THE MATHEMATICAL MODELS FOR THE MULTISTAGE INVENTORY CONTROL PROCESSES

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Abstract. In this paper two mathematical models of multistage inventory control processes with continuous and discrete density functions of demands are investigated. These processes are modelled by recursion equations of the dynamic programming. For inventory control problem with the continuous density function there was created a new continuous optimal control problem, which is equivalent to the given one. Applying the maximum principle solves this new problem. The optimal policy ordering policy is defined. Also, we have found the optimal policy for ordering of products in the multistage inventory control problem with the discrete density function of demands. In this case such number of moments of time was found that the demands are satisfied without extra products.

Keywords. Optimal inventory processes, dynamic programming, Maximum principle.

1. Indroduction

The inventory control theory is one of the newest operation research branches. The formulation of the inventory control problem depends on the concrete situation. However, there exist many common factors whose give an opportunity to create a sufficiently common model for the inventory control. In 1951 economists. Arrow (laureate of the Nobel prize in 1972), Harris and Marshak formulated the creation principles of the mathematical model for the inventory control problem. Latterly, many mathematicians and economists in close cooperation with other scientists work on solving the inventory control problems. A lot of publications on the general inventory control theory have been published [3, 4, 5]. Though, in many cases, only the inventory control processes with linear or convex product order functions are examined. In this paper, the inventory control models, generalizing the classical ones, are analyzed. In the developing these models, some restrictions, so peculiar for classical models, have been discarded. For instance, linearity and convexity conditions for the products order function. The models are constructed under general conditions for the warehouse order receiving.

1. The stochastic inventory control problem with continuous demands

We study infinite inventory control problem. We solve it using approximation method for discrete processes to control optimally the continuous processes [5, 6]. This allows us to solve the inventory control problem with general conditions for warehouse order receiving.

Let x be the initial quantity of products stores in a warehouse. We also assume, that the products are ordering at the discrete moments of time t = 0, 1, 2, …, the orders fulfill at the same moments of time with the probability p, and they are fulfill one moment later with probability $q = 1 - p$.

Suppose given functions: $\varphi(s)$ – density function of demands; $c(z)$ – the order's price function (z – quantity of products' ordered); $h(z)$ – function of products stored in warehouse (price of storing z products); $p(z)$ – penalty function (penalty for shortage of z products); α $(0 < \alpha < 1)$ – discount coefficient.

Let's denote $f(x)$ minimal expected value of expenses of the optimal inventory problem in infinite time, as x – the initial number of inventory. Our multistage inventory process is described by dynamic programming equations as follows:

$$f(x) = \min_{y \geq x}\Big(c(y-x) + pL(y) + qL(x) + \alpha\int_0^\infty f(y-s)\varphi(s)ds\Big); \quad (1)$$

here

$$L(y) = \begin{cases} \int_y^\infty p(s-y)\varphi(s)ds + \int_0^y h(y-s)\varphi(s)ds, & \text{as } y > 0, \\ \int_0^\infty p(s-y)\varphi(s)ds, & \text{as } y \leq 0. \end{cases}$$

We are analyzed the solution to the equation (1). It has the following form

$$f(x)=J_1(x\{u_t\})=\sum_{t=0}^{\infty}z_t\left(\int_0^{\infty}c(x_t)(u_t-1)+pL(u_t\cdot x_t)+\right.$$

$$\left.+qL(x_t)\prod_{i=0}^{t-1}\varphi(s_i)ds_i\right)\to\inf \qquad (2)$$

Suppose

$$\begin{aligned} x_{t+1}&=u_t\cdot x_t-s_t, & x_0&=x,\\ z_{t+1}&=z_t\cdot\alpha, & z_0&=1,\end{aligned} \qquad (3)$$

here s_t – random variables with density $\varphi(s_t)$.

We construct of the optimal continuous problem in the case of:

$$\int_0^{\infty}f(y-s)\varphi(s)ds=f(y-\mu),$$

here μ is mean of s_t, defined as

$$\mu=\int_0^{\infty}s_t\varphi(s_t)ds_t.$$

In this case, the criterion of quantity for the initial process is expressed as

$$J(x,\{u_t\})=\sum_{t=0}^{\infty}\big(c(x_t(u_t-1))+pL(x_t\cdot u_t)+L(x_t)\big). \qquad (4)$$

The equation of motion are:

$$\begin{aligned} x_{t+1}&=u_t\cdot x_t-\mu, & x_0&=x,\\ z_{t+1}&=z_t\cdot\alpha, & z_0&=1,\end{aligned} \qquad (5)$$

the control $u_t\geq 1$ at $t=0,1...$

Let's find a trajectory for the continuous problem of the optimal control problem (4)-(5). We are going to study the case, when the differential equation for the trajectory of the continuous control process according to the first recursion relation of the system (5) has the following

$$\frac{dy}{ds}=uay+b\mu, \qquad y(0)=x. \qquad (6)$$

Since the solution of this equation satisfies the following condition in interval $[t,t+1)$,

$$y(s)=e^{ua(s-t)}y(t)+e^{ua(s-t)}\int_t^s e^{ua(t-\tau)}b\mu d\tau,$$

by using the definition of the optimal continuous problem [5], we get the following relations:

$$u=e^{ua},$$

$$e^{ua}\int_t^{t+1}e^{ua(t-\tau)}bd\tau=-1.$$

From the relations we have obtained follows

$$a=\ln u,$$

$$b=-\frac{\ln u}{u-1}.$$

It's not difficult to check, that the differential equation according to the second recursion relation of the system (5) has the following

$$\frac{dz}{ds}=z\ln\alpha, \qquad z(0)=1.$$

We are going to write the quality's criteria for the continuous problem of the problem (4)-(5) in the following way

$$\bar{J}(x,u(s))=\int_0^{\infty}z(s)\big(\bar{c}(y(s)(u(s)-1))+p\bar{L}_1(u(s)\cdot y(s))+$$

$$+q\bar{L}_2(y(s))\big)ds, \quad u(t)=u_t.$$

Since $z_t=\alpha^t$, by using the definition of the continuous analogue [5], we get the following relations:

$$\int_t^{t+1}z(s)\bar{c}(y(s)(u_t-1))ds=\alpha^t c(x_t(u_t-1)), \qquad (7)$$

$$\int_t^{t+1}z(s)\bar{L}_1(u_t\cdot y(s))ds=\alpha^t L(u_t\cdot x_t), \qquad (8)$$

$$\int_t^{t+1}z(s)\bar{L}_1(u_t\cdot y(s))ds=\alpha^t L(u_t\cdot x_t), \qquad (9)$$

for all $u_t\geq 1,\quad t=0,1,....$

Therefore, we get the optimal control problem

$$\begin{aligned}\bar{J}(y(s),u(s))&=\int_0^{\infty}z(s)\bar{c}(y(s)(u(s)-1))+\\ &+p\bar{L}_1(u(s)\cdot y(s))+q\bar{L}_2(y(s))ds\to\min_u,\end{aligned} \qquad (10)$$

$$\begin{aligned}\frac{dy}{ds}&=\ln u(s)\left(y(s)-\frac{\mu}{u(s)-1}\right), & y(0)&=x,\\ \frac{dz}{ds}&=z(s)\ln\alpha, & z(0)&=1.\end{aligned} \qquad (11)$$

We will find the optimal control by using the maximum principle for (10) – (11) problems. For the discussing problem the Hamilton's function has the following form

$$\begin{aligned}H(\psi(s),y(s),u(s))&=\psi_0(z(s)(\bar{c}y(s)(u(s)-1))+\\ &+p\bar{L}_1(u(s)\cdot y(s))++q\bar{L}_2(y(s))+\psi(s)\ln u(s)\cdot\\ &\cdot\left(y(s)-\frac{\mu}{u(s)-1}\right)=\psi_0\alpha(y,u)++\psi(s)\beta(y,u),\end{aligned} \qquad (12)$$

here $\psi_0\leq 0$.

Theorem 1. Let the following three conditions be satisfied:

1) Functional $\bar{J}(y(s),u(s))$, defined by the equation (10), is bounded;
2) Piecewise partially continuous control of the problem (10) - (11) exist;
3) Functions $\bar{c}(y)$, $\bar{L}_1(y)$, $\bar{L}_2(y)$ are continuous and have continuous derivatives.

Then the optimal control $u^0(s)$ of the problem (10)-(11) satisfies the maximum condition

$$\begin{aligned}&\max_{u\geq 1}H(\psi^0(s),y^0(s),u(s))=\\ &=H(\psi^0(s),y^0(s),u^0(s)),\end{aligned} \qquad (13)$$

$$H\left(\psi^0(s), y^0(s), u^0(s)\right) = 0, \quad \forall s \geq 0. \tag{14}$$

here function $H(\psi(s), y(s), u(s))$ is defined by formula (12), $\psi^0(s)$ is the solution of the differential equation

$$\frac{d\psi^0(s)}{ds} = -\frac{\partial H\left(\psi^0(s), y(s), u^0(s)\right)}{\partial y} \tag{15}$$

and $\psi_0 \neq 0$.

Proof. We are going to study the process with fixed end of trajectory:

$$\int_0^T f_0(z(s), y(s), u(s))ds \to \inf, \tag{16}$$

as the conditions (11) and $z(T) = z^0(T)$, $y(T) = y^0(T)$ are satisfied. Here $\left(z^0(s), y^0(s), u^0(s)\right)$ is the optimal process of the problem (10)-(11);

$$f_0(z(s), y(s), u(s)) = z(s)\left(\bar{c}(y(s)(u(s)-1)) + p\bar{L}_1(u(s)y(s)\right) + q\bar{L}_2(y(s)).$$

The maximum of the Hamilton's function, defined by formula (12), is reached at:

$$\max_{u\geq 1} H\left(\psi^0(s), y^0(s), u(s)\right) = H\left(\psi^0(s), y^0(s), u^0(s)\right),$$

here $y^0(s)$ is the trajectory of the corresponding control $u^0(s)$.

We know, that $u^0(s)(0 \leq s \leq T)$ is the optimal control of (16). Besides, the maximum principle and condition

$$H\left(\psi^0(s), y^0(s), u^0(s)\right) = 0, \ (0 \leq s \leq T)$$

are satisfying for this problem.

Since this equation is valid for all *T*, we have:

$$H\left(\psi^0(s), y^0(s), u^0(s)\right) = 0, \ \forall s \geq 0. \tag{17}$$

The only statement we have to prove is $\psi_0 \neq 0$. Basically, if $u^0(y) > 1$, then from the relation (12) it follows, that the point $u^0(s)$ is the root of the equation

$$\psi_0 \alpha_u'(y(s), u^0(s)) + \psi^0(s)\beta_u'(y(s), u^0(s)) = 0. \tag{18}$$

If $\psi_0 = 0$, then from (18) it follows, that either $\psi(s) = 0$ or $\beta_u' \neq 0$, but this contradicts the maximum principle. So, if $u^0(s) > 1$, then $\psi_0 \neq 0$. The only case that remains to consider is $u^0 = 1$. By substituting $u^0 = 1$ in the formula (12), we get the equation

$$\psi_0 z(s)\bar{c}(0) + p\bar{L}_1(y) + q\bar{L}_2(y) + \psi(s)\mu = 0.$$

From this equation it follows $\psi_0 \neq 0$.

Theorem is proved.

The solution the equation (10) – (11) describing the optimal control (10-11) is formulated in the following theorem.

Theorem 2. Let the following two conditions be satisfied:

1) Functional $\bar{J} = \bar{J}(y(s), u(s))$ defined by the formula (10) is bounded;
2) Functions $\bar{c}(y)$, $\bar{L}_1(y)$, $\bar{L}_2(y)$ are continuous and have continuous derivatives.

Then the optimal control of the problem (10) - (11) either satisfies the equation

$$\begin{aligned} &xc'(x(u-1)) + p\bar{L}_1{}'(u \cdot x) - (x(u-1))^2 + \\ &+ \mu(u \ln u - u + 1)\left(\bar{c}x(u-1) + p\bar{L}_1(u \cdot x) + q\bar{L}_2(x)\right)) : \\ &: (u(u-1) + \ln u(ux - x - \mu)) = 0 \end{aligned} \tag{19}$$

and conditions $u(x) > 1$, $u(x) \neq \frac{\mu}{x} + 1$, or $u = 1$ and condition $\max\limits_u H(\psi(s), y(s), u(s)) = H(\psi(s), y(s), u(s))$, holds true.

Proof. To analyze the optimal control of the problem (10)-(11) we use the maximum principle [3]. From Theorem 1 it follows, that the Hamilton's function with optimal control is equal to zero, it means

$$H\left(\psi^0(s), y^0(s), u^0(s)\right) = 0, \ \forall s \geq 0.$$

Since $z(s) = \alpha^s$, the Hamilton's function for this problem has such form

$$\begin{aligned} &H\left(\psi^0(s), y^0(s), u^0(s)\right) = \\ &= \psi_0 \alpha^s \left(\bar{c}(y^0(s)(u^0(s)-1))\right) + \\ &+ p\bar{L}_1(u^0(s)y^0(s)) + q\bar{L}_2(y^0(s)) + \\ &+ \psi(s)\left(y^0 - \frac{\mu}{u^0(s)-1}\right)\ln u^0 = 0. \end{aligned} \tag{20}$$

According to the maximum principle, function $\psi^0(s)$ satisfies the differential equation

$$\begin{aligned} &\frac{d\psi}{ds} = -\frac{\partial H}{\partial y} = -\alpha^s \psi_0 \bar{c}'(y^0(s)(u^0(s)-1) + \\ &+ p\bar{L}_1'(u^0(s)y^0(s))u^0(s) + q\bar{L}_2'(y^0(s)) - \\ &-\psi^0(s)\ln u^0(s). \end{aligned} \tag{21}$$

Since

$$H(\psi^0(0), u^0(0), x) = 0,$$

assuming that $\psi_0 = -1$, from the last relation we get

$$\begin{aligned} &H\left(\psi^0(0), y^0(0), u^0(0)\right) = \max_u H(\psi(0), u(0), x) = \\ &= -\alpha^s\left(\bar{c}(y^0(0)(u^0(0)-1))\right) + p\bar{L}_1(u^0(0)y^0(0)) + \\ &+ q\bar{L}_2(y^0(0)) + \psi^0(0)\left(y^0 - \frac{\mu}{u^0(0)-1}\right)\ln u^0 = 0. \end{aligned} \tag{22}$$

So

$$\psi^0(0) = \left(\alpha^s \left(u^0(0)-1\right)\right)\bar{c}(x(u^0(0)-1) + p\bar{L}_1(u^0(0)x) + q\bar{L}_2(x)\Big): \\ :\left(x(u^0(0)-1) - \mu \ln u^0\right) \quad (23)$$

If $u^0 > 1$, then

$$H_u^{'}\left(\psi^0(0), u^0(0), x\right) = \\ = -\alpha^s \left(\bar{c}^{'} \left(x(u^0(0)-1)\right)x + xp\bar{L}_1(xu^0(0)) + \right. , \\ + \psi^0(0)\left(\left(x - \frac{\mu}{u^0(0)-1}\right)\frac{1}{u^0(0)} + \frac{\mu \ln u^0}{\left(u^0(0)-1\right)^2}\right) = 0$$

$$u^0(0) = u(x). \quad (24)$$

From the equations (23) and (24) it follows, that the optimal control $u(x)$ satisfies the equation (19). If $u(0)=1$, then from the condition (17) we get the proof of the theorem.

Theorem is proved.

2. The inventory control problem with discrete demands.

Let an initial inventory i of the i-th product be equal to s_i $\left(s_i \geq 0,\ i = \overline{1,n}\right)$ and at each moment of time t (t=0, 1,...) the lot of products $x = \left(x_1, x_2, .., x_n\right)$ can be ordered. The lot order's cost x is equal to $\varphi(x)$, here

$$\varphi(x) = c_0 + \sum_{i=1}^{n}\left(c_i + x_i \alpha_i\right),$$

where $c_i > 0$, $\alpha_i > 0$, c_0 is general management expenses, $c_i + \alpha_i x_i$ is expenses of the order x_i of i-th product. Let $\varphi(\mathbf{0}) = 0$, $\mathbf{0} = (0,...,0)$. Furthermore, the following data is given: r_i $\left(i = \overline{1,n}\right)$ – demands for the *i-th* product; d_i $\left(i = \overline{1,n}\right)$ – stock-price for the *i-th* product; λ – discount coefficient for one period of time ($0 < \lambda < 1$).

In the single case the order function $\varphi(x)$ is

$$\varphi(x) = \begin{cases} c_1' + \alpha_1 x_1, & \text{as } x_1 > 0, \\ 0, & \text{as } x_1 = 0, \end{cases}$$

here $c_1' = c_0 + c_1$.

If $s_1 > r_1$, then it is obvious that the optimal policy is not to order the products till the inventory is larger than r_1. If $s_1 < r_1$, then the optimal policy has the following structure: to order such quantity of product that it would be possible to satisfy the demands at certain time moments and also that all inventory after these time moments would be used up.

Let's find the number of time moments, during which the demands are satisfied, but the products are not ordered. We denote the number of these moments by $N_0(s_1)$ and minimal expenses when the initial inventory is $s_1 < r_1$ – by $f(s_1)$. Let's discuss the case when $s_1 = 0$. We obtain, that the function $f(0)$ satisfies the equation of dynamic programming

$$f(0) = \min_{N\geq 0}\left(c_1 + \alpha_1(N+1)r_1 + \right. \\ \left. + \lambda d_1 r_1\left(N\lambda + \lambda^2(N-1) + ... + \lambda^N\right) + \lambda^{N+1} f(0)\right).$$

Therefore, we have

$$f(0) = \min_{N\geq 0}\left(\frac{c_1 + \alpha_1(N+1)r_1}{1-\lambda^{N+1}} + \right. \\ \left. + d_1 r_1 \lambda \frac{N(1-\lambda) - \lambda\left(1-\lambda^N\right)}{(1-\lambda)^2\left(1-\lambda^{N+1}\right)}\right) = \\ = \min_{N\geq 0}\varphi(N) = \varphi(N_0).$$

Let's find $N_0(s_1)$, when $0 < s_1 < r_1$. We have

$$f(s_1) = \min_{N\geq 0}\left(c_0 + \alpha_1(Nr_1 - s_1) + \right. \\ \left. + \lambda d_1 r_1\left(N + (N-1)\lambda + ... + \lambda^{N-1}\right) + \lambda^{N+1}\varphi(N_0)\right) = \\ = c_0 - \alpha_1 s_1 + \min\left(\frac{\lambda d_1 r_1\left(N(1-\lambda) - \lambda\left(1-\lambda^N\right)\right)}{(1-\lambda)^2\left(1-\lambda^{N+1}\right)} + \right. \\ \left. + \alpha_1 N_1 r_1 + \lambda^{N+1}\varphi(N_0)\right) = c_0 - \alpha_1 s_1 + \alpha_1 N_0(s_1) r_1 + \\ + \frac{\lambda d_1 r_1\left(N_0(s_1)(1-\lambda) - \lambda\left(1-\lambda^{N_0(s_1)}\right)\right)}{(1-\lambda)^2\left(1-\lambda^{N_0(s_1)+1}\right)} + \lambda^{N_0(s_1)+1}\varphi(N_0).$$

Let's study the multistage case, with the condition $s_i = 0$ $\left(i = \overline{1,n}\right)$. Then $s = 0 = (0,...,0)$ and

$$f(\mathbf{0}) = \min_{N\geq 0}\sum_{i=1}^{N+1}\left(c_i + \alpha_i(N+1)r_i\right) + c_0 + \\ + \lambda\sum_{i=1}^{n}\left(d_i r_i\left(\lambda N + ... + \lambda^N\right) + \lambda^{N+1} f(\mathbf{0})\right)$$

From the last relation we find

$$f(\mathbf{0}) = \min_{N\geq 0}\left(\frac{c_0 + \sum_{i=1}^{n}\left(c_i + \alpha_i(N+1)r_i\right)}{1-\lambda^{N+1}} + \right. \\ \left. + \lambda\sum_{i=1}^{n} d_i r_i \frac{N(1-\lambda) - \lambda\left(1-\lambda^N\right)}{(1-\lambda)^2\left(1-\lambda^{N+1}\right)}\right) = \\ = \min_{N\geq 0} F(N) = F(N_0).$$

Let's study the case, when $s_i < r_i$ $\left(i = \overline{1,m}\right)$ and $s_i \geq r_i$ $\left(i = m+1,...,n;\ 1 \leq m \leq n\right)$.

Suppose, $m = n$. We are going to find the number of time moments, during which the demands are satisfied, but the products are not ordered. Denote this number by $N_0(s')$, here $s' = (s_1,...,s_n)$. We have

$$f(s') = \min_{N \geq 0}\left(c_0 + \sum_{i=1}^{n}(c_i + \alpha_i (N+1) r_i - s_i) + \frac{\lambda}{(1-\lambda)^2}\sum_{i=1}^{m} d_i r_i \left(N(1-\lambda) - \lambda\left(1-\lambda^N\right)\right) + \lambda^{N+1} f(\mathbf{0}) \right) == Q(N_0(s')).$$

Suppose, $1 \leq m \leq n$. Consider an inventory control problem, where the initial inventory (of products) is specified by $s' = (s_1,...,s_n)$. Let us denote the time period, the demand conditions are satisfied and no new product orders are needed, by $N_0(s')$. Now, $N_0(s') + 1$ is fixed, and the minimal time period, associated with the very first depletion is found. Let us denote indices of products, whose inventory is too small to satisfy demand conditions, by $m+1, m+2,..., m+t \quad (m+t \leq n)$.

Consider an inventory control problem, where the initial inventory is specified by $s'' = (s_1, s_2,..., s_{t+m})$.

We designate the time period, the demand conditions are satisfied and no new product orders are needed as $N_0(s'')$. The latter quantity gives minimum to the following expression:

$$f(s'') = \min\left(\min_{(N+1)r_i > s_i}\left(c_0 + \sum_{i=1}^{t+m}(c_i + \alpha_i (N+1) r_i - s_i) \right) + \lambda \sum_{i=1}^{t+m} d_i r_i \left(N(1-\lambda) - \lambda\left(1-\lambda^N\right)\right) + \lambda^{N+1} f(\mathbf{0}) \right),$$

It some products, whose indices fall into the set $\{t+m+1,...,n\}$, are depleted during the time period $N_0(s') + 1$, we come to the earlier discussed situation, with the initial inventory $s'' = (s_1, s_2,..., s_{t+m})$. If products, whose indices fall into the set $\{m+1,...,n\}$ are not depleted during the time period $N_0(s') + 1$, then the only optimal policy is to make a new order with the following product amounts:

$$(N_0(s') + 1) r_i - s_i, \quad \left(i = \overline{1,m}\right).$$

If the demand is satisfied in the time period $N_0(s') + 1$, we pass to the earlier case, with the earlier discussed inventory control policy.

3. Conclusions

The inventory control theory is one of the newest branches of the operation research. In this paper, the two multistage inventory control models are analysed. These processes are modelled by the dynamic programming equations. There is a new continuous optimal control process for the inventory control problem with the continuous density function of demands, equivalent to the investigated one, created. In addition, for the new continuous optimal and multistage inventory control process with the discrete density function of demands the optimal policy for ordering the products is determined. For the discrete process amount of time moment, during which the demands are satisfied without additional product restocking, are found. While creating these models, some restrictions of the classical models are discarded. The models using more general conditions of warehouse inventory replenishment are constructed.

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DOI: 10.5755/j01.itc.33.4.11950