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WORD ERROR PROBABILITY OF ASK SIGNALS IN THE PRESENCE OF NAKAGAMI FADING

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Abstract. In this paper we give the derivation of the Nakagami distribution and then we calculate the error probability of digital system with ASK signals in the presence of Nakagami fading as the dominant interference. The Gaussian noise is not taken into consideration because of that.

1. Indroduction

In this paper we will calculate the error probability of digital system with ASK signals. We will consider the system with Nakagami fading as dominant interference. In spite of this condition the Gaussian noise is not taken into consideration. This case can happen in mobile telecommunications in urban environment (places) when the envelope of useful signal at the input of the receiver has Nakagami density distribution [5, 6]. We consider the case with noncoherent datection.

The Rayleigh and Nakagami-*m* distributions are frequently used in communications systems analysis, for example, to model fading in wireless environments [6]. Some problems in wireless systems involve analysis using the bivariate (correlated) Rayleigh and Nakagami-*m* distributions. Determining the effect of correlation in fading between diversity branches in dual-diversity systems [4], and finding transition probabilities for a first-order Markov chain that models the fading process are two examples of problems requiring these distributions.

2. The Nakagami distribution

The random vector (r_1, r_2) has a bivariate Nakagami-*m* distribution if the joint pdf is given by:

$$p_{r_1 r_2}(r_1,\Omega_1,r_2,\Omega_2/m,\rho)=\frac{4(r_1r_2)^m e^{-(\Omega_2 r_1^2+\Omega_1 r_2^2)/\Omega_1\Omega_2(1-\rho)}}{\Gamma(m)\Omega_1\Omega_2(1-\rho)\left(\sqrt{\Omega_1\Omega_2\rho}\right)^{m-1}}\cdot I_{m-1}\left\{\frac{2\sqrt{\rho}\,r_1r_2}{\sqrt{\Omega_1\Omega_2}(1-\rho)}\right\} \tag{1}$$

where $r_1 \geq 0$, $r_2 \geq 0$, $\Omega_1=\frac{\overline{r_1^2}}{m}$, $\Omega_2=\frac{\overline{r_2^2}}{m}$, $\rho=\operatorname{cov}(r_1^2 r_2^2)\sqrt{\operatorname{var}(r_1^2)\operatorname{var}(r_2^2)}, \rho\neq 0{,}1$ and m is any positive number not less than 1/2, ($m\geq\frac{1}{2}$) [5]. The bivariate probability density function (pdf) of two correlated Rayleigh random variables is derived in [7] and can be considered a special case of the bivariate Nakagami-*m* distribution when $m=1$. Practical values of *m* for wireless applications typically range from $m=1$ to $m=15$, depending on the direct path component and other model parameters.

We consider the probability $\Pr(R_1<r_1, R_2<r_2)$ denoted by $F_{r_1,r_2}(r_1,\Omega_1;r_2,\Omega_2/m,\rho)$, that is:

$$F_{r_1,r_2}(r_1,\Omega_1;r_2,\Omega_2/m,\rho)=\int_0^{r_1}\int_0^{r_2} p_{r_1,r_2}(p_1,\Omega_1;p_2,\Omega_2/m,\rho)\,dp_2\,dp_1 \tag{2}$$

Using the definition of the incomplete gamma function [1]:

$$\gamma(\alpha,x)=\int_0^x e^{-t}t^{\alpha-1}dt, [\operatorname{Re}\alpha|>0]$$

the result is [9]:

$$F_{r_1,r_2}(r_1,\Omega_1;r_2,\Omega_2/m,\rho)=\frac{(1-\rho)^m}{\Gamma(m)}\cdot\sum_{k=0}^{\infty}\rho^k\cdot\frac{\gamma\left(m+k,\frac{r_1^2}{\Omega_1(1-\rho)}\right)\gamma\left(m+k,\frac{r_2^2}{\Omega_2(1-\rho)}\right)}{k!\,\Gamma(m+k)}. \tag{3}$$

The important special case of the Rayleigh distribution can be determined by setting $m=1$ in (3), so that

$$F_{r_1,r_2}\left(r_1,\Omega_1;r_2,\Omega_2/\rho\right)=\left(1-\rho\right)\cdot \sum_{k=0}^{\infty}\rho^k P\left(k+1,\frac{r_1^2}{\Omega_1\left(1-\rho\right)}\right)P\left(k+1,\frac{r_2^2}{\Omega_2\left(1-\rho\right)}\right) \quad (4)$$

where $P(\alpha,x)=\left(1/\Gamma(\alpha)\right)\int_0^x e^{-t}t^{\alpha-1}dt\ \left[\mathrm{Re}\alpha>0\right]$ is another common form of the incomplete gamma function [1].

3. The two symbols word error probability

In this paper we consider the telecommunication system where the signal is amplitude modulated (ASK). The detection of signals is no coherent. The receiver of this system is shown in Figure 1.

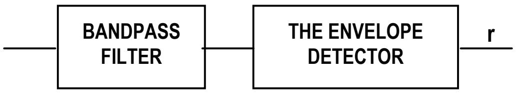

Figure 1. The receiver of the telecommunication system

The transmitter sends the signals A_0 and A_1. For the hypothesis H_0 we have:

$$H_0 : A_0=\frac{r_0^2}{m} \quad (5)$$

and for the hypothesis H_1:

$$H_1 : A_1=\frac{r_1^2}{m}. \quad (6)$$

When the transmitter send symbols H_0, H_0, the symbols at the output of the receiver $\left(r_1,r_2\right)$ have the joint cumulative probability density function:

$$F\left(r_1,A_0,r_2,A_0/m,\rho\right)=\frac{\left(1-\rho\right)^m}{\Gamma(m)}\cdot \sum_{k=0}^{\infty}\rho^k\cdot\frac{\gamma\left(m+k,\frac{r_1^2}{A_0\left(1-\rho\right)}\right)\gamma\left(m+k,\frac{r_2^2}{A_0\left(1-\rho\right)}\right)}{k!\,\Gamma\left(m+k\right)} \quad (7)$$

The probabilities of events: $P\left(D_0,D_0\right)$, $P\left(D_0,D_1\right)$, $P\left(D_1,D_0\right)$ and $P\left(D_1,D_1\right)$ are then:

$$P\left(D_0,D_0\right)=\int_0^a dr_1\int_0^a p_{r_1,r_2}\left(r_1,A_0,r_2,A_0/m,\rho\right)dr_2= F\left(a,A_0,a,A_0/m,\rho\right), \quad (8)$$

$$P\left(D_0,D_1\right)=\int_0^a dr_1\int_a^{\infty} p_{r_1,r_2}\left(r_1,A_0,r_2,A_0/m,\rho\right)dr_2= F\left(a,A_0,\infty,A_0/m,\rho\right)-F\left(0,A_0,a,A_0/m,\rho\right), \quad (9)$$

$$P\left(D_1,D_0\right)=\int_a^{\infty} dr_1\int_0^a p_{r_1,r_2}\left(r_1,A_0,r_2,A_0/m,\rho\right)dr_2= F\left(\infty,A_0,a,A_0/m,\rho\right)-F\left(a,A_0,a,A_0/m,\rho\right), \quad 10)$$

$$P\left(D_1,D_1\right)=\int_a^{\infty} dr_1\int_a^{\infty} p_{r_1,r_2}\left(r_1,A_0,r_2,A_0/m,\rho\right)dr_2= F\left(\infty,A_0,\infty,A_0/m,\rho\right)-F\left(\infty,A_0,a,A_0/m,\rho\right)- F\left(a,A_0,\infty,A_0/m,\rho\right)+F\left(a,A_0,a,A_0/m,\rho\right). \quad (11)$$

When the transmitter send symbols H_0, H_1, the symbols at the output of the receiver $\left(r_1,r_2\right)$ have the joint cumulative probability density function:

$$F\left(r_1,A_0,r_2,A_1/m,\rho\right)=\frac{\left(1-\rho\right)^m}{\Gamma(m)}\cdot \sum_{k=0}^{\infty}\rho^k\cdot\frac{\gamma\left(m+k,\frac{r_1^2}{A_0\left(1-\rho\right)}\right)\gamma\left(m+k,\frac{r_2^2}{A_1\left(1-\rho\right)}\right)}{k!\,\Gamma\left(m+k\right)}. \quad (12)$$

The probabilities of events: $P\left(D_0,D_0\right)$, $P\left(D_0,D_1\right)$, $P\left(D_1,D_0\right)$ and $P\left(D_1,D_1\right)$ are in this case:

$$P\left(D_0,D_0\right)=\int_0^a dr_1\int_0^a p_{r_1r_2}\left(r_1,A_0,r_2,A_1/m,\rho\right)dr_2= F\left(a,A_0,a,A_1/m,\rho\right), \quad (13)$$

$$P\left(D_0,D_1\right)=\int_0^a dr_1\int_a^{\infty} p_{r_1,r_2}\left(r_1,A_0,r_2,A_1/m,\rho\right)dr_2= F\left(a,A_0,\infty,A_1/m,\rho\right)-F\left(a,A_0,a,A_1/m,\rho\right), \quad (14)$$

$$P\left(D_1,D_0\right)=\int_a^{\infty} dr_1\int_0^a p_{r_1,r_2}\left(r_1,A_0,r_2,A_1/m,\rho\right)dr_2= F\left(\infty,A_0,a,A_1/m,\rho\right)-F\left(a,A_0,a,A_1/m,\rho\right), \quad (15)$$

$$P\left(D_1,D_1\right)=\int_a^{\infty} dr_1\int_a^{\infty} p_{r_1,r_2}\left(r_1,A_0,r_2,A_1/m,\rho\right)dr_2= F\left(\infty,A_0,\infty,A_1/m,\rho\right)--F\left(\infty,A_0,a,A_1/m,\rho\right)- F\left(a,A_0,\infty,A_1/m,\rho\right)-F\left(a,A_0,a,A_1/m,\rho\right). \quad (16)$$

When the transmitter send symbols H_1 and H_0, the symbols at the output of the receiver $\left(r_1,r_2\right)$ have the joint cumulative probability density function:

$$F\left(r_1,A_1,r_2,A_0/m,\rho\right)=\frac{\left(1-\rho\right)^m}{\Gamma(m)}\cdot \sum_{k=0}^{\infty}\rho^k\cdot\frac{\gamma\left(m+k,\frac{r_1^2}{A_1\left(1-\rho\right)}\right)\gamma\left(m+k,\frac{r_2^2}{A_0\left(1-\rho\right)}\right)}{k!\,\Gamma\left(m+k\right)}. \quad (17)$$

The probabilities of events: $P(D_0,D_0)$, $P(D_0,D_1)$, $P(D_1,D_0)$ and $P(D_1,D_1)$ are now:

$$P(D_0,D_0)=\int_0^a dr_1\int_0^a p_{r_1r_2}(r_1,A_1,r_2,A_0/m,\rho)dr_2 = F(a,A_1,a,A_0/m,\rho), \quad (18)$$

$$P(D_0,D_1)=\int_0^a dr_1\int_a^\infty p_{r_1r_2}(r_1,A_1,r_2,A_0/m,\rho)dr_2 = F(a,A_1,\infty,A_0/m,\rho)-F(a,A_1,a,A_0/m,\rho), \quad 19)$$

$$P(D_1,D_0)=\int_a^\infty dr_1\int_0^a p_{r_1,r_2}(r_1,A_1,r_2,A_0/m,\rho)dr_2 = F(\infty,A_1,a,A_0/m,\rho)-F(a,A_1,a,A_0/m,\rho), \quad (20)$$

$$P(D_1,D_0)=\int_a^\infty dr_1\int_0^a p_{r_1,r_2}(r_1,A_1,r_2,A_0/m,\rho)dr_2 = F(\infty,A_1,a,A_0/m,\rho)-F(a,A_1,a,A_0/m,\rho)- F(a,A_1,\infty,A_0/m,\rho)-F(\infty,A_1,a,A_0/m,\rho)+ F(a,A_1,a,A_0/m,\rho). \quad (21)$$

Finally, the transmitter send symbols H_1 and H_1. The symbols at the output of the receiver (r_1,r_2) have the joint cumulative probability density function:

$$F(r_1,A_1,r_2,A_1/m,\rho)=\frac{(1-\rho)^m}{\Gamma(m)}\cdot \sum_{k=0}^{\infty}\rho^k\cdot\frac{\gamma\left(m+k,\frac{r_1^2}{A_1(1-\rho)}\right)\gamma\left(m+k,\frac{r_2^2}{A_1(1-\rho)}\right)}{k!\,\Gamma(m+k)}. \quad (22)$$

The probabilities of events: $P(D_0,D_0)$, $P(D_0,D_1)$, $P(D_1,D_0)$ and $P(D_1,D_1)$ are:

$$P(D_0,D_0)=\int_0^a dr_1\int_0^a p_{r_1,r_2}(r_1,A_1,r_2,A_1/m,\rho)dr_2 = F(a,A_1,a,A_1/m,\rho). \quad (23)$$

$$P(D_0,D_1)=\int_0^a dr_1\int_a^\infty p_{r_1,r_2}(r_1,A_1,r_2,A_1/m,\rho)dr_2 = F(a,A_1,\infty,A_1/m,\rho)-F(a,A_1,a,A_1/m,\rho), \quad (24)$$

$$P(D_1,D_0)=\int_a^\infty dr_1\int_0^a p_{r_1,r_2}(r_1,A_1,r_2,A_1/m,\rho)dr_2 = F(\infty,A_1,a,A_1/m,\rho)-F(a,A_1,a,A_1/m,\rho), \quad (25)$$

$$P(D_1,D_1)=\int_a^\infty dr_1\int_a^\infty p_{r_1,r_2}(r_1,A_1,r_2,A_1/m,\rho)dr_2 = F(\infty,A_1,\infty,A_1/m,\rho)-F(\infty,A_1,a,A_1/m,\rho)- F(a,A_1,\infty,A_1/m,\rho)+F(a,A_1,a,A_1/m,\rho). \quad (26)$$

4. Conclusion

The Rayleigh and Nakagami-*m* distributions are frequently used in communications systems analysis to model fading in wireless environments. Some problems in wireless systems involve analysis using the bivariate (correlated) Rayleigh and Nakagami-*m* distributions. Because of that in this paper we give the derivation of the Nakagami distribution and then we calculate the error probability of digital system with ASK signals in the presence of Nakagami fading as the dominant interference. The Gaussian noise is not taken into consideration because of that. Our results have useful practical application in mobile telecommunications in urban environment.

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