

DEFINING TABU TENURE FOR THE QUADRATIC ASSIGNMENT PROBLEM

Alfonsas Misevičius, Armantas Ostreika

*Kaunas University of Technology, Department of Multimedia Engineering
Studentu St. 50, LT-51368 Kaunas, Lithuania*

Abstract. Tabu search (TS) algorithms are among the most efficient heuristic techniques in combinatorial optimization. Within these algorithms, it is important that the proper policies for maintaining the tabu tenure (tabu list size) are applied. In this paper, we discuss the mechanisms of defining the tabu tenure for the famous combinatorial optimization problem – the quadratic assignment problem (QAP). Several variants of maintaining the tabu tenure are implemented. They are examined on the instances taken from the QAP instances library – QAPLIB. The results from the experiments demonstrate that the deterministic strategies of defining the tabu tenure are preferable to the corresponding random strategies.

Keywords: combinatorial optimization, heuristics, tabu search, tabu tenure, quadratic assignment problem.

Introduction

The quadratic assignment problem (QAP) is one of the most difficult combinatorial optimization problems. Formally, it can be formulated as follows [11]. Let two matrices $A = (a_{ij})_{n \times n}$ and $B = (b_{kl})_{n \times n}$ and the set Π of all possible permutations of $\{1, 2, \dots, n\}$ be given. The goal is to find a permutation $\pi = (\pi(1), \pi(2), \dots, \pi(n)) \in \Pi$ that minimizes

$$z(\pi) = \sum_{i=1}^n \sum_{j=1}^n a_{ij} b_{\pi(i)\pi(j)}. \quad (1)$$

The quadratic assignment problem is NP-hard [17] and cannot be actually solved to optimality for larger n ($n > 36$) [2]. Therefore, heuristic algorithms are commonly used to get near-optimal (or possibly pseudo-optimal) solutions within reasonable computation time. (For a survey of heuristics for the QAP, see, for example, [3, 5, 16].) Among the heuristic approaches, the tabu search (TS) method has been proven to be a powerful technique, in particular, for solving the quadratic assignment problem [1,6,12,13,19]. The tabu search based algorithms seem to be obligatory – in one or another form – if one seeks for high-quality solutions for this challenging problem.

The remaining part of this paper is organized as follows. We start with some preliminary concepts. In Section 2, we discuss several variants of maintaining the tabu tenure for the quadratic assignment problem. The results of the computational experiments with the various variants of the tabu tenure are presented in Section 3. Finally, Section 4 completes the paper with concluding remarks.

1. Preliminary concepts

Let $\Theta_2: \Pi \rightarrow 2^\Pi$ be a 2-exchange neighbourhood function that assigns for every π the set of neighbouring solutions such that $\Theta_2(\pi) = \{\pi' \mid \pi' \in \Pi, | \{i \mid \pi(i) \neq \pi'(i)\} | = 2\}$. The solution $\pi' \in \Theta_2(\pi)$ can be obtained from π by a corresponding move $\phi(\pi, i, j): \Pi \times N \times N \rightarrow \Pi$, which swaps the i -th and j -th element in the given permutation, i.e. $\pi'(i) = \pi(j)$, $\pi'(j) = \pi(i)$, $i, j = 1, 2, \dots, n$. The solution $\pi^* \in \Pi$ is said to be locally optimal with respect to the neighbourhood Θ_2 if $z(\pi^*) \leq z(\pi')$ for every $\pi' \in \Theta_2(\pi^*)$.

The tabu search method is based on the neighbourhood search with local-optima avoidance. The central idea is allowing climbing moves when no improving neighbouring solution exists, i.e. a move is allowed even if a new solution is worse than the current one. The reverse moves must be forbidden. This is done by storing the corresponding solution (or some attribute associated with this solution) in a tabu list (T). The tabu list keeps information on the last $h = |T|$ solutions (attributes) which have been considered during the search process. The parameter h is called a tabu tenure (tabu list size).

With the QAP, a common way is organizing the tabu list as an $n \times n$ integer matrix $T = (t_{ij})_{n \times n}$. At the beginning, all the entries of T are set to zero. As the algorithm progresses, the entry t_{ij} stores the current iteration number plus the tabu tenure h , i.e. the number of the future iteration starting at which the move $\phi(\cdot, i, j)$ may again be performed. So, the move

$\phi(\cdot, i, j)$ is tabu if the value of t_{ij} is equal or greater than the current iteration number.

Formally, TS starts from an initial solution π in Π . The process is then continued in an iterative way – moving from the current solution to a neighbouring one. At each step of the procedure, the neighbourhood $\mathcal{O}_2(\pi)$ of the given solution is explored and the non-tabu move to the solution $\pi' \in \mathcal{O}_2(\pi)$ that improve most the objective function value is chosen. The best so far solution is recorded. The process stops as soon as a termination criterion is satisfied.

A more thorough discussion on the principles as well as the specific facets of the tabu search (like target analysis, aspiration criterion, long-term-memory, etc.) can be found in [8, 9, 10, 18 (p. 159–165)].

2. Variants of maintaining the tabu tenure for the QAP

The main difficulties within the tabu search algorithms are the cycles of the search trajectories and convergence speed. The cycling trajectories (and, consequently, stagnation of the search) take place when the tabu tenure (tabu list size) is too small. On the other hand, the convergence rate of the search process tends to be low when the tabu tenure is too large. In both cases, it might be impossible to find high-quality solutions within reasonable computation time. Therefore, defining the proper (optimal) value of the tabu tenure is one of the key aspects of the tabu search algorithms.

In this section, we will describe some variants of defining the tabu tenure for the quadratic assignment problem. We will focus only on the modifications related to the tabu tenure. The algorithm itself is identical to the combined enhanced tabu search (CETS) algorithm [13], except the modifications actually discussed. For the more details and the template of CETS, see [13].

2.1. Fixed tabu tenure

In the simplest case, the tabu tenure is fixed at a constant value. Typically, this value depends on the problem size, n , i.e. $h = cn$ (or more precisely, $h = \lceil cn \rceil$), where c is an empirical parameter (for the QAP, c is somewhere between 0.01 and 2.0 for ordinary n 's ($n \leq 150$)).

By having the fixed value of the tabu tenure, there is a risk that the phenomenon known as "deterministic chaos" will occur [1]. In this case, "short-term" cycles are absent, but the search trajectories are still confined in a limited part of the solution space – this is like having "long-term" (hidden) cycling. One of the possible ways to try to avoid this cycling is to apply variable tabu tenure instead of the fixed one. Below, we briefly describe several variants of using the non-fixed tabu tenure where the tenure is varied in some deterministic or random manner during the search process.

2.2. Periodic tabu tenure

The periodic (cyclic) tabu tenure is an example of the deterministic strategy of maintaining the tabu tenure. In this case, the tabu tenure range $[h_{\min}, h_{\max}]$ is considered (here, $h_{\min} = c_1n$ and $h_{\max} = c_2n$, $c_1 < c_2$; the appropriate values of c_1 and c_2 may be determined experimentally). So, at the beginning, h is equal to the minimum value h_{\min} ; further, h is increased gradually, step by step, until the maximum value h_{\max} is reached; once the maximum value h_{\max} has been achieved (or possibly a new better locally optimal solution has been encountered), the actual value of h is dropped to h_{\min} , and so on. In our experimentation, $h_{\min} \in [0.07n, 0.3n]$ and $h_{\max} \in [0.15n, 0.4n]$, depending on the problem size, n .

2.3. Random tabu tenure

The random (probabilistic) tabu tenure is similar to the periodic one, except that the value of the tenure is picked randomly from the interval $[h_{\min}, h_{\max}]$. During the search process, h is renewed (regenerated) every $2h_{\max}$ iterations (see also the robust tabu search algorithm [19]). In [19], h_{\min} and h_{\max} are fixed at $0.9n$ and $1.1n$; while, in [7], the range is extended to $[0.2n, 1.8n]$. In this work, h_{\min}, h_{\max} vary from $0.07n$ to $0.4n$, depending on n .

2.4. Dynamic periodic tabu tenure: basic variant

We also propose two additional strategies for maintaining the tabu tenure. They are based on dynamic changing of the value of the tabu tenure. In fact, the tabu tenure is individually defined for the particular move $(\phi(\cdot, i, j), i = 1, 2, \dots, n - 1, j = i + 1, \dots, n)$; that is, the tabu tenure is tuned for each pair of indices (i, j) (see Figure 1). We use the additional matrix $H = (h_{ij})_{n \times n}$ for storing these dynamic values of the tabu tenure. Then, the tabu list is updated according to the following formula:

$$t_{ij} = q + h_{ij},$$

where t_{ij} denotes the corresponding entry of the tabu list, q is the current iteration number, and h_{ij} denotes the current value of the tabu tenure for the move $\phi(\cdot, i, j)$. The values h_{ij} are maintained according to the rule described in Section 2.2.

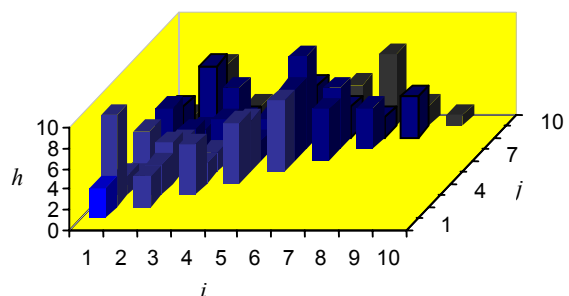


Figure 1. Dynamic tabu tenures

The above strategy adds some flexibility and robustness to the search process. It can be viewed as a special type of diversification mechanism.

2.5. Dynamic periodic tabu tenure: modified variant

The difference between the basic and modified variant of the dynamic strategy is that, in the modified version, one starts from zero tabu tenure instead of h_{min} . The remaining process is identical to that described in Section 2.4. The illustration of the modified tabu tenure for the separate pair of indices is shown in Figure 2.

The motivation of this modification is based on the assumption that there is no necessity to apply the large tabu tenure at the initial phase of the search (see also Section 2.6).

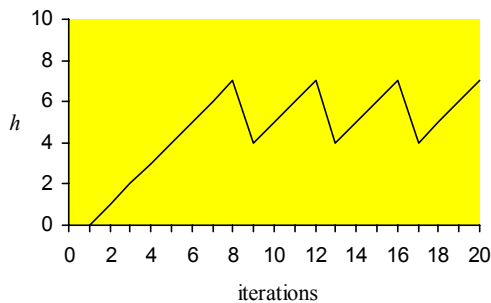


Figure 2. Illustration of the modified variant of the tabu tenure

2. 6. Using tabu search with delay

It could be conjectured that, in the early stages of the search (provided that the process starts from the statistically-average-quality solution), the cycles are unlikely. For this reason, we do not need to take care of the tabu moves and restrict the exploration of the promising regions of the solution space at the initial iterations of the algorithm. This is what we call the tabu search with delay [13]. In this case, the tabu tenure is equal to zero at the beginning of the search process. After w iterations are performed, the tabu search is continued in the ordinary way. The parameter w is defined as $\lfloor \delta \cdot n \rfloor$, where δ is a delay factor (we used $\delta=1$).

3. Computational experiments

We have carried out a number of computational experiments in order to test the performance of the different variants of defining the tabu tenure. In the experiments, we used the QAP instances taken from the publicly available QAP library – QAPLIB [4]. We examined the following types of the QAP instances:

- a) random instances (these instances are randomly generated according to a uniform distribution; in QAPLIB, they are denoted by tai20a, tai25a,

- tai30a, tai35a, tai40a, tai50a, tai60a, tai80a, and tai100a);

- b) quasi-random instances (in these instances, the values of the matrix A are generated pseudo-randomly and the values of the matrix B are "Manhattan" distances between grid points; the following are the grid-based instances: sko42, sko49, sko56, sko64, sko72, sko81, sko90, sko100a–f, tho30, tho40, tho150, wil50, and wil100);
- c) real-life like instances (they are generated in such a way that the entries of the matrices A and B resemble a distribution from real world problems; these instances are denoted by tai20b, tai25b, tai30b, tai35b, tai40b, tai50b, tai60b, tai80b, tai100b, and tai150b).

The variants of the tabu tenure used in the experimentation are as follows: the fixed tabu tenure (FTT), the periodic tabu tenure (PTT), the random tabu tenure (RTT), the basic variant of the dynamic periodic tabu tenure (DPTT-B), and the modified variant of the dynamic periodic tabu tenure (DPTT-M). All these variants have been tried in both the standard (non-delay) mode and the delay mode. The delay variants are denoted by FTT^d, PTT^d, RTT^d, DPTT-B^d, and DPTT-M^d, respectively. Two sets of the fixed values of the tabu tenure were examined: set 1 ($h=0.35n$ ($n < 50$), $h=0.2n$ ($50 \leq n < 100$), $h=0.1n$ ($n \geq 100$)); set 2 ($h=0.7n$ ($n < 50$), $h=0.6n$ ($50 \leq n < 100$), $h=0.5n$ ($n \geq 100$)). The corresponding variants are entitled as FTT-1 (FTT-1^d) and FTT-2 (FTT-2^d). Regarding the non-fixed tenures, we used the following ranges of tabu tenure: a) $h_{min}=0.3n$ and $h_{max}=0.4n$ ($n < 50$), b) $h_{min}=0.15n$ and $h_{max}=0.3n$ ($50 \leq n < 100$), c) $h_{min}=0.07n$ and $h_{max}=0.15n$ ($n \geq 100$).

The performance criteria for the algorithms are as follows: a) the average deviation from a (pseudo-)optimal (best known) solution – $\bar{\delta}$ ($\bar{\delta} = 100(\bar{z} - \bar{z})/\bar{z}$ [%], where \bar{z} is the average objective function value over 10 restarts and \bar{z} is the best known value (BKV) of the objective function); b) the number of solutions that are within 1% optimality – $C_{1\%}$; c) the number of the best known solutions – C_{BKS} .

The results of comparison of the different variants of the tabu tenure are presented in Tables 1–9. As can be seen, the computation times are different for the random and real-life like instances. We used relatively larger runs for the random instances and shorter runs for the real-life like instances. This is to get more fairness of the experiments.

From Tables 1–9, it can be observed that the deterministic strategies of maintaining the tabu tenure (in particular, the fixed and periodic tabu tenures) are slightly superior to the random strategy. Surprisingly, the fixed tabu tenure in combination with the delay mechanism yielded the best results (this is especially evident for the quasi-random instances). The reason

may be that the value of the tabu tenure was especially tuned for this type of instances. However, in general it may be risky to use "tweaked" values of the tabu tenure – see the results obtained with the increased tabu tenure (columns "FTT-2"/"FTT-2^d" in Tables

1–9). On the whole, the tuning of the tabu tenure may be time-consuming. We therefore guess that it is probably better to use the range of tabu tenure instead of the fixed value (however, the adjusting of the range of the tabu tenure is still important).

Table 1. Results of comparison of the tabu tenure variants for the random QAP instances (I).
(CPU times per restart are given in seconds. 3 GHz Pentium computer was used in the experiments)

Instance	n	BKV	$\bar{\delta}, C_{1\%}/C_{BKS}$												CPU time
			FTT-1		FTT-2		PTT		RTT		DPTT-B		DPTT-M		
tai20a	20	703482 ^a	0.183	10/4	0.276	10/2	0.183	10/4	0.152	10/5	0.061	10/8	0.183	10/4	0.3
tai25a	25	1167256 ^a	0.254	10/4	0.189	10/5	0.203	10/5	0.180	10/5	0.320	10/1	0.168	10/5	0.7
tai30a	30	1818146 ^a	0.085	10/8	0.095	10/4	0.058	10/7	0.078	10/8	0.112	10/7	0.072	10/8	1.3
tai35a	35	2422002 ^a	0.267	10/4	0.278	10/4	0.300	10/3	0.078	10/7	0.178	10/5	0.370	10/2	2.3
tai40a	40	3139370 ^a	0.478	10/0	0.590	10/0	0.398	10/0	0.430	10/0	0.460	10/0	0.454	10/0	3.8
tai50a	50	4941410 ^a	0.489	10/0	0.684	9/0	0.519	10/0	0.584	10/0	0.598	10/0	0.676	10/0	8.9
tai60a	60	7205962 ^b	0.630	10/0	0.789	10/0	0.715	10/0	0.666	10/0	0.615	10/0	0.704	10/0	18.5
tai80a	80	13526696 ^c	0.525	10/0	0.651	10/0	0.571	10/0	0.480	10/0	0.545	10/0	0.523	10/0	60
tai100a	100	21071540 ^d	0.432	10/0	0.605	10/0	0.437	10/0	0.488	10/0	0.422	10/0	0.447	10/0	147

^a comes from [4]; ^b comes from [12]; ^c comes from [14]; ^d comes from [15].

Table 2. Results of comparison of the tabu tenure variants for the quasi-random QAP instances (I).
(CPU times per restart are given in seconds. 3 GHz Pentium computer was used in the experiments)

Instance	n	BKV	$\bar{\delta}, C_{1\%}/C_{BKS}$												CPU time
			FTT-1		FTT-2		PTT		RTT		DPTT-B		DPTT-M		
sko42	42	15812 ^a	0.015	10/6	0.023	10/4	0.032	10/6	0.015	10/6	0.015	10/6	0.111	10/5	1.0
sko49	49	23386 ^a	0.058	10/2	0.036	10/4	0.068	10/1	0.048	10/2	0.044	10/2	0.062	10/2	1.7
sko56	56	34458 ^a	0.196	10/2	0.237	10/3	0.158	10/2	0.282	10/0	0.152	10/1	0.245	10/1	2.5
sko64	64	48498 ^a	0.140	10/3	0.042	10/5	0.082	10/3	0.096	10/5	0.230	10/1	0.050	10/3	4.8
sko72	72	66256 ^a	0.132	10/0	0.207	10/1	0.163	10/0	0.196	10/0	0.212	10/0	0.217	10/0	6.6
sko81	81	90998 ^a	0.285	10/0	0.157	10/0	0.220	10/0	0.177	10/0	0.220	10/0	0.210	10/0	10.0
sko90	90	115534 ^a	0.273	10/0	0.204	10/0	0.267	10/0	0.203	10/0	0.205	10/0	0.253	10/0	13.8
sko100a	100	152002 ^a	0.102	10/1	0.142	10/0	0.113	10/1	0.133	10/0	0.081	10/2	0.137	10/0	32
sko100b	100	153890 ^a	0.197	10/1	0.148	10/2	0.193	10/0	0.132	10/1	0.196	10/0	0.183	10/0	33
sko100c	100	147862 ^a	0.138	10/0	0.085	10/1	0.075	10/0	0.306	10/1	0.113	10/0	0.099	10/0	32
sko100d	100	149576 ^a	0.232	10/0	0.173	10/0	0.263	10/0	0.216	10/0	0.233	10/0	0.177	10/0	33
sko100e	100	149150 ^a	0.344	10/0	0.145	10/0	0.216	10/1	0.280	10/0	0.325	10/0	0.257	10/1	33
sko100f	100	149036 ^a	0.228	10/0	0.218	10/0	0.366	10/0	0.344	10/0	0.271	10/0	0.231	10/0	33
tho30	30	149936 ^a	0.567	7/4	0.230	9/7	0.277	9/4	0.377	9/4	0.283	9/5	0.314	9/3	0.4
tho40	40	240516 ^a	0.149	10/0	0.154	10/0	0.118	10/0	0.120	10/0	0.183	10/1	0.041	10/0	0.9
tho150	150	8133398 ^a	0.274	10/0	0.239	10/0	0.259	10/0	0.214	10/0	0.299	10/0	0.245	10/0	174
wil50	50	48816 ^a	0.056	10/0	0.054	10/1	0.045	10/1	0.073	10/0	0.065	10/2	0.073	10/1	1.9
wil100	100	273038 ^a	0.123	10/0	0.095	10/0	0.084	10/0	0.089	10/0	0.141	10/0	0.139	10/0	33

^a comes from [4].

Unexpectedly, the actual results do not confirm the advantage of the delay modification over the standard variant. The exception is the results for the quasi-

random instances obtained by using the fixed and dynamic tenures as well as the results for the random instances obtained by using the dynamic tenures.

Table 3. Results of comparison of the tabu tenure variants for the real-life like QAP instances (I).
(CPU times per restart are given in seconds. 3 GHz Pentium computer was used in the experiments)

Instance	<i>n</i>	BKV	$\bar{\delta}, C_{1\%}/C_{BKS}$								CPU time				
			FTT-1		FTT-2		PTT		RTT			DPTT-B	DPTT-M		
tai20b	20	122455319 ^a	0.181	10/6	0.181	10/6	0.226	10/5	0.221	10/5	0.136	10/7	0.136	10/7	0.1
tai25b	25	344355646 ^a	0.007	10/9	0.081	10/7	0.569	9/8	0.045	10/8	0.008	10/9	0		0.2
tai30b	30	637117113 ^a	1.376	4/3	1.553	2/2	1.824	3/2	1.844	2/2	1.361	4/4	1.860	2/1	0.3
tai35b	35	283315445 ^a	0.800	8/2	1.555	6/1	1.204	6/4	1.078	6/1	0.920	8/3	0.392	8/3	0.6
tai40b	40	637250948 ^a	2.156	3/3	1.989	4/4	0.894	6/6	1.464	5/5	1.693	4/4	2.510	2/2	0.9
tai50b	50	458821517 ^a	0.437	10/3	0.309	10/3	0.220	10/6	0.359	10/1	0.373	10/2	0.505	8/3	1.9
tai60b	60	608215054 ^a	0.772	10/0	0.661	6/1	0.835	5/1	1.012	4/2	0.643	6/1	0.798	5/1	4.1
tai80b	80	818415043 ^a	0.759	8/0	0.771	8/0	0.938	6/0	0.991	3/0	0.820	6/0	1.007	4/0	13.0
tai100b	100	1185996137 ^a	0.684	8/0	1.041	6/1	0.516	9/1	0.605	8/2	0.988	7/1	0.921	7/0	33
tai150b	150	498896643 ^b	0.503	10/0	0.602	10/0	0.561	10/0	0.540	10/0	0.528	10/0	0.618	10/0	101

^a comes from [4]; ^b comes from [20].

Table 4. Results of comparison of the tabu tenure variants for the random QAP instances (II).
(CPU times per restart are given in seconds. 3 GHz Pentium computer was used in the experiments)

Instance	<i>n</i>	BKV	$\bar{\delta}, C_{1\%}/C_{BKS}$								CPU time				
			FTT-1 ^d		FTT-2 ^d		PTT ^d		RTT ^d			DPTT-B ^d	DPTT-M ^d		
tai20a	20	703482	0.213	10/3	0.213	10/3	0.229	10/3	0.155	10/6	0.091	10/7	0.299	10/2	0.3
tai25a	25	1167256	0.316	10/2	0.118	10/7	0.231	10/4	0.195	10/5	0.159	10/6	0.246	10/4	0.7
tai30a	30	1818146	0.132	10/4	0.078	10/8	0.174	10/5	0.072	10/8	0.039	10/9	0.047	10/4	1.3
tai35a	35	2422002	0.154	10/6	0.251	10/4	0.202	10/5	0.212	10/4	0.213	10/4	0.124	10/6	2.3
tai40a	40	3139370	0.435	10/0	0.558	10/0	0.477	10/0	0.273	10/0	0.395	10/0	0.471	10/0	3.8
tai50a	50	4941410	0.592	10/0	0.762	10/0	0.561	10/0	0.671	10/0	0.606	10/0	0.586	10/0	8.9
tai60a	60	7205962	0.702	10/0	0.787	10/0	0.722	10/0	0.672	10/0	0.636	10/0	0.650	10/0	18.5
tai80a	80	13526696	0.543	10/0	0.741	10/0	0.526	10/0	0.537	10/0	0.573	10/0	0.506	10/0	60
tai100a	100	21071540	0.468	10/0	0.690	10/0	0.424	10/0	0.454	10/0	0.399	10/0	0.425	10/0	147

Table 5. Results of comparison of the tabu tenure variants for the quasi-random QAP instances (II).
(CPU times per restart are given in seconds. 3 GHz Pentium computer was used in the experiments)

Instance	<i>n</i>	BKV	$\bar{\delta}, C_{1\%}/C_{BKS}$								CPU time				
			FTT-1 ^d		FTT-2 ^d		PTT ^d		RTT ^d			DPTT-B ^d	DPTT-M ^d		
sko42	42	15812	0.039	10/5	0.019	10/7	0.039	10/5	0.011	10/7	0.022	10/4	0.022	10/4	1.0
sko49	49	23386	0.061	10/3	0.071	10/1	0.085	10/0	0.079	10/0	0.044	10/3	0.044	10/3	1.7
sko56	56	34458	0.187	10/3	0.248	10/1	0.278	10/0	0.180	10/1	0.183	10/2	0.151	10/0	2.5
sko64	64	48498	0.122	10/2	0.136	10/3	0.075	10/3	0.211	10/3	0.199	10/3	0.240	10/3	4.8
sko72	72	66256	0.139	10/1	0.069	10/2	0.129	10/0	0.241	10/0	0.177	10/0	0.125	10/0	6.6
sko81	81	90998	0.225	10/0	0.192	10/0	0.269	10/0	0.248	10/0	0.268	10/0	0.244	10/0	10.0
sko90	90	115534	0.247	10/0	0.180	10/0	0.181	10/0	0.245	10/0	0.302	10/0	0.235	10/0	13.8
sko100a	100	152002	0.145	10/0	0.157	10/0	0.129	10/1	0.134	10/0	0.152	10/0	0.204	10/0	32
sko100b	100	153890	0.141	10/0	0.096	10/2	0.136	10/0	0.230	10/0	0.120	10/1	0.109	10/0	33
sko100c	100	147862	0.200	10/0	0.156	10/3	0.195	10/0	0.042	10/0	0.156	10/0	0.115	10/0	32
sko100d	100	149576	0.285	10/0	0.193	10/2	0.293	10/0	0.220	10/0	0.243	10/0	0.251	10/0	33
sko100e	100	149150	0.302	10/0	0.163	10/0	0.285	10/1	0.316	10/0	0.243	10/0	0.171	10/0	33
sko100f	100	149036	0.260	10/0	0.230	10/0	0.274	10/0	0.339	10/0	0.231	10/0	0.249	10/0	33
tho30	30	149936	0.069	10/7	0.023	10/9	0.214	9/8	0.255	9/5	0.149	9/8	0.121	10/5	0.4
tho40	40	240516	0.172	10/0	0.181	10/1	0.136	10/1	0.198	10/0	0.113	10/1	0.207	10/1	0.9
tho150	150	8133398	0.187	10/0	0.222	10/0	0.343	10/0	0.396	10/0	0.263	10/0	0.330	10/0	174
wil50	50	48816	0.039	10/2	0.039	10/3	0.084	10/1	0.055	10/1	0.065	10/1	0.056	10/1	1.9
wil100	100	273038	0.112	10/0	0.096	10/0	0.106	10/0	0.131	10/0	0.116	10/0	0.096	10/0	33

Table 6. Results of comparison of the tabu tenure variants for the real-life like QAP instances (II).
(CPU times per restart are given in seconds. 3 GHz Pentium computer was used in the experiments)

Instance	<i>n</i>	BKV	$\bar{\delta}, C_{1\%}/C_{BKS}$										CPU time		
			FTT-1 ^d		FTT-2 ^d		PTT ^d		RTT ^d		DPTT-B ^d			DPTT-M ^d	
tai20b	20	122455319	0.181	10/6	0.091	10/8	0.136	10/7	0.192	10/6	0.136	10/7	0.181	10/6	0.1
tai25b	25	344355646	0.060	10/7	0.138	9/6	0.014	10/8	0.022	10/7	0.573	9/7	0.604	9/5	0.2
tai30b	30	637117113	1.590	3/2	1.763	3/3	2.020	2/2	1.310	4/3	1.628	3/1	1.797	2/2	0.3
tai35b	35	283315445	0.800	8/4	2.129	4/3	0.909	7/3	1.503	6/1	0.843	8/2	1.535	6/1	0.6
tai40b	40	637250948	0.840	7/6	2.636	2/2	1.278	6/5	1.972	3/2	1.972	3/3	0.846	7/7	0.9
tai50b	50	458821517	0.336	10/2	0.218	10/3	0.308	9/2	0.420	9/2	0.713	9/1	0.352	10/3	1.9
tai60b	60	608215054	0.796	5/1	1.138	3/1	0.809	5/1	1.011	4/1	0.689	6/1	0.725	6/0	4.1
tai80b	80	818415043	0.930	5/0	0.949	4/0	1.027	3/0	1.058	4/0	0.976	3/0	1.027	6/0	13.0
tai100b	100	1185996137	0.668	7/0	0.555	8/2	0.841	7/0	1.074	6/1	0.900	7/1	0.745	7/1	33
tai150b	150	498896643	0.572	10/0	0.568	10/0	0.604	10/0	0.530	10/0	0.588	10/0	0.510	10/0	101

Table 7. Summary results of comparison of the tabu tenure variants (I)

Instance type	Deviation from BKV averaged over different types of instances					
	FTT-1	FTT-2	PTT	RTT	DPTT-B	DPTT-M
tai*a	0.371	0.462	0.376	0.348	0.368	0.400
sko*,tho*,wil*	0.195	0.144	0.167	0.183	0.182	0.169
tai*b	0.768	0.874	0.779	0.816	0.747	0.875
<i>n</i> < 100	0.433	0.461	0.426	0.433	0.402	0.459
<i>n</i> ≥ 100	0.296	0.318	0.280	0.304	0.327	0.314
	0.393	0.419	0.383	0.394	0.380	0.416

Table 8. Summary results of comparison of the tabu tenure variants (II)

Instance type	Deviation from BKV averaged over different types of instances					
	FTT-1 ^d	FTT-2 ^d	PTT ^d	RTT ^d	DPTT-B ^d	DPTT-M ^d
tai*a	0.395	0.466	0.394	0.360	0.346	0.373
sko*,tho*,wil*	0.163	0.137	0.181	0.196	0.169	0.165
tai*b	0.677	1.019	0.795	0.909	0.902	0.832
<i>n</i> < 100	0.382	0.528	0.427	0.461	0.452	0.440
<i>n</i> ≥ 100	0.304	0.284	0.330	0.351	0.310	0.291
	0.358	0.456	0.398	0.429	0.410	0.396

Table 9. Resulting ranking of the tabu tenure variants

	FTT-1*	FTT-2*	PTT*	RTT*	DPTT-B*	DPTT-M*
Deviation from BKV averaged over all instances	0.376	0.437	0.391	0.412	0.395	0.406
Resulting rank	1	6	2	5	3	4

* the deviations are averaged over the non-delay and delay modifications.

It could also be viewed that, for smaller problems (*n* < 100), it is quite easy to reach the (pseudo-)optimal solutions, while the statistically average quality of solutions is rather bad. On the other hand, it is very difficult to achieve (pseudo-)optimal solutions for larger problems (*n* ≥ 100) (if the computation time is limited); however, the average quality of solutions tends to be relatively high.

Our variants of defining the tabu tenure allow to find the near-optimal solutions very quickly (see Tables 1–6). For example, for the large real-life like instance tai150b (*n* = 150), approximately 100 seconds are enough to get the solution that is only 0.5 percent above the pseudo-optimum (see column "FTT-1" of Table 3). The results from Tables 1–6 may be improved considerably by increasing the number of ite-

rations (the run time), or by a more accurate adjusting of the tabu tenures and other control parameters. Thus, we have performed additional extensive experiments on the challenging QAP instance tai100a by using the basic variant of the dynamic tabu tenure with delay. After this long-lasting experimentation – it took several weeks (!) on a 3 GHz computer – we were successful in finding new solution that is slightly better than that announced in [15]. The new objective function value is equal to **21071498**.

4. Concluding remarks

In this paper, several variants of defining the tabu tenure for the quadratic assignment problem are discussed. In particular, the fixed tabu tenure, the periodic tabu tenure, the random tenure as well as some variants of the dynamic tabu tenure were investigated. All these variants were examined on various types of the QAP instances taken from the QAP instances library – QAPLIB.

The results from the computational experiments show that is important that the proper values of the tabu tenure (the range of tabu tenure) are maintained. If the tabu tenure is too small, the process returns to the previous solutions time after time. On the other hand, too large tabu tenure may forbid certain promising moves. In any case, the efficiency of the algorithm is lessened significantly. Therefore, a balance between these opposites is needed.

It may also be seen that the deterministic variants of the tabu tenure (in particular, the fixed and periodic tenures) are in most cases superior to the random tabu tenures. The non-fixed (periodic) tabu tenures could possibly be considered to be preferable to the fixed tabu tenures, although the results of experiments with the quasi-random instances indicate that also the fixed tabu tenures hide some potential. So, the further investigation of the fixed tabu tenure as well as other new variants of the tabu tenure (for example, adaptive (self-tuning) tabu tenure) might be interesting. In addition, trying these variants for other combinatorial optimization problems would be worthwhile.

References

- [1] **R. Battiti, G. Tecchioli.** The reactive tabu search. *ORSA Journal on Computing*, 1994, Vol.6, 126–140.
- [2] **N.W. Brixius, K.M. Anstreicher.** The Steinberg wiring problem. *Working Paper, University of Iowa, USA*, 2001.
- [3] **R.E. Burkard, E.Çela, P.M. Pardalos, L. Pitsoulis.** The quadratic assignment problem. In *D.Z. Du, P.M. Pardalos (eds.), Handbook of Combinatorial Optimization*, Kluwer, Dordrecht, 1998, Vol.3, 241–337.
- [4] **R.E. Burkard, S. Karisch, F.Rendl.** QAPLIB – a quadratic assignment problem library. *Journal of Global Optimization*, 1997, Vol.10, 391–403. [See also <http://www.seas.upenn.edu/qaplib/>]
- [5] **E. Çela.** The Quadratic Assignment Problem: Theory and Algorithms. *Kluwer, Dordrecht*, 1998.
- [6] **Z. Drezner.** The extended concentric tabu for the quadratic assignment problem. *European Journal of Operational Research*, 2005, Vol.160, 416–422.
- [7] **Z. Drezner, G.A. Marcoulides.** On the range of tabu tenure in solving quadratic assignment problems. *Under review*, 2007.
- [8] **M. Gendreau.** Recent advances in tabu search. In *C.C. Ribeiro, P. Hansen (eds.), Essays and Surveys in Metaheuristics*, Kluwer, Boston, 2001, 369–377.
- [9] **F. Glover, M. Laguna.** Tabu Search. *Kluwer, Dordrecht*, 1997.
- [10] **A. Hertz, E. Taillard, D. de Werra.** Tabu search. In *E.H.L. Aarts, J.K. Lenstra (Eds.), Local Search in Combinatorial Optimization*, Wiley, Chichester, 1997, 121–136.
- [11] **T. Koopmans, M. Beckmann.** Assignment problems and the location of economic activities. *Econometrica*, 1957, Vol.25, 53–76.
- [12] **A. Misevičius.** A tabu search algorithm for the quadratic assignment problem. *Computational Optimization and Applications*, 2005, Vol.30, 95–111.
- [13] **A. Misevičius, J. Blonskis.** Experiments with tabu search for random quadratic assignment problems. *Information Technology and Control*, 2005, Vol.34, 237–244.
- [14] **A. Misevičius, A. Lenkevičius, D. Rubliauskas.** Iterated tabu search: an improvement to standard tabu search. *Information Technology and Control*, 2006, Vol.35 (No.3), 187–197.
- [15] **A. Misevičius, A. Tomkevičius, J. Karbauskas.** Stagnation-protected tabu search variants for unstructured quadratic assignment problems. *Information Technology and Control*, 2006, Vol.35 (No.4), 363–370.
- [16] **P.M. Pardalos, F. Rendl, H. Wolkowicz.** The quadratic assignment problem: a survey and recent developments. In *P.M. Pardalos, H. Wolkowicz (eds.), Quadratic Assignment and Related Problems. DIMACS Series on Discrete Mathematics and Theoretical Computer Science*, Vol.16, AMS, Providence, 1994, 1–41.
- [17] **S. Sahni, T. Gonzalez.** P-complete approximation problems. *Journal of ACM*, 1976, Vol.23, 555–565.
- [18] **S. Salhi.** Heuristic search methods. In *G. Marcoulides (ed.), Modern Methods for Business Research*, Lawrence Erlbaum, Mahwah, 1998, 147–175.
- [19] **E. Taillard.** Robust taboo search for the QAP. *Parallel Computing*, 1991, Vol.17, 443–455.
- [20] **E. Taillard, L.M. Gambardella.** Adaptive memories for the quadratic assignment problem. *Tech. Report IDSIA-87-97, Lugano, Switzerland*, 1997.

Received October 2007.