# AUTONOMOUS VEHICLE FOLLOWING-PERFORMANCE COMPARISON AND PROPOSITION OF A QUASI-LINEAR CONTROLLER

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**Abstract**. This paper studies longitudinal vehicle following problem, where two control strategies i.e. LQR optimal synthesis and a novel quasi-linear controller are adopted for a linearized vehicle model. In case of LQR, the results are enhanced by inclusion of a minimum order state observer and a feed-forward controller. Since the dynamics of the feedback linearized system correspond to a certain class of linear systems, i.e. a system having more than one pole in excess to zeros, a simpler order quasi-linear feedback controller is applied. The designed controller guarantees asymptotic tracking of the desired trajectories under the constraints of physical limitations inherent in the system. A comparative simulation study of the two approaches reveals the system's performance under various conditions.

Keywords: Autonomous vehicles, Integrated Vehicle Highway Systems, Cruise Control, Quasi-linear Control.

## 1. Introduction

Continuously increasing number of vehicles on road has put the pressure on the research of intelligent transportation systems (ITS). Recurrent congestion during rush hours is a common problem in major cities and on highways around the globe. The ITS systems incorporate vehicles which are equipped with onboard lateral and longitudinal controllers. The ultimate aim has been to enhance the traffic capacity with the available infra-structure. Furthermore, safety has been of prime concern since most of the traffic accidents are the result of human error. The automation of the driving system can preclude such errors, as it compensates for human limitations in sensing the environment and reacting to unexpected events. This paper addresses the autonomous longitudinal vehicle following problem where a follower vehicle maintains a constant headway distance with a leader.

Recent research in intelligent vehicles on highways varies in the types of controllers used for automated driving; examples are classical PID control [1, 2], LQR based approach [3, 4, 5], control based on a Lyapunov function [6], H-infinity control [7], fuzzy logic [8], neural network [9], model predictive control [10] etc. Each of the methods used therein offer convincing results but generally some practical issues like control saturation, sensing error and plant parameter variation, oscillatory nature of control response, inherent limitations of the plant etc, were not simultaneously considered. For example [3] considers optimal LQR synthesis but establishes longitudinal control with very high gain control vector, whereas [11] considers acceleration and deceleration limits but does not address model uncertainty and [7] attempts to damp out longitudinal oscillations by using feed forward and feedback H-infinity control. This paper investigates a framework where the performance criteria of the system are firstly met by applying nominal control using LQR optimization and establish bounds of stable operation. The regulation and tracking aspects of a constant distance and velocity are addressed under the constraint of limitations imposed by the plant. Then a minimum order state estimator is emploved in order to avoid noise in the measurements of velocity and acceleration. To further refine the control response, a feed-forward compensator is incorporated. The recent research in the field of linear feedback control has led to a new idea termed as Quasi-Linear Control [12], which is particularly useful for a class of systems with more than one pole in excess to zeros. The linearized dynamics of the system is applied with Quasi-Linear Feedback, as a novel control approach. A great improvement of performance is obtained with simpler compensation and using fewer measurements in contrast to [3, 6] etc. The rest of the paper is organized as follows: In Section 2, the vehicles' dynamic model is described. In Section 3 we propose controller synthesis by LQR method with a reduced order state observer. This section mainly comprises the extract of our work [13]. In section 4, a novel Quasi-linear controller is implemented on the linearized plant and again the regulation and tracking aspects of a constant distance and velocity are addressed in Section 5. The simulation results reveal a comparison of the control methods and we establish that

the new approach is superior from different viewpoints. Finally we draw conclusions in Section 6.

- The motion of the vehicles is constrained to (i) translations only.
- (ii) The movement of vehicles is smooth.

## 2. Vehicle dynamics modelling

To describe modeling of the system, it is appropriate to state the following simplifying assumptions.



Figure 1. A follower vehicle equipped with onboard sensor and controller to keep a fixed distance from the leader

The vehicles' longitudinal motion is modeled with the choice of the following variables as listed below:

 $\xi_I$  – Distance of the leader from a reference point

 $\xi_f$  – Distance of the follower from a reference point

 $\xi_d$  – Desired distance to be maintained between the vehicles

 $\xi_{\rho}$  – Error in the desired distance to be maintained

 $v_l$  – Velocity of the leader

 $v_f$  – Velocity of the follower

 $a_I$  – Acceleration of the leader

 $a_f$  – Acceleration of the follower

 $m_f$  – Mass of the vehicle

 $\delta_f$  – Engine input of the vehicle

 $\tau_f$  – Engine time constant of the vehicle

 $K_{ad}$  – Aerodynamic drag coefficient of the vehicle

 $K_{md}$  – Mechanical drag on the vehicle

From Fig.1  $\xi_l - \xi_f = \xi_d \pm \xi_e$ ,  $\xi_d$  is the desired spacing that is to be strictly maintained between the two vehicles and  $\xi_e$  is the amount of error or drift from this desired spacing. Consider that the dynamics of vehicles on road is described by the following model [5]:

$$\xi_e = v_f - v_l$$

$$v_f = a_f$$

$$a_f = g_f(v_f, a_f) + h_f(v_f)\delta_f$$
(1)

Functions  $g_f(.,.)$  and  $h_f(.)$  are given by:

$$g_{f}(v_{f}, a_{f}) = -\frac{2K_{ad}}{m_{f}} - \frac{1}{\tau_{f}} \left[ a_{f} + \frac{K_{ad}}{m_{f}} v_{f}^{2} + \frac{K_{md}}{m_{f}} \right], \quad (2)$$
$$h_{f}(v_{f}) = \frac{1}{m_{f}\tau_{f}}. \quad (3)$$

If the parameters in equation (2) and (3) are exactly known, the following feedback linearizing control law could be adopted:

$$\delta_f = m_f \mu_{fc} + K_{ad} v_f^2 + K_{md} + 2\tau_f K_{ad} v_f a_f , \qquad (4)$$

where  $\delta_f$  is the engine input and  $\mu_{fc}$  is the control applied by the follower that makes the closed loop system to satisfy certain performance criteria. The drag coefficients  $K_{ad}$  and  $K_{md}$  are important parameters for the dynamics of vehicles; their values are assumed to be known, however their values and variations are not the focus of this study. As discussed above, if the strong assumption about exactly known parameters is invalid, the feedback linearization process would be inexact. Such a case, where variations in these parameters are considered, is described in [18].

Manipulating equation (1) through equation (4), the  $3^{rd}$  equation in (1) becomes:

$$a_f = \frac{1}{\tau_f} \left( \mu_{f_c} - a_f \right), \tag{5}$$

where  $\tau_{f}$  is the engine time constant, a single parameter that describes the dynamics of propulsion system, transmission and internal disturbances. The system thus takes the form which can be described by the following standard set of linear equations:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\boldsymbol{\mu}_{fc}(t) \text{ where } \mathbf{A} \in \mathfrak{R}^{3x3},$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\boldsymbol{\mu}_{fc}(t)$$

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/\tau_f \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1/\tau_f \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T \text{ and }$$

$$\mathbf{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T \text{ where } \boldsymbol{\mu}_{f_c} \text{ can be viewed as the throttle/}$$

0 brake input causing acceleration/decelerations in the

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controlled vehicle. The model is formulated with the choice of the following states variables;

$$x_1(t) = \xi_e, x_2(t) = v_f$$
 and  $x_3(t) = a_f$  where  $x_1(t)$ 

and  $x_2(t)$  represent relative velocity and acceleration of the follower, respectively.

### 3. Optimal controller synthesis

Different approaches for controller synthesis exist in the literature, each having its own pros and cons. The vehicle following problem imposes a trade-off between applied acceleration and transient response time. For this kind of problem, controller synthesis by optimal control theory is one of the better choices, although classical PID control may also be adopted as in [1, 2]. The desired spacing  $\xi_d$  can be determined by using the so-called spacing policy. A spacing policy is important because it directly affects vehicle safety, traffic flow on the highway: smaller inter-vehicle distances can lead to a denser-flow utilization of the highway and driver's comfort. The desired spacing could be a constant value or it can be speed dependent; the latter, being more promising, is adopted in this work as in [14], however some other spacing policies [15, 16, 17] may also be adopted. As given in [14] it is shown that vehicle following stability can be recovered in an autonomous operation if a speed dependent spacing policy is adopted, which incorporates constant time headway in addition to the constant distance. This takes the form  $\xi_d = \lambda v_f + \lambda_o$ , where  $\lambda$  and  $\lambda_o$  are suitably chosen positive constants. The parameter  $\lambda$  is the time headway and its effect is to introduce more spacing between the two vehicles, as the velocity of the follower increases, which intuitively makes sense. Setting  $\lambda_{a}$  to be zero, allows

for minimum desired distance between two vehicles to be zero provided the follower vehicle has zero velocity. Another interesting way of describing the speed dependent spacing policy is given by California Rule of Thumb [14], which suggests a spacing one vehicle length for every 10mph.

The system described by equation (6) is controllable since the controllability matrix  $\begin{bmatrix} B & AB & A^2B \end{bmatrix}$  is of full rank, hence the poles of the system can be placed arbitrarily, provided  $\mu_{f_c}$  is unconstrained. However, here we assign the poles such that the resultant closed loop system would have  $\mu_{f_c}$  constrained in accordance with the physical limitations. Thus the control law would take the form:

$$\mu_{f_{e}} = f\{\xi_{e}(t), v_{f}(t), a_{f}(t)\} \text{ where } v_{f}(t) \ge 0.$$
(7)

Using LQR optimal synthesis, the control vector K is obtained in such a way that the performance index

$$J = \int_{0}^{\infty} \{x^{*}(t)Qx(t) + \mu^{2}_{f_{c}}(t)\}dt \text{ is minimized, where } Q$$

is a suitably chosen positive-semi definite Hermitian or real symmetric matrix. Simulations are performed to achieve the following performance objectives: (i) Longitudinal maneuvers shall not result in an overshoot more than 10% of the error in desired spacing. (ii) The settling time of the spacing and velocity response shall be appropriate such that in successive maneuvers, the control action is completed well in time without collisions. (iii) The throttle or break action shall result in acceleration and deceleration, not exceeding 0.2g and -0.51g respectively [4]. After choosing a suitable Q, the reduced matrix Riccati equation is solved for matrix P, and then K is computed as below:

$$A^{T}P + PA - PBB^{T}P + Q = 0.$$
 (8)  
$$K = B^{T}P$$

The response of the system is observed with the choice of initial conditions as:

$$\begin{bmatrix} x_1^0 & x_2^0 & x_3^0 \end{bmatrix} = \begin{bmatrix} 20 & 2.4 & 0 \end{bmatrix}$$

The chosen initial conditions are twice in magnitude as compared to the initial conditions chosen as the worst case in [3], where LQR synthesis is applied on the same model and the resultant control vector is of the form: K = [40.01, 55.78, 24.45], which shows a states' amplification to high values. In contrast, our approach yields: K= [1.0488, -3.0106, -2.7966]. Although in this case, the designed controller has a higher challenge to cope with as compared to [3], still the rate of convergence is quite higher as shown in Figure 2; therefore the control approach adopted seems superior to [3], where the response of the decay of error is oscillating and too delayed, which can potentially result in the loss of ride comfort and possible initiation of another maneuver by leader during the process of settling an earlier maneuver by the follower. The control scheme developed by [6] is promising where the idea of "expected spacing" is introduced, however the control law developed therein requires leader vehicle's control input information at all times in addition to leader's velocity and acceleration. Such a vehicle following strategy requires establishing high efficiency communication link between vehicles, whereas in this work an unknown vehicle can be followed, which does not transmit any information to the follower. The relative distance, velocity and acceleration could be measured using onboard sensors (radar, laser or infrared).

We also consider the velocity-following case, where the leader vehicle performs a longitudinal maneuver and follows a kind of trapezoidal trajectory. The controller is at work unless there is an error in safe distance and there exists some relevant velocity. With the same selection of system and control parameters, the velocity following case is demonstrated in Figure 4. It is shown that the controller

efficiently tends the velocity profile of the follower to track the leader's velocity. As shown in Figure 3, a very small delay in follower's velocity with respect to the leader is observed which also means that error in safe distance would remain potentially small even if the leader takes sudden maneuvers. The small ripple in the initial duration of follow up is the result of chosen initial conditions. If the vehicles are moving with a speed of 20 m/s (72 km/hr), the small lag between two profiles is of 0.4 sec, which corresponds to an approximate error of 9 m in distance. As it is obvious from Figure 2, this amount of error can be easily handled by the controller in about 8 seconds, so with this demonstration, we can establish a bound on the stable operation of our system that the methodology is perfectly valid if the successive maneuvers of the leader are separated by 8 seconds.



Figure 2. The response of the follower vehicle in terms of error in desired spacing, relative velocity and acceleration



Figure 3. The velocity follow-up of the follower as the leader increases/decreases its velocity

The constraint  $v_f(t) \ge 0$  is an important issue in

the design of vehicle follower systems, because generally the control exhibits overshoot in tracking leader's velocity. Since physically, negative velocity is not realizable so that the designed system must be able to handle this limitation. The simulation analyses show that in the worst case scenario, i.e. if the leader applies hard brakes while coming to a sudden stop by applying maximum deceleration of -5 m/s<sup>2</sup>, the resulting overshoot is fractional and negligibly small, however a limiter is included in the dynamics, which prevents  $v_f(t)$  to take a negative value. The tracking response for such a case is given in Figure 4.



Figure 4. The velocity follow-up case, as the leader makes a sudden stop and then accelerates with max acceleration



Figure 5. Simulation diagram of close loop control system with a minimum order state observer and a feed-forward compensator

It is important to note that differentiating a state variable to generate another one has to be avoided, since differentiation of a signal always decreases signal to noise ratio, because noise generally fluctuates more rapidly than the command signal. Sometimes the signal-to-noise-ratio may be decreased by several times by a single differentiation process. Methods are available to estimate unmeasured state variables without going through a differentiation process. In this work, we measure the relative distance between the vehicles and then estimate the relative velocity and acceleration of the follower vehicle. We design a high gain minimum-order state observer, which provides the estimates of unmeasured states. We compare the performance results and observe the effects of employing an observer. The control signal in the present case will be thus defined as:

$$\mu_{f_c} = f\{\xi_e(t), x_2(t), x_3(t)\}, \qquad (9)$$

where  $x_2(t)$  and  $x_3(t)$  are the estimated states. The complete simulation diagram for the system with estimated states is given as Figure 5. Here the state vector of (5) is partitioned into two parts  $x_a$  (a scalar) and  $x_b$  (a 2x1 vector).  $x_a$  is the output that can be directly measured and  $x_b$  is the unmeasured portion. After partitioning the linearized model of (1) through (4), the equations for the measured and unmeasured states are:

$$x_a = A_{aa} x_a + A_{ab} x_b + B_a \mu_{fc}$$

$$x_b = A_{ba} x_a + A_{bb} x_b + B_b \mu_{fc}$$
(10)

With a choice of suitable poles for the observer, the gain matrix  $K_e$  is computed by pole placement method. The simulations for similar set of conditions are performed on the new system. As expected, a slightly degraded performance (in terms of overshoot and settling time) is observed as a result of using estimated states for feedback as shown in Figure 6 (dashed lines).

#### 3.1. Use of a Feed-Forward Compensator

In Figure 6 (dashed lines), it is clearly seen that the overshoot from the desired state is larger as compared to the previous case (Figure 3). This problem may lead to reduction in ride comfort and invite a potential collision scenario. To overcome this difficulty and to

ensure further safety, a feed-forward compensator is used to improve the performance. The transfer function of the compensator is a first order system as given below:  $K_{ffc}=1/(as+b)$ . The tuning of parameters *a* and *b* to values 1 and 0.15, respectively, yields the desired behavior of the system, which leads to two folds reduction in overshoot and in an appreciable convergence rate. The results of this simulation study are shown in Figure 6 (dark continuous lines). This response is more desirable as it guarantees safety and ride comfort which are the main objectives of this study. Table 1 summarizes the performance of all techniques applied so far in this study.



Figure 6. Response with state estimator only (dashed) and then added with feed-forward controller (dark continuous lines)

Table 1.					
	% overshoot	Rise time(sec)	Settling time(sec)		
Full State feedback with initial error of					
10m	1.25	4.55	7.0		
20m	1.25	4.57	7.2		
30m	1.25	4.57	7.5		
40m	1.25	5.20	7.9		
50m	1.25	5.30	8.4		
Reduced State feedback with estimator and with initial error of					
10m	13	2.25	7.2		
20m	13	2.30	7.7		
30m	13	2.34	8.4		
40m	13	2.35	8.9		
50m	13	2.35	9.7		

Table 1.

State feedback with estimator and feed- forward controller				
10m	5.7	2.10	6.4	
20m	5.7	2.13	6.9	
30m	5.7	2.16	7.4	
40m	5.7	2.18	7.9	
50m	5.7	2.19	8.4	

# 4. Quasi linear controller design

The transfer function of the feedback-linearized system has two integrators and one stable pole in excess to zeros. The output of a single-input singleoutput linear feedback system with more than one pole in excess over the zeros in the loop transmission cannot track arbitrarily fast its input [12]. This is because of the fact that the linear compensator cannot secure any phase margin when the gain increases unboundedly because it has two poles more than zeros in the loop transmission. This fact does not prevent the quasi-linear compensator to achieve high performance because the destabilizing lag effect is pushed towards larger frequencies by the pole which wanders farther away with the increase of the gain. The linear compensator might achieve the same performance but only augmenting indefinitely its chain of lead-lag compensation. One example is presented here which motivates the use of such a controller and hence illustrates the results of [12]. It also shows that this good performance can be obtained with a reduced control effort. The general structure of the quasi-linear compensator is explained as follows: Consider a plant described by:  $G_p(s) = \frac{1}{s^2}$ . A possible frequency domain feedback compensator design for a closed loop system to control the plant output is of the form:

$$G_c(s) = K \frac{s+1}{s+2}$$
 (11)

The value of the gain K is tuned until a good performance of the control is achieved, so we raise the gain K > 0 to increase the performance of the closed loop and use the lead-lag compensation for ensuring some phase margin. At the value of K = 1000, a quite slow and highly oscillatory response to a step input is obtained as shown in Figure 7. This is because of the fact that raising the value of K with a compensator as in (11), the frequency component of eigenvalues of the closed loop system remains unchanged whereas the imaginary part changes drastically. As a rough measure, the imaginary part is scaled by a factor of 3 for every rise in scaling of K by 10. So the oscillations in response grow larger with increasing K in this fashion. However, with a slightly different compensator as below,

$$G_k(s) = K \frac{(s+a)}{(s+2K^{0.6})}.$$
 (12)

Again choosing a=1 and with K=1000, as in (12), the results are completely different as shown in Figure 8. This great improvement of performance was obtained without any change in the order of compensation (which is already minimal), but by letting the pole of the compensator depend on the gain K itself: the exponent of K was f=0 in Figure 7 and f=0.6 in Figure 8. The first compensator is a linear one and the compensator for the latter case is termed as `quasi-linear'.





Figure 8. Response of the closed loop system with a quasilinear compensator

With this example we come back to the vehiclefollowing problem. The resulting system of equations (1) to (4) has two poles in excess to zeros, thus the same approach could be potentially adopted with certain constraints. Since the plant considered has a limited power to exhibit acceleration or braking action, the gain K as in the example, cannot be applied arbitrarily large to reach a suitable performance of tracking. Physically the gain of controller amounts to acceleration/ deceleration of the plant to track velocity and safe distance. Most of the research works in automated driving deal with this constraint of available acceleration by placing a rate limiter in the dynamics, which limits the rate of change of velocity of the plant according to inherent limits. However this approach causes instabilities and delays in controlled response. In our work we show that the controller has the property that, when the leader car accelerates and creates an error in the safe distance, the controlled response of the follower never exceeds the value of leader acceleration. This implies that if the leader performs a maneuver with maximum acceleration, the controlled response of the follower shall never exceed that acceleration.

The feedback-linearized system has a stable pole whose location is governed by the parameter  $\tau_f$ . For improved performance, the zero of the compensator (parameter *a*) has to be chosen according to this stable pole. A comparable value of *a* with respect to the location of stable pole may result in pole-zero cancellation and therefore an unacceptable response, whereas a larger value results in instability, however a sufficiently smaller value leads to a desired behavior. In this work, a suitable value of the stable zero is found to be at -1. Here a quasi-linear design is presented which has been obtained after considerable search and tuning over the parameter space. It is given by:

$$G_{k(ql)}(s) = K \frac{(s+a)}{(s+2K^{f})}$$
 where K=100 and f=0.58.

With the above mentioned set of conditions, the closed loop system proved to be stable as given below:

After feedback linearization

$$G_{plant}(s) = \frac{1}{s^2(s + \tau_f)}$$

$$G_{closedloop}(s) = \frac{G_{k(ql)}(s)G_{plant}(s)}{1 + G_{k(ql)}(s)G_{plant}(s)}.$$
(13)

Using the chosen controller parameters and a suitable engine time constant of 500msec, the transfer function of the close loop system will be:

$$G_{closedloop}(s) = \frac{100(s+1)}{s^4 + 19.45s^3 + 72.25s^2 + 100s + 100} \cdot (14)$$

It is not difficult to see that characteristic equation will be Hurwitz for any chosen value of engine time constant. Thus it is proved that the resulting system is asymptotically stable and is good for all types of vehicles.

The performance of the system firstly requires that the initial errors in desired spacing should be brought to zero in reasonable time. Secondly any difference in the velocities of leader and follower should also be brought to zero, but since the spacing policy is speed dependent, the value of the desired spacing will not be fixed and will be changing with the velocity of the leader. Therefore we will consider each of these two cases separately.

## 5. Scenario and performance analysis

## 5.1. Tracking a desired spacing

The following scenario will be used to establish the baseline for transitional maneuvers problem, the scenario assumed here is such that the initial difference in the desired spacing is 10 m, the new desired spacing as a result of reduced velocity requires a change of spacing from 70 m to 20 m. Later, because of the leader's increased velocity, the desired spacing is set to 40 m. This scenario and the follow up of the controlled car, is illustrated in Figure 9. This scenario has been adopted to show the tracking of the resultant desired spacing as a consequence of varying speeds.



Figure 9. Response of the controlled system while tracking desired spacings

#### 5.2. Velocity following case

In this case the scenario adopted is similar but here we address the issue of tracking the leaders' velocity by the follower. Resorting to the worst case, the assumed scenario is a hard maneuver that can only be encountered when the leader accelerates with full throttle or decelerates after applying hard brakes. Initially both vehicles are moving with 40 m/sec with no difference of velocities, however the leader applies maximum brakes and in 5 seconds, it comes down to 15 m/sec and then gets steady at this value for another 15 seconds. After this maneuver it accelerates at 20<sup>th</sup> instant and increases velocity to get steady at 35 Km/hr. The complete maneuver of the leader and controlled follow up of the follower is illustrated in Figure 10.

The slope of the trajectory, depicting leader velocity in Figure 10, shows that in the initial phase, the leader car is applying maximum deceleration of -5  $m/s^2$  and then later it applies an acceleration of  $2m/s^2$ , except in the steady velocity situation where the acceleration is zero. The follower tracks the velocity of leader, which is changing rapidly as a result of hard maneuver of leader. In doing so, the acceleration profile of follower as given in Figure 10 (dotted lines), shows that it catches up with the leader velocity without crossing the available accelerations/braking limits. This performance of the controller necessarily rules out the use of any rate limiter that may exhibit instability. It is shown that the acceleration comes to zero when there exists no difference of velocities. The hard maneuvers are handled by the follower car in 15 seconds which seems a reasonable settling time from practical point of view.



Figure 10. Response of the controlled vehicle

#### 5.3. Robustness to parameter variation of the plant

The parameter  $\tau_f$  (engine time constant) determines the plant performance which represents the characteristics of vehicle propulsion system including the engine, transmission, tires and wheels, and any other internal controllers. Considering different vehicles with their engine time constants as in Section 4, simulations have been performed in the range of time constants covering the entire range of vehicles and it is observed that the designed controller is robust to handle the uncertainties of the plant dynamics. For the same set of conditions as in the previous sub-section, a zoomed version of the scenario is shown in Figure 11.

#### 5.4. Robustness to external disturbances

The closed loop system with the quasi-linear feedback controller exhibits robustness to external disturbances. As an intuitive example, the feedback loop requires exact measurement of output, and if the measurements are not accurate due to sensor noise, the controller's input would be noise-corrupted and hence it would be desirable that the controller still exhibit stability with uncertain measurements. To model uncertain measurements, a white noise of maximum amplitude of 0.5 is applied to the output measurement and the controller's performance is observed as shown in Figure 12. The steady state errors, as observed in Section 3 of this article and in some earlier referenced work, are a potential threat to safety aspect, especially in the close formations moving at slower speeds. We observed that the novel quasi-linear control exhibits extremely low steady state errors, so it can be regarded as superior to the classical approaches.



Figure 11. Velocity-following response for vehicles with different engine time constants



Figure12. Response of the controlled system with noisy measurements

#### 6. Conclusion

In this paper we have adopted different control approaches for maintaining a fixed desired distance to a leading vehicle. The performance of the system is limited by the exact feedback linearization of the nonlinear model. The control strategies are based on LQR combined with a minimum order state observer, and Quasi-linear theories. In each case, various issues of vehicle following problem are addressed. In the case of LQR synthesis, the key factor limiting the controller performance is the tradeoff between rate of convergence of tracking error and physical limitations of the plant. We achieve both objectives by suitable choice of the optimal values. The degradation of performance due to the induction of state observer is effectively tackled by introducing a feed-forward compensator. The simulation results show that the proposed control law can be used to exponentially stabilize the vehicle movement. In the design of novel control strategy based on arbitrarily fast and robust tracking by Quasilinear control, a minimal order controller is designed which is able to produce desirable results. The controllers require fewer measurements as compared to some other well-known control approaches and earlier work on the subject. The controllers are also capable to handle plant model parametric uncertainties, with good disturbance rejection properties, under the constraints of physical limitations. By design, the controller is robust enough to operate on nearly any vehicle. Simulated responses reveal that the approaches offer better prospects than earlier work on similar issues. Our future goal is to extend this work to the vehicles platooning problem, where many vehicles move in a fashion such that they maintain a tight inter-vehicular spacing to make maximum use of the available roads and highways.

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