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DEFINITION, EVALUATION AND TASK-ORIENTED APPLICATION OF IMAGE SMOOTHNESS ESTIMATES

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Abstract Throughout the last decade development and implementation of digital data processing technologies was an area of increasing interest. Special and constant attention was paid, mainly, to the processing of graphical information, i.e. to the analysis, efficient encoding (compression) and rendering of digital images. Scientific inquiry into the area was stipulated by the necessity and strong desire to make great quantities of visual information more intelligible and more attractive for a widening round of users.

In the paper, the task-oriented use of image smoothness estimates is analysed. Firstly, the notion of smoothness of a generalized digital image is introduced; secondly, two highly valuable properties (invariance and continuity) of image smoothness estimates are formulated and proved; finally, a new objective computational procedure for the determination of image smoothness estimates is proposed. In parallels, some interesting digital image compression ideas (strategies), based on the direct application of image smoothness estimates, are described.

Key words: digital images, discrete transforms, image smoothness estimates, image compression.

1. Indroduction

Determination of smoothness parameters for the digitized real world images as well as development of rational computational procedures for finding of their numerical values (smoothness estimates) occupies a due place in the class of problems associated with digital image processing.

Those, who are closely bound up with digital images, have an intuitive sensation of the influence of image smoothness level to the results obtained from the application of one or another image processing technique. The higher smoothness class of an image, the better processing (filtering, encoding etc.) outcomes are derived.

On the other hand, everybody, who is immersed in solving a particular, say, digital image encoding (compression) problem, is well awared of difficulties, impediments and urgent goals that confront a researcher. Videlicet, enhancement of the quality of a restored image, with the compression ratio being fixed, and vice versa, settlement of the asymmetry problem consisting in disproportion of time expenditure associated with image encoding and image decoding processes, adaptation of some specialized image compression techniques and ideas to digital images of different dimensionality, and others.

To facilitate overcoming of the above difficulties and to achieve the planned ahead goals, the smoothness notion of an image (one of the main digital image characterizing parameters), to our mind, should undergo not only a well founded analysis and practical applicability ascertainment, but also should come into prompt usage to maximal extent.

The detailed interpretation of the smoothness level of a generalized digital image together with a new rational procedure for finding image smoothness estimates is presented below. At the same time, some interesting digital image compression techniques (ideas, strategies), based on the direct application of image smoothness estimates, are briefly brought to light in the paper.

2. Understanding of smoothness level of an image

Let $(S^d(n), \delta)$ stand for a finite metric space of generalized (d-dimensional) digital images, identified with d-dimensional data arrays such that:

$$S^d(n) = \{[X(m)] \mid m = (m_1, ..., m_d) \in I^d\}, \qquad (1)$$

$I = \{0, 1, ..., N-1\}$, $N = 2^n$, $n \in \mathrm{N}$, $d \in \{1, 2, 3\}$; $X(m) \in \{0, 1, ..., 2^p - 1\}$, for all $m \in I^d$; p ($p \geq 1$) indicates the number of bits, attached to encode the pixel values in $[X(m)]$, namely: $p = 1$, for binary (black and white) digital images; $p > 1$, for gray-scale digital images. The distance (metrics) δ between any two elements of the set $S^d(n)$ – the images $[X_1(m)]$ and $[X_2(m)]$ is specified by the formula

$$\delta = \delta(X_1, X_2) = \sqrt{\frac{1}{N^d} \sum_{m \in I^d} \left(X_2(m) - X_1(m)\right)^2} \, . \quad (2)$$

The latter expression comes into the usage every time when it is necessary to estimate the quality of a restored image against that of the original one, or to establish the (dis)similarity fact between any two images.

In general, the d-dimensional discrete spectrum (Fourier, Walsh-Hadamard, cosine etc., [1]) $[Y(k)]$, $k = (k_1, ..., k_d) \in I^d$, of $[X(m)] \in S^d(n)$ is defined as follows:

$$Y(k) = \frac{1}{N^d} \sum_{m \in I^d} X(m) \cdot \Phi(k, m), \quad (3)$$

for all $k \in I^d$; here $\{\Phi(k, m)\}$ is a finite system of d-dimensional discretized orthogonal functions (Fourier, Walsh, cosine etc.). In most cases, these functions are presented in terms of corresponding one-dimensional orthogonal functions, i.e. $\Phi(k, m) = \prod_{i=1}^{d} \Phi(k_i, m_i)$, $k, m \in I^d$; consequently, the multidimensional discrete transform (expression (3)) can be found applying consecutively (with respect to each spatial coordinate of the image) one-dimensional discrete transforms of the same type.

The initial image $[X(m)]$ can be restored uniquely using the inverse d-dimensional discrete transform:

$$X(m) = \sum_{k \in I^d} Y(k) \cdot \overline{\Phi(k, m)}, \quad (4)$$

$m = (m_1, \dots, m_d) \in I^d$; $\overline{\Phi(k, m)}$ is a complex conjugate of $\Phi(k, m)$.

The main property of the d-dimensional discrete spectrum $[Y(k)]$ is associated with Parseval's theorem for d-dimensional number series, and can be formulated this way – spectral coefficients $Y(k)$ ($k = (k_1, ..., k_d) \in I^d$) decrease in absolute value, as their serial numbers k (indices $k_1, ..., k_d$) increase, provided the base vectors of the discrete transform in use (expression (3)) are presented in a frequency order.

The latter circumstance implies (serves as the necessary precondition) that there exists a d-dimensional hyperbolic "surface"

$$z = z(x_1, ..., x_d) = C / (x_1 \cdot \ldots \cdot x_d)^\alpha \quad , \quad (5)$$

($C \geq 0,\ \alpha \geq 0$), which approximates the ordered array $\{|Y(k)| \,\big|\, k \in I^d,\ k_1^2 + ... + k_d^2 \neq 0\}$ in the mean-square error sense, i.e.

$$\sqrt{\frac{1}{N^d - 1} \sum_{\substack{k \in I^d \\ (k_1^2 + ... + k_d^2 \neq 0)}} \left(|Y(k)| - \frac{C}{\left(\overline{k_1} \cdot \ldots \cdot \overline{k_d}\right)^\alpha}\right)^2} \to \min;$$

here $\overline{k_i} = \max\{k_i, 1\}$, $i = 1, \ldots, d$.

The quantity α, characterizing the decrease ("decay") tendency of spectral coefficients, i.e. the shape of the hyperbolic "surface" $z = z(x_1, ..., x_d)$ (expression (5)), is assumed to be the smoothness parameter (level, class) of the (generalized) image $[X(m)] \in S^d(n)$.

A few interesting procedures (approaches) for the determination of image smoothness parameter values (smoothness estimates) as well as some exceptionally important their properties (from the standpoint of practical applicability) are presented in the sections below.

2.1. Evaluation of image smoothness estimates

As it was mentioned above, the image smoothness parameter α and the real coefficient C (expression (5)) both characterize the shape of the hyperbolic "surface" $z = z(x_1, ..., x_d)$, introduced to approximate the ordered set of spectral coefficients of the image $[X(m)] \in S^d(n)$.

We are interested in the numerical values of α and C. What "mathematics" should be applied to estimate them? Not going into details, we signify that many approaches are possible. To wit, the least squares method, successive coordinate optimization procedures, special techniques etc.

Evidently, the direct application of the least squares method is fairly problematic, since both the data array $\{|Y(k)| \,\big|\, k \in I^d,\ k_1^2 + ... + k_d^2 \neq 0\}$ and the sought-for approximating "surface" $z = z(x_1, ..., x_d)$ are nonlinear. The use of "linearization" procedures (logarihmization) is far from being effective either, because the most part of spectral coefficients, especially those corresponding to high frequencies, are equal to zero. Beyond doubt, the use can be made only of nonzero coefficients, and, sometimes, it serves the purpose – the very first approximation (estimate) α_0 of the image smoothness parameter α is obtained.

Let us denote the set of indices of nonzero spectral coefficients in the discrete spectrum $[Y(k)]$ of the image $[X(m)]$ by H, i.e.

$$H = \{k = (k_1, ..., k_d) \in I^d \,\big|\, Y(k) \neq 0,\ k_1^2 + ... + k_d^2 \neq 0\}.$$

Then, application of the "linearization" procedure mentioned above and the method of least squares lead to the following result:

$$\alpha_0 = \frac{1}{A_N} \cdot \sum_{k \in H} \left(B_N - s \cdot P(k)\right) \cdot \log\left|Y(k)\right| , (6)$$

where: $A_N = s \cdot C_N - B_N^2$; $B_N = \sum_{k \in H} P(k)$; $C_N = \sum_{k \in H} P^2(k)$; $P(k) = \log(\bar{k}_1 \cdot ... \cdot \bar{k}_d)$, for all $k \in H$; besides, $A_N = 0$ if and only if the set H is empty, i.e. the digital image $[X(m)] \in S^d(n)$ is absolutely smooth.

To make the estimate more precise, various optimizing techniques can be applied. One straightforward approach – an iterative procedure for two-dimensional images – is described in [2]. Preliminary experimental results show that the real world image smoothness parameter values (estimates), found using the said iterative approach, fall into the interval [0; 3) (Figure 1).

Image smoothness parameter values, obtained for the same digital image with the use of different discrete transforms (DHT and DCT), slightly differ – in the case of DCT, the said values are higher (by 0.05-0.15); besides, the time expenditure, associated with the application of DCT, in general, triples the time expenditure for WHT.

Finally, the iterative procedure (approach) itself has limitations to real-time applications, [2].

(a) (b) (c) (d)

Figure 1. Digital image smoothness analysis (with DCT in use): (a) image "Acura" 256x256, α = 1.44; (b) image "Lena" 256x256, α = 0.69; (c) image "Forest" 256x256, α = 0.37; (d) image "Dissolve" 256x256, α = 0.04

With this end in view, we have developed a new, more efficient, computational procedure (algorithm) for the determination of image smoothness estimates. The procedure employs both the coordinate optimization approach and the specially compiled constant fields. One version of the algorithm, oriented to process two-dimensional gray-scale images, is presented below.

A l g o r i t h m. Let $[X(m)] \in S^2(n)$ and $[Y(k)]$ be its discrete spectrum; also, let the set of indices of nonzero coefficients in $[Y(k)]$ be given by H.

1. $\alpha := 0$; $\delta := \delta_{\max}$.

2. Compute: $Y_\Sigma = \sum_{k \in H} \left|Y(k)\right|^2$.

3. Find:

$$Z_Y(\alpha) = \sum_{k \in H} \left|Y(k)\right| \cdot A(\alpha, k),$$

$$W_Y(\alpha) = Z_Y(\alpha) / A(\alpha) ;$$

$$\tau = \tau(\alpha) = Y_\Sigma - W_Y(\alpha) \cdot Z_Y(\alpha);$$

here: $A(\alpha, k) = 1/(\bar{k}_1 \cdot \bar{k}_2)^\alpha$; $A(\alpha) = \sum_{k \in H} A^2(\alpha, k)$; to improve the overall performance, constant fields for $A(\alpha, k)$ and $A(\alpha)$ ($\alpha \in [0; \alpha_{\max}]$, $k \in H$) are compiled beforehand.

4. If $\tau < \delta$, then $\delta := \tau$; otherwise, pass to 6.

5. If $\alpha < \alpha_{\max}$, then $\alpha := \alpha + h$ (h is a step size; $h \in (0; 0.1)$), and pass to 3.

6. The end. The image smoothness estimate α is obtained.

The proposed algorithm is fast enough, and can be used in real time applications.

Finally, we observe that if one is interested only in the difference between the smoothness classes of any two digital images (or, any two fragments of the same image), then the very first image smoothness level approximations (expression (6)) can be put into action.

2.2. The main properties of image smoothness estimates

Consider an image $[X(m)] \in S^d(n)$, whose smoothness level is characterized by the parameter (estimate) α . We are going to formulate and prove some exceptionally important properties of the smoothness estimate α , namely:

1) Invariance of α with respect to the isometric transformations (rotation, reflection, inversion, luminance change), acting upon the image $[X(m)]$;

2) Continuity of $\alpha : S^d(n) \to \mathrm{R}$, grasped in the way that small (discrete) changes in $[X(m)]$ correspond to small (discrete) changes in α.

First of all, let us characterize the above isometric transformations – rotation, reflection, inversion and the image luminance change.

Rotation of the image is considered to be an action (transformation) with an outer outcome such that the mutual position of spatial coordinate axes of the image is left unchanged. There are 3 different ways to perform rotation in a two-dimensional image space and 23 different ways – in a three-dimensional space; besides, there are no rotations in a one-dimensional image space.

Reflection transformation is identified with an action, when the mutual position of the image spatial coordinate axes is changed. There are 3 different reflections in a three-dimensional image space and only 1 reflection in image spaces of lower dimensionality.

In both cases (rotation, reflection), the relationship between the initial image $[X(m)] \in S^d(n)$ ($d \in \{1, 2, 3\}$) and the transformed one $[\hat{X}(m)] \in S^d(n)$ can be established this way: $\hat{X}(m) = \hat{X}(m_1, ..., m_d) = X(\hat{m})$, where the index $\hat{m} = (\hat{m}_1, ..., \hat{m}_d)$, depending on a particular action, equals either $\hat{m}_s = N - m_t - 1$ or $\hat{m}_s = m_t$, for all $s = 1, ..., d$; here $t \in \{1, ..., d\}$. For instance, in the case of a counter-clockwise rotation (by the angle of 90°) of the image $[X(m)] \in S^2(n)$, the newly obtained (transformed) image $[\hat{X}(m)] \in S^2(n)$ satisfies relationships: $\hat{X}(m_1, m_2) = X(\hat{m}_1, \hat{m}_2)$, where $\hat{m}_1 = N - m_2 - 1$, $\hat{m}_2 = m_1$, for all $(m_1, m_2) \in I^2$. At the same time, reflection of $[X(m)] \in S^2(n)$ (with respect to the principal diagonal of the image) gives: $\hat{m}_1 = m_2$, $\hat{m}_2 = m_1$, for all $(m_1, m_2) \in I^2$.

Inversion of $[X(m)] \in S^d(n)$ is understood to be an action, which produces a new image $\hat{X}(m)$ such that $\hat{X}(m) = 2^p - X(m) - 1$, for all $m \in I^d$; here p stands for the number of bits attached to encode pixel values in the image. Finally, the luminance change is realized by multiplying all the elements (pixel values) of $[X(m)]$ by a scalar λ $(\lambda > 0)$, so that the transformed image $[\hat{X}(m)] = [\lambda X(m)] \in S^d(n)$.

The earlier mentioned invariance property of the image smoothness parameter value (estimate) α is proved referring to the following facts:

1) The discrete image spectrum $[Y(k)]$ approximating hyperbolic "surface" (expression (5); Section 2) is symmetric with respect to spatial coordinates of the image, i.e. the image smoothness value (estimate) α doesn't change, if the discrete spectral coefficients, falling into the set $\{Y(k) \,|\, Y(k) = Y(k_1, ..., k_d),\ \bar{k}_1 \cdot ... \cdot \bar{k}_d = const\}$ and satisfying condition $|Y(k_1, ..., k_d)| = |Y(k_{i_1}, ..., k_{i_d})|$ (for all possible permutations $\{i_1, ..., i_d\}$ of $\{1, ..., d\}$) are interchanged;

2) The base vectors (expression (3); Section 2) Φ_i ($i \in \{1, ..., d\}$) of the discrete transform in use satisfy the following condition:

$$\Phi_i(k_i, m_i) = (-1)^{k_i} \Phi_i(k_i, N - m_i - 1),$$

for all $m_i = 0, 1, ..., N/2 - 1$; $k_i \in \{0, 1, ..., N-1\}$.

We note, for instance, that the discrete cosine transform (DCT) as well as the discrete Walsh-Hadamard transform (WHT) possesses the indicated property; by the way, any "wavelet" type discrete transform is an exception.

Based on this understanding, the following results are derived (we shall confine ourselves with a two-dimensional image $[X(m)] \in S^2(n)$):

– in the case of counter-clockwise rotation of $[X(m)]$ by the angle of 90°, we get

$$\hat{Y}(k_1, k_2) = \\ = \frac{1}{N^2} \sum_{(m_1, m_2) \in I^2} \hat{X}(m_1, m_2) \Phi_1(k_1, m_1) \Phi_2(k_2, m_2) = \\ = \frac{(-1)^{k_2}}{N^2} \sum_{(m_1, m_2) \in I^2} X(N - m_2 - 1, m_1) \Phi_1(k_1, m_1) \cdot \\ \cdot \Phi_2(k_2, N - m_2 - 1) = (-1)^{k_2} Y(k_2, k_1),$$

for all $(k_1, k_2) \in I^2$; thus, the image smoothness parameter value (estimate) α is left unchanged, because $|\hat{Y}(k_1, k_2)| = |Y(k_2, k_1)|$ and $\bar{k}_1 \cdot \bar{k}_2 = \bar{k}_2 \cdot \bar{k}_1$, for all $(k_1, k_2) \in I^2$;

– in the case of reflection (with respect to the principal diagonal of the image), we get

$$\hat{Y}(k_1, k_2) = \\ = \frac{1}{N^2} \sum_{(m_1, m_2) \in I^2} \hat{X}(m_1, m_2) \Phi_1(k_1, m_1) \Phi_2(k_2, m_2) = \\ = \frac{1}{N^2} \sum_{(m_1, m_2) \in I^2} X(m_2, m_1) \Phi_1(k_1, m_1) \cdot \Phi_2(k_2, m_2) = \\ = Y(k_2, k_1),$$

for all $(k_1, k_2) \in I^2$;

– in the case of inversion,

$$\hat{Y}(k_1,k_2)=$$
$$=\frac{1}{N^2}\sum_{(m_1,m_2)\in I^2}\hat{X}(m_1,m_2)\Phi_1(k_1,m_1)\Phi_2(k_2,m_2)=$$
$$=\frac{1}{N^2}\sum_{(m_1,m_2)\in I^2}(2^p-X(m_1,m_2)-1)\Phi_1(k_1,m_1)\cdot$$
$$\cdot\Phi_2(k_2,m_2)=\frac{2^p-1}{N^2}\sum_{(m_1,m_2)\in I^2}\Phi_1(k_1,m_1)\cdot$$
$$\cdot\Phi_2(k_2,m_2)-\frac{1}{N^2}\sum_{(m_1,m_2)\in I^2}X(m_1,m_2)\Phi_1(k_1,m_1)\cdot$$
$$\cdot\Phi_2(k_2,m_2)=$$
$$=\begin{cases}2^p-1-Y(0,0), & k_1=k_2=0,\\ -Y(k_1,k_2), & (k_1,k_2)\in I^2,\ k_1^2+k_2^2\neq 0,\end{cases}$$

i.e. $\left|\hat{Y}(k_1,k_2)\right|=\left|Y(k_1,\ k_2)\right|$, for all $(k_1,k_2)\in I^2$, $k_1^2+k_2^2\neq 0$;

– after the luminance of the image $[X(m)]$ is changed, we obtain

$$\hat{Y}(k_1,k_2)=$$
$$=\frac{1}{N^2}\sum_{(m_1,m_2)\in I^2}\hat{X}(m_1,m_2)\Phi_1(k_1,m_1)\Phi_2(k_2,m_2)=$$
$$=\frac{\lambda}{N^2}\sum_{(m_1,m_2)\in I^2}X(m_1,m_2)\Phi_1(k_1,m_1)\cdot\Phi_2(k_2,m_2)=$$
$$=\lambda\cdot Y(k_1,k_2),$$

for all $(k_1,k_2)\in I^2$; now, it sufficies to take in that the simultaneous multiplication of all spectral coefficients (in $[Y(k)]$) by the same scalar λ doesn't influence the final result – the image smoothness parameter value (estimate) α (Section 2.1).

Now, we are going to show that the image smoothness estimate α, being a function defined on the space of digital images $S^d(n)$, continuously depends on $[X(m)]\in S^d(n)$, i.e. small changes in $[X(m)]$ correspond to small changes in α. Really, let us assign an increment $[\Delta X(m)]$ to the image $[X(m)]$, so that $[\widetilde{X}(m)]=[X(m)+\Delta X(m)]$ is left in $S^d(n)$.

Suppose,

$$\delta(X,\widetilde{X})=\sqrt{\frac{1}{N^d}\sum_{m\in I^d}(\Delta X(m))^2}\leq\varepsilon_0$$

(ε_0 is a small positive number).

If we denote discrete spectra of $[X(m)]$, $[\Delta X(m)]$ and $[\widetilde{X}(m)]$ by $[\Delta Y(k)]$ and $[\widetilde{Y}(k)]$, respectively, we'll obtain

$$\left|\Delta Y(k)\right|=\left|\widetilde{Y}(k)-Y(k)\right|=$$
$$=\left|\frac{1}{N^d}\sum_{m\in I^d}\Delta X(m)\Phi(k,m)\right|\leq$$
$$\leq\frac{1}{N^d}\sum_{m\in I^d}\left|\Delta X(m)\right|\cdot\left|\Phi(k,m)\right|\leq t_N\cdot\varepsilon_0;$$

here $t_N=\max_{k,m\in I^d}\{\left|\Phi(k,m)\right|\}$ is a constant, for a fixed value of N and for a particular discrete transform (for instance, in the case of WHT, $t_N=1$).

Thus, small changes in the image correspond to small changes in its discrete spectrum.

The main factor, having influence on the image smoothness parameter value α (Algorithm; Section 2.1) is an auxiliary variable $\tau=\tau(\alpha)$.

Let us estimate the increment $\Delta\tau$, corresponding to the increment $[\Delta Y(k)]$ (we confine ourselves with the case $d=2$):

$$\left|\Delta\tau\right|=\left|\Delta Y_\Sigma-Z^2_{\Delta Y}(\alpha)/A(\alpha)\right|=$$
$$=\left|\sum_{k\in H}\left|\Delta Y(k)\right|^2-\left(\sum_{k\in H}\left|\Delta Y(k)\right|\cdot A(\alpha,k)\right)^2\Big/A(\alpha)\right|\leq$$
$$\leq\left|\sum_{k\in H}\left|\Delta Y(k)\right|^2-\sum_{k\in H}\left|\Delta Y(k)\right|^2\cdot A^2(\alpha,k)\Big/A(\alpha)\right|=$$
$$=\sum_{k\in H}\left|\Delta Y(k)\right|^2\left(1-A^2(\alpha,k)/A(\alpha)\right)\leq\left(N^2-2\right)\cdot t_N^2\varepsilon_0^2$$

(here $H=\{k\,|\,k=(k_1,k_2)\in I^2,\ k_1^2+k_2^2\neq 0\}$), i.e. small changes in $[Y(k)]$ are associated with small changes in the value of τ.

Thus, continuous dependence of τ on $[\Delta Y(k)]$ as well as finiteness of the number of steps in the evaluation procedure (Algorithm; Section 2.1) implies that the smoothness parameter value (estimate) α continuously depends on $[\Delta X(m)]$, i.e.

$$(\delta(X,\widetilde{X})\leq\varepsilon_0)\Rightarrow(\left|\alpha_X-\alpha_{\widetilde{X}}\right|\leq\mu_0);$$

here α_X and $\alpha_{\widetilde{X}}$ indicate smoothness classes of $[X(m)]$ and $[\widetilde{X}(m)]$, respectively; by the way, the relationship $\varepsilon_0\leftrightarrow\mu_0$ can be established only experimentally.

3. Task-oriented application of image smoothness estimates

The intuitively perceptible notion of smoothness of an image, expressed in terms of image smoothness estimates (Section 2), appears to be an interesting and perspective means in solving diversified digital image processing problems. Some of them are presented briefly below.

3.1. Adaptive encoding of two-dimensional images

The maiden successful implementation of image smoothness estimates came to light with the recently developed strategy for the efficient encoding (compression) of two-dimensional gray-scale images, [3]. The structural (block) scheme of the developed adaptive image encoding strategy is presented in Figure 2. Smoothness estimates of the image under processing were (for the first time) straightforwardly and internally bound up with the image encoding process itself.

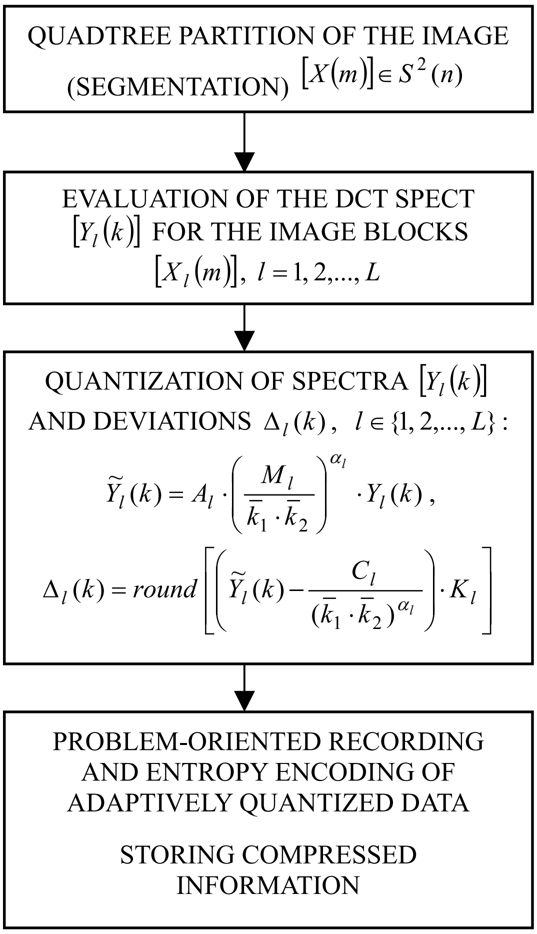

Figure 2. Block-scheme of the developed adaptive image encoding strategy

As it can be seen (Figure 2), the smoothness parameter values (estimates) α_l ($l = 1, 2, ..., L$) are employed twice. Firstly, they are used to quantize DCT coefficients of the image blocks (quadtree elements) $[X_l(m)]$. Secondly, the discrete spectrum $[Y_l(k)]$ approximating hyperbolic surface $z = z_l(x_1, x_2) = C_l / (x_1 \cdot x_2)^{\alpha_l}$, characterized by the value α_l, is applied to record efficiently a nonzero deviation $\left| \widetilde{Y}_l(k) - C_l / (\bar{k}_1 \cdot \bar{k}_2)^{\alpha_l} \right|$; by the way, quantization parameters A_l, M_l and K_l ($l = 1, 2, ..., L$) are introduced to maximize the overall image compression effect, under a priori prescribed condition $\delta(X, \widetilde{X}) \le \varepsilon_0$.

Undeniable advantage of the proposed adaptive image encoding strategy – restored images distinguish themselves by “soft” texture and absence of Gibb’s phenomenon, resulting from attempting to approximate square wave by a trigonometric polynomial (so peculiar to JPEG standard). To say more, the strategy appeared to be exclusively efficient at higher image compression ratios, [3].

The detailed description of the developed approach and a number of comparative experimental analysis results are presented in [4].

3.2. Achieving fractal image compression speed gains via image smoothness estimates

The block based fractal image encoding idea (in its simplest form – Jacquin’s approach, [5]) can be described this way: the image under processing $[X(m)] \in S^2(n)$ is partitioned at two scales (one twice the other), i.e. into the so-called range blocks $[U(m)] \in S_1^2(3) \subset S^2(3)$ and domain blocks $[V(m)] \in S_1^2(4) \subset S^2(4)$. The former (range) blocks are non-overlapping and contain every pixel. The latter ones (domain blocks) may overlap and not necessarily contain every pixel. The essence of the approach is the pairing of each range block $[U(m)]$ to a domain block $[V(m)]$ such that $\delta = \delta(U, V)$ is minimal. To improve performance, additional transformations (rotation, reflection, luminance change) are applied to the image (block) $[V(m)]$. Evidently, the computation required is enormous. An obvious way to achieve compression speed gains is to limit the search region of domain blocks for the current range block.

We have proposed an idea which explores the necessary image (block) similarity condition, [6]. The latter condition follows directly from the continuity property of image smoothness estimates (Section 2.2), namely:

$$(\delta(U,V) \le \varepsilon_0) \Rightarrow (|\alpha_U - \alpha_V| \le \mu_0);$$

here α_U and α_V signify smoothness parameter values (estimates) for images (blocks) $[U(m)]$ and $[V(m)]$, respectively; ε_0 is a priori fixed small positive number, ensuring similarity of $[U(m)]$ and $[V(m)]$. In other words, two images can not be similar if their smoothness classes differ significantly ($|\alpha_U - \alpha_V| > \mu_0$).

In addition, the invariance property of image smoothness estimates makes the above necessary condition adaptable with the earlier introduced isometric transformations, acting upon the image $[V(m)]$ (Section 2.2).

Compression time savings are achieved, mainly, owing to the following two factors: firstly, for the determination of the level of smoothness each domain block $[V(m)]$ as well as each range block $[U(m)]$ is looked over only once; secondly, candidate domain blocks (forming a "pool") and a related range block, roughly speaking, fall into the same class of smoothness. Thus, to ensure optimal pairing ($\delta = \delta(U,V) \to \min$), it is quite sufficient to analyse only those pairs "range block $U \leftrightarrow$ domain block V", for which $|\alpha_U - \alpha_V| \leq \mu_0$ (practically, $\mu_0 < 0.15$).

Theoretical investigations and preliminary experimental analysis results confirm vitality and usefulness of the proposed image compression time accelerating approach, [6, 12]. In particular, for the image "Lena" 256x256 (Fig. 1, (b)) the following image compression speed gains have been obtained (Table 1; $|\alpha_U - \alpha_V| \leq \mu_0$).

Table 1. (Fractal compression time savings)

μ_0	Compression time (sec)	The error (δ)
Jacquin's approach	88.31	7.48
0.2	27.76	7.94
0.15	21.98	8.14
0.025	6.87	9.51
0.001	3.87	13.2

Also, we have found out that application of the cosine discrete transform (DCT), for the evaluation of image (block) smoothness estimates, was more preferable (from the standpoint of image compression time savings) than the discrete Walsh-Hadamard transform (WHT) (Fig. 3; image "Lena" 256x256 analysis results).

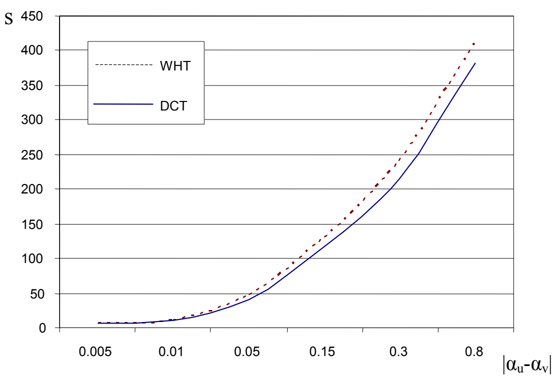

Figure 3. Fractal image compression speed gains (DCT and WHT)

3.3. Problem-oriented change of image dimensionality

One of the most general leading principles, which are at the helm in drawing up digital image encoding and analysis strategies, says – the digital image processing should always be performed in the task-oriented image space, which either gives optimum to the objective function (final result) or facilitates the most rational use of a particular specialized image processing algorithm, acting in the chosen image space.

The above principle needs to be explained in more detail. Suppose, a two-dimensional digital image $[X_2(m)] \in S^2(n_2)$ allows representations in image spaces $S^1(n_1)$ and $S^3(n_3)$ too, i.e. dimensions of the image $[X_2(m)]$ are such that $1 \cdot n_1 = 2 \cdot n_2 = 3 \cdot n_3$. To generate one-dimensional and three-dimensional analogues (images $[X_1(m)] \in S^1(n_1)$ and $[X_3(m)] \in S^3(n_3)$ of $[X_2(m)]$, one or another image scanning trajectory should be applied. What is the efficiency of the application of a particular scanning trajectory (curve)? Special criteria are needed. In general, those criteria may be very specific. But, if digital image processing is linked with its efficient encoding (compression), then the only criterion – preservation of maximal smoothness of the image. So, in the case of necessity, choose a trajectory (scan line ordering, Hilbert curves, Peano trajectories etc.) that gives maximum to the image smoothness parameter value in a newly chosen space.

Some interesting developments (in this field) are presented in [10, 11]. In the publication [10], special criteria, based on the image smoothness analysis results, have been introduced to pick up an optimal space for hyperbolic image filtering. In particular, it has been shown that hyperbolic filtering (image compression ratio being fixed) of the image $[X(m)]$ in the space $S^{d_1}(n_{d_1})$ is more efficient than in $S^{d_2}(n_{d_2})$, provided $\alpha_{d_1} / \alpha_{d_2} < \log M_{d_2} / \log M_{d_1}$; here: α_{d_1} and α_{d_2} signify smoothness estimates of the image analogues in spaces $S^{d_1}(n_{d_1})$ and $S^{d_2}(n_{d_2})$, respectively; M_{d_1} and M_{d_2} stand for the filtering levels in the latter spaces; $d_1, d_2 \in \{1, 2, 3\}$.

No doubt, the use of image smoothness estimates in task-oriented image dimensionality change procedures forms a new platform for the development and successful further analysis of mathematical digital image processing (encoding, filtering etc.) techniques, [12].

4. Conclusion

The notion of smoothness of a generalized digital image is presented in the paper. Smoothness of the image is understood to be a real nonnegative number, which characterizes the manifestation of high frequency harmonics in the image. Two highly interesting and valuable properties of image smoothness estimates are stated and proved, namely: invariance with respect to the isometric transformations (rotation, reflection etc.), acting upon the image, and continuity of the response to small changes in the image. These two properties form a theoretical basis for the direct and successful application of image smoothness estimates to the development of new digital image processing (encoding, filtering, synthesizing etc.) technologies.

In addition to this, a new algorithm (procedure) for the determination of image smoothness estimates is proposed. The algorithm employs the coordinate optimization approach and can be used in real-time applications.

Some areas of practical applicability of image smoothness estimates, associated with our recent developments, are elucidated in the paper. Among the latter – adaptive encoding of two-dimensional gray-scale images, achievement of fractal image compression speed gains, problem-oriented change of image dimensionality.

We are going to focus our future research on the completion of the following problems: fractal image encoding (the new strategy), efficiency analysis of hyperbolic image filtering in spaces of different dimensionality.

References

[1] **N. Ahmed, K.R. Rao**. Orthogonal Transforms for Digital Signal Processing. *Springer-Verlag, Berlin, Heidelberg – New York,* 1975.

[2] **J. Valantinas, N. Morkevičius**. Smoothness Analysis of Two-Dimensional Gray-Level Images. *Information Technology and Control, No* 1(14), *Technologija, Kaunas*, 2000, 15 – 24.

[3] **J. Valantinas, N. Morkevičius.** On the application of image smoothness estimates to adaptive coding of gray-level images. *Information Technology and Control, No*.1(22), *Technologija, Kaunas*, 2002, 13–21.

[4] **N. Morkevičius**. Development and analysis of multidimensional image modelling and efficient coding methods. *Doctoral Thesis (Assoc. prof. J.Valantinas). Kaunas University of Technology*, 2002.

[5] **A. Jacquin**. Fractal Image Coding: A review. *Proceedings of the IEEE, Vol*.81, *No*.10, 1993, 1451-1465.

[6] **J. Valantinas, N. Morkevičius, T. Žumbakis**. Accelerating Compression Times in Block-Based Fractal Image Coding Procedures. *Proceedings of the* 20th *Eurographics UK Conference. Leicester (De Montfort university, UK): IEEE Computer Society Press (Los Alamitos, California),* 2002, 83-88.

[7] **M.J. Turner, J.M. Blackledge, P.R. Andrews.** Fractal Geometry in Digital Imaging. *Academic Press, Cambridge*, 1998.

[8] **Y. Fisher.** Fractal Image Compression – Theory and Application. *Springer-Verlag, New York*, 1994.

[9] **J. Valantinas**. A new approach to hyperbolic filtering of gray-level images. *Information Technology and Control, No*.1(7), *Technologija, Kaunas*, 1998, 35–42.

[10] **J. Valantinas**. Hyperbolic image filtering under the influence of image dimensionality. *Information Technology and Control, No*.3(24), *Technologija, Kaunas*, 2002, 7-13.

[11] **J. Valantinas, R. Valantinas.** Problem-Oriented Change of Image Dimensionality. *Proceedings of the Third International Symposium on Image and Signal Processing and Analysis, Rome (Italy), Universita degli Studi ROMA TRE*, 2003, 228-232.

[12] **J. Valantinas.** Development of the basic principles and mathematical techniques for the objectively-oriented encoding and analysis of digital images. *Information Technology and Control, No*.2(27), *Technologija, Kaunas*, 2003, 21–33.