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ALGORITHMIC METHODS OF QUASI-OPTIMAL FILTER SYNTHESIS

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Abstract. The aim of the work is to create and to refine methods of algorithmic filter synthesis allowing designing quasi-optimal filters in cases when statistical characteristics of effective signal and interference – non-stationary stochastic processes – are unknown and application of classic synthesis methods, e.g. Kalman-Busy method, is impossible. Paper presents the basics of algorithmic quasi-optimal filter synthesis, the essence of which is the formulation of the filter synthesis task in the form of searching optimization and its solution with application of simplex search methods. Examples of algorithmic quasi-optimal filter synthesis are presented. Parameters of quasi-optimal filter transfer function found by algorithmic method correspond to theoretical parameters calculated on the basis of statistical characteristics of input signals.

1. Indroduction

Non-stationary stochastic processes with zero means are applied into filter input: $g(t)$ – effective signal and $\eta(t)$ – "white noise" disturbance.
The entrance signal is the sum:

$$\varphi(t) = g(t) + \eta(t) .$$

The purpose of the system is to restore useful signal $g(t)$, or a certain function from this signal.

The function $g(t)$ sometimes can be expressed by the first order differential equation with variable coefficient $A(t)$ that depends on statistical characteristics of the signal [3]:

$$\frac{dg(t)}{dt} = A(t)g(t) + V(t) , \qquad (1)$$

where $V(t)$ is non-stationary "white noise" stochastic process with zero mean.

Correlation functions of non-stationary stochastic signals $V(t)$ and $\eta(t)$ can be expressed as follows:

$$R_V(t,\tau) = L(t)\delta(t-\tau), , \qquad (2)$$

$$R_\eta(t,\tau) = N(t)\delta(t-\tau), , \qquad (3)$$

where $L(t)$ and $N(t)$ are continuous differentiable functions. The signals $V(t)$ and $\eta(t)$ are non-correlated, i.e. $R_{V\eta}(t,\tau) = 0$.

In case of filtration task, given value of exit signal is effective signal, i.e. $y_0(t) = g(t)$.
The error of the system is:

$$\Delta y(t) = y_0(t) - y(t) . \qquad (4)$$

Kalman and Busy have determined [3] that optimal system which guarantees reproduction of effective signal and minimal mean square error $\overline{\Delta y^2}$ is defined by the following differential equation

$$\frac{dy}{dt} = Q(t)y(t) + C(t)\varphi(t) , \qquad (5)$$

where $Q(t)$ and $C(t)$ are time functions wich can be found from the condition of mean square error minimum:

$$M\left\{[\Delta y(t)]^2\right\} \to \min . \qquad (6)$$

If effective signal can be presented by the form (1), methodics for detection of functions $C(t)$ and $Q(t) = A(t) + C(t)$ [3] require statistical characteristics of stochastic signals $V(t)$ and $\eta(t)$ given by (2) and (3).

When statistical characteristics (2) and (3) are unknown, the problems of filter synthesis stated in works [3] and [4] can be solved with application of algorithmic stochastic systems synthesis methods [1].

The aim of this work is to create and enhance algorithmic methods for solution of the filter synthesis problems enabling to design quasi-optimal filters in cases when statistical characteristics of signals $g(t)$ and $\eta(t)$ are unknown and it is impossible to apply the classical synthesis methods (see [3] and [4]).

2. Fundamentals of algorithmic quasi-optimal filter synthesis

The system must reproduce as much precisely as possible the effective signal $g(t)$ or certain function

$$y_0(t) = H(t)g(t) \qquad (7)$$

of the effective signal $g(t)$.

In this case the system's quality index mean square error

$$\overline{\Delta y^2} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left[y_0(t) - y(t) \right]^2 dt \qquad (8)$$

can be evaluated during searching optimization by discrete analogues of this index (moving average of Δy^2 etc.). For example, signal's $y(t)$ deviation from determined value $y_0(t)$ can be evaluated after performing N measurements of the exit signal:

$$S_N = \frac{1}{N} \sum_{i=1}^{N} w_i \left[y_0(t) - y(t) \right]^2 , \qquad (9)$$

where w_i is tuning factor.

We will insert filter with excessive transmission function into system's contour:

$$W(p) = \frac{a_1 p^m + a_2 p^{m-1} + ... + a_{m+1}}{a_{m+2} p^n + a_{m+3} p^{n-1} + ... + a_{m+p+2}} \qquad (10)$$

where $n > m$.

We will input vector $\boldsymbol{x} = (x_i, \ldots, x_k)$, components of which will be equal to coefficients of filter transmission function ($x_1 = a_1, x_2 = a_2, \ldots, x_{k-1} = a_{m+p+1}, x_k = a_{m+p+2}$). In general case, when effective signal $g(t)$ and interference $\eta(t)$ are non-stationary processes, as time passes, depending on signals' characteristics changes, the parameters and structure of the filter are changing, and the vector $\boldsymbol{x}$ is a function of time $\boldsymbol{x} = \boldsymbol{x}(t)$.

The output signal $y(t)$ of the system and various indices of filter quality $\overline{\Delta y^2}$, S_N etc. depend upon vector $\boldsymbol{x}$.

After selection of the objective function – index of quality $J(\boldsymbol{x})$, we can formulate the problem of searching optimization.

It is necessary to find a vector $\boldsymbol{x}$, upon which filter's transmission function's $W(p)$ structure and parameters are dependant, and which guarantees the minimum of the objective function

$$J(\boldsymbol{x}) = J\left[y(\boldsymbol{x}) \right] \qquad (11)$$

that under the restriction

$$\boldsymbol{x} \in \Omega_{\boldsymbol{x}} . \qquad (12)$$

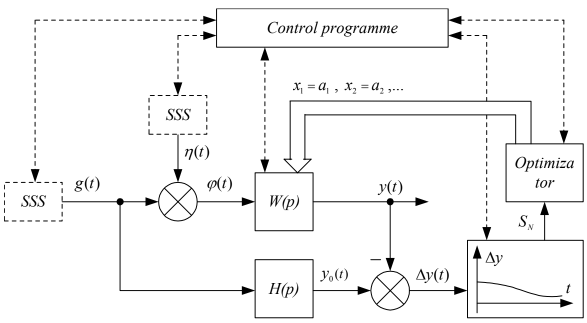

Figure 1. Flowchart of quasi-optimal filter algorithmic synthesis

The problem (11), (12) is solved with application of simplex search methods [2]. A schematic drawing of the solution is presented in Figure 1. Sources of stochastic signals (SSS), shown in Figure 1, can be real or composed for modeling purposes using special computer programs.

3. Examples of quasi-optimal filter algorithmic synthesis

The problem for filter algorithmic synthesis is formulated as one with the objective function (11) and constraint (12). The problem was solved with application of algorithmic filter synthesis method, using software package Matlab and Simulink according to schematic drawing of Figure 1, into which sources of non-stationary stochastic signals $g(t)$ and $\eta(t)$ were plugged in. During solution of the problem the statistical characteristics of those signals were not used. The result of problem solution is the quasi-optimal filter transfer function

$$W(p) = \frac{k(t)}{T(t)p + 1} , \qquad (13)$$

the parameters of which $k(t)$ and $T(t)$ are changing with time. Function (13) was obtained from function (10) during the process of optimization.

Filter's time constant dependency upon time is depicted in Figure 2 (first curve), amplification coefficient's dependency upon time – in Figure 3 (first curve). Changes of filter's parameters are influenced by changes of statistical characteristics of input signals.

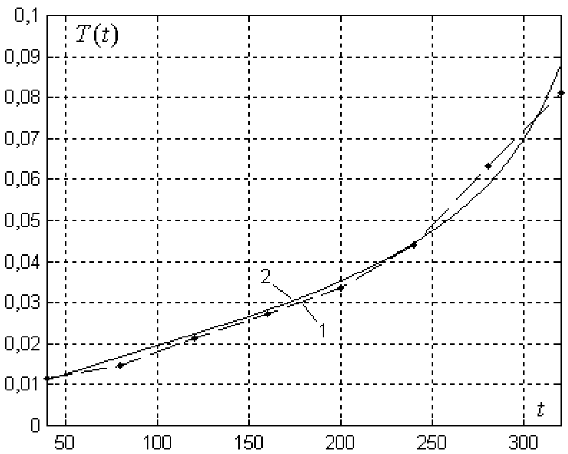

Figure 2. Quasi-optimal filter's time constant's experimental (first curve) and theoretical (second curve) dependencies upon time

During formation of sources of stochastic non-stationary signals $g(t)$ and $\eta(t)$, spectral densities of signals were used:

$$S_g(\omega, t) = \frac{2D(t)\alpha}{\alpha^2 + \omega^2} , \qquad (14)$$

$$S_\eta(\omega, t) = 0{,}005t , \qquad (15)$$

where $D(t)$ – dispersion of the effective signal, $D(t) = 450 - 1{,}25t$, α – parameter of the effective signal correlation function, $\alpha = 2$.

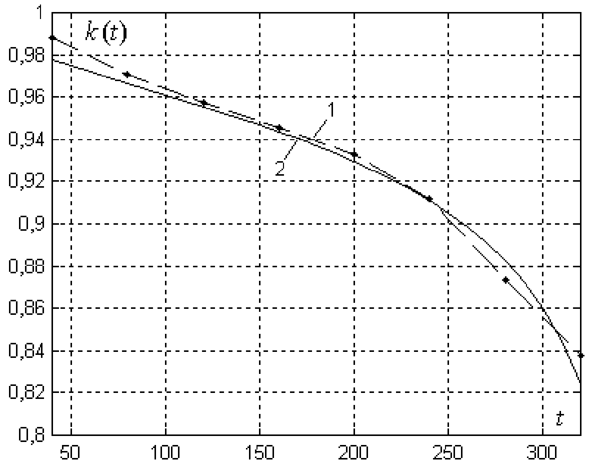

Figure 3. Quasi-optimal filter's amplification coefficient's experimental (first curve) and theoretical (second curve) dependencies upon time

Characteristics given by 14 and 15 enabled us, using the methodology of optimal filter synthesis [4], to calculate theoretical values of filter transmission function, which are shown in Figures 2 and 3 as the second curves, and to compare with results of algorithmic filter synthesis.

At the same time optimal parameters of the filter were calculated, when input signals are stationary stochastic processes, spectral densities of which are

$$S_g(\omega) = \frac{400}{4+\omega^2}, \tag{16}$$

$$S_\eta(\omega) = 1{,}4 . \tag{17}$$

In this case, parameters of optimal filter are close to parameters of quasi-optimal filter (13) and the transfer function of the quasi-optimal filter is

$$W(p) = \frac{0{,}874}{0{,}0635p+1} .$$

This working regime of quasi-optimal filter is presented in Figures 4, 5 and 6, where time-series diagrams of the effective signal, filter input signal and output signal are shown.

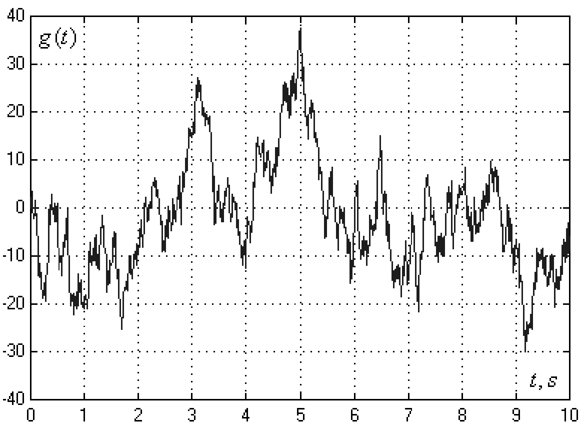

Figure 4. Time-series diagram of system's effective signal

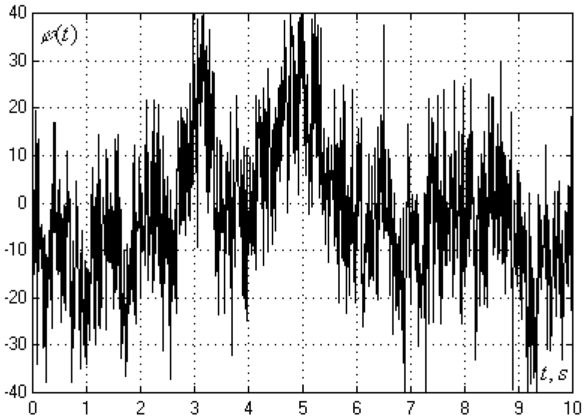

Figure 5. Time-series diagram of filter's input signal

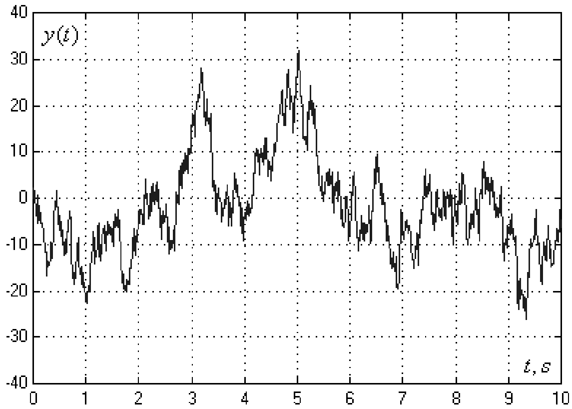

Figure 6. Time-series diagram of quasi-optimal filter's input signal

4. Conclusions

Parameters of quasi-optimal filter delivered by the algorithmic method correspond to theoretical parameters calculated according to statistical characteristics of input signals.

Algorithmic method was designed which gives a possibility to perform filter synthesis in cases when statistical characteristics of input signals are unknown and application of classic synthesis methods is impossible.

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