_C i_b) \sin(\varphi - 120^\circ)]. \end{aligned} \quad (3)$$

On a base of power equivalency in rotating and translating motors an expression to calculate the force developed by LIM is obtained:

$$F_{em} = \frac{\pi}{\tau} \cdot \frac{M_{em}}{p}, \quad (4)$$

$$\varphi = \frac{\pi}{\tau} \cdot x, \quad (5)$$

where  $\tau$  is pole pitch;  $p$  is the number of pole pairs,  $x$ - excursion of the secondary.

Figure 2 shows a scheme of linear induction motor, where the angle of rotation  $\varphi$  is transformed to excursion of secondary  $x$ .

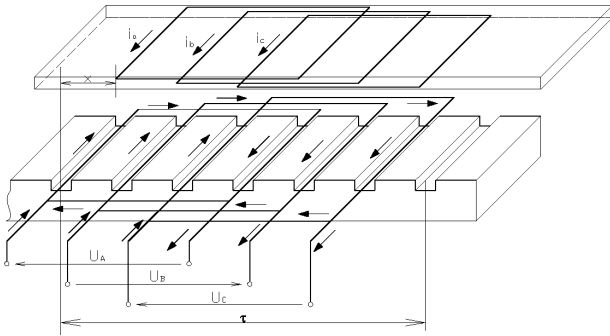


Figure 2. Electrical circuit of linear induction motor windings

The dynamic model is supplemented by a movement equation.

### 3. Mathematical model of a motor with Y connected windings without neutral wire

In motors with symmetrical windings supplied with balanced voltage system current does not flow in neutral wire. They can be modelled by the method

described above even if they are connected without neutral wire.

If phase windings of a motor are non-symmetrical and connected without neutral wire or if they are symmetrical but supplied by unbalanced voltage system due to absence of neutral wire they must be modelled considering supply by line voltages.

A scheme of a three-phase motor with Y connected windings without neutral wire is given in Figure 3.

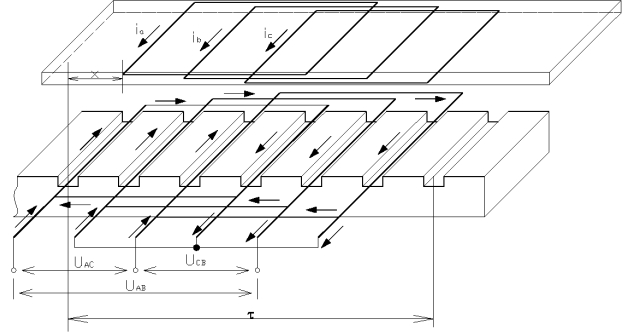


Figure 3. Scheme of a three-phase LIM with Y connected windings without neutral wire

Equations for an inductor and secondary are:

$$\begin{cases} U_{AC} = R_A i_A + \frac{d\Psi_A}{dt} - R_C i_C - \frac{d\Psi_C}{dt} \\ U_{BC} = R_B i_B + \frac{d\Psi_B}{dt} - R_C i_C - \frac{d\Psi_C}{dt} \\ U_{ac} = R_a i_a + \frac{d\Psi_a}{dt} - R_c i_c - \frac{d\Psi_c}{dt} \\ U_{bc} = R_b i_b + \frac{d\Psi_b}{dt} - R_c i_c - \frac{d\Psi_c}{dt}, \end{cases} \quad (6)$$

where  $U_{AC}, U_{BC}, U_{ac}, U_{bc}$  are the line voltages of stator and rotor.

There are two independent currents in Y connection. By Ohm's law the third one is equal to the sum of these two currents taken with minus sign. Therefore, one current of equation (6) can be expressed as a sum others two. This statement is valid for flux linkages also. If two independent quantities are chosen, for example, currents of a and b windings, then current and flux linkages of phase c are calculated as:

$$\begin{cases} i_C = -(i_A + i_B), \quad \Psi_C = -(\Psi_A + \Psi_B) \\ i_c = -(i_a + i_b), \quad \Psi_c = -(\Psi_a + \Psi_b). \end{cases} \quad (7)$$

Substituting (3) to (2) yields:

$$\begin{cases} U_{AC} = (R_A + R_C) i_A + R_B i_B + \frac{d(2\Psi_A + \Psi_B)}{dt} \\ U_{BC} = R_C i_A + (R_B + R_C) i_B + \frac{d(\Psi_A + 2\Psi_B)}{dt} \\ U_{ac} = (R_a + R_c) i_a + R_c i_b + \frac{d(2\Psi_a + \Psi_b)}{dt} \\ U_{bc} = R_c i_a + (R_b + R_c) i_b + \frac{d(\Psi_a + 2\Psi_b)}{dt}. \end{cases} \quad (8)$$

Flux linkages of windings are calculated according to formulas, presented in [6]. Substituting these formulas to (8) and considering (7) yields:

$$\begin{aligned} u_{AC} = & (R_A + R_C)i_A + R_B i_B + \\ & + 2(L_A - M_S)\frac{di_A}{dt} + (L_B - M_S)\frac{di_B}{dt} + \\ & + 3M_{SR} \left[ \cos\left(\frac{\pi}{\tau}x\right)\frac{di_a}{dt} + \cos\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right)\frac{di_b}{dt} \right] - \\ & - 3M_{SR} \frac{\pi}{\tau} \left[ i_a \sin\left(\frac{\pi}{\tau}x\right) + i_b \sin\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right) \right] \frac{dx}{dt}; \end{aligned} \quad (9)$$

$$\begin{aligned} u_{BC} = & R_C i_A + (R_B + R_C)i_B + \\ & + (L_A - M_S)\frac{di_A}{dt} + 2(L_B - M_S)\frac{di_B}{dt} + \\ & + 3M_{SR} \left[ \cos\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right)\frac{di_a}{dt} + \cos\left(\frac{\pi}{\tau}x\right)\frac{di_b}{dt} \right] - \\ & - 3M_{SR} \frac{\pi}{\tau} \left[ i_a \sin\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right) + i_b \sin\left(\frac{\pi}{\tau}x\right) \right] \frac{dx}{dt}; \end{aligned} \quad (10)$$

$$\begin{aligned} U_{ac} = & (R_a + R_c)i_a + R_c i_b + 3M_{SR} \cos\frac{\pi}{\tau}x \frac{di_A}{dt} + \\ & + 3M_{SR} \cos\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right)\frac{di_B}{dt} + \\ & + 2(L_a - M_R)\frac{di_a}{dt} + (L_b - M_R)\frac{di_b}{dt} - \\ & - 3M_{SR} \frac{\pi}{\tau} \left[ i_A \sin\frac{\pi}{\tau}x + i_B \sin\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right) \right] \frac{dx}{dt}; \end{aligned} \quad (11)$$

$$\begin{aligned} U_{bc} = & R_c i_a + (R_b + R_c)i_b + 3M_{SR} \cos\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right)\frac{di_A}{dt} + \\ & + 3M_{SR} \cos\frac{\pi}{\tau}x \frac{di_B}{dt} + \\ & + (L_a - M_R)\frac{di_a}{dt} + 2(L_b - M_R)\frac{di_b}{dt} - \\ & - 3M_{SR} \frac{\pi}{\tau} \left[ i_A \sin\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right) + i_B \sin\frac{\pi}{\tau}x \right] \frac{dx}{dt}. \end{aligned} \quad (12)$$

The system of equations (9) – (12) describes induction motor with Y connected winding without neutral wire. To solve it in Matlab, it is necessary to express them in the following way:

$$\mathbf{M}(t)\mathbf{x}' = \begin{pmatrix} A_1 & A_2 & A_3 & A_4 & A_5 & 0 \\ B_1 & B_2 & B_3 & B_4 & B_5 & 0 \\ C_1 & C_2 & C_3 & C_4 & C_5 & 0 \\ D_1 & D_2 & D_3 & D_4 & D_5 & 0 \\ 0 & 0 & 0 & 0 & \frac{\tau}{\pi} & 0 \\ 0 & 0 & 0 & 0 & 0 & m \end{pmatrix} \begin{pmatrix} di_A/dt \\ di_B/dt \\ di_a/dt \\ di_b/dt \\ dv/dt \\ dv^2/d^2t \end{pmatrix} \quad (13)$$

where the following notations are used:

$$\begin{aligned} A_1 &= 2(L_A - M_S); B_2 = 2(L_B - M_S); \\ C_3 &= 2(L_a - M_R); D_4 = 2(L_b - M_R); \\ A_2 &= L_B - M_S; B_1 = L_A - M_S; \end{aligned}$$

$$C_4 = L_b - M_R; D_3 = L_a - M_R;$$

$$A_3 = B_4 = C_1 = D_2 = 3M_{SR} \cos\frac{\pi}{\tau}x;$$

$$D_1 = A_4 = 3M_{SR} \cos\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right);$$

$$C_2 = B_3 = 3M_{SR} \cos\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right);$$

$$A_5 = -3M_{SR} \left[ i_a \sin\left(\frac{\pi}{\tau}x\right) + i_b \sin\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right) \right];$$

$$B_5 = -3M_{SR} \left[ i_a \sin\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right) + i_b \sin\frac{\pi}{\tau}x \right];$$

$$C_5 = -3M_{SR} \left[ i_A \sin\frac{\pi}{\tau}x + i_B \sin\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right) \right];$$

$$D_5 = -3M_{SR} \left[ i_A \sin\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right) + i_B \sin\frac{\pi}{\tau}x \right];$$

The left-hand side of the equation is expressed as:

$$\mathbf{F}(t, \mathbf{x}) = \begin{pmatrix} U_{AC} - (R_A + R_C)i_A - R_C i_B \\ U_{BC} - R_C i_A - (R_B + R_C)i_B \\ U_{ac} - (R_a + R_c)i_a - R_c i_b \\ U_{cb} - R_c i_a - (R_b + R_c)i_b \\ v \\ F_{em} - F_{st} \end{pmatrix}. \quad (13)$$

Electromagnetic force is calculated from the expression:

$$\begin{aligned} F_{em} = & \frac{\pi}{\tau} \left\{ -pM_{SR} \left[ (i_A i_a + i_B i_b + i_C i_c) \sin\frac{\pi}{\tau}x \right] - \right. \\ & - pM_{SR} \left[ (i_A i_b + i_B i_c + i_C i_a) \sin\left(\frac{\pi}{\tau}x + 120^\circ\right) \right] - \\ & \left. - pM_{SR} \left[ (i_A i_c + i_B i_a + i_C i_b) \sin\left(\frac{\pi}{\tau}x - 120^\circ\right) \right] \right\}. \end{aligned} \quad (14)$$

#### 4. Mathematical model of an induction motor with $\Delta$ connected windings

A scheme of an induction motor with  $\Delta$  connected windings is given in Figure 4.

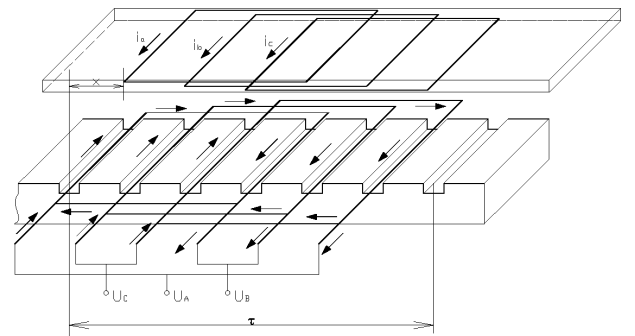


Figure 4. Scheme of an induction motor with  $\Delta$  connected windings

Equations of voltage equilibrium for this circuit are:

$$\begin{cases} u_{AB} = \frac{d\Psi_A}{dt} + R_A i_A \\ u_{BC} = \frac{d\Psi_B}{dt} + R_B i_B \\ u_{CA} = \frac{d\Psi_C}{dt} + R_C i_C. \end{cases} \quad (14)$$

All other equations such as rotor voltage equilibrium, magnetic flux linkages for stator and rotor are the same, as for Y connection with neutral wire. Therefore, a motor is modelled according to the same equations, only the right-hand side vector is expressed as:

$$\mathbf{F}(t, x) = \begin{pmatrix} U_{AB} - R_A i_A \\ U_{BC} - R_B i_B \\ U_{CA} - R_C i_C \\ -R_a i_a \\ -R_b i_b \\ -R_c i_c \\ v \\ F_{em} - F_{st} \end{pmatrix}. \quad (15)$$

### 5. Modelling of a linear induction drive

Equations describing behaviour of a motor are programmed in Matlab program language. The developed program is generalized and fits to model operation of induction motors with different connection of windings. The program can be controlled by Command window row or by user’s graphical interface. It is recommended to use the command line to investigate complex operation modes as reverse or to attach the developed motor model to other programs. User’s graphical interface is applied to investigate the simplest operating modes as starting, steady state because users freedom in graphical interface is limited.

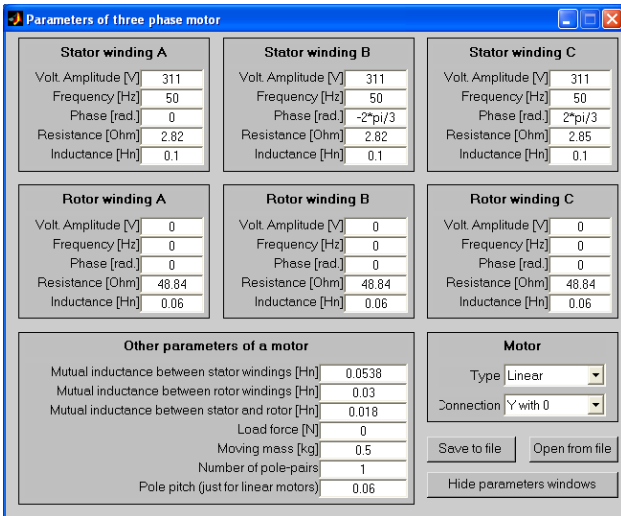


Figure 4. User’s graphical interface window

User’s graphical interface window is shown in Figure 4. All parameters of winding: resistance, inductance, supply voltage are to be set for each winding separately. It is required to the type of a motor (rotating or linear) and connection of stator (inductor) windings.

Figure 5 presents a force developed by LIM at starting when windings are Y connected without neutral wire.

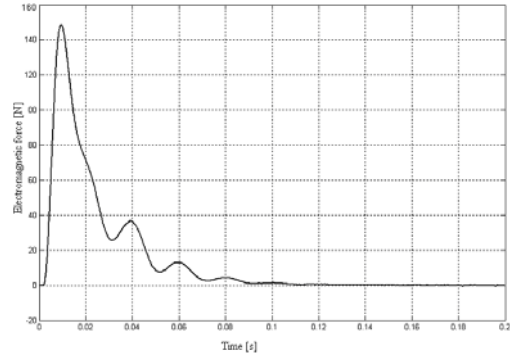


Figure 5. Developed electromagnetic force at starting process

The program gives a possibility to obtain other characteristics, such as speed, inductor and secondary currents, excursion and so on. Currents of secondary at starting no-load LIM are presented in Figure 6.

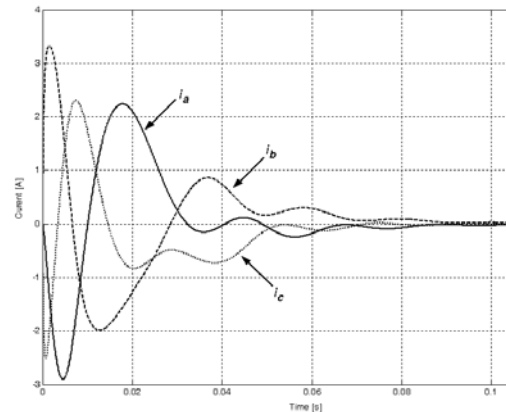


Figure 6. Transient currents of secondary at starting of no-load linear induction motor

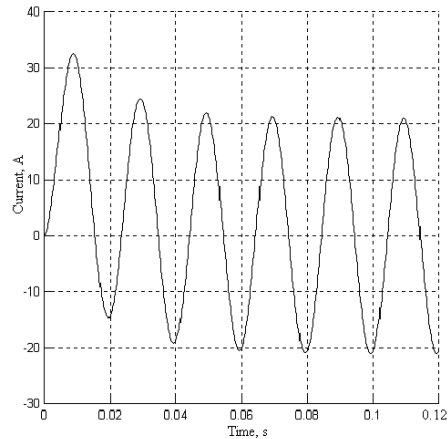


Figure 7. A phase current during starting process at Y connection of windings

All data of simulation are obtained in command line, therefore they can be easily processed and used in new calculations.

## 6. Conclusions

1. The developed model of linear induction motor considers electric and magnetic asymmetry of LIM due to open magnetic circuit of LIM.
2. The developed algorithm and programs give a possibility to investigate transients in the drives with rotating and translating motors at symmetrical and asymmetrical supply voltages, different phase angles between them and different modes of winding connection.
3. Created program gives a possibility to investigate transients of three phase linear drive with consideration of asymmetry of LIM windings.
4. Due to absence transform of coordinates the developed model facilitates investigation of LIM control modes.

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