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DESIGN OF NOVEL SCALAR QUANTIZER MODEL FOR GAUSSIAN SOURCE

Zoran Peric, Jelena Nikolic, Zlatan Eskic, Srbislava Krstic

Faculty of Electronic Engineering, Nis, Serbia

Nebojsa Markovic

Telecom, Serbia

Abstract. This paper proposes the novel model of scalar quantizer that combines two classical models, the model of scalar compandor and the model of Lloyd-Max's scalar quantizer. Particularly, the proposed model utilizes the advantages of the both models while tending to minimize their deficiencies. The performance analysis of the novel quantizer is carried out assuming the Gaussian source of unit variance. It is demonstrated that with the novel model of quantizer near to the Lloyd-Max's optimal performances can be achieved. Moreover, we showed that for a fixed number of quantizaton levels, the average complexity of the novel quantizer is significantly smaller than that of the Lloyd-Max's scalar quantizer.

Keywords: Compandor, Gaussian source, Lloyd-Max's quantizer, Novel model, Scalar quantizer.

1. Introduction

Speech, as the most natural way of communication among people, has a precious value for man, hence, a great attention is paid to secure and reliable digital transmission of speech signals [1, 5, 6, 10]. Speech coding is a procedure to represent a digitalized speech signal using as few bits as possible, maintaining at the same time a reasonable level of signal quality [1, 5, 6, 10]. Although the great number of compression techniques have been developed [1, 5, 6, 10], an evident increase of speech communication subscriber demands, in public digital telephony, mobile telephony, multimedia communications, audio techniques, voice over IP, satellite and military communications, still instigates on the new quantizer models explorations. It is important to point out that the development of the novel quantizer models is usually focused on the maximization of the received speech quality for the particular transmission rate, when high level of the signal to quantization noise ratio (*SQNR*) is needed, or on the minimization of the transmission rate when the high data compression is needed [1, 2, 5, 6, 10].

We decided to focus our novel model development to provide as maximum as possible the signal to quantization noise ratio and as low as possible the average complexity of the quantizer model. Since it is well known that Lloyd-Max's quantizer model of scalar quantizer provides maximal optimal performances for the unit variance case of the input speech signal [1, 2, 6, 8], and since we demonstrated that the scalar compandor model has very low average complexity [3], we decided to start our research from these two classical models of scalar quantizers. Let us recapitulate that the average complexity was defined as an arithmetic mean of arithmetic, memory and implementation complexities [3]. We decided to use here the same definition of the average complexity in order to make the novel model average complexity comparable with the average complexity of the Lloyd-Max's quantizer model and the scalar compandor model. Moreover, in order to point out the vantages of the novel scalar quantizer model, its performances will be compared with the optimal performances (*SQNR*) that correspond to the model of Lloyd-Max's quantizer [6].

Let us recall that the non-optimal performances are the main deficiency of the scalar compandor. It is well known that non-optimality of the scalar compandor is caused by the rough approximation of the input signal in the region of high amplitudes [1, 6, 10]. However, the vantage of this quantizer model repose in its low complexity, which as we demonstrated in [3], is significantly lower than the average complexity of Lloyd-Max's quantizer model. Considering the fact that Lloyd-Max's quantizer model provides optimal performances, we get an idea to use a suitable combination of both models in order to develop a novel model representing the compromise between average complexity and relative distortion error. Finally, since the short-term statistics of speech signals are modeled by Gaussian source [1, 6, 10], we decided to design novel scalar quantizer assuming mentioned source, as well as to explore if the performances improvement of the novel scalar quantizer model over the compandor model can be achieved.

2. Scalar Quantization

The function of scalar quantizer is to transform instantaneous value of an input signal, which belongs in general to countless set of values of continual amplitude range, to the nearest allowed value from the discrete set of N amplitudes [1, 6]. Particularly, scalar quantizer Q with N quantization levels can be defined with Q: $R \rightarrow Y$, as a functional mapping of the set of real numbers R onto the set of the output representation [1, 6]. The set of the output representation constitutes the code book Y having the size N:

$$Y \equiv \{y_1, y_2, y_3, ..., y_N\} \subset R . \quad (1)$$

It is well known that scalar quantizer is unique determined with output values y_j, j=1, 2, ..., N also called output representation levels, and with partition of the input range of values onto N cells, i.e., intervals α_j, j=1, 2, ..., N [1, 6]. These cells are defined with the decision thresholds $\{t_0, t_1, \ldots, t_N\}$, such that $\alpha_j=(t_{j-1}, t_j]$, j=1, 2, ..., N. Hence, a quantized signal has value y_j when the original signal belongs to the quantization cell α_j [1, 6]:

$$Q(x) = y_j, \quad x \in \alpha_j . \quad (2)$$

Note that symmetrical probability density function of the source signal results in symmetry of the decision thresholds and the representation levels [1, 6]. This fact will be considered when performing novel scalar quantizer design.

2.1. Scalar Compandor

The model of a nonuniform scalar quantizer consisting of a compressor, a uniform quantizer and expandor in cascade, is called a scalar compandor [1, 6]. Particularly, the non-uniform quantization is achieved by compressing the input signal x, then quantizing it with a uniform quantizer and expanding the quantized version of the compressed signal using a non-uniform transfer characteristic inverse to that of the compressor.

We decided to define compresor function $c(x)$ at decision thresholds t_j, j=0, 1, ..., N, similarly as it was done in [7]:

$$c(t_j) = -1 + 2\frac{\int_{-\infty}^{t_j} p^{1/3}(x)dx}{\int_{-\infty}^{+\infty} p^{1/3}(x)dx}, \; j = 0,...,N . \quad (3)$$

Note that compresor function $c(x)$, at decision thresholds t_j, j=0, 1, ..., N can also be defined as follows [6]:

$$c(t_j) = -1 + \frac{2j}{N} . \; j = 0,...,N . \quad (4)$$

From the last two equations, it is obvious that, using such defined compresor function the compresson of an input signal x from range (-∞, ∞) to range [-1, 1] is enabled. Furthermore, combining Eqs. (3) and (4), the following set of equations is derived:

$$\frac{\int_{-\infty}^{t_j} p^{1/3}(x)dx}{\int_{-\infty}^{\infty} p^{1/3}(x)dx} = \frac{j}{N}, \; j = 0,...,N , \quad (5)$$

from which it is posible to determine decision thresholds t_j, j=1, 2, ..., N-1. Since we assume infinity range of an input signal it is obvious that thresholds t_0 and t_N should have the infinity values t_0=-∞ and t_N=∞. Finally, we can determine representation levels from the following set of equations:

$$\frac{\int_{-\infty}^{y_j} p^{1/3}(x)dx}{\int_{-\infty}^{\infty} p^{1/3}(x)dx} = \frac{\left(j - \frac{1}{2}\right)}{N}, \; j = 1,...,N . \quad (6)$$

Observe that finding of the mentioned decision thresholds and representation level values is actually equivalent to scalar compandor design.

2.2. Lloyd-Max's Algorithm

Lloyd and Max independently proposed an algorithm to compute optimal quantizers using mean-square error distortion measure [8]. Particularly, Lloyd and Max have shown that necessary conditions of decision thresholds and representation levels optimality can be given as follows [6]:

Condition 1. Every representation level y_j, j=1, ..., N should be centroid of the probability density function in the appropriate interval $\alpha_j=(t_{j-1},t_j]$:

$$y_j = \frac{\int_{t_{j-1}}^{t_j} xp(x)dx}{\int_{t_{j-1}}^{t_j} p(x)dx}, \; j = 1,...,N . \quad (7)$$

Condition 2. Decision thresholds t_j, j=1, ..., N-1 should be halfway between the neighbouring representation levels:

$$t_j = \frac{(y_j + y_{j+1})}{2}, \; j = 1,...,N-1 . \quad (8)$$

Namely, Lloyd-Max's algorithm is based on searching the best code book correspondig to the best partition of quantizer range. Note that Lloyd-Max's algorithm starts with an estimate of the decision thresholds and the representation levels, and the convergence is better if the estimate is better [4, 11]. Here we decided to provide better convergence of the Lloyd-Max's algorithm by using the idea proposed in [4, 11]. In general, Lloyd-Max's algorithm is comprised of the following steps:

Step 1: The code book and distortion initializations.

Step 2: For the given code book, application of the decision thresholds and the representation levels optimality conditions while generating the improved code book.

Step 3: Estimation of the quantizer distortion and checking the stopping criteria.

3. Novel Model of Scalar Quantizer

Let us assume that the novel model of scalar quantizer divides the quantizer amplitude range into three mutually disjoint regions denoted as R_1, R_2 and R_3. Further, let us assume that union of regions R_1 and R_3 represents outer quantizer region which consists of $2L$ outer cells $\alpha_1,...,\alpha_L$ and $\alpha_{N-L+1},\ldots,\alpha_N$. Moreover, let us suppose that region R_2 represents inner quantizer region which comprises range $(-t_{N-L}, t_{N-L})$ and contains N-$2L$ inner cells, $\alpha_{L+1},\ldots,\alpha_{N-L}$. Observe that $L<<N$ is the number of cells in the region R_1 as well as in the region R_3. Particularly, in this paper we propose applying of the compandor model on the inner region design and the Lloyd-Max's model on the outer region design. It is important to notice that the novel model is actually the generalized model since it presents the compandor model for L=0 while for L=N/2 it presents the Lloyd-Max's quantizer model.

Here, we destined to initialize the Lloyd-Max's algorithm with the code book obtained from Eq. (6). Namely, we have already demonstrated in [4, 11] that Lloyd-Max's algorithm stops after only one iteration when the compandor's code book is used as initial.

Note that the one of the paper goals is to provide the novel scalar quantizer model that reduces average complexity of the Lloyd-Max's scalar quantizer. Recall that we alredy performed the average complexity analysis of the compandor model and the model of Lloyd-Max's quantizer [3], where we defined average complexity as arithmetic mean value of arithmetic, memory and implementation complexities. Moreover, the arithmetic complexity was defined by the number of arithmetical/logical operations per sample needed for implementation of encoding and decoding procedure (quantation procedure), and the memory complexity was defined by the memory size needed for storing parameters of the considered scalar quantizer model, and finally, the implementation complexity was defined by the number of digital circuits needed for hardware realization of the considered scalar quantizer model, i.e. defined by the number of instructions needed for software realization of scalar quantizer. Recall that we demonstrated in [3] that the average complexity of the compandor model with N quantization levels, denoted by K^C, can be determined from the following expression:

$$K^C = \frac{1}{3}(8N + 10), \tag{9}$$

while the average complexity of Lloyd-Max's quantizer with N quantization levels, denoted by K^{LM}, can be determined from:

$$K^{LM} = \frac{1}{3}(10N + 3). \tag{10}$$

Owing to the fact that the novel model presents a combination of compandor model with N-$2L$ quantization levels and Lloyd-Max quantizer model with $2L$ quantization levels, its average complexity, denoted by K^N, can be simply determined by combining Eqs. (9) and (10):

$$K^N = \frac{1}{3}(8N + 4L + 13), \tag{11}$$

and therefore comparision with the mentioned classical models can be provided.

4. Quantizer Performances

Performances of quantizers are often specified in terms of $SQNR$ (signal to quantization noise ratio), that can be evaluated from [1, 6]:

$$SQNR = 10\log_{10}\left(\frac{\sigma^2}{D}\right), \tag{12}$$

where σ^2 denotes the variance of the input signal x and D is distortion introduced during quantization process. In order to make performances of the designed novel quantizer comparable with theoretical optimal performances, we assume unit variance case of the signal to be quantized. In the mentioned case, signal to quantization noise ratio actually depends only on distortion:

$$SQNR = 10\log_{10}\left(\frac{1}{D}\right). \tag{13}$$

Let us define now deviation of the qunatizer's distortion in respect to optimal distortion, i.e. let us introduce relative distortion error δ [1, 6]:

$$\delta = \frac{D - D^{opt}}{D^{opt}}, \tag{14}$$

where D^{opt} represents the optimal value of distortion corresponding to the Lloyd-Max's quantizer having N quantization levels. It is obvious that the relative distortion error can also be expressed as:

$$\delta = 10^{-\frac{\Delta SQNR}{10}} - 1, \tag{15}$$

where:

$$\Delta SQNR = SQNR - SQNR^{opt}, \quad (16)$$

and $SQNR^{opt}$ denotes the optimal value of the signal to quantization noise ratio.

Considering that the novel model presents a combination of compandor model with N-2L quantization levels and Lloyd-Max's quantizer with 2L quantization levels, we can now provide expression for determination of the total distortion introduced by the novel model during quantization procedure:

$$D = \frac{1}{12(N-2L)^2}\left(\int_{-t_{N-L}}^{t_{N-L}} p^{1/3}(x)dx\right)^3 + 2\sum_{j=N-L}^{N-1}\int_{t_j}^{t_{j+1}}(x-y_{j+1})^2 p(x)dx, \quad (17)$$

where the first term represents distortion of the compandor model while the second term represents Lloyd-Max's quantizer distortion. Since the good approximation of the compandor distortion can be achieved by using Bennett's integral, here we use such integral assuming the range [$-t_{N-L}$, $+t_{N-L}$] for the N-2L compandor [9]. Finally, since we decided to consider Gaussian source of unit variance, distortion as well as signal to quantization noise ratio will be evaluated taking into acount the following probability density function [1, 6]:

$$p(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right). \quad (18)$$

5. Numerical Results

Numerical values for *SQNR* determined for the different quantization level numbers N (N=32 and N=64) with linear change of value L within range 0 to 4, are shown in Table 1. Comparing these values with the theoretical optimal values of signal to quantization noise ratio shown in Table 2 [6], one can notice that performances of the novel scalar quantizer approach optimal performances by increasing the number of the outer cells. In order to provide better insight in the performances of the novel quantizer, the numerical values of relative distortion error are listed in Table 3. Introducing a criterion which states that practically acceptable relative distortion error has to be less than 0.01, from Table 3 it is easy to notice that such a criterion is satisfied for L=4. Combining this fact with the results given in Table 4, from which one can notice that the novel model provides significant complexity reduction over Lloyd-Max's quantizer model, we can reasonably believe that quantizer solution with L=4 can easily find the way to its practical implementation.

Table 1. Numerical values of *SQNR* for different number of quantizer levels N with L within range 0 to 4

SQNR	N=32	N=64
L=0	25.756	31.777
L=1	25.898	31.848
L=2	25.932	31.865
L=3	25.947	31.872
L=4	25.956	31.877

Table 2. Optimal values of *SQNR* for different number of quantizer levels (N=32, N=64)

	N=32	N=64
$SQNR^{opt}$	26.01	31.89

Table 3. Relative distortion error δ

δ	N=32	N=64
L=0	0.060	0.026
L=1	0.026	0.010
L=2	0.018	0.006
L=3	0.015	0.004
L=4	0.010	0.003

Table 4. Comparison of average complexities for the Lloyd-Max quantizer model, scalar compandor model and novel scalar quantizer model

	K^{LM}	K^{C}	K^{N}			
			L=1	L=2	L=3	L=4
N=32	107.7	88.7	91	92.3	93.7	95
N=64	214.3	174	176.3	177.7	179	180.3

6. Conclusion

Combining the Lloyd-Max's scalar quantizer model with the scalar compandor model we developed the novel scalar quantizer model which presents a compromise between design complexity and deviation of the qunatizer's distortion in respect to optimal distortion. Importance of this model development stands in the fact that it represents generalized scalar quantizer model, which for L=0 presents compandor model while for L=N/2 presents Lloyd-Max's quantizer model. Moreover, the importance of the novel model development is additionaly propound via complexity reduction over the Lloyd-Max's scalar quantizer model while keeping the performances near to the optimal. In respect to the near optimal performances, we believe that the novel quantizer model can reasonably find the way to its practical implementation.

Consequently, the novel model provides justification for near-optimal and complexity-reducing design strategies of scalar quantizers.

References

[1] **A. Gersho, R. M. Gray.** Vector Quantization and Signal Compression. *Kluwer Academic Publishers, Boston, Dordrecht, London*, 1992.

[2] **J. Max.** Quantizing for minimum distortion. *IEEE Transactions on Information Theory, Vol.*6, *No.*1, 1960, 7-12.

[3] **J. Nikolic, Z. Peric, D. Pokrajac.** Average complexity analysis of scalar quantizer design. *Proc. of the* 6*th WSEAS International Conference on Telecommunications and Informatics* (*TELE-INFO* '07), *Dallas, Texas, USA*, 22-24 *March*, 2007, 22-27.

[4] **J. Nikolic, Z. Peric.** Lloyd-Max's algorithm implementation in speech coding algorithm based on forward adaptive technique. *Informatica, Vol*. 19, *No*.2, 2008, 255-270.

[5] **M.H. Johnson, A. Alwan.** Speech coding: Fundamentals and applications. *In J. Proakis (Ed.), Wiley Encyclopedia of Telecommunications* 5, *John Wiley* & *Sons, New York*, 2002, 2340-2359.

[6] **N. Jayant, P. Noll.** Digital Coding of Waveforms, Principles and Applications to Speech and Video. *Prentice Hall Secondary Education Division, New Jersey*, 1984.

[7] **N. Judell, L. Scharf.** A Simple Derivation of Lloyd's Classical Result for the Optimum Scalar Quantizer. *IEEE Transactions on Information Theory, Vol.*32, *No*.2, 1986, 326-328.

[8] **S. Lloyd.** Least squares quantization in PCM. *Unpublished memo., Bell Lab*.,1957; *IEEE Transactions on Information Theory, Vol.*28, *No*.2, 1982, 129-137.

[9] **S. Na, D.L. Neuhoff.** Bennett's Integral for Vector Quantizers. I*EEE Transactions on Information Theory*, *Vol*.41, *No*.4, 1995, 886-900.

[10] **W. C. Chu.** Speech coding algorithms: Foundation and Evolution of Standardized Coders. *John Wiley & Sons, New Jersey*, 2003.

[11] **Z. Peric, J. Nikolic.** An effective method for initialization of Lloyd-Max's algorithm of optimal scalar quantization for Laplacian source. *Informatica, Vol*. 18, *No*.2, 2007, 279-288.

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