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THE PERFORMANCE OF THE IM-DD SYSTEMS IN THE PRESENCE OF QUANTUM NOISE AND GAUSSIAN NOISE IN THE FIBER

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AbstractIn this paper the performances of an IM-DD optical telecommunication system in the presence of quantum noise at the terminals of photodiode and Gaussian noise in the fiber are determined. We give the expressions for the probability of events: both symbols are correctly detected; one is correct, one is wrongly and both are wrong detected.

1. Indroduction

The quantum noise appears because of quantum natural of light [1]. It is important to determine the probability of the quaint number in the some time interval. Always we could make the time interval so small that no one quant of light appears in that or only one appears. It can be assumed that the probability of appearance one pulse in the time interval Δt is proportional to this interval. If it is valid, the quant number has Poisson's distribution [1]. It is important to determine the probability density function of the quantum noise current amplitudes. In some cases this probability density function is Gaussian. At quantum noise the variance of this distribution is proportional to the average value of the quantum noise. Because of that the quantum noise is the Winer random process. When the quantum noise is significant, and binary hypothesis have equal probability, the threshold is not at the medium.

In the optical IM-DD systems [1, 2] interference with Gaussian probability density function can appear in the fiber. The interferences with this distribution originate from optical amplifiers along the fiber. They are 50-100km for away from each other and compensate the reducing of light. The noise appeares because of the spontaneous emission of lights(radiation) has Gaussian probability density function. The Gaussian noise formed in the fiber can be the result of mixing the modes. Some new modes can be formed at one connection of the optical fibers, but at the next connection that is modal noise. The modal noise exists because of nonlinear transformations along the fiber and it is similar to the interferometar noise. The amplitudes of this noise can have Gaussian pdf [3]. Also, several interferences with uniform pdf of phases can occure in the optical fiber. When the number of interferences is more than 10, it can be assumed that the noises have Gaussian pdf.

2. System analysis

The intensity of the light exiting the photodiode, for the hypotesis H_0 is:

$$\lambda_0 = C(A_0 + x)^2 \tag{1}$$

and for the hypothesis H_1 is:

$$\lambda_1 = C(A_1 + x)^2 . \tag{2}$$

The Gaussian noise x appears in the fiber and has the probability density function:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}} \tag{3}$$

where σ_x^2 is the variance of the Gaussian noise.

The number of quants exiting the photodiode depends on the intensity of light and its conditional probability in a time interval is [4]:

$$p(n/\lambda) = \frac{\lambda^n}{n!} e^{-\lambda} = \frac{C^n (A + x)^{2n}}{n!} e^{-C(A+x)^2} . \tag{4}$$

The probability of quant number is obtained by averaging the expression (4):

$$p(n) = \int_{\lambda} p(n/\lambda) p(\lambda) d\lambda = \int_{-\infty}^{\infty} \frac{C^n (A + x)^{2n}}{n!} e^{-C(A+x)^2} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}} dx \tag{5}$$

because of: $p(\lambda) d\lambda = p(x) dx$

The probabilities of quant number for hypotheses H_0 and H_1, are:

$$P_0(n) = \int_{-\infty}^{\infty} \frac{\lambda_0^n}{n!} e^{-\lambda_0} \cdot p(x) dx =$$

$$= \int_{-\infty}^{\infty} \frac{C^n (A_0 + x)^{2n}}{n!} e^{-C(A_0+x)^2} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}} dx \,, \quad (6)$$

$$P_1(n) = \int_{-\infty}^{\infty} \frac{C^n (A_1 + x)^{2n}}{n!} e^{-C(A_1+x)^2} \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}} dx \,. \quad (7)$$

The decision threshold is n_T.

The equations (6) and (7) can be written in the form:

$$P_0(n) = \frac{1}{n!\sigma\sqrt{2\pi a}} e^{-d_0} \sum_{i=0}^{2n} \binom{2n}{i} \frac{s_0^{2n-i}}{\sqrt{a^i}} J(i) \,, \quad (8)$$

$$P_1(n) = \frac{1}{n!\sigma\sqrt{2\pi a}} e^{-d_1} \sum_{i=0}^{2n} \binom{2n}{i} \frac{s_1^{2n-i}}{\sqrt{a^i}} J(i) \,. \quad (9)$$

3. Calculation of the detection probability

The probabilities of correct detection $P(D_0/H_0)$ and $P(D_1/H_1)$ are:

$$P(D_0/H_0) = \sum_{n=0}^{n=n_T} P_0(n) =$$

$$= \sum_{n=0}^{n=n_T} \frac{1}{n!\sigma\sqrt{2\pi a}} e^{-d_0} \sum_{i=0}^{2n} \binom{2n}{i} \frac{s_0^{2n-1}}{\sqrt{a^i}} J(i) \,, \quad (10)$$

$$P(D_1/H_1) = \sum_{n=n_T+1}^{\infty} P_1(n) =$$

$$= \sum_{n=n_T+1}^{\infty} \frac{1}{n!\sigma\sqrt{2\pi a}} e^{-d_1} \sum_{i=0}^{2n} \binom{2n}{i} \frac{s_1^{2n-1}}{\sqrt{a^i}} J(i) \,. \quad (11)$$

The probabilities of wrong detection $P(D_1/H_0)$ and $P(D_0/H_1)$ are:

$$P(D_1/H_0) = \sum_{n=n_T+1}^{\infty} P_0(n) =$$

$$= \sum_{n=n_T+1}^{\infty} \frac{1}{n!\sigma\sqrt{2\pi a}} e^{-d_0} \sum_{i=0}^{2n} \binom{2n}{i} \frac{s_0^{2n-1}}{\sqrt{a^i}} J(i) \quad (12)$$

$$P(D_0/H_1) = \sum_{n=0}^{n=n_T} P_1(n) =$$

$$= \sum_{n=0}^{n=n_T} \frac{1}{n!\sigma\sqrt{2\pi a}} e^{-d_1} \sum_{i=0}^{2n} \binom{2n}{i} \frac{s_1^{2n-1}}{\sqrt{a^i}} J(i) \quad (13)$$

The joint probability of quant number in two time moments for transmitting symbols (0,0) $P_{00}(n_1,n_2)$ is:

$$P_{00}(n_1,n_2) = \sum_{i=0}^{\infty} \frac{R^i}{i!} a_{10} a_{20} \quad (14)$$

where:

$$a_{10} = \frac{1}{n_1!} \int_{-\infty}^{\infty} \left(x_1^2 + A_0\right)^{n_1} e^{-\left(x_1^2 + A_0\right)} e^{-\frac{x_1^2}{2\sigma_x^2}} \cdot H_i\left(\frac{x_1}{\sigma_x}\right) dx_1 \,, (15)$$

$$a_{20} = \frac{1}{n_2!} \int_{-\infty}^{\infty} \left(x_2^2 + A_0\right)^{n_2} e^{-\left(x_2^2 + A_0\right)} e^{-\frac{x_2^2}{2\sigma_x^2}} \cdot H_i\left(\frac{x_2}{\sigma_x}\right) dx_2 \,. (16)$$

The joint probability of quant number in two time moments for transmitting symbols (0,1) $P_{01}(n_1,n_2)$ is:

$$P_{01}(n_1,n_2) = \sum_{i=0}^{\infty} \frac{R^i}{i!} a_{10} a_{21} \,, \quad (17)$$

where:

$$a_{10} = \frac{1}{n_1!} \int_{-\infty}^{\infty} \left(x_1^2 + A_0\right)^{n_1} e^{-\left(x_1^2 + A_0\right)} e^{-\frac{x_1^2}{2\sigma_x^2}} \cdot H_i\left(\frac{x_1}{\sigma_x}\right) dx_1 \,, \quad (18)$$

$$a_{21} = \frac{1}{n_2!} \int_{-\infty}^{\infty} \left(x_2^2 + A_1\right)^{n_2} e^{-\left(x_2^2 + A_1\right)} e^{-\frac{x_2^2}{2\sigma_x^2}} \cdot H_i\left(\frac{x_2}{\sigma_x}\right) dx_2 \,. \quad (19)$$

The joint probability of quant number in two time moments for transmitting symbols (1,0) $P_{10}(n_1,n_2)$ is:

$$P_{10}(n_1,n_2) = \sum_{i=0}^{\infty} \frac{R^i}{i!} a_{11} a_{20} \,, \quad (20)$$

where:

$$a_{11} = \frac{1}{n_1!} \int_{-\infty}^{\infty} \left(x_1^2 + A_1\right)^{n_1} e^{-\left(x_1^2 + A_1\right)} e^{-\frac{x_1^2}{2\sigma_x^2}} \cdot H_i\left(\frac{x_1}{\sigma_x}\right) dx_1 \,, (21)$$

$$a_{20} = \frac{1}{n_2!} \int_{-\infty}^{\infty} \left(x_2^2 + A_0\right)^{n_2} e^{-\left(x_2^2 + A_0\right)} e^{-\frac{x_2^2}{2\sigma_x^2}} \cdot H_i\left(\frac{x_2}{\sigma_x}\right) dx_2 \,.(22)$$

The joint probability of quant number in two time moments for transmitting symbols (1,1) $P_{11}(n_1,n_2)$ is:

$$P_{11}(n_1,n_2) = \sum_{i=0}^{\infty} \frac{R^i}{i!} a_{11} a_{21} \,, \quad (23)$$

where:

$$a_{11} = \frac{1}{n_1!} \int_{-\infty}^{\infty} \left(x_1^2 + A_1\right)^{n_1} e^{-\left(x_1^2 + A_1\right)} e^{-\frac{x_1^2}{2\sigma_x^2}} \cdot H_i\left(\frac{x_1}{\sigma_x}\right) dx_1 \,, (24)$$

$$a_{21} = \frac{1}{n_2!} \int_{-\infty}^{\infty} \left(x_2^2 + A_1\right)^{n_2} e^{-\left(x_2^2 + A_1\right)} e^{-\frac{x_2^2}{2\sigma_x^2}} \cdot H_i\left(\frac{x_2}{\sigma_x}\right) dx_2 \,.(25)$$

The probability of correct detection $P(D_0 D_0/H_0 H_0)$ is:

$$P(D_0 D_0/H_0 H_0) = \sum_{n_1=0}^{n_T} \sum_{n_2=0}^{n_T} \cdot P_{00}(n_1,n_2) =$$

$$= \sum_{n_1=0}^{n_T} \sum_{n_2=0}^{n_T} \cdot \sum_{i=0}^{\infty} \frac{R^i}{i!} a_{10} a_{20} \,. \quad (26)$$

The probabilities of wrong detection $P(D_0D_1/H_0H_0)$, $P(D_1D_0/H_0H_0)$ and $P(D_1D_1/H_0H_0)$ are:

$$P(D_0D_1/H_0H_0)=\sum_{n_1=0}^{n_T}\sum_{n_2=n_T+1}^{\infty}\cdot P_{00}(n_1,n_2)=$$

$$=\sum_{n_1=0}^{n_T}\sum_{n_2=n_T+1}^{\infty}\cdot\sum_{i=0}^{\infty}\frac{R^i}{i!}a_{10}a_{20}\,, \qquad (27)$$

$$P(D_1D_0/H_0H_0)=\sum_{n_1=n_T+1}^{\infty}\sum_{n_2=0}^{n_T}\cdot P_{00}(n_1,n_2)=$$

$$=\sum_{n_1=n_T+1}^{\infty}\sum_{n_2=0}^{n_T}\cdot\sum_{i=0}^{\infty}\frac{R^i}{i!}a_{10}a_{20}\,, \qquad (28)$$

$$P(D_1D_1/H_0H_0)=\sum_{n_1=n_T+1}^{\infty}\sum_{n_2=n_T+1}^{\infty}\cdot P_{00}(n_1,n_2)=$$

$$=\sum_{n_1=n_T+1}^{\infty}\sum_{n_2=n_T+1}^{\infty}\cdot\sum_{i=0}^{\infty}\frac{R^i}{i!}a_{10}a_{20}\,. \qquad (29)$$

In a similar way we can obtain the probabilities of events for transmitting symbols: (0,1), (1,0) and (1,1).

The results can be applied in the design of the optical IM-DD systems.

4. Conclusion

The probability of error for an optical IM-DD system in the presence of quantum noise and Gaussian noise in the fiber is calculated in this paper. When more independent interferences (appearing by reflection) exist in the fiber then the interferometer noise can be approximated by Gaussian noise. In that way we determine the performances of the IM-DD systems in the presence of interferences. The quant number at quantum noise at the terminals of the photodiode, is independent in two non overlapping time intervals, but Gaussian noise is correlated at two digital interval distance. Because of that it is important to determine the probability of events of two symbol code word. We give the expressions for the probability of events: both symbols are correctly detected; one is correct, one is wrong and both are wrongly detected.

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