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SYSTEM DEGRADATION FACTOR FOR NETWORKED CONTROL SYSTEMS

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Abstract. The main differences between networked control systems (NCS) and classical directly wired, (non-NCS) are phenomena such as packet dropouts and packet delays. In general, dynamical properties of NCS and non-NCS are different and understood as degradation of the system. Mainly, the considerations concern on the estimating of the degradation degree of the NCS with linear time-invariant components (plant, actuator, sensor, controller) in respect to non-NCS realization of the same system. It is shown that the problem can be reduced to analysis of the nonstationarity degree of NCS. The degradation degree is calculated in frequency domain using SVD-DFT transformation of system operator defined on finite time horizon. The method enables to make a quantitative comparison of different control strategies, under given probability of packet dropouts, time delays, etc. Theoretical considerations of computational algorithms are complemented with numerical examples.

Keywords: networked control systems, discrete-time systems, time varying systems, communication networks.

1. Introduction

Networked control systems (NCS) are defined as control systems with at least one link (interconnection between elements) being a real-time network [1, 2]. It means that some information (plant input, output or reference input etc.) is exchanged through the network. Real time networks can be classified into two categories: internal fast networks represented by local area networks (LAN) and external slow networks (internet), usually called wide area networks (WAN). In comparison with LAN, WAN has longer time delays, higher probability of packet dropouts and a longer range. Most often, in many networks the common set of communication protocols is used, namely, the Internet protocol suite or TCP/IP protocol suite. The name is obtained from the two most important protocols: Transmission Control Protocol (TCP) and Internet Protocol (IP). During the last decade the technology of general computer networks has progressed very rapidly. Ethernet networks are major competitors to the industrial control networks [3, 4] and the most important reasons are: the well established structure, fast transmission speed (still increasing), widespread usage, numerous software applications, and relatively low costs. Control applications may exploit LAN and WAN to perform remote control at long distances without investing into the whole infrastructure. In comparison to the traditional point to point connections or earlier non-ethernet based industrial networks, the main advantages of global communication networks are wire reduction, low cost and easy maintenance. Until now there exist some general methods [5, 6, 7] as well as some successful applications of NCS in such fields as [3]: dc motors automobiles, aircrafts [8], mobile robots, robotic manipulators, teleoperation [9, 10], and distance learning [11].

Computer networks introduce several problems unexpected in conventional control systems, for instance, packet dropouts, connected with non-guaranteed datagram delivery – User Datagram Protocol (UDP), multiple transmissions, time-delays – caused not only by A/D, D/A converters and the computation time but also by the waiting time (access to the communication medium) and the propagation time. Such effects occur simultaneously and influence the general properties of the system or even lead to instability. On the other hand, changes in the system may be so small that they are negligible. Therefore it would be useful to analyze and assess how strong the changes in the system are, for example, by introducing the degradation degree of the system. The maximal tolerable degradation degree might be an additional design parameter. If the degradation of NCS system due to packet dropouts or time delays is too large, the designer should consider corrections at least in one of following media:

- changes in hardware – a faster network in the case of large traffic, shielding or better MAC protocol in the case of large occurrences of errors;

- changes in software – an optimized router configuration, higher priority to the critical process, optimized coding for the transmission;
- changes in the controller and structure of the system – a different algorithm, hierarchical structure instead of a direct structure.

The paper develops a systematic method for analysis of the degree of degradation of NCS in comparison to non-NCS. The degradation degree of a linear time invariant (LTI) system is defined as the tendency of NCS to have properties similar to a linear time varying (LTV) system rather than to LTI one. The degradation degree of a system with time invariant components is equivalent to the non-stationarity degree defined previously for time-varying systems [12, 13]. The system under consideration (non-NCS one) must be linear and time invariant.

The main contribution of the paper is: the concept of the degradation degrees with four computational methods derived by originally by the author. Two of them (WBA and WRD) base on, presented briefly in the conference paper, system variability degree [13]. This paper extends and utilizes the time-variability degree to specific class of control systems – networked control systems and allows to quantify for them the impact of network effects. The proposed methods make use of operator description for linear systems introduced previously e.g. in [12, 13]. To facilitate reading, the most important results from these papers are quoted in Section 3.

2. Networked Control Systems

The most simple NCS without feedback loop is shown in Figure 1. The system consists of a digital controller/filter, network channel and a digital (digitalized) plant. The equivalent digital plant model may be obtained from the analog plant as shown in Figure 2 and Figure 3. If the digital plant does not contain an internal controller (Figure 2), the system is called a direct structure NCS. In the case where the digital plant has internal feedback loop with both analog or digital controller, for example as shown in Figure 3, the whole NCS has a hierarchical structure.

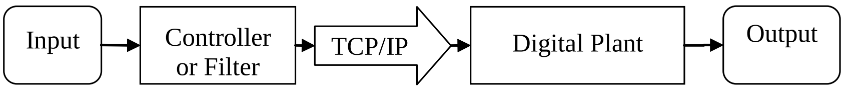

Figure 1. NCS without feedback loop: the simplest configuration

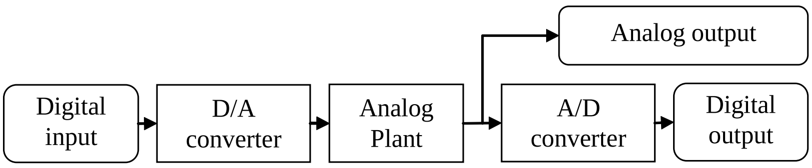

Figure 2. Equivalent diagram for digital plant in direct structure NCS. The digital plant consists of analog plant and A/D, D/A converters

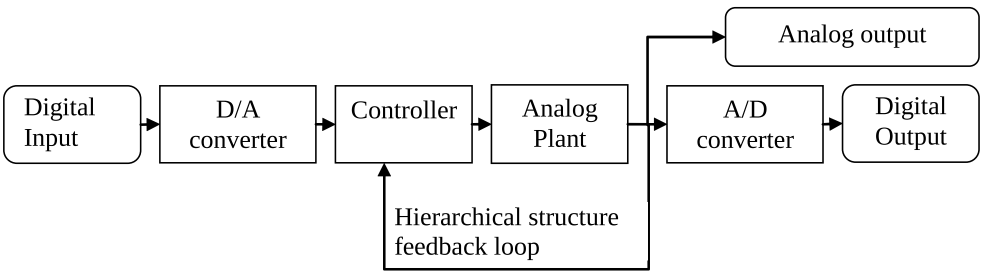

Figure 3. Equivalent diagram for digital plant in hierarchical structure NCS. The structure is hierarchical if the plant has at least one internal feedback loop

The most general NCS configuration is shown in Figure 4. It consists of at least 2 network channels both for controlling and monitoring. Two, more specific, feedback configurations with only one network channel are shown in Figures 5 (networked controlling) and 6 (networked monitoring). Networked monitoring is much more often used than network controlling, which may be rather economic justified to minimize components with network interface for systems with large amount of sensors connected directly to the controller or controller implemented in the intelligent sensor.

The common feature of all NCS configurations is that the control and/or the measurement signals are encapsulated in a frame or packed and sent by means of the network. The direct structure NCS may be used for simple plants and most often in dedicated LAN. The hierarchical structure NCS is rather recommended for fast, autonomous or critical systems and communication by means of WAN. The analysis methodology developed in this paper can be used for both the direct and the hierarchical structure by treating the remote feedback loop as a pure digital plant.

The total delay in NCS is composed of at least two components [14, 15, 3]:

- *waiting time delay* – packets have to wait in queues (sensor, controller queue) before getting access to the media;
- *propagation delay* – time taken by packets travelling over physical media.

Packet losses are connected not only with communication errors but also with time delays. Ethernet allows to lose packets only in selected lower network protocols such as UDP. Higher network protocols such as TCP may require retransmission when an error occurs in a packet or the switch/router drops the packet. Retransmissions in fast NCS may be undesirable because delays affected by the packet loss may be several times greater than the standard (expected, mean) total transmission delay, including waiting and propagation delays. In this case, the data from overdue packet are outdated and, independently, in the UDP/TCP protocol the very long delay is equivalent to the packet loss. Vice versa, packet loss is equivalent to infinite transmission delay.

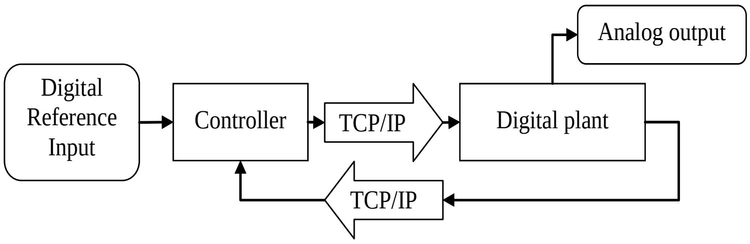

Figure 4. NCS with a remote controller and remote plant. The configuration is the most often used for remote monitoring and control. It consists of two unidirectional or one bidirectional network channels

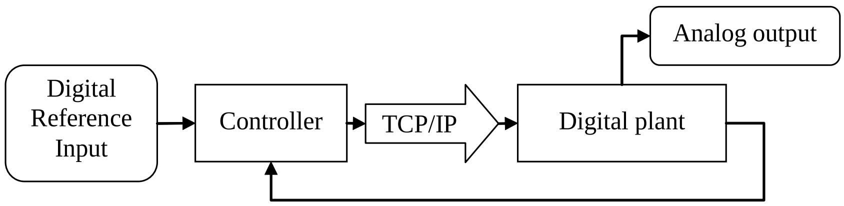

Figure 5. NCS with a remote actuator. Simplified network configuration is mainly used when the measure is available in the controller, however the actuator is situated at a relatively long distance

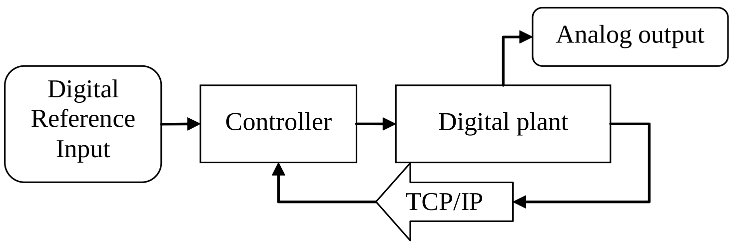

Figure 6. NCS with a remote sensor. Simplified network configuration is used when the measure is transmitted over the network channel and the actuator is available directly at the controller

3. System description

In order to describe NCS one can use the generalized description of time varying discrete-time systems in the form of the state space model with time-dependent matrices. While the non-NCS system matrices are time-invariant ($\mathbf{A}(k) = \mathbf{A}$, $\mathbf{B}(k) = \mathbf{B}$, $\mathbf{C}(k) = \mathbf{C}$), the characteristic NCS behaviour (packet dropout, time delays etc.) can be modelled as variations of these system matrices. The input-output relationships in all real systems are always featured by a non-zero time delay. In such a case the system matrix $\mathbf{D}(k) \equiv \mathbf{0}$ and the term $\mathbf{D}(k)\mathbf{u}_p(k0$ in equation (0) can be omitted. Hence

$$\mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{u}_p(k), \tag{1}$$

$$\mathbf{y}(k) = \mathbf{C}(k)\mathbf{x}(k) + \mathbf{D}(k)\mathbf{u}_p(k), \quad k \in \mathbf{N}, \quad \mathbf{x}(0) = \mathbf{0}, \tag{2}$$

where $\left\{\mathbf{x}(k) \in \mathbb{R}^n, \mathbf{u}_p(k) \in \mathbb{R}^m, \mathbf{y}(k) \in \mathbb{R}^p,\ k \in \{0, \ldots, N-1\}\right\}$ is the state, the input and the output, respectively, and

$\left\{\mathbf{A}(k)\in\mathbb{R}^{n\times n},\mathbf{B}(k)\in\mathbb{R}^{n\times m},\mathbf{C}(k)\in\mathbb{R}^{p\times n},\ k\in\{0,...,N-1\}\right\}$ are the system matrices with perturbations.

In order to employ operators description of the system with finite dimensional matrices it is assumed that the system is defined on finite time horizon. Such an assumption does not restrict applicability of proposed methods. For stable systems system defined on infinite time horizon can be sufficiently approximated even with relatively short time horizons e.g. 100 steps. The minimal length of time horizon can be estimated from system impulse response. The model of the system may be rewritten using matrix operators:

$$\hat{\mathbf{L}}=\begin{bmatrix}\mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}\\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}\\ \mathbf{A}(1) & \mathbf{I} & \mathbf{0} & \vdots & \vdots\\ \vdots & \ddots & \mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{A}(N-2)\cdot\ldots\cdot\mathbf{A}(1) & \cdots & \mathbf{A}(N-2) & \mathbf{I} & \mathbf{0}\end{bmatrix},\quad \hat{\mathbf{N}}=\begin{bmatrix}\mathbf{I}\\ \mathbf{A}(0)\\ \vdots\\ \mathbf{A}(N-2)\cdot\ldots\cdot\mathbf{A}(0)\end{bmatrix}, \tag{3}$$

$$\hat{\mathbf{B}}=\begin{bmatrix}\mathbf{B}(0) & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \ddots & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{B}(N-1)\end{bmatrix},\quad \hat{\mathbf{C}}=\begin{bmatrix}\mathbf{C}(0) & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \ddots & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{C}(N-1)\end{bmatrix},\quad \hat{\mathbf{u}}=\begin{bmatrix}\mathbf{u}_p(0)\\ \vdots\\ \mathbf{u}_p(k-1)\end{bmatrix} \tag{4}$$

where the operators $\hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$ have block diagonal forms, and the state $\hat{\mathbf{x}}$, the output $\hat{\mathbf{y}}$ and the input $\hat{\mathbf{u}}$ have block column vector forms. The operator $\hat{\mathbf{C}}\hat{\mathbf{L}}\hat{\mathbf{B}}$ is a compact, Hilbert-Schmidt operator from l_2 into l_2 and boundedly maps signals $\mathbf{u}_p(k)\in\mathcal{U}=l_2[0,N]$ into signals $y\in\mathcal{Y}$.

The output trajectory of the *nominal* system can be given in as

$$\hat{\mathbf{y}}=\hat{\mathbf{C}}\hat{\mathbf{N}}\mathbf{x}_0+\hat{\mathbf{C}}\hat{\mathbf{L}}\hat{\mathbf{B}}\hat{\mathbf{u}}\cdot \tag{5}$$

It is assumed in this paper that initial conditions are equal to zero and thus the system response is determined by the term $\hat{\mathbf{y}}_u=\hat{\mathbf{C}}\hat{\mathbf{L}}\hat{\mathbf{B}}\hat{\mathbf{u}}$.

In further considerations we use the Singular Value Decomposition (SVD)

$$\mathbf{USV}^{\mathrm{T}}=\hat{\mathbf{C}}\hat{\mathbf{L}}\hat{\mathbf{B}}, \tag{6}$$

where $\mathbf{U\,S\,V}^{\mathrm{T}}$ is SVD product of the system input-output operator $\hat{\mathbf{C}}\hat{\mathbf{L}}\hat{\mathbf{B}}$, $\mathbf{S}$ = diag(σ_i) is a diagonal matrix with singular values, and **U, V** are orthonormal matrices composed of column vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ respectively.

Linear dynamic systems can be variable in the frequency domain – linear time invariant, in the time domain – linear frequency invariant or both in the frequency and time domain – linear time-varying. The main difference between time and frequency invariant systems can be obtained by comparison of the output functions. The output function of time-invariant system is a time domain convolution or a frequency domain multiplication, whereas the output function of frequency invariant system is a time domain multiplication or a frequency domain convolution. In the most general case a linear dynamic system can be variable in both the frequency and the time domain. From the mathematical point of view, a dichotomous classification of time or frequency variant systems is clearly defined. However, from the practical viewpoint, the degree of time non-stationarity on a continuous scale plays a key role in the quantitative assessment of properties of NCS.

NCS with LTI components (plant, actuators, sensors, controller) and the specific phenomena of packet losses and variable time delays become LTV, where the time-variability of the system (non-stationarity) results from networking. For such a system, the degradation degree of LTI-NCS is equivalent to the non-stationarity degree of the LTV system.

The degradation degree can be defined in various ways. Our approach is based on SVD of the system operator $\hat{\mathbf{C}}\hat{\mathbf{L}}\hat{\mathbf{B}}$. This spectral decomposition is a generalisation of SVD of a matrix. For discrete-time systems and finite time horizons the system operator is finite dimensional. As known from linear algebra, the SVD of a matrix leads to a set of singular values σ_i and the corresponding sets of singular input vectors $\mathbf{v}_i$ and output vectors $\mathbf{u}_i$. Every real or complex matrix $\mathbf{X}$ can be written in the form $\mathbf{X}=\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^*$, where $\boldsymbol{\Sigma}=\mathrm{diag}\{\sigma_i\}$ and $\mathbf{U}$ and $\mathbf{V}$ are composed of $\mathbf{u}_i$ and $\mathbf{v}_i$, respectively.

The relation between the input and output power spectral density and an approximated amplitude diagram have been introduced in [12]. The most important result of [7] says that the magnitude-frequency response $|G(\omega_k)|$ can be uniquely defined by

$$|G(\omega_k)|=\sqrt{\frac{1}{N}\sum_{j=1}^{N}\sigma_j^2\left|\mathrm{DFT}_k[\mathbf{u}_j]\right|^2}=\sqrt{\frac{1}{N}\sum_{j=1}^{N}\left|\sigma_j\sum_{n=1}^{N}u_{n,j}e^{-2\pi\sqrt{-1}(k-1)(n-1)/N}\right|^2}\ . \tag{7}$$

The classical magnitude diagram for LTI systems on an infinite time horizon, obtained by substitution $z = \exp\left(j\omega T_p\right)$ in the discrete transfer function, is almost identical to the above approximation. Parameters N, Tp should be chosen in order to ensure the desired resolution $\Delta f = 1/NT_p$ and range $f \in \left[0, 1/2T_p\right)$ in frequency domain.

4. The degradation Degree of NCS

It is well known that LTI system does not modulate an input, output and state. The differences between input and output signals are reflected only in the amplitude and phase. Alternatively, this idea can be formulated using SVD-DFT of the system operator**.** For LTI system the amplitude spectra of the corresponding singular vectors of LTI system are identical for **U** and **V,** that is

$$\left|\mathrm{DFT}_k[\mathbf{u}_j]\right| = \left|\mathrm{DFT}_k[\mathbf{v}_j]\right| \tag{8}$$

for all k, j or equivalently

$$\left|\sum_{n=1}^{N} u_{n,j} e^{-2\pi\sqrt{-1}(k-1)(n-1)/N}\right| = \left|\sum_{n=1}^{N} v_{n,j} e^{-2\pi\sqrt{-1}(k-1)(n-1)/N}\right| \tag{9}$$

is satisfied for all k, j**.**

The relationship (9) is not satisfied for general LTV systems because of the splitting effect of the amplitude spectra, as typical for LTV systems. Let us assume the input signal for LTV system to be sinusoidal with frequency ω_0, and the frequency of the time varying parameters is ω_p. In the normalized output spectrum the side bands $\omega_0 - \omega_p, \omega_0 + \omega_p$ appear and additionally, the amplitude of the main band ω_0 diminishes. As the measure of the system time-variability, one can use both the main band attenuation and the distance between the side bands and the main band.

(I) *Weighted band attenuation (WBA)*

Definition of this coefficient follows directly from property (8). This definition takes advantage of the differences between the input and the output amplitude spectra of the system operator. First of all, this measure is sensitive to the main band attenuation (amplitude modulation). On the other hand, the distance between the main band and the side bands (frequency modulation) is of secondary importance. The less the output is affected by the parameter nonstationarity the smaller is *WBA*. Here, the rate of parameter changes is of secondary importance.

WBA can be evaluated numerically as a sum of squared differences between consecutive discrete values of the power spectral density of normalized eigenvectors for the input and the output spectra

$$WBA = \sum_{i=1}^{N} \frac{\sigma_i}{\sigma_1} \left\| \left|\mathrm{DFT}[\mathbf{v}_i]\right| - \left|\mathrm{DFT}[\mathbf{u}_i]\right| \right\|_2 . \tag{10}$$

(II) *Weighted relative distance between corresponding bands (WRD)*

While WBA is sensitive to differences in the magnitude and the rate, the main aim of WRD is to introduce a factor sensitive mainly to the distance between the main and the side bands. To employ this idea, one has to know indexes of maximal values in the transformed singular vectors. Let us assume that only the largest magnitude of each singular vector is important and the remaining ones can be neglected. Thus this coefficient can be computed by means of the formula

$$S_{WRD} = \Delta f \sum_{i=1}^{N} \left| k_{mv}(i) - k_{mu}(i) \right| \frac{\sigma_i}{\sigma_1}, \tag{11}$$

where $k_{mv}(i)$ denotes the index of maximal value in the vector $\mathbf{M}_V(i)$=|DFT($\mathbf{v}_i$)|, $k_{mu}(i)$ is the index of maximal value in the vector $\mathbf{M}_U(i)$=|DFT($\mathbf{u}_i$)|, the resolution in frequency domain $\Delta f = \dfrac{1}{T_p N}$ is the normalisation factor in eq. (11).

This coefficient depends on the rate of parameter changes. The slower the changes, the smaller the coefficient. The main disadvantage of this measure is the assumption that only the largest magnitude of each singular vector is important. In fact, all magnitudes should be taken into consideration. The coefficient can be improved by taking into account not only the first maximal value of vectors **U**, **V** but also the consecutive values sorted in decreasing order and with weights. Hence,

$$WRD = \Delta f \sum_{j=1}^{N} \left[\frac{\sigma_j}{\sigma_1} \sum_{i=1}^{N/2} \left(\left| I_V(i,j) - I_U(i,j) \right| \frac{A_V(i,j) - A_U(i,j)}{A_V(1,j) - A_U(1,j)} \right) \right], \tag{12}$$

where $\mathbf{M}_V$ is the magnitude matrix of 1-dimensional DFT of the matrix $\mathbf{V} = \left[\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_N\right]$, that is $\mathbf{M}_V = \left|\mathrm{DFT}(\mathbf{V})\right|$, $\mathbf{A}_V$=column matrix $\mathbf{M}_V$ sorted in decreasing order, that is $A_V(i,j) \geq A_V(i+1,j)$ and $\mathbf{I}_V$ is the index matrix such that $A_V\left(I_V(i,j), j\right) = M_V(i,j)$, where bold capitals denote matrices and italic capitals denote entries of the corresponding matrices, that is $\mathbf{A}_V = \left\{A_V(i,j) : i,j = 1,2,\ldots,N\right\}$. Similar relations hold for matrices $\mathbf{U}, \mathbf{A}_U, \mathbf{M}_U$ and $\mathbf{I}_U$, respectively.

(III) *Difference amplitude diagram (DAD)*

Definition of this measure takes advantage of differences in the magnitude spectra between the diagrams for NCS and the nominal, non-NCS. The measure is calculated as the mean absolute error between two magnitude diagrams in linear scale

$$DAD = \frac{1}{N} \sum_{k=1}^{N} \left\| \left|G_{NCS}(j\omega_k)\right| - \left|G_{non-NCS}(j\omega_k)\right| \right\|. \tag{13}$$

It is sensitive to differences in magnitude diagrams. The main disadvantage of the definition is the fact that one has to know the reference nominal system and to perform SVD-DFT analysis also for this system. On the other hand, the main advantage of

DAD is good linearity and results concentration. These features make the method the most useful out the four measures described in this section.

(IV) ***Difference impulse transform (DIT)***

The DIT degradation degree is similar to DAD with the only difference that DIT does not take an advantage of the SVD-DFT transform. It uses the transformed impulse response of NCS (h_{NCS}) and the nominal ($h_{non\text{-}NCS}$) non-NCS instead of the approximated Bode diagrams.

$$DIT = \frac{1}{N}\sum_{k=1}^{N}\left|A_{NCS}(\omega_k) - A_{non-NCS}(\omega_k)\right|, \tag{14}$$
$$A_{NCS}(\omega_k) = \left|\mathrm{DFT}\left(h_{NCS}(t)\right)\right|, A_{non-NCS}(\omega_k) = \left|\mathrm{DFT}\left(h_{non-NCS}(t)\right)\right|.$$

The definition has all the disadvantages of DAD together with a weak concentration, which makes it much less useful than DAD.

For time-invariant systems all these coefficients should be equal to zero. Value of the coefficients, except DIT does not depend whether the system is stable/unstable. However for unstable systems the algorithm is weak conditioned. Especially for longer N usually approach infinity and exceed the range of real numbers.

From practical point of view, DIT degradation degree is not very useful. Comparison of the properties of the DIT and the three degrees given above (WBA, WRD and DAD) shows the weakness of the direct impulse response transformation and the prevalence of operators based degrees (WBA, WRD and DAD). The main reason for including DIT is to show the difference.

More details concerning stability of feedback NCS can be found in e.g. [16, 17, 18, 19, 20, 21]. Now we shortly consider the following two cases of stability. First when both non-NCS and NCS are stable and the degradation degrees are applicable. The second one when network channel follows to instability e.g. non-NCS is stable while NCS is unstable. There are two possible reasons: large number of packets are lost or real delay caused by network is larger than assumed in the model. In such case the NCS system should be redesigned to ensure stability, for example, to convert direct structure into hierarchical structure or to design the controller to work with larger delays.

In the next section we attempt to check whether the given relations present good measures of the system degradation degree.

5. Numerical examples

In this section the properties of NCS degradation degrees (WBA, WRD, DAD, DIT) are explored on the basis of numerical examples. To carry out a numerical analysis, two models of NCS with a specified probability of the packet loss/ time delay are presented and discussed.

For the purpose of comparison, the graphical form of step responses is used. It is also assumed that the system is described by the discrete state space model, defined on a finite time horizon. The sampling period is taken equal to T_s=0.04 s, k=1,2,…,N, where N=256 is the time horizon length, and is scaled to improve resolution in the time domain.

5.1. System without feedback

The diagram of the system is shown in Figure 1. The P controller has gain equal to 1 and as the digital plant the first order time lag with a delay is assumed. This delay represents the total expected delay of the system. The discrete transfer function of the equivalent non-NCS control system has the form:

$$G(z) = \frac{1-a}{(z-a)z^2}\ . \tag{15}$$

Matrices in the state space model of the system are

$$\mathbf{A} = \begin{bmatrix} a & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1-a \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{C}_n = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}^T, \mathbf{C}_d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}^T, \tag{16}$$

where vector $\mathbf{C}_n$ represents the nominal (expected) delay and vector $\mathbf{C}_d$ represents the maximal delay under consideration. $\mathbf{C}_n$ may be substituted for $\mathbf{C}_d$ at each time instant in model (1)-(2) with a given probability of the delay or with a given number of delayed packets during the considered time interval.

The output vector $\mathbf{C}$ is described as

$$\mathbf{C}(k) = \begin{cases} \mathbf{C}_n & r(k) > p \\ \mathbf{C}_d & r(k) \le p \end{cases}, \tag{17}$$

where $\mathbf{r} = \{r(k) \in [0,1], k = 1,2,...N\}$ is the random, uniform distribution vector with values at each time sample on the simulation horizon and p is the probability of packet delay.

NCS is analysed with two different values of the time lag constant. In the first case the system is slow with a=0.9, while in the second case it is relatively fast with a=0.1. Three diagrams are presented in Figures 7-9. Circles correspond to the fast system and dots to the slow one. The computed values of the WBA degradation factor versus the relative number of delayed packets (the number of delayed packets by the total number of packets) for 400 simulations are shown in Figure 7. In turn, the values of DAD and WRD degradation degree are shown in Figures 8, and 9, respectively. All the diagrams grow with the increase of the relative number of delayed packets. The degradation factor for the fast system has much higher level than for the slow one, which is consistent with our intuition. The difference between the fast and the slow system is biggest on the DAD diagram. Of course, the average levels for all degradation degrees are different. Two step responses with similar WRD degradation factors (approximately, 0.008) are presented in Figure 10, the slow system for p=0.49 and the fast system for p=0.016.

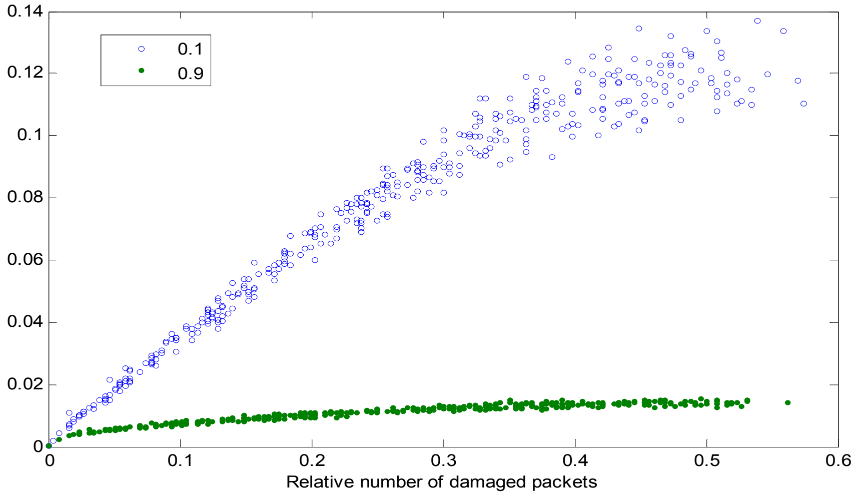

Figure 7. WBA degradation factor for fast (circles) and slow (dots) time lag systems with packet delays vs. the packet delay probability. WBA grows along with the relative number of delayed packets and it grows faster for the fast system which is consistent with intuition

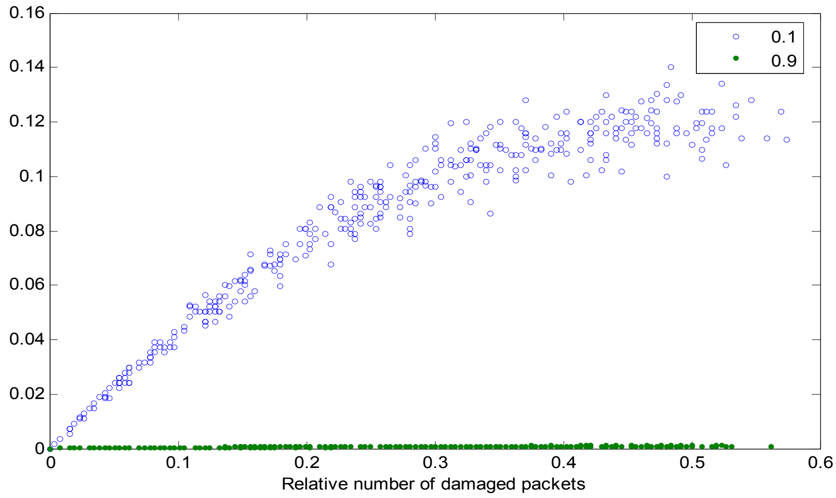

Figure 8. DAD degradation factor for fast (circles) and slow (dots) time lag systems with packet delays vs. the packet delay probability. The differences between the fast and the slow system are largest in DAD diagram. Of course, average levels for all degradation degrees are different

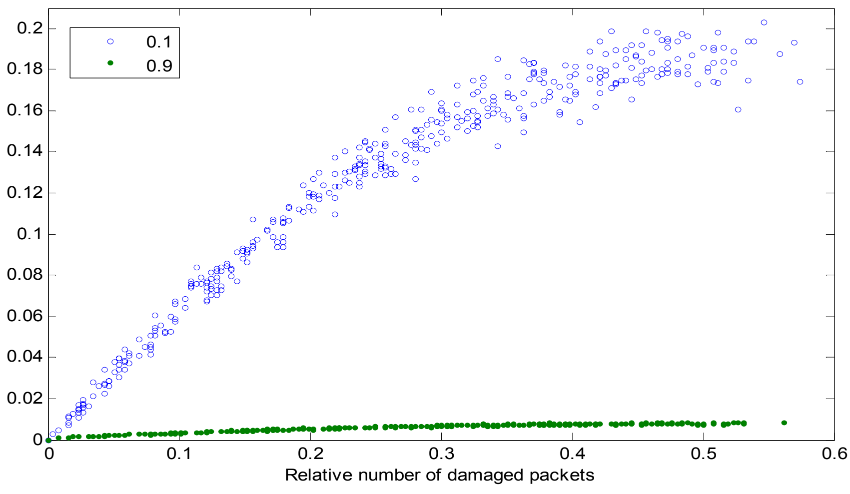

Figure 9. WRD degradation factor for fast (circles) and slow (dots) time lag systems with packet delays vs. the relative number of delayed packets. The diagram is similar to that of DAD, however it does not mean that the diagrams are almost identical. The definitions are quite different

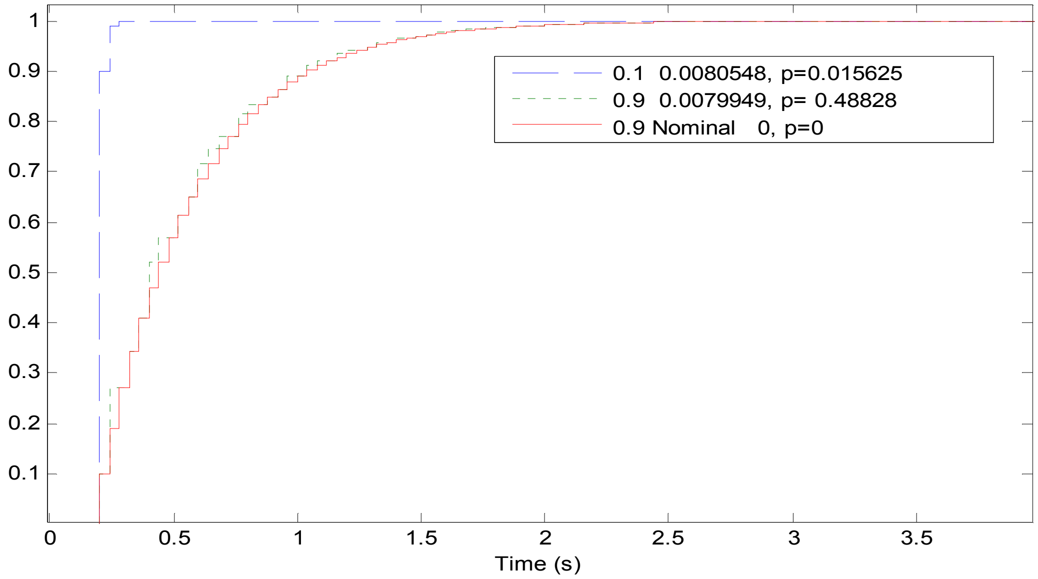

Figure 10. Step responses for fast (solid line) and slow (dotted line) time lag systems with packet delays. The degradation of the step responses for approximately the same WRD are rather similar however the definitions are based on the frequency domain, and the differences in the time domain do not need to correspond to the computed degradation degree

It should be underlined that the step response is not a good indicator of the network system properties. First of all the step response as well as impulse response applied to stable NCS is sensitive mostly at the beginning of the response. It mean that impact on the output is strongly dependent on time shifts, i.e. packet lost have higher impact at the beginning of the response.

NCS systems are usually characterised rather by given probability of packed damage than by a fixed time moments of packet lost or delays. The main aims of control are in general: to minimize tracking error for a given reference signal and to provide good disturbance attenuation. Both of the tasks are time independent, thus any performance or degradation index should not be time dependent. Single signal analysis for time-varying systems can always cause undetectable changes in output signal (compensated itself by the signal/system) and may be inaccurate. Proposed method takes into account full set of orthogonal vectors which guarantee good detectability of possible time-varying disturbances and is insensitive to possible time-shifts. Step responses on Figure 10 are shown only to illustrate dynamical properties of analysed systems, not for evaluation properties of the NCS.

5.2. System with feedback

We now analyze properties of the degradation degree for NCS system with feedback control loop. The system consists of three components. The controller G_c is assumed to be the real proportional system with the gain factor k and time lag 0.1. The integral action G_i may be assigned either to the controller or to the plant. The digital plant is the first order time lag system with a delay representing the total expected delay of the system. The transfer functions of all components are

$$G_c(z)=\frac{k}{(z-0.1)},\; G_i(z)=\frac{1}{z-1},\; G_p(z)=\frac{b}{(z-a)z^2}. \quad (18)$$

The equivalent non-NCS, open loop transfer function is given by

$$G(z)=\frac{kb}{(z-0.1)(z-1)(z-a)z^2}. \quad (19)$$

The matrices of the state space model of the system are given as

$$\mathbf{A}_0=\begin{bmatrix}0.1&0&0&0&-k\\1&1&0&0&0\\0&1&a&0&0\\0&0&1&0&0\\0&0&0&1&0\end{bmatrix},\mathbf{A}_1=\begin{bmatrix}0.1&0&0&0&-k\\0&1&0&0&0\\0&1&a&0&0\\0&0&1&0&0\\0&0&0&1&0\end{bmatrix},\mathbf{A}_2=\begin{bmatrix}0.1&0&0&0&-k\\1&1&0&0&0\\0&0&a&0&0\\0&0&1&0&0\\0&0&0&1&0\end{bmatrix},\mathbf{B}=\begin{bmatrix}k\\0\\0\\0\\0\end{bmatrix},\mathbf{C}=\begin{bmatrix}0\\0\\0\\0\\T_pb\end{bmatrix}^{\mathrm{T}} \quad (20)$$

where matrix $\mathbf{A}_0$ represents the nominal system without packet dropouts, $\mathbf{A}_1$ and $\mathbf{A}_2$ represent packet dropouts in NCS with a remote actuator before and after the integral action, respectively, a=0.9 is the time lag.

The network channel can have two different configurations. The matrix $\mathbf{A}_0$ may be substituted either for $\mathbf{A}_1$ (case 1) or $\mathbf{A}_2$ (case 2) at each time instant in the model (1)-(2) with a given packet dropout probability or with a given number of lost packets within the considered time horizon.

Case 1 of NCS with an integral action in the plant $G_{plant} = G_i G_p$ is shown in Figure 5, where the system matrix $\mathbf{A}$ is given by

$$\mathbf{A}(k) = \begin{cases} \mathbf{A}_0 & r(k) > p \\ \mathbf{A}_1 & r(k) \le p \end{cases}. \tag{21}$$

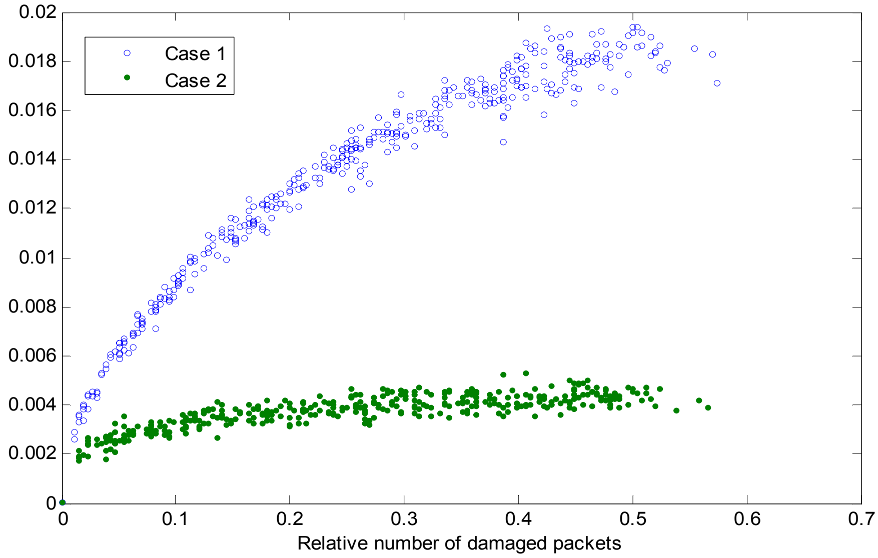

Figure 11. WBA degradation factor for two NCS configurations: Case 1 – circles and Case 2 – dots of a closed loop control system with packets loss vs. the relative number of lost packets. The definition gives concentrated results however the WBA is less linear than DAD

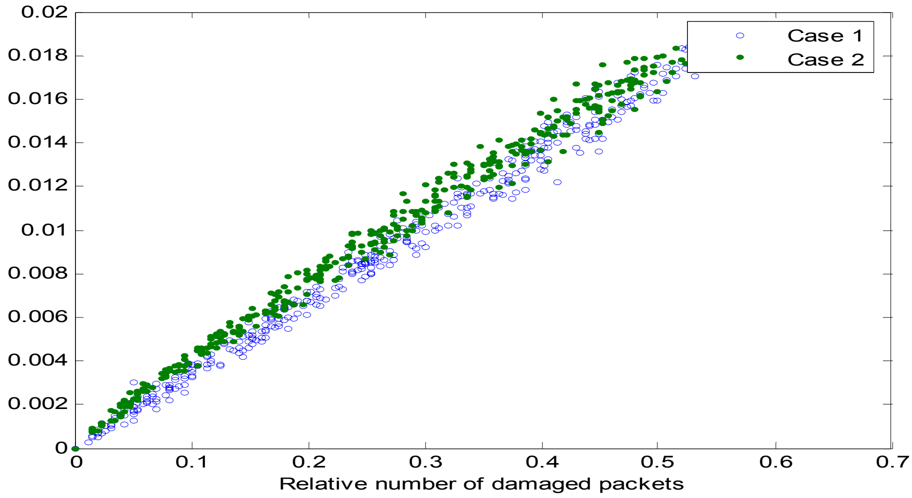

Figure 12. DAD degradation factor for two NCS configurations: Case 1 – circles and Case 2 – dots of a closed loop control system with packets loss vs. the relative number of lost packets. DAD also gives best results. Unfortunately, it requires computation of two approximated SVD-DFT diagrams for NCS and non-NCS case. If the nominal model is known, DAD is probably the best degradation degree out of the four introduced in this paper

Case 2 of NCS with an integral action in the controller $G_{controller} = G_c G_i$ is shown in Figure 5 with the matrix $\mathbf{A}$ defined as

$$\mathbf{A}(k) = \begin{cases} \mathbf{A}_0 & r(k) > p \\ \mathbf{A}_2 & r(k) \le p \end{cases}, \tag{22}$$

where $\mathbf{r} = \{r(k) \in [0,1], k = 1, 2, \ldots N\}$ is the random, uniform distribution vector with values at each time sample from the simulation time horizon and p is the probability of the packet loss.

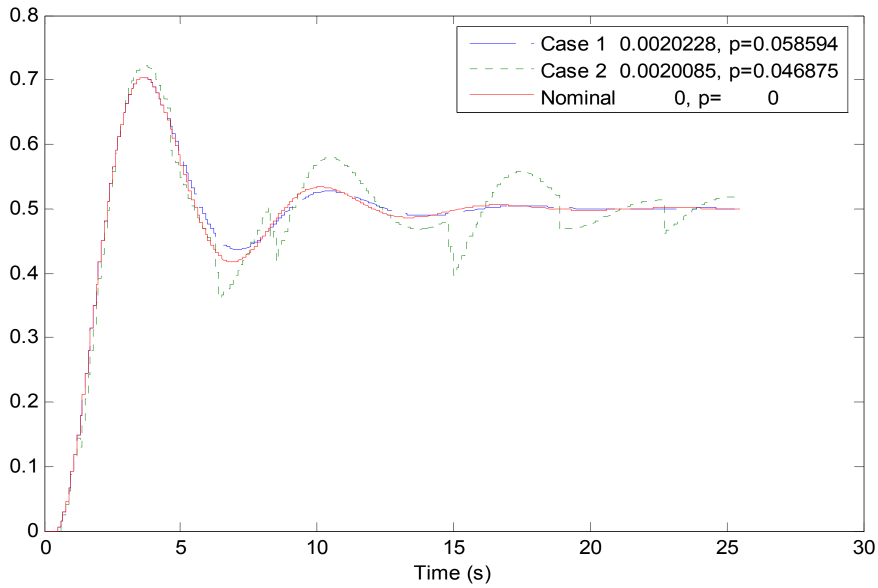

Figure 13. Step responses for two closed control loop NCS configurations with packets loss and similar degradation factors: Case 1 – solid line, Case 2 – dotted line and the system without packets loss – dashed line

Figures 11-12 show two different degradation degrees: WBA and DAD against the relative number of lost packets (the number of lost packets by the total number of packets) for case 1 and case 2. The total number of simulations equals 400 in each case. Each of the computed degrees shows different properties. WBA gives concentrated results however is less linear than DAD. Moreover, WBA has higher level for case 1 than for case 2 which does not go along with our intuition and the step response. DIT gives unconcentrated results and cannot be used for evaluation of degradation degree. It seems that in this example the best properties are shown by DAD, though it requires two approximated SVD-DFT diagrams for NCS and non-NCS. If the nominal model is known, DAD is probably the best measure out of the four introduced in Section 4. For this reason, it is used in the further analysis illustrated in Figures 13 and 14.

The step responses for similar DAD are shown in Figure 13, the solid line corresponds to the network channel before the integral (case 1 – p=0.0586, DAD=0.00202), the dotted line – to the network channel after the integral (case 2 – p=0.0469, DAD=0.00201) and the dashed line – to the reference system without packet loss (non-NCS – p=0, DAD=0). For a given nonzero degradation degree value the both systems in case 1 and 2 are different from the nominal one. As it can be seen from Figures 13 and 14, similar level of the degree results in a similar range of degradation (deviation) of the step response and the frequency error. The degradation degree is calculated in the frequency domain. The high frequency disturbances which are visible in the step response for case 2 correspond to higher levels of the difference frequency response for high frequencies (Figure 14). Nevertheless, the error for low frequencies is larger in case 1 which is due to similar levels of DAD. For WBA, the frequency effects such as pole shifting and dynamics modulation have more significant impact than a simple signal modulation etc., and in case 1 this results in higher values of WBA – Figure 11.

6. Conclusion

Proposed method based on the defined degradation degree is attractive for the networked control systems analysis. Alternative direct simulation of the control system performance under specific transfer conditions returns results strongly dependent on the input signal (usually step or impulse). The impulse response of stable NCS is sensitive mostly at the beginning of the response and it follows e.g. to different effects for time shifted moments of packet lost, what is in practise unexpected. Better results may be achieved for flat magnitude-frequency spectrum input signal.

Nevertheless single signal analysis for time-varying systems can always cause undetectable changes in output signal (compensated itself by the signal/system). Since the network effects are modelled by the time-varying dynamics and the real control systems have to attenuate disturbances, the single signal analysis may be inaccurate. Proposed method in order to precisely analyse the most important properties take into account full set of the orthogonal vectors of the transfer operator. The approach guarantee detectability of all possible time-varying disturbances practically without sensitivity to time-shifts of packed lost and packet delays.

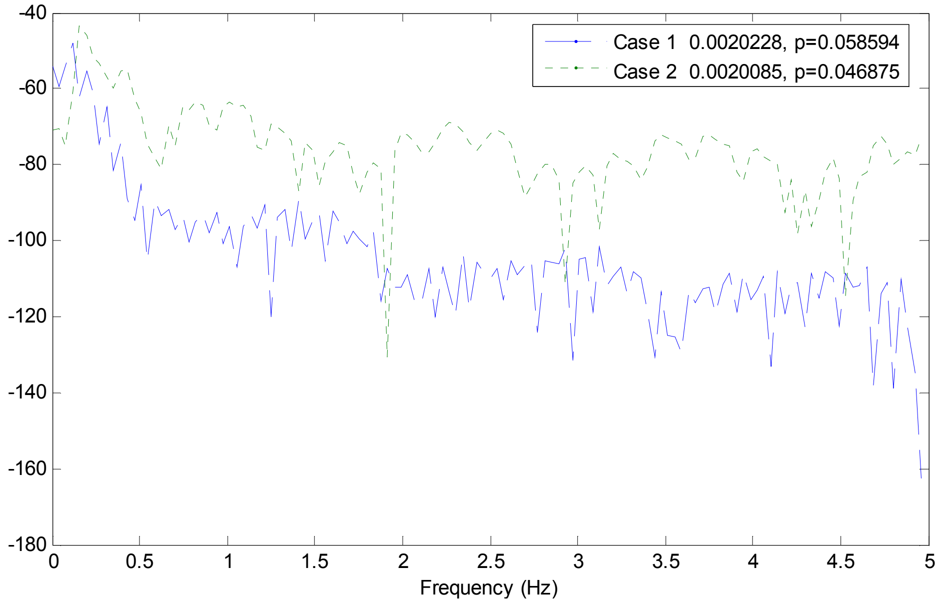

Figure 14. Difference between transformed impulse responses (magnitude only) for NCS with similar degradation factors: Case 1 – solid line, Case 2 – dotted line. Similar level of DD results in the similar range of the frequency error. High frequency disturbances which are visible in the step response in case 2 correspond to higher levels of the difference frequency response for high frequencies. Nevertheless error for low frequencies is larger in case 1 which is due to similar levels of DAD

The developed method shows how to quantify the degradation of NCS in comparison to non-NCS LTI systems. This method can be used in the case of packets loss and variable time delays. The degradation factor is a measure which can be helpful in designing and validating NCS. The designers very often assume that NCS works almost identically as the corresponding non-NCS and for this reason the methodology for LTI systems can be applied to NCS. In our paper we try to answer the question whether a chosen controller and a system configuration with network channels and a given plant dynamics have properties similar to LTI systems. In the case of negative answer the designing methodology must be verified to meet the design requirements. The degradation degree method suggests not only changes in control loop in the classical sense (controller, compensator etc.) but also gives information whether a chosen routers configuration and the coding algorithm for data through the communication media are suitable. The degradation degree algorithm requires that all the system components and the control loop without network channels must be LTI. However it does not restrict possible applications of the method only to LTI systems. Since the possible changes in the dynamics are the most important factors for evaluation purposes, the system can be linearized.

As can be seen from above examples, computed value of any degradation degree is dependent on assumed realization of packet delays and packet dropouts. In practise it is possible to use alternatively derivative measures defined as: upper bound (or maximal value) of any defined previously degree evaluated for each relative number of damaged packets. Instead of the maximal value can be used the mean value with corresponding standard deviation, which is easier to estimate than the upper bound.

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