ISSN 1392 – 124X INFORMATION TECHNOLOGY AND CONTROL, 2005, Vol.34, No.1

HYBRID FUZZY MODEL OF NONLINEAR PLANT

Raimundas Liutkevičius, Saulius Dainys

Vytautas Magnus University
Vileikos 8, LT-3035 Kaunas, Lithuania

Abstract. This paper presents predictive fuzzy modeling of nonlinear plant. Three different fuzzy models are presented, where the first one is synthesized using Mamdani type rule base, the second – Takagi-Sugeno type rule base. The experimental results show advantages and disadvantages of the presented models and their adequacy to the nonlinear plant. A hybrid modeling approach was offered to eliminate the disadvantages of the Mamdani and the Takagi-Sugeno fuzzy models. The advantages of the hybrid model are experimentally proved and presented in this paper.

1. Indroduction

Conventional system modeling methods are not easy to use for nonlinear processes because it is difficult to describe properly all their nonlinearities. One of the alternative ways to model nonlinear process is to use fuzzy methods. There are many advanced modeling techniques such as white box modeling, black box modeling, fuzzy logic modeling etc. [1] and in this paper fuzzy modeling technique with implicit use of prior knowledge is analyzed. In this paper two well known approaches are compared together to show their advantages and disadvantages and according to them a third approach is proposed to eliminate the disadvantages of the first two. The proposed algorithm is also invertible and locally liner so it can be used in the model based control, model predictive control tasks. The model can also be used in the diagnostics of the processes to identify the faulty operation conditions.

2. Nonlinear plant

The plant's scheme is shown in Figure 1. Its central part is a close tank with adjustable water level within the range from 0 to 25 cm. The "level" variable of the process can be varied using water pump (1). The pump is the actuator and has an electrical input-range of 0 to 10 V. The tank has two outlets for water flow. The manual valve (3) and/or the combination of the magnetic valve (2) and manual valve (2a) control the exit water flow. These valves and the control of the water pump manipulate the stationary condition of water flow. The water flows in and out of the tank through rubber hoses, what are circled in rings. This water flow peculiarity increases plant's nonlinear characteristics. The pumps have dead zones of different magnitudes and saturation non-linearity; they introduce electrical noises and delays into the system. The water flow also depends on the water temperature and its softness, what makes the modeling task more difficult.

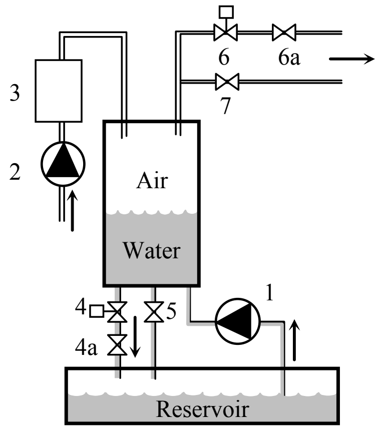

Figure 1. Plant's structure

3. Fuzzy modeling

Nonlinear object's model is synthesized using fuzzy modeling methods. In general case fuzzy model has these main components: fuzzification of inputs, inference mechanism with rule base that relates inputs to outputs and defuzzification of output fuzzy set for crisp output calculation, Figure 2 [4].

3.1. Fuzzification

Fuzzification maps the crisp values of the preprocessed inputs of the model into fuzzy sets, represented by membership functions. The degree of membership of a single crisp variable to a single fuzzy set is evaluated using membership function and can get the values from an interval [0, 1]. Each input variable in most cases is described by more than three fuzzy sets.

Because of simple calculations, triangular [2] membership functions are usually used in fuzzy systems. In this paper for the fuzzification of inputs, triangular membership functions are used. Other types like *Gaussian*, *trapezoidal*, *S-type* membership functions sometimes have the advantages over triangular but the choice depends on the application.

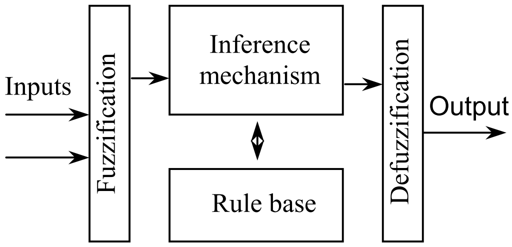

Figure 2. SISO fuzzy system

3.2. Rule base

The relationship between input and output variables are described in a rule base, composed of IF… THEN form rules. Usually fuzzy systems are synthesized using two types of rules that differ in the consequent (THEN part) proposition form: *Mamdani*, or standard [4] and *Takagi-Sugeno*, or functional [1].

The rules in standard fuzzy systems (*Mamdani*) have the form:

R_i: IF x_1 is A^j_1 and…and x_n is A^j_n THEN y is B^j (1)

where R_i denotes the i-th rule, $i = 1 \ldots N_r$, where N_r is the number of rules. x_n is the n-th input to the fuzzy system, A^j_n and B^j are fuzzy sets described by membership functions $\mu_{Ai,j}(x_i){:}\rightarrow[0,1]$ and $\mu_{Bi}(y){:}\rightarrow[0,1]$. The propositions in the *IF* part of the rule are combined by applying minimum operators. Sometimes product is calculated, but it mostly depends on the situation. The number of prepositions in the consequence part of the rule depends on the number of outputs of the fuzzy system.

The rules in functional fuzzy systems (*Takagi-Sugeno*) have the form:

R_i: IF x_1 is A^j_1 and…and x_n is A^j_n THEN y is $f_j(x)$ (2)

where $f_j(y)$ is a crisp function of the input variables, rather than a fuzzy proposition [6]. For the particular application the effectiveness of the fuzzy system in most cases depends on the order of the function.

When the number of rules and their form in a *Mamdani* type fuzzy system are fixed, the parameters that have to be tuned are input and output membership functions, their universes of discourse and the shape of membership functions. In case of *Takagi-Sugeno* fuzzy system, the parameters are the membership functions of the inputs and the coefficients of the function in the rule's consequent part. In this paper both types will be used in the fuzzy models and compared together.

3.3. Inference engine

Inference mechanism calculates the degree to which each rule fires for a given fuzzified input by considering the rules. A rule fires when the degree of membership of the *IF* part is higher than 0. The firing strength, β_j of the rule in this paper is determined using the product of degrees of memberships calculated according to the formula:

$$\beta_j = \prod_{i=1}^{n} A_{i,j} , \tag{3}$$

where $A_{i,j}$ defines the membership function on input i, used in a rule j. Logical *and* operator is also widely used because it is easier to calculate the result.

3.4. Defuzzification

A defuzzifier compiles the information provided by each of the rules and makes a decision from this basis. The are different methods for the calculation of crisp output of fuzzy system like Centroid average (CA), Center of gravity (COG), Maximum center average (MCA), Mean of maximum (MM), Smallest of maximum, etc. but in this paper Center of gravity defuzzification methods for *Takagi-Sugeno* and *Mamdani* fuzzy systems are used. The crisp output of *Takagi-Sugeno* fuzzy system is calculated according to the formula:

$$u^{crisp} = \frac{\sum_{i=1} f_i(x) \prod_j A_{i,j}}{\sum_i \prod_j A_{i,j}} , \tag{4}$$

where f_j(x) used in this paper is defined as:

$$f_i(x) = a_{i,0} + a_{i,1} \frac{1}{(e^{x_1})^2} + a_{i,2} \frac{1}{(e^{x_2})^2} , \tag{5}$$

where a is the vector of polynomial coefficients, x_p – inputs to fuzzy system. It can be seen that the COG method of defuzzification takes a weighted sum of the designated consequences of the rules according to firing strengths of the rules.

The crisp output for *Mamdani* type fuzzy system is calculated according to formula:

$$u^{crisp} = \frac{\sum_{i=1} b_i \cdot \prod_j A_{i,j}}{\sum_i \prod_j A_{i,j}} , \tag{6}$$

where b_i is the center of the output's i-th membership function.

4. Fuzzy models of nonlinear plant

In this paper a predictive fuzzy model is synthesized and trained using identification. When the prediction error overpasses the defined limits, the process of parameter identification of the model is reapplied. New parameters of the changed process are learned using the recursive least square method. This method is selected because [3]:

- Measurement data in adaptive models are acquired periodically.
- It is desirable that calculations are done recursively, for time saving
- Estimations $\theta(t)$ are dependent on the previous estimations result.
- Recursive estimators track time varying parameters.
- Can be used in fault tracking algorithms.

The recursive least squares method is defined as:

$$P(k) = \frac{1}{\lambda}\Big(I - P(k-1)\xi(x^k)(\lambda I + (\xi(x^k))^T \cdot P(k-1)\xi(x^k))^{-1}(\xi(x^k))^T\Big)P(k-1) \tag{7}$$

$$\theta(k) = \theta(k-1) + P(k)\xi(x^k)(y^k - (\xi(x^k))^T\theta(k-1)), \tag{8}$$

where vector $\xi(x)$ for *Mamdani* type fuzzy systems is:

$$\xi(x) = [\xi_1(x)...\xi_k(x)] \tag{9}$$

for *Takagi-Sugeno* type fuzzy systems is [4]:

$$\xi(x) = [\xi_1(x)...\xi_R(x), x_1\xi_1(x)...x_1 \xi_R(x),...,x_n\xi_1(x)...x_n\xi_R(x)], \tag{10}$$

where

$$\xi_i(x) = \frac{\prod_{j=1}^{N}\mu_j}{\sum_{i=1}^{R}\prod_{j=1}^{N}\mu_j} \tag{11}$$

θ is a vector of estimated parameters, where θ = [$a_{1,0}$, $a_{2,0}$, … $a_{R,0,...}$ $a_{1,n}$, $a_{2,n}$, … $a_{R,n}$] for *Takagi-Sugeno* type fuzzy systems, and θ = [b_1, b_2, … , b_3] for *Mamdani* type fuzzy systems [5].

N is the number of inputs, R – the number of fuzzy rules, I – unity matrix, and λ – obliteration factor, which is not taken into consideration in this paper. The initial conditions for the parameter estimation method are set to zeros. The recursive least squares method allows to identify the appropriate coefficients of the function $f(x)$, equation (5).

Fuzzy models of the selected nonlinear plant have two inputs: voltage, u, applied to the water pump and the predicted water level y'_{t-1} at time $t-1$ and one output – the predicted current water level y' in the tank, Figure 3.

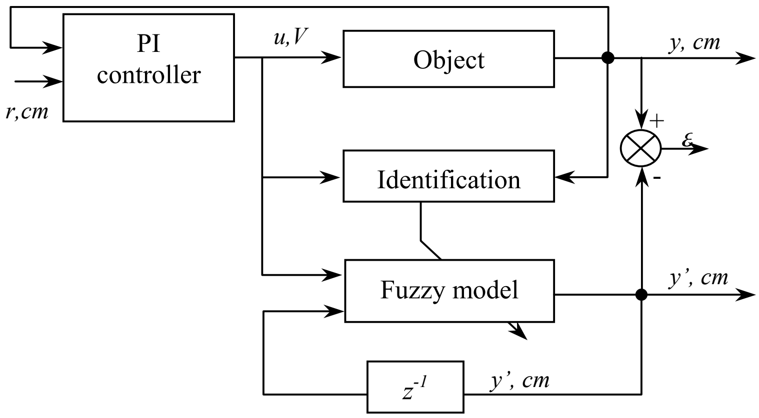

Figure 3. Scheme of fuzzy model

4.1. *Mamdani* type fuzzy model

The structure of the *Mamdani* type fuzzy model is presented in Figure 4. The processes in the water tank are separated into three modes – the inflow of water to the tank, the outflow of water from the tank and the balance of the desired water level in the tank as the water always flow out of the tank through the holes at the bottom of the tank so in order to balance the water at the desired level continuous water inflow is required. These modes are evaluated in fuzzy model as separate fuzzy sub-models. Separation of modes decreases the number of rules in the fuzzy controller and increases the quality of the model. The output of the fuzzy model is the sum of the outputs of the sub-models.

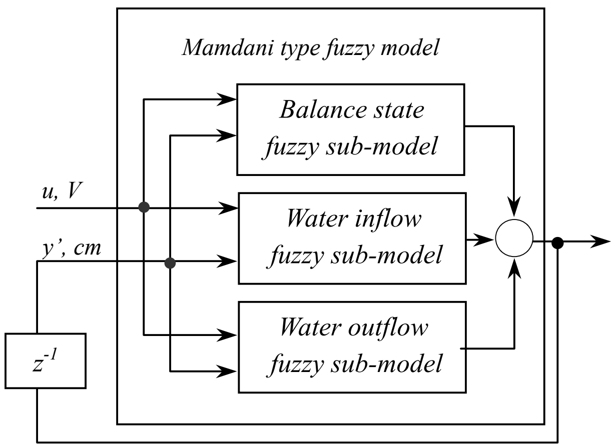

Figure 4. *Mamdani* type fuzzy model structure

Fuzzy sub-model of the water level balance at the desired set point

This sub-model has two inputs one output. The universe of discourse of the first input is defined by 11 triangular membership functions, and the universe of discourse of the second input – by 201 triangular membership functions, uniformly spread across the universe of discourse. According to this, the rule base is a matrix with dimensions 11x201 and has total 2211 *Mamdani* type rules. Each rule has its own output triangular membership function whose center has to be identified. COG defuzzification method is used for the output calculation. The number of rules was chosen experimentally with the aim to retain high prediction rate of the sub-model.

Fuzzy sub-model of water inflow

Water inflow sub-model has two inputs and one output. The universe of discourse of the first input is covered by 11 triangular membership functions and the universe of discourse of the second input – by 41 triangular membership functions. The rule base is a matrix with dimensions 11x41 and has 451 rules. Each rule has its own output triangular membership function whose center has to be identified. COG defuzzification method is used for the output calculation.

Fuzzy sub-model of water outflow

Water inflow sub-model has two inputs and one output. The universe of discourse of the first input is covered by 11 triangular membership functions and the universe of discourse of the second input – by 21 triangular membership functions. The rule base is a matrix with dimensions 11x21 and has 231 rules. Each rule has its own output triangular membership function whose center has to be identified. COG defuzzification method is used for the output calculation.

As can be noticed from the number of rules, that was selected experimentally, the process of the balance of water level at the desired set point with the sufficient accuracy is the most complicated and requires much more membership functions for the evaluation of inputs. The results of the experiments with this type of fuzzy model are presented in the next section.

4.2. Takagi-Sugeno type fuzzy model

The structure of the *Takagi-Sugeno* type fuzzy model in general is similar to *Mamdani* type fuzzy model, see Figure 4. Here instead of *Mamdani* type sub-models *Takagi-Sugeno* type sub-models are used. This means that the fuzzy rules in sub-models have different structure, different defuzzification and identification processes. *Takagi-Sugeno* type fuzzy models have fewer rules but computationally are more difficult. The rule base of such models is very difficult to tune in a manual way, using expert knowledge about the process. The processes in the water tank are also separated into three modes – the inflow of water to the tank, the outflow of water from the tank and the balance of the desired water level in the tank.

Fuzzy sub-models

All three sub-models have the same structure: two inputs one output. The universes of discourse of the first input of these sub-models are defined by 21 triangular membership functions, and the universes of discourse of the second input of sub-models – by 41 triangular membership functions, uniformly spread across the universe of discourse. The rule bases of these fuzzy sub-models each has 861 rules but the number of parameters that have to be identified this time is a vector of six elements, instead of 861 as it would be in case of *Mamdani* type model. *Takagi-Sugeno* fuzzy models require less memory for the rule base storage but the quality of the model is dependent only on the six parameters that are quite difficult to exactly identify. COG defuzzification method is used for the output calculation.

4.3. Hybrid fuzzy model

The processes in the water tank are separated into three modes as in the previous models: the water level balance at the desired set point, the water level inflow and the water level outflow. From the experiments it was noticed that *Mamdani* type fuzzy model is more accurate than *Takagi-Sugeno*, but it has one big drawback – huge amount of parameters that need to be identified. The process of identification is time consuming so it would not be possible to implement this model in real time control or fault detection system. In order to maintain the accuracy of the *Mamdani* type fuzzy model and to reduce the number of parameters, a hybrid model was synthesized. The structure of the hybrid fuzzy model is presented in Figure 5.

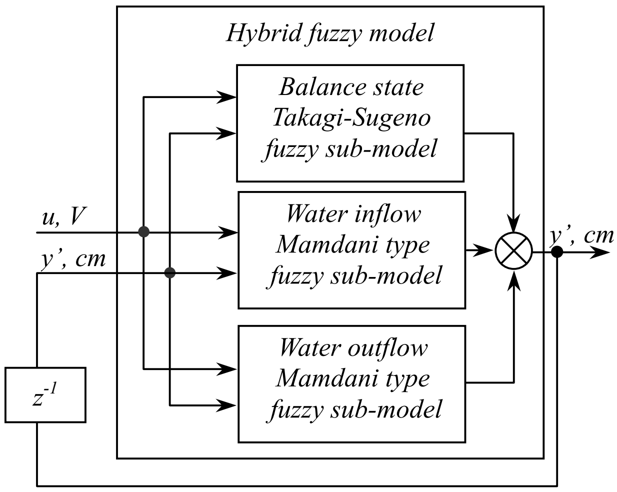

Figure 5. The structure of hybrid model

Fuzzy sub-model of the water level balance at the desired set point

This sub-model has two inputs one output. The universe of discourse of the first input is defined by 21 triangular membership functions, and the universe of discourse of the second input – by 41 triangular membership functions, uniformly spread across the universe of discourse. According to this, the rule base is a matrix with dimensions 21x41 and has total 861 *Takagi-Sugeno* type rules. The consequent part of all rules is a function that is defined by the vector of 6 elements that need to be identified. COG defuzzification method is used for the output calculation.

Fuzzy sub-model of water inflow

Water inflow sub-model has also two inputs and one output. The universe of discourse of the first input is covered by 11 triangular membership functions and the universe of discourse of the second input – by 41 triangular membership functions. The rule base is a matrix with dimensions 11x41 and has 451 rules. Each rule has its own output triangular membership function whose center has to be identified. COG defuzzification method is used for the output calculation.

Fuzzy sub-model of water outflow

Water inflow sub-model has two inputs and one output. The universe of discourse of the first input is covered by 11 triangular membership functions and the universe of discourse of the second input – by 41 triangular membership functions. The rule base is a matrix with dimensions 11x41 and has 451 rules. Each rule has its own output triangular membership function whose center has to be identified. COG defuzzification method is used for the output calculation.

The first sub-model was changed to *Takagi-Sugeno* type because it has the biggest amount of parameters of all sub-models – 2211 to maintain precise predictions. Takagi-Sugeno type fuzzy model for the modeling of the same process has only 6 parameters and the prediction accuracy was almost the same. So the change of the type of the sub-model reduces the number of parameters and retains almost the same accuracy, see the next section.

5. Analysis of models

For the analysis of the synthesized models the data from the control system were used, see Figure 3. PI controller was downloaded into Quantum controller that is connected to the sensors and actuators of the water tank. The set point signal for the PI controller was selected of stair form, composed from 8000 values, the signal values were changed every 2000 values. The data were acquired and set point values were changed in 1second intervals.

Fuzzy models were compared according to the following criteria:

- The number of parameters used in the model that need to be identified
- The number of times the re-identification of parameters was required expressed in %. (The process of identification of the model parameters is reapplied then the model's prediction error exceeds the defined limits)
- The accuracy of the prediction. (Mean, mean quadric deviation and standard deviation of prediction error was calculated.)

The results of experiments are presented in Table 1 and Figures 6, 7, 8.

Table 1. Comparison data of fuzzy models

Fuzzy model	Amount of parameters	Number of re-identification of model parameters (%)	Prediction error		
			Mean	Mean quadratic deviation	Standard deviation
Mamdani type fuzzy model	2892	7.07	0.0267	0.0005	0.0228
Takagi-Sugeno type fuzzy model	18	8.26	0.0250	0.00059	0.0243
Hybrid fuzzy model	908	7.69	0.0257	0.0006	0.0235

From the results of the experiments it can be seen that the best prediction is reached with *Mamdani* type fuzzy model, but this model is defined by 2892 parameters that need to be identified. Such a model might not be possible to use in controllers because of big memory requirements to store the rules. An alternative is the *Takagi-Sugeno* fuzzy model that has much less parameters, only 18, but less parameters influence the lower prediction precision. So in the cases when the prediction quality is very important and the memory amount is not – *Mamdani* type fuzzy model has an advantage over the other two, but if the memory amount is important – *Takagi-Sugeno* type fuzzy model should be used. The hybrid approach, offered in this paper, noticeably reduces the number of parameters and maintains the prediction accuracy similar to *Mamdani* type fuzzy model.

6. CONCLUSIONS

In this paper a hybrid fuzzy modeling of the nonlinear process has been proposed. Hybrid fuzzy model was experimentally compared to *Mamdani* type and *Takagi-Sugeno* type fuzzy models and its advantages and disadvantages were determined. *Mamdani* type fuzzy models are more accurate than *Takagi-Sugeno* type fuzzy models but they have much more parameters what sometimes might be a drawback. A hybrid approach was offered to reduce the number of parameters and to maintain the prediction accuracy.

The number of parameters in all three models was also reduced by splitting the fuzzy models into sub-models, so during the synthesis of the model if it is possible always try to split the model into sub-models.

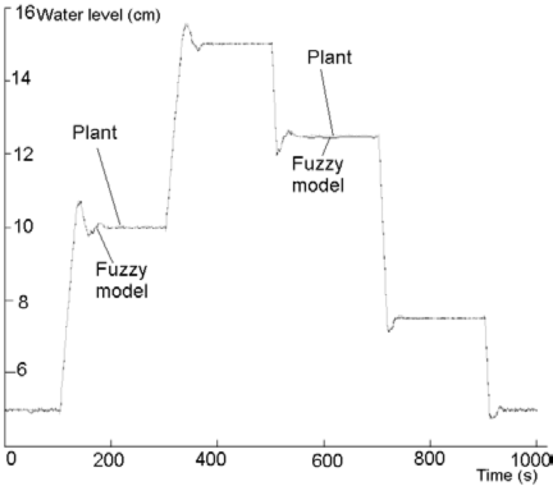

Figure 6. Output of the plant and the *Mamdani* type fuzzy model

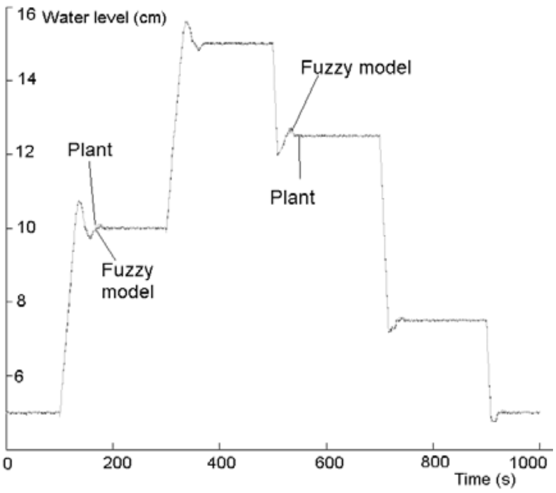

Figure 7. Output of the plant and the *Takagi-Sugeno* type fuzzy model

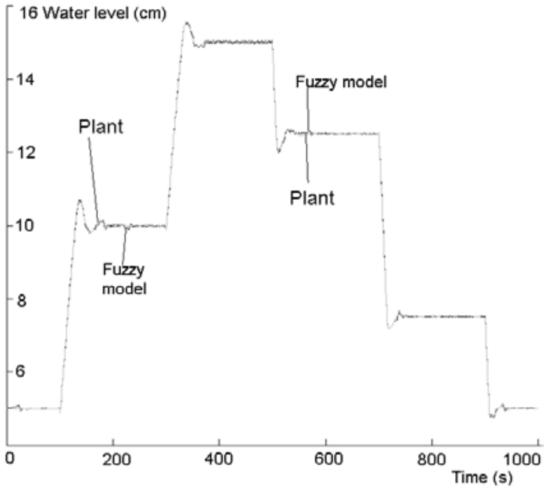

Figure 8. Output of the plant and the *Hybrid* fuzzy model

References

[1] **J. Abonyi.** Fuzzy model identification for control. *Birkhauser, Boston, USA*, 2003.

[2] **R. Babuška.** Construction of Fuzzy Systems – Interplay between Precision and Transparency. *ESIT*, 2000, *Achen, Germany*.

[3] **V.M. Becerra, R.K.H. Galvao, J. Calado. P. Silva.** Nonlinear system identification using linear-wavelet models. *Applied Mathematics and Computer Science.* 14(2), 2004, 221-232.

[4] **K.M. Passino, S. Yurkovich.** Fuzzy Control. *Addison Wesley Longman*, 1998.

[5] **A. Riid.** Transparency analysis of first-order Takagi-Sugeno systems. *Proc.*10*th International Symposium on System-Modeling-Control* 2001, *Zakopane, Poland, Vvol.* 2, 165-170.

[6] **T. Takagi, M. Sugeno.** Fuzzy identification of systems and its application to modeling and control. *IEEE Transactions on Systems*, *Man and Cybernetics*, 15(1), 1985, 116-132.

DOI: 10.5755/j01.itc.34.1.11972