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REDUCTION OF FINITE ELEMENT MODELS BY PARAMETER IDENTIFICATION

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Abstract. The study deals with the problem among to formally reduce a complex finite element structural model to a simpler one. As a sample task, the reduction of a girder model to the simpler equivalent membrane model has been investigated. The coincidence of model displacements at given loading conditions is employed as a criterion of mutual adequacy of the two models. Both static and dynamic displacements at selected reference points have been used in the expression of the penalty-type target function, the minimum of which indicates the best fit between the original and reduced models. The target function has been minimized by using the geometrical and physical parameters of a typical membrane element as optimization variables. The calculations have been performed in MATLAB environment. The validity of obtained parameters of the membrane model has been tested by investigating the original and reduced structures of different geometrical shapes at complex loadings.

Keywords: Finite element models, reduction, parameter identification.

1. Introduction

Finite element techniques in principle enable to analyse structures of any level of complexity, including their essentially non-linear behaviour, peculiarities of internal texture, etc. [11]. The models are often generated by the expense of very complex structures, huge dimensionalities and internal interactions, which require very large and often prohibitive amounts of computational resources. Building simplified (reduced) computational models is a common practice enabling to obtain solutions with practically acceptable costs.

Modest size computational models of bodies the internal structure of which is complex and heterogeneous are generally obtained by using appropriate constitutive equations describing the material behavior. In structural analysis, constitutive relations connect applied stresses or forces to strains or deformations. More generally, in physics, a constitutive equation is a relation between two physical quantities that is specific to a material or substance, and does not follow directly from physical law. It is combined with other equations that do represent physical laws to solve physical problems [10]. Some constitutive equations are simply phenomenological; others are derived from first principles. Most commercial finite element systems possess large libraries of constitutive models. For example, LS-DYNA program accepts a wide range of material and equation of state models, each with unique number of history variables. Approximately 100 material models are implemented in LS-DYNA. Some materials include strain-rate sensitivity, failure, equations of state and thermal effects [7].

The concept of multi-scale finite elements has been introduced to describe the generalization of the traditional finite element by prescribing it to more complex behavior laws than could be possible by using traditional constitutive equation models [4].

Multiscale finite element can be regarded as scaleable, mathematical macro-model, which captures the response of a sub-region of the computational domain, in a manner seamlessly compatible with the finite element modeling infrastructure. Such generalized finite elements are implemented by using domain decomposition methodology. Domain decomposition methods solve a boundary value problem by splitting it into smaller boundary value problems on subdomains and iterating to coordinate the solution between the subdomains. The problems on the subdomains are independent, which make domain decomposition methods suitable for parallel computing. Domain decomposition is an active, interdisciplinary research area concerned with the development, analysis, and implementation of coupling and decoupling strategies in mathematical and computational models of natural and engineering systems. The simplest applications of the idea of domain decomposition are super elements in linear static problems, which are implemented in most finite element programs such as ANSYS, NASTRAN, etc. However, there are numerous studies of application of domain decomposition in dynamics, such as component mode synthesis methods. Generally, computational model reduction methodologies are important in research area leading to efficient and reasonably adequate models of various engineering systems [6].

Numerous engineering approaches to construction of multiscale finite element approaches have been reported in the scientific literature. As an example, the very high velocity contact interaction problems are among the most complicated in computational mechanics. The failure processes that follow the interaction are initiated in micro-volumes considerably smaller than the measurements of the interacting bodies. It is practically impossible to model the behavior of the material at the micro-level – the number of degrees of freedom of such a model would be too large and unrealistic for computer resources nowadays and probably in the nearest future as well. In practice, usually the computations are performed by using macroscopic material models that approximately describe real processes taking place in the material. In [5] real and numerical shooting-through experiments are presented for the Nextel fabrics and the Kevlar-epoxy shield. The micro- and mezzo-mechanical models have been used to simulate the behavior of small specimens, and in comparison of the numerical results with experimental data the material model characteristics have been found. The stiffness coefficients were used for determining the deviatoric stresses and by means of state equation of the relationship between the pressure and volume change was established. Further computations have been performed by using the macro-mechanical model where a layered structure has been presented as the porous continuum. It enabled to disregard the real geometry of the weave and to present the averaged strength parameters of fabrics. The resulting model was axisymmetric, of reasonable dimension, and the obtained results were satisfactorily close to the experimental ones.

As an example, a woven textile structure can be represented by using models of different levels of detalization. A woven structure composed of shell elements [2], simpler and more efficient combined particles model [3], orthotropic membrane models [1], have been employed in order to represent the dynamic behavior of textile cloths under conditions of mechanical impact and penetration. A special attention and prospective deserve models, in which central and distant zones of the same structure are presented by different models. As in [1], the zone of ballistic interaction of textile structure has been modeled by using the complex contact model of a woven structure, meanwhile the distant zones have been presented by membrane elements. The coupling between the zones has been implemented by means of tie constraint. The main purpose of this combination was to implement the “almost infinite” surrounding.

As a rule, such combined models are obtained by using a lot of engineering intuition and basing on profound knowledge of physical properties of the investigated phenomena. More regular approaches are necessary, which enable to synthesize simplified or reduced models of internally complex structures. The parameters of the reduced model can be adjusted by performing the minimization of error functions, quantitatively indicating the non-coincidence of the response between the simplified and reference models. An alternative approach can be based on neural network techniques in order to synthesize the models exhibiting the required structural response [9].

Generally, simplified (reduced) models of complex structures can be obtained on the base of comparison of their response to appropriate set of excitations against the response of a more detailed model exposed to the same excitations. In this work, we demonstrate probably the simplest approach to the construction of reduced models based on minimization of penalty-type target function representing the residual between the two sets of responses. The work presents the procedure and results of synthesis of the continuous membrane model, which imitates the behavior of girder structure under static, as well as, dynamic loadings.

2. Problem Formulation

The analyzed source structure is a 2D girder, which is composed of tiny beam elements, and the approximating reduced model is a planar membrane. The girder consists of rods of uniform width and thickness. It is necessary to find the parameters of the equivalent orthotropic membrane. Assume that the membrane model presents a satisfactory approximation of the girder if the displacements of nodes at the same loading are obtained nearly the same by using both models.

Consider rectangular plate and rectangular girder, which have identical dimensions (Figure 1). The girder geometry is described by rod width h, rod thickness b and N number of cells along the side of the girder. Physical parameters used in the small displacement elasticity model are Young module E_g and mass density ρ_g. The membrane model is characterized by thickness s_m and orthotropic material parameters: Young module E_m, Poisson ratio ν_m, shear module G_m and mass density ρ_m.

Membrane parameters ν_m, E_m, G_m and s_m have to be established, which enable the membrane to exhibit the same or similar behaviour in terms of displacements of respective element nodes at a given loading.

As a measure of quality of the approximation of the girder model by equivalent membrane model we employ the minimum of a penalty-type target function expressed as a sum of squares of differences between the displacements of corresponding nodes of each model. The static as well as dynamic behaviour of the two structures has been analyzed. The schemes of two static loading cases are presented in Figures 2 a) and b), where both structures have been exposed to static load F and the differences of displacements of 4 selected nodes have been included into the penalty function expression.

The obtained parameters of the equivalent membrane shall be tested by using several freely selected test models (loading cases), two of which are presented in Figures 2 c) and d).

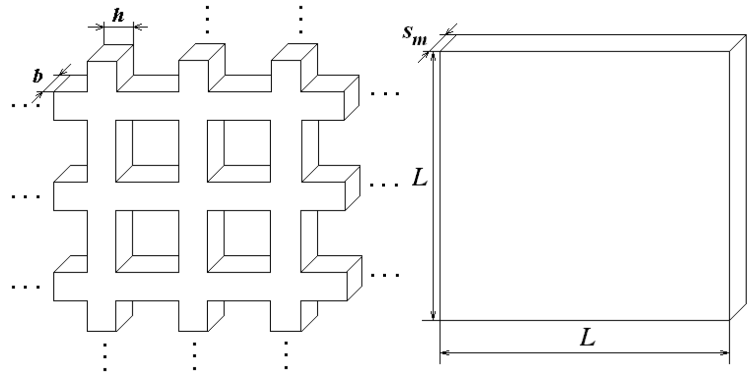

Figure 1. The girder (a) and equivalent steel membrane (b)

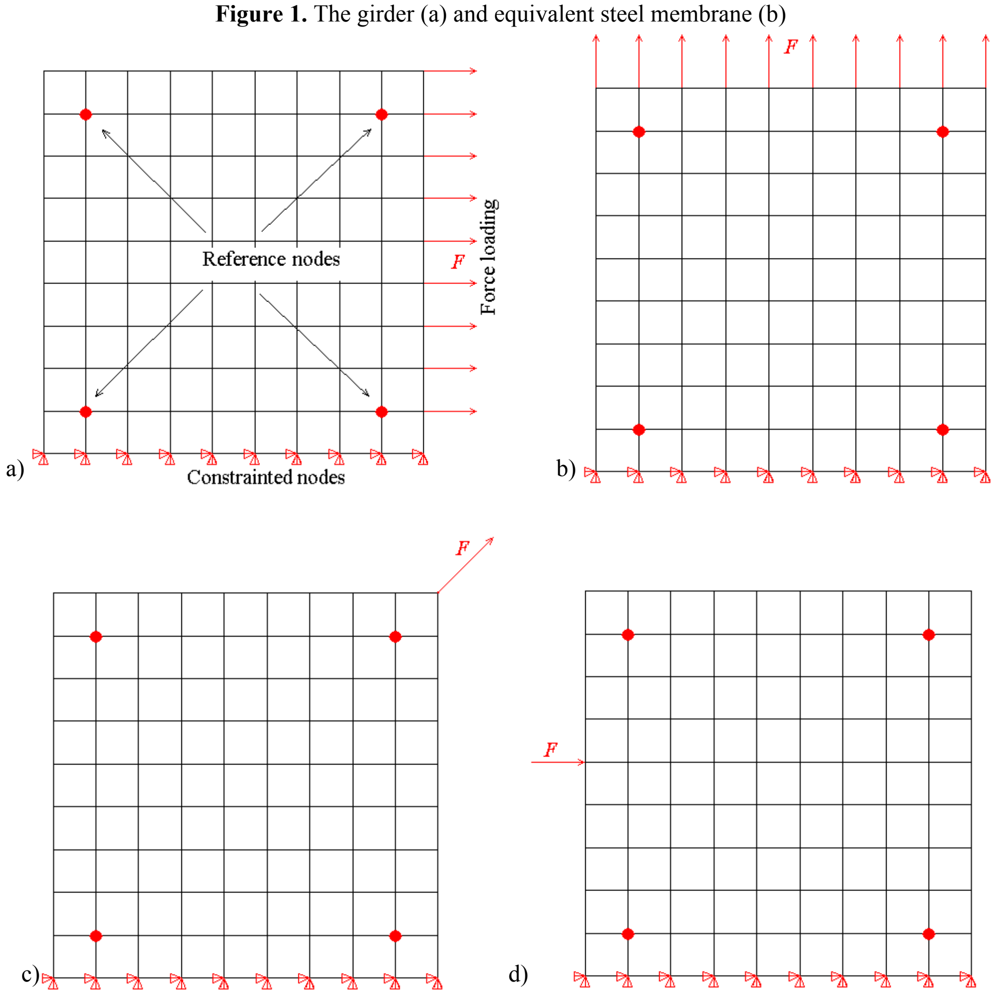

Figure 2. The finite element models: a) 1st analysis model; b) 2nd analysis model; c) 1st test model; d) 2nd test model

In the case of dynamic analysis, the differences between displacements of nodes are minimized at selected time moments. The analysis has been performed by using ANSYS and MATLAB software. The displacements obtained in ANSYS have been used when forming the target function, which subsequently has been minimized by employing MATLAB function FMINCON().

3. Analysis of Results

3.1. Static analysis

Further, the analysis of the statics of selected girder as well as parameters of membrane resulted in the optimization, are presented. In order to facilitate the optimization problem assume $E_g = E_m$, $\nu_m = 0$, $\rho_g = \rho_m$ and $h = b$. Assume the girder rods being thin enough to maintain the mechanical features of the girder:

$$\frac{1}{20} \le \frac{b}{L_m} \le \frac{1}{8}, \quad L_m = \frac{L}{N} \quad \Rightarrow \quad \frac{L}{20 \cdot N} \le b \le \frac{L}{8 \cdot N}, \quad (1)$$

where L is the side length of the rectangular element and N – number of divisions of the side, which is equal to the number of cells along the side of the girder.

In this example, numbers of divisions of the membrane and the girder are selected the same, however, generally the grid spacing may be much smaller than side length of the membrane element.

Consider the models in Figure 3. In the first load case (LC1) (Figures 3 a), b)), all nodes of the bottom side are fixed meanwhile all nodes of the right hand side are exposed to forces imitating distributed loading along the *Ox* direction. In the second load case (LC2) (Figures 3 c), d)) the top side is exposed to distributed loading along the *Oy* direction.

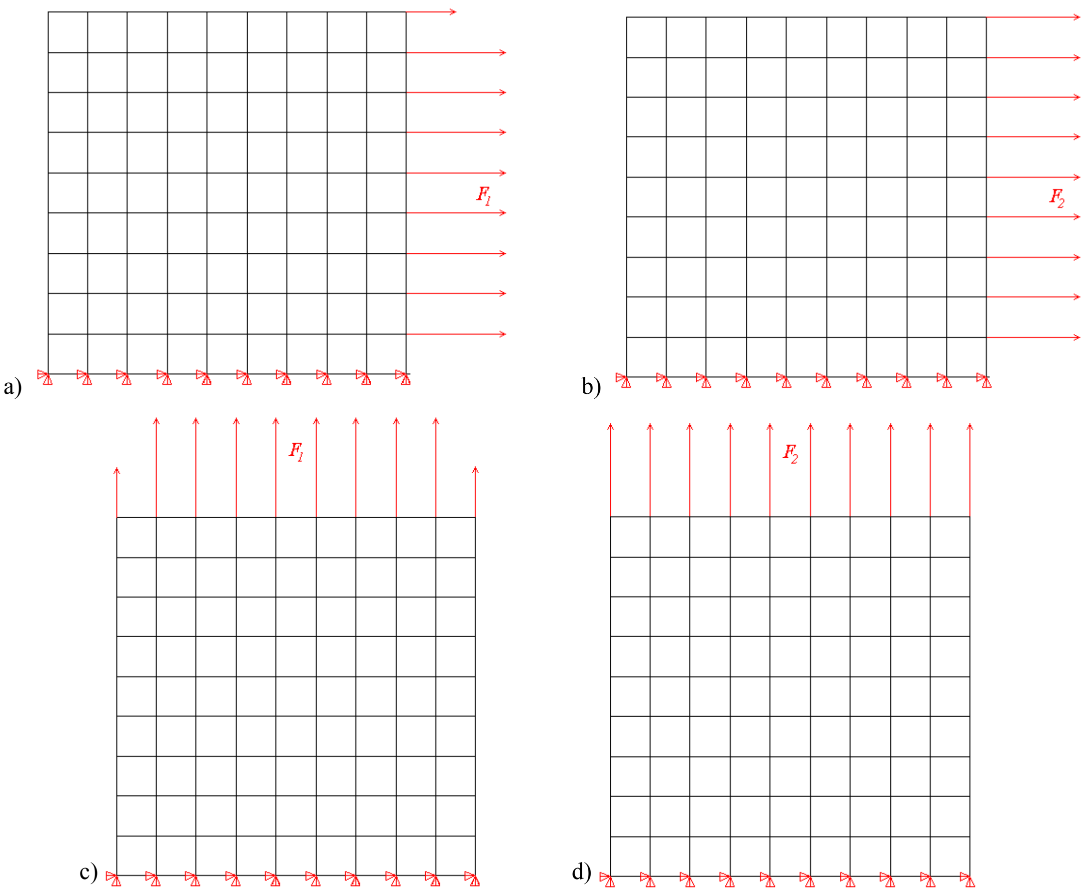

Figure 3. The finite element mesh and loads of the : 1st analysis model of the membrane (a) and the girder (b); 2nd analysis model of the membrane (c) and the girder(d)

The target function is read as follows:

$$T(p_1^i, p_2^i, q_1^i, q_2^i) = \frac{\sum_{i=1}^{n} (\vec{p}_1^i - \vec{q}_1^i)^2}{\sum_{i=1}^{n} (\vec{p}_1^i)^2 + \sum_{i=1}^{n} (\vec{q}_1^i)^2} + \frac{\sum_{i=1}^{n} (\vec{p}_2^i - \vec{q}_2^i)^2}{\sum_{i=1}^{n} (\vec{p}_2^i)^2 + \sum_{i=1}^{n} (\vec{q}_2^i)^2}, \quad (2)$$

where p_1^i, p_2^i are the vectors of i-node displacements of the 1st and 2nd models of membrane, q_1^i and q_2^i are the vectors of i-node displacements of the 1st and 2nd models of girder, $n = (N+1)^2$ – total number of the nodes of each model.

After the minimization of (2) we obtained the relationship of optimum thickness s_m of the equivalent membrane against the girder rod thickness b and against the shear module G_m. By applying the least squares approximation (LSA), the quadratic and linear relationships between the geometric parameters of the girder and the equivalent membrane have been established as in Figure 4.

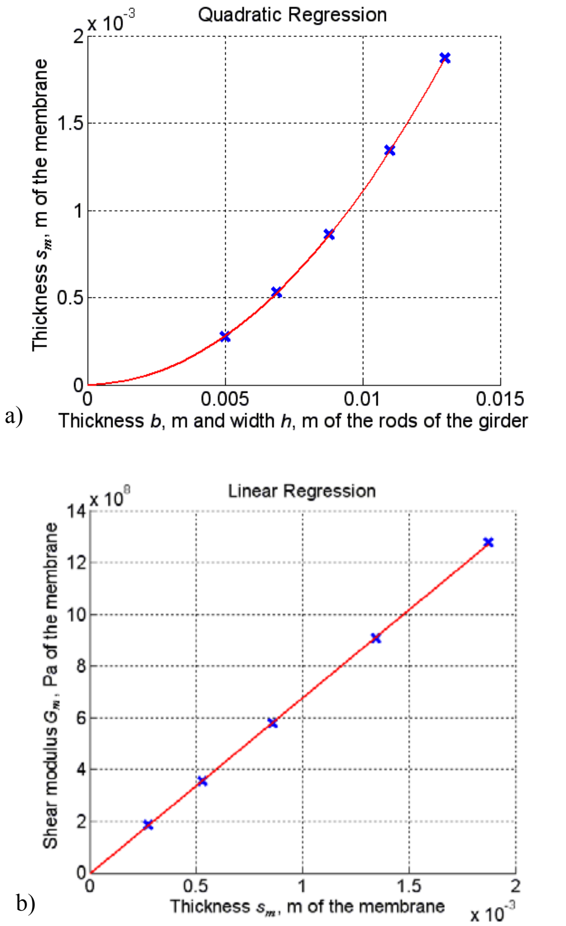

Figure 4. The pairs of optimal parameters of the girder and the equivalent membrane (points marked by crosses) and the regression curves,(a) – square fit; (b) – linear fit

The analytical expressions of the regression curves presented in Figure 4 are read as

$$
\begin{aligned}
&s_m = s(N,b) = b^2 \cdot (1{,}1275 + 1{,}1088 \cdot N),\\
&G_m = G(N,b) = N \cdot 10^3 (293{,}09 - 3{,}6260 \cdot N\,) \; +\\
&+ b \cdot 10^8 (1{,}6818 - 2{,}0013 \cdot N) \; + N^2 \cdot b^2 \cdot 10^{10} (9{,}4502\,)
\end{aligned}
\qquad (3)
$$

The estimation of derived formulas (3) against calculated pairs of optimum parameters at different values of girder parameter b and side division N is presented in Figure 5.

It follows from Figure 5 that the increase of the mesh division parameter N, makes each rod of the girder thinner, see formula (1). The corresponding values of the thickness of the equivalent membrane and its shear module tend to decrease. The deviations of calculated optimum values of the membrane parameters with respect to the obtained regression (3) have been evaluated as

$$
\Delta_i = \frac{n \cdot \sqrt{(p_x^i - q_x^i)^2 + (p_y^i - q_y^i)^2}}{\sum_{i=1}^{n} (|\,q_x^i\,| + |\,q_y^i\,|)}, \qquad (4)
$$

where n – total number of nodes of each model, p_x^i, p_y^i are x and y displacements of node i of the membrane, q_x^i, q_y^i are x and y displacements of node i of the girder.

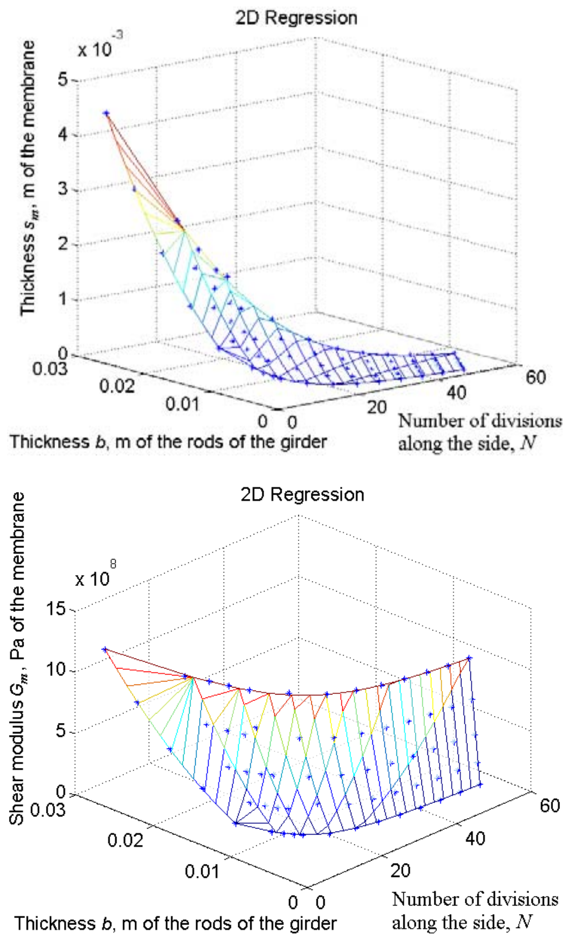

Figure 5. 2D regressions of depending parameters

Further, we present the evaluation of the derived dependencies (3). The number of divisions along the side of tested models was $N = 38$, and the values b of the girder was chosen in accordance with the formula $b = \dfrac{L}{8 \cdot N}$. The parameters of equivalent membrane have been calculated according to formula (3). The same loading of the model has been used in all investigated cases as in Figures 2 a) and b). The estimation of the deviations of membrane displacements from the reference displacements of the girder is presented in Figure 6.

The largest deviations of the first model (Figure 6 a)) are at the nodes in the vicinity of the constrained side of the membrane. On the contrary, in the second model (Figure 6 b)) the deviations at the nodes in the vicinity of the constrained side of the membrane are the smallest.

The next numerical experiment is performed by loading the same girder and equivalent membrane by means of the force applied in the plane xOy at the corner at angle 45^0, (model Figure 2 c), Figure 7 a)).

Figure 7 b) is obtained using model (model Figure 2 d)) by applying the force at the mid-side node.

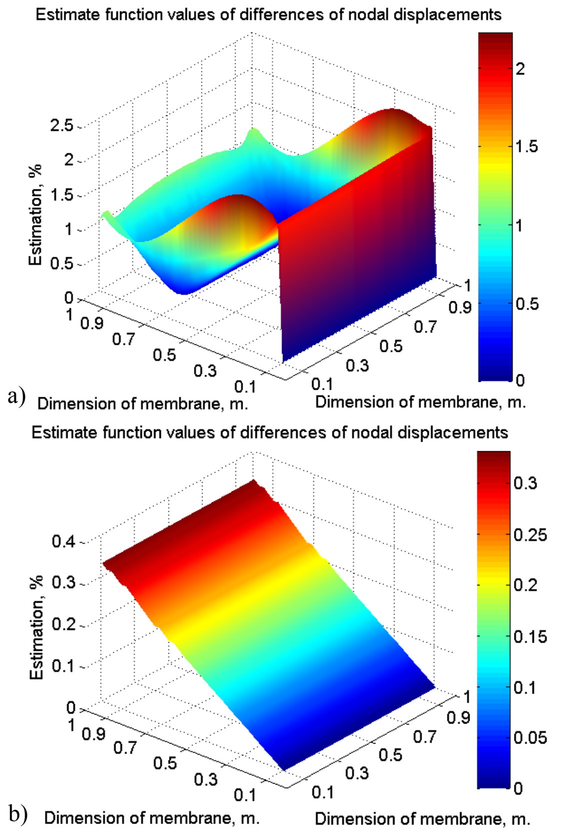

Figure 6. The estimated values of differences of displacements of the corresponding nodes of membrane and girder by using 1st (a) and 2nd (b) models (Figure 2)

We have found the equivalent membrane for the selected girder by considering the extra loading cases (models Figure 2 c) and d)) and determined that the deviations of relevant nodes have increased up to 6 times. The maximum value of relative displacement deviation between the two models was equal to approximately 16%. The biggest deviations of displacements appear at the nodes affected by the force. In order to reduce the deviations we should include the displacements of nodes of the two models into the target function to be optimized. Formula (2) is to be modified by adding the summation terms representing the reference nodes of all investigated models. As a consequence, the regression formulas (3) could have been altered.

The approximation of the reference girder model by the equivalent membrane has practical importance only if linear dimensions of membrane elements are much bigger than the distance between the adjacent rods of the girder. The parameter optimization procedure remains essentially the same. The only difference is that the reference points on the girder may correspond to the points between the nodes of the membrane and therefore the interpolation of membrane displacements is necessary.

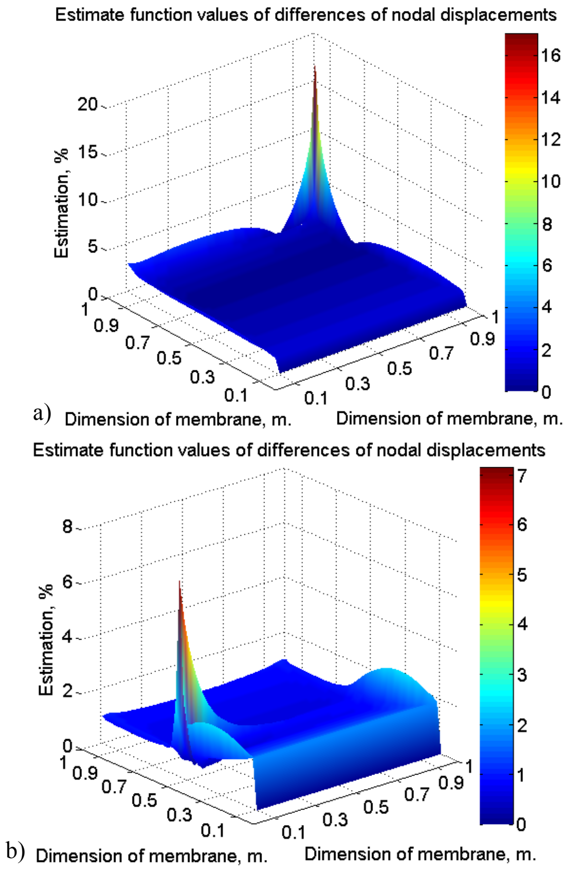

Figure 7. The estimated values of differences of displacements of the corresponding nodes of membrane and girder with free selected models

The estimations of displacement differences between the two models in the first loading case is presented in Figures 8 a), b) for the coincident (*N=38*) and non-coincident (*N=24*) mesh, respectively.

It can be observed that the maximum estimation values did not change, however, the estimations at individual nodes may change significantly (up to 6 times in this case).

3.2. Dynamic analysis

Here we extend the regression formulas determined in section 3.1 for the static analysis to the dynamic analysis. Time is introduced as continuous variable $t \in [0; T]$ and optimization is performed by using a new target function, obtained by integrating expression (2) over time. Now P and Q are three-dimensional matrices, in which nodal displacements are stored at all time steps t_k. The integration over time is performed numerically by using the 5th order Newton – Cottes quadrature formula [8].

The time law of the loading is read as $T \approx L\left(\frac{E_g}{\rho_g}\right)^{-\frac{1}{2}}$. We chose the time interval of the transient dynamic analysis equal to the time necessary for the longitudinal elastic wave to travel distance L equal to the side length of the model. The sine-pulse shaped by time law of force F we assumed to have period $T_F \approx \frac{T}{2}$, Figure 9 b). Forces at individual nodes have been applied $F = \frac{2 \cdot F_{\max} \cdot \sin(\pi \cdot t)}{2 \cdot N - 1}$. In order to imitate the distributed loading of the side of the membrane, the nodes at vertices of the rectangular membrane are affected by only half of the force, which is applied to other nodes of the membrane boundary.

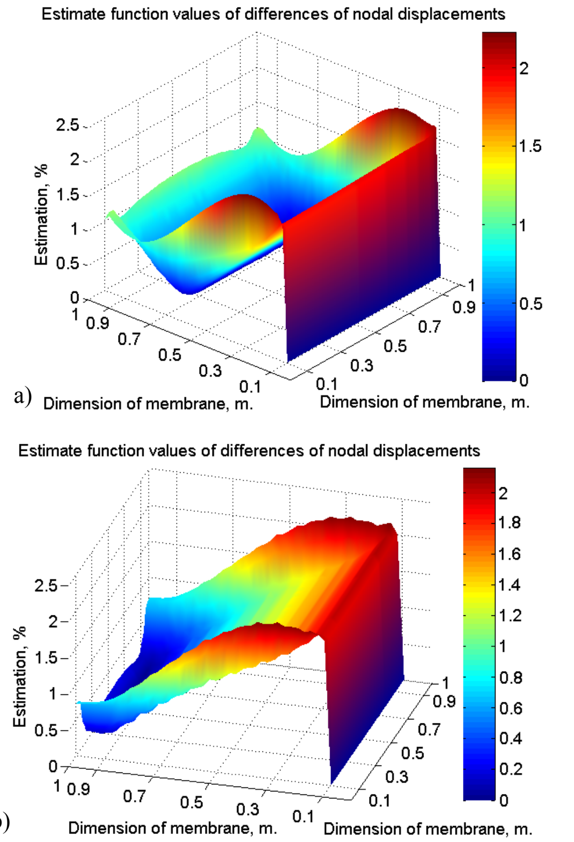

Figure 8. The estimated values of differences of displacements of respective nodes of equivalent membrane and reference girder at coincident meshing N=38 (a) and non-coincident meshing N=24 (b)

We select 8 reference nodes (Figure 9 a)), at which displacement time laws of both structures are compared against each other.

In order to perform the minimization of the penalty type target function, the parameters of the membrane are calculated by using formula (3) derived for the static analysis case. In the simplest case we are using only one optimization variable as equivalent mass density ρ_m of the membrane. The mesh 48×48 in both structures is employed. The numerical values of the parameters are presented in Table1. First column presents the parameter names, second column – formulas, by which they are calculated, and the third – the values of the parameters.

We use FMINCON() function in order to determine ρ_m^* value. The minimization process is shown in Figure 10.

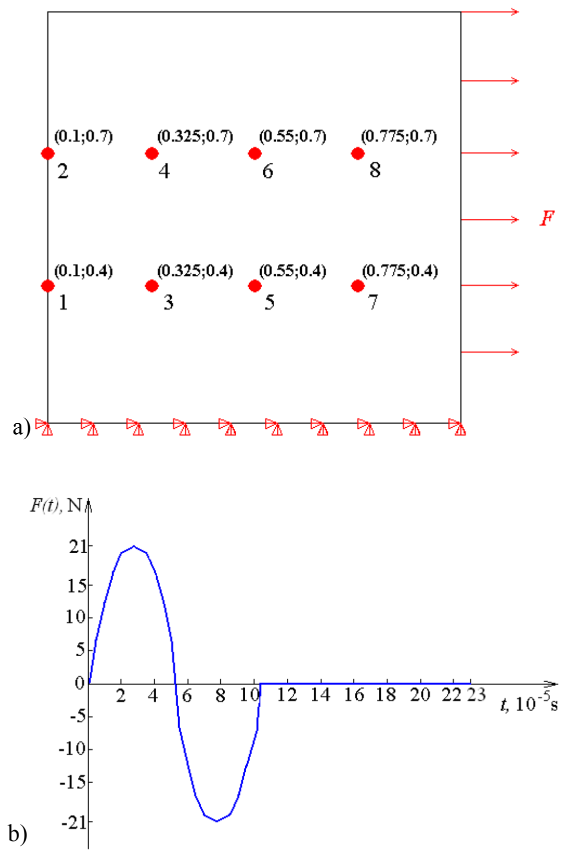

Figure 9. a) The reference nodes of the model, b) The time law of the loading force

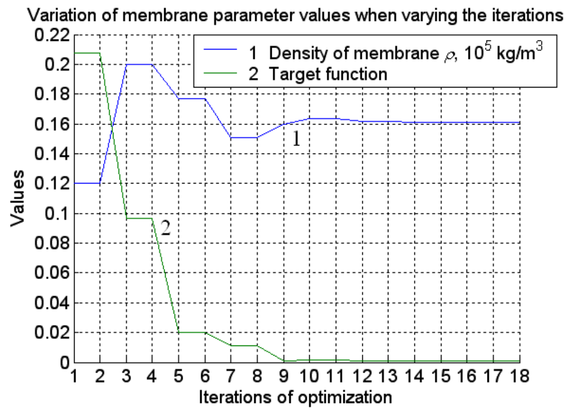

Figure 10. The variation of the value optimization parameter during iterations

We get the equivalent membrane by having the density $\rho_m^* = 16125 \frac{kg}{m^3}$, $\frac{\rho_m^*}{\rho_g} \approx 2{,}067$. The obtained density value of the equivalent membrane does not necessarily correspond to the density of a real material. It is rather a mathematical approximation of the density of equivalent membrane, which provides good coincidence of displacements of both structures. On the other hand, when estimating the "reality" of the equivalent density we should take into account the surface density ($\rho_m \cdot s_m$) rather than volumetric density ρ_m, because the planar behavior of the structure is investigated.

Table 1. Parameters of the girder and equivalent membrane obtained as a result of target function minimization

Parameter name	Formula, by which parameter is calculated	Parameter value
E_g ,Pa.	–	$1.5 \cdot 10^{11}$
b, m.	$\frac{L}{8 \cdot N_g}$	0.00234375
h, m.	b	0.00234375
ρ_g, $\frac{\text{kg}}{\text{m}^3}$.	–	7800
N_g	–	48
ν_m	–	0
E_m, Pa.	E_g	$1.5 \cdot 10^{11}$
s_m, m.	$s(N,b)$	0.000298551116
G_m, Pa.	$G(N,b)$	1179631860
ρ_m, $\frac{\text{kg}}{\text{m}^3}$.	ρ_m^*	16125
N_m	–	48

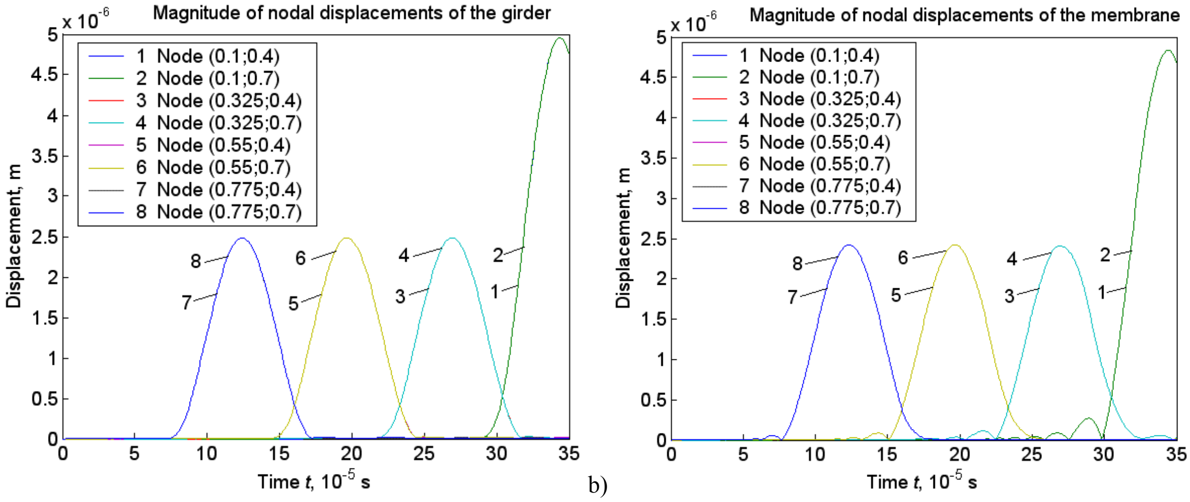

Figure 11. The magnitudes of displacements at several selected time moments in the girder (a) and equivalent membrane structure (b)

Figure 11 presents the magnitudes of displacements caused by the transient vibration process (the wave traveling along the *Ox* direction, Figure 2 a)) at several selected time moments in the girder (a) and equivalent membrane structure (b). The two graphs are practically equivalent to each other. Figure 12 presents errors of displacements of reference nodes.

The obtained errors are quite small indicating that the regression formulas (3) are suitable for employing them for the dynamic analysis only with the equivalent density value of the membrane being adjusted properly.

4. Conclusions

A formal approach to the reduction of a complex finite element structural model to the simpler one has been proposed. The procedure is based on the penalty type target function minimization in the space of parameters of the reduced model. As a sample task, the synthesis of the reduced equivalent continuous membrane model, which imitates the behavior of the girder structure, has been solved.

For the static analysis case the equivalent membrane parameter set has been determined at which the models worked satisfactorily in the case of coincident, as well as, non-coincident meshes of the reference and reduced equivalent structure and different loading configurations. Regression formulas for obtaining the equivalent parameters have been derived.

The model based on the equivalent parameters obtained for static analysis has been demonstrated to work also in the dynamic analysis case, provided that a proper equivalent mass density value of the equivalent reduced structure is selected.

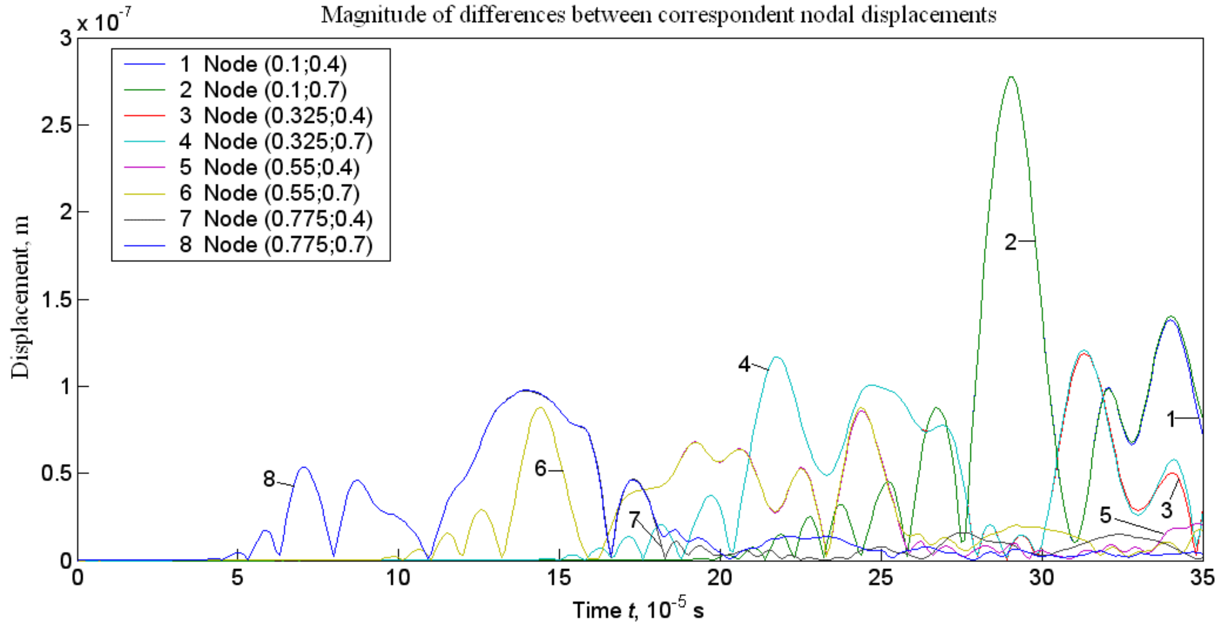

Figure 12. Differences between corresponding displacements of reference nodes of the girder and membrane model

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