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PROCEDURE FOR THE CONSTRUCTION OF DIGITAL SHADOW MOIRE IMAGES FOR THE ANALYSIS OF BENDING VIBRATIONS OF A PLATE

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Abstract. The procedure for the generation of digital stroboscopic shadow moiré images for the eigenmodes of bending vibrations of a plate is developed. It is based on the methods of computer graphics and the method of finite elements for the analysis of bending vibrations of a plate. The construction of digital shadow moire images builds the ground for hybrid numerical-experimental procedures and enables to analyse the experimental results with greater precision.

Keywords: digital image, computer graphics, finite elements.

1. Indroduction

The problem of bending vibrations is common in different engineering and physical applications. Bending vibrations of centrally clamped rotating circular disks play crucial role in functionality of hard disk drives [1, 2]. Lots of efforts are spent for dynamic stabilisation, control and measurement of bending vibrations in such micromechanical systems [3].

Nevertheless, measurement of microscopic deflections from the state of equilibrium is a challenging problem. Different optical measurement techniques are developed for experimental investigation of bending vibrations [4]. Shadow moire is one of the popular methods for experimental analysis of bending vibrations of structures.

Unfortunately, interpretation of experimental measurement results is a nontrivial inverse engineering problem often having non-unique solutions [5]. Therefore there exists a definite need for hybrid numerical – experimental techniques [6, 7] that could help to interpret the measurement results. Such techniques usually comprise a numerical model of the system coupled with optical and geometrical parameters of the measurement set-up. Then the predicted response of experimental optical measurement system can be mimicked in virtual numerical environment when the dynamical parameters of the analysed object are predefined.

Visualisation techniques of the results from finite element analysis procedures are important due to several reasons. First of all meaningful and accurate is representation of processes taking place in the analysed structures [8]. Secondly, and perhaps even more important, is developing the background for hybrid numerical – experimental techniques. A typical example of FEM application in developing a hybrid technique is presented in [7].

2. Optical scheme of measurement

The principle of the shadow moire analysis [4] is shown in Fig. 1. x, y and z are the orthogonal Cartesian axes of coordinates (y axis is not shown in the figure for the sake of simplicity). The surface of the plate in the status of equilibrium is in the plane $z = -d$, here d is the distance between the moire grating and the plate in the status of equilibrium. Moire grating is in the plane $z = 0$ and the photographic plate is parallel to this plane. The deflection of the plate is w. α is the angle between the z axis and the direction of the parallel incident rays of light. u is the shift of the shadow moiré grating in the direction of the x axis with respect to the initial moiré grating.

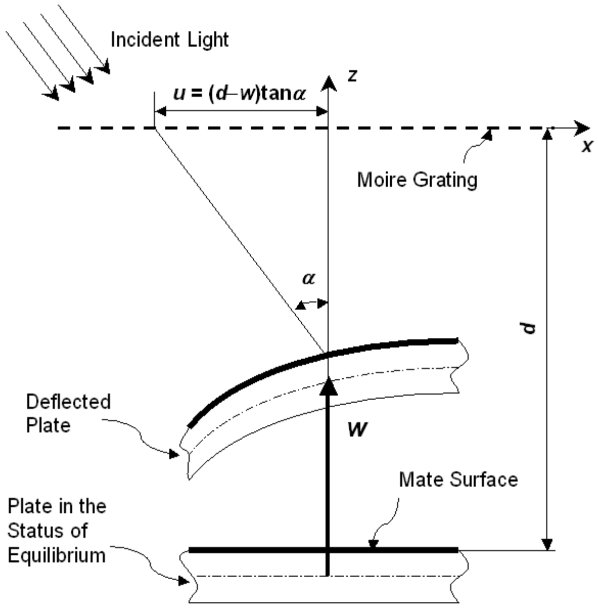


Figure 1. The schematic of vibration analysis by shadow moiré.

External excitation of linear elastic structures leads to dynamic response, which is directly related to the eigenmodes of the structure. Therefore analysis of natural vibrations taking place according to the eigenmode of the structure is important.

The developed technique for the construction of digital images is applicable in hybrid numerical - experimental shadow moire analysis using the stroboscopic method.

3. Procedure for construction of the digital images

First of all the eigenmodes for the structure are calculated by using the displacement formulation common in the finite element analysis. The plate bending element with the independent interpolation of the displacement w and the rotations about the appropriate axes θ_x and θ_y is used [9]. It is assumed that the structure performs vibrations according to the eigenmode (the frequency of excitation is about equal to the eigenfrequency of the corresponding eigenmode and the eigenmodes are not multiple). The vibrations of the structure are registered stroboscopically when the structure is in the state of extreme deflections according to the eigenmode.

Then the moiré images are produced [4] assuming that the shadow displacement in the direction of the x axis u takes the following value:

$$u = (d - w)\tan\alpha\,. \tag{1}$$

For the more general case it is assumed that the grating makes an angle β with the x axis. The parallel rays of light turn together with the grating. Then the intensity of the moiré image takes the form:

$$I = \frac{1}{2}\left(\begin{array}{l}\cos^2\left(\dfrac{2\pi}{\lambda}(x\cos\beta + y\sin\beta)\right) + \\ + \cos^2\left(\dfrac{2\pi}{\lambda}\left(\begin{array}{l}x\cos\beta + y\sin\beta - \\ -(d-w)\tan\alpha\end{array}\right)\right)\end{array}\right), \tag{2}$$

where λ is the constant describing the distance between the lines of the grating.

The nodal variables of the plate bending element are the deflection of the plate w, the rotation of the plate about the x axis Θ_x and the rotation of the plate about the y axis Θ_y (it is assumed that $v = -z\Theta_x$ and $u = z\Theta_y$, where u and v are the displacements of the plate in the x and y directions). The stiffness matrix of the plate bending element takes the form:

$$[K] = \iint\left([B]^T[D][B] + [\overline{B}]^T[\overline{D}][\overline{B}]\right)dxdy\,, \tag{3}$$

where:

$$[D] = \frac{h^3}{12}\begin{bmatrix}\dfrac{E}{1-\nu^2} & \dfrac{E\nu}{1-\nu^2} & 0 \\ \dfrac{E\nu}{1-\nu^2} & \dfrac{E}{1-\nu^2} & 0 \\ 0 & 0 & \dfrac{E}{2(1+\nu)}\end{bmatrix},$$

$$[\overline{D}] = \frac{Eh}{2(1+\nu)k_s}\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix},$$

$$[B] = \begin{bmatrix}0 & 0 & \dfrac{\partial N_1}{\partial x} & \dots \\ 0 & -\dfrac{\partial N_1}{\partial y} & 0 & \dots \\ 0 & -\dfrac{\partial N_1}{\partial x} & \dfrac{\partial N_1}{\partial y} & \dots\end{bmatrix},$$

$$[\overline{B}] = \begin{bmatrix}\dfrac{\partial N_1}{\partial y} & -N_1 & 0 & \dots \\ \dfrac{\partial N_1}{\partial x} & 0 & N_1 & \dots\end{bmatrix}, \tag{4}$$

and E is the modulus of elasticity; ν – the Poisson's ratio; h – the thickness of the plate; k_s=1.2 – the shear correction factor [9]; N_i – the i-th shape function of the finite element.

The mass matrix takes the following form:

$$[M]=\iint [N]^T \begin{bmatrix} \rho h & 0 & 0 \\ 0 & \frac{\rho h^3}{12} & 0 \\ 0 & 0 & \frac{\rho h^3}{12} \end{bmatrix} [N] dxdy\,, \qquad (5)$$

where:

$$[N]=\begin{bmatrix} N_1 & 0 & 0 & \dots \\ 0 & N_1 & 0 & \dots \\ 0 & 0 & N_1 & \dots \end{bmatrix}, \qquad (6)$$

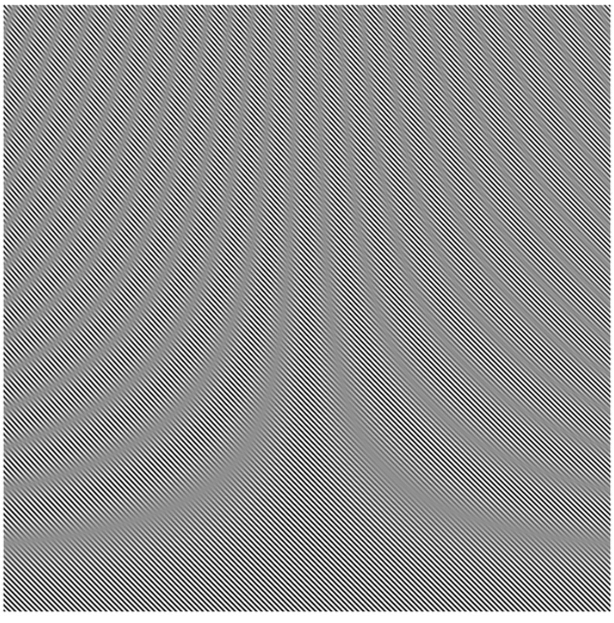
and ρ is the density of the material of the plate.

The relationships presented above form the basis for the generation of the images described below. Other steps of the numerical procedure for the construction of digital images are described in detail in [10].

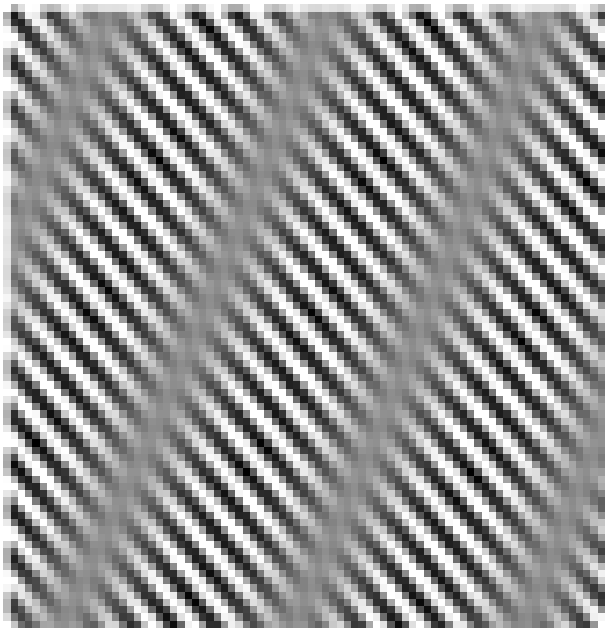
4. Numerical results

The most often analysed problem is that of the bending vibrations of a plate. Here the plotting procedure is developed for shadow moiré images of the eigenmodes of a plate.

The rectangular cantilever plate with the fixed lower edge is analysed. The digital stroboscopic shadow moire image of the second eigenmode when $\beta=\frac{\pi}{4}$, $d = 1$ and $\tan\alpha = 1$ is shown in Fig. 2. This rectangular plate serves as a good model problem for the validation of the results of analysis.



***a*)**



***b*)**

Figure 2. The digital stroboscopic shadow moire image of the second eigenmode of the cantilever plate: a) full field view; b) zoomed image of the upper left corner

Further a more complicated problem is analysed: a circular plate with the fixed internal radius. The perspective projection of the finite element mesh for the tenth eigenmode is shown in Fig. 3. The mesh in the status of equilibrium is grey; and deflected according to the eigenmode is black.

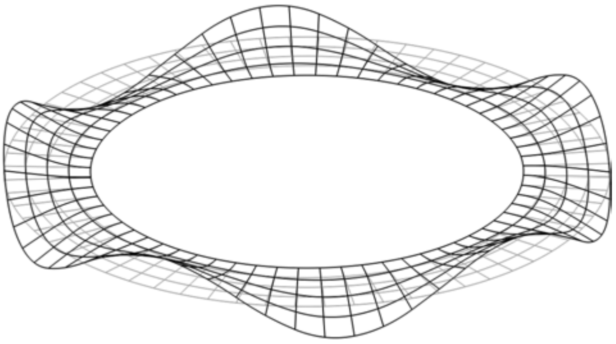


Figure 3. The perspective projection of the tenth eigenmode of the circular plate (the mesh in the status of equilibrium is grey and deflected according to the eigenmode is black).

Isolines of the displacement of the plate for the tenth eigenmode are shown in Fig. 4.

The digital stroboscopic shadow moire image of the tenth eigenmode is shown in Fig. 5. The correspondence of the character of shadow moiré fringes with the isolines of the previous drawing is evident.

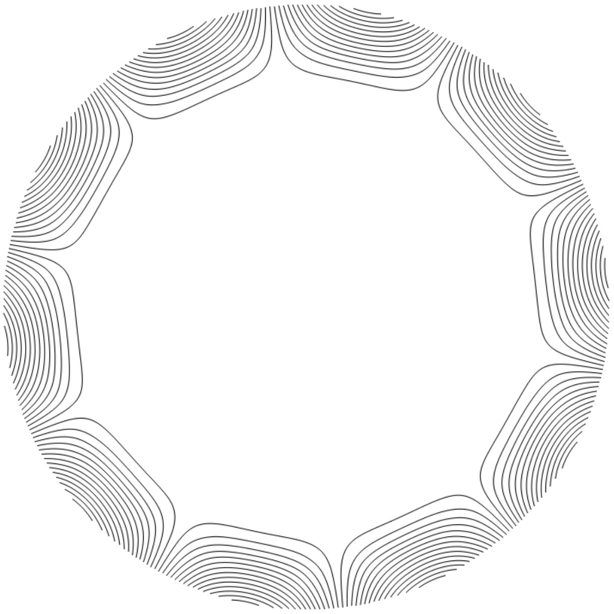


Figure 4. Isolines of the displacement of the circular plate for the tenth eigenmode

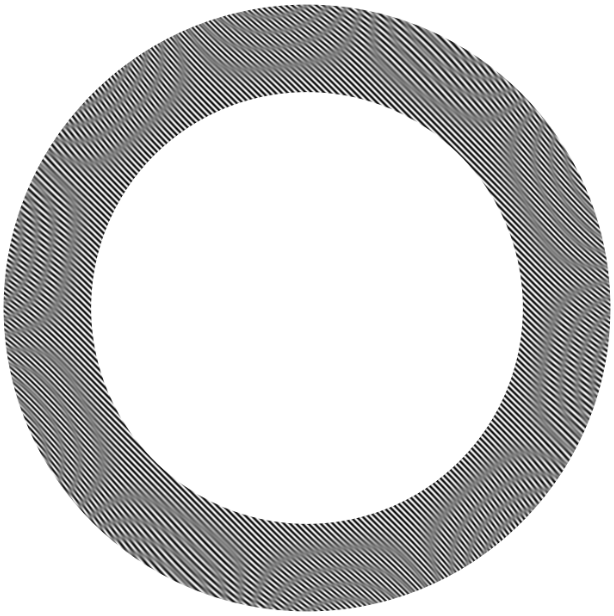


Figure 5. The digital stroboscopic shadow moire image of the tenth eigenmode of the circular plate

5. Conclusions

The construction of digital shadow moire images builds the ground for hybrid numerical -experimental procedures for effective solution of inverse optical engineering problems. The procedure for the generation of digital stroboscopic shadow moiré images of the eigenmodes of bending vibrations of a plate is developed. Shadow moiré analysis has the advantage over other types of moiré techniques for the analysis of bending behaviour of structural elements because it represents the displacement (not its derivative) as the analysed quantity.

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