ISSN 1392 – 124X INFORMATION TECHNOLOGY AND CONTROL, 2007, Vol.36, No.2

ALGORITHMS FOR LPTV SYSTEM PARAMETERS ESTIMATION

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Abstract. A nonrecursive and recursive algorithms are proposed for estimation of the linear periodically time-varying (LPTV) system parameters. Using a block approach, the parameter estimation problem is transformed into one-dimensional minimization problem for L set parameters of the LPTV system. The algorithm can be performed in parallel at L time moments for each set of parameters to reduce the computation time.

1. Introduction

The linear periodically time-varying (LPTV) systems are a generalization of the linear time-invariant (LTI) systems and are used in control systems, signal processing and digital communications [1-3]. There are many ways to represent LPTV systems: common ones include difference equations or state-space models with periodically time-varying coefficients, equivalent linear time-invariant models obtained by the blocking technique [4-6]. The problem of estimating parameters from noisy observed data has a long history in engineering and experimental science [7-9]. The autoregressive moving average (ARMA) model is one of the most effective and practical. The main reason for the popularity of ARMA modeling is that it is possible to obtain good estimates of the unknown parameters by solving a simultaneous set of linear equations. In this paper, the problem of estimation of LPTV system parameters is considered. When system parameters vary rapidly with time, least squares algorithms are not capable of tracking the changes satisfactory. The paper describes nonrecursive and recursive parallel algorithms for fast LPTV system parameters estimation based on the blocking technique.

2. Estimation problem

A LPTV system with input sequence $x(k)$ and output sequence $u(k)$, k = 0,1,…, is described by the equation

$$u(k)=\sum_{i=0}^{N}\beta_i(k)x(k-i)-\sum_{j=1}^{M}\alpha_j(k)u(k-j) \cdot \quad (1)$$

The output $y(k)$ is assumed to be contaminated with the noise $n(k)$

$$y(k) = u(k)+n(k).$$

Parameters of LPTV system satisty: $\alpha_j(k)=\alpha_j(k+L)$, $\beta_i(k)=\beta_i(k+L)$ for all k = 0, 1, …; L is periodicity, L = 2, 3, …; $N \le M$. A noise $n(k)$ is nonmeasurable, normally distributed, statistically independent with $E\{n(k)\}=0$, $E\{n(k)n(k+\tau)\}=\sigma_n^2\delta(\tau)$, where $E\{n(k)\}$ is a mean value, σ_n^2 is the variance and $\delta(\tau)$ is the Kronecker delta function. The objective of parameter estimation is to estimate the LPTV system parameters in the equation (1) based on the measured sequences $x(k)$ and $y(k)$. It is assumed that the M and N are known.

3. Parallel algorithm

Consider that the input sequence $x(k)$ and the output sequence $y(k)$ of the LPTV system are measured for $k = 0,1,\dots,T$. The model of the LPTV system is described by equation

$$y(k)+\sum_{j=1}^{M}a_j(k)y(k-j)-\sum_{i=0}^{N}b_i(k)x(k-i)=e(k) \;,$$

where $a_j(k) = a_j(k+L)$ and $b_i(k) = b_i(k+L)$ – parameters of the model. The equation error $e(k)$ arises from the noise contaminated output $y(k)$ and from erroneous parameter estimates.

Replacing k by $l+mL$ and blocking the input sequence $x(k)$ and the output sequence $y(k)$, we obtain the equation of the model

$$y(l+mL)=\sum_{i=0}^{N}b_i(l)x(l+mL-i)-\sum_{j=1}^{M}a_j(l)y(l+mL-j)+e(l+mL), \quad (2)$$

where $l = M, \dots, M+L-1$; $m = 0, 1, \dots, p-1$; $p=fix((T-M+1)/L)$ – rounds the value of $(T-M+1)/L$ to the nearest integer towards zero.

The system of linear equations (2) is expressed in a matrix form:

$$\bar{y}(l) = A(l)\ \theta(l) + \bar{e}(l),\ l = M,...,M+L-1, \tag{3}$$

where

$$\bar{y}(l) = [y(l),..., y(l+mL),..., y(l+(p-1)L)]^T, \tag{4}$$

$$\theta(l) = [b_0(l),..., b_i(l),..., b_N(l), a_1(l),..., a_j(l),..., a_M(l)]^T, \tag{5}$$

$$\bar{e}(l) = [e(l),..., e(l+mL),..., e(l+(p-1)L)]^T$$

$$A(l) = \begin{bmatrix} x(l),..., x(l-N), -y(l-1),..., -y(l-M) \\ . \ . \ . \\ x(l+mL),..., x(l+mL-N), \\ -y(l+mL-1),..., -y(l+mL-M) \\ . \ . \ . \\ x(l+(p-1)L),..., x(l+(p-1)L-N), \\ -y(l+(p-1)L-1),..., -y(l+(p-1)L-M) \end{bmatrix}, \tag{6}$$

Define $x_l(m) = x(l+mL)$, $y_l(m) = y(l+mL)$, and $e_l(m) = e(l+mL)$, (m = 0, 1, …; l = M, …, M+L–1). Then errors of the parallel model (2) are as follows

$$e_l(m) = y_l(m) + \sum_{j=1}^{M} a_j(l) y_l(m-j) - \sum_{i=0}^{N} b_i(l) x_l(m-i) \ . \tag{7}$$

Minimization of the loss functions

$$V_l = \min_{\theta(l)} \bar{e}^T(l)\bar{e}(l) = \min_{\theta(l)} \sum_{m=0}^{p-1} e^2(l+mL) \ ,$$

$$l = M, \ldots, M+L-1, \tag{8}$$

gives the nonrecursive optimal parameter estimates

$$\hat{\theta}(l) = (A^T(l)A(l))^{-1} A^T(l)\bar{y}(l),\ l = M, \ldots, M+L-1, \tag{9}$$

where

$$\hat{\theta}(l) = [\hat{b}_0(l),..., \hat{b}_i(l),..., \hat{b}_N(l), \hat{a}_1(l),..., \hat{a}_j(l),..., \hat{a}_M(l)]^T.$$

The parameter estimation problem has been transformed into one-dimensional minimization problem (8) for L set parameters of the LPTV system.

Writing the nonrecursive estimation equations (9) for $\hat{\theta}(m+1,l)$ and $\hat{\theta}(m,l)$, (m = 0, 1, …; l = M, …, M+L–1) and subtracting one from the other, results in the recursive parallel parameter estimation algorithm:

$$\hat{\theta}(m+1,l) = \hat{\theta}(m,l) + \gamma(m,l)[y(m+1,l) - A^T(m+1,l)\hat{\theta}(m,l)]),$$

$$\gamma(m,l) = \frac{P(m,l)A(m+1,l)}{A^T(m+1,l)P(m,l)A(m+1,l)+1},$$

$$P(m+1,l) = [I - \gamma(m,l)A^T(m+1,l)]P(m,l).$$

To start the recursive algorithm, one sets some initial values of the vectors $\hat{\theta}(0,l)$ and matrices $P(0,l)$ [8].

Using the time-shift operator z^{-1} such that $z^{-1}y(k) = y(k-1)$, the parallel model (2) is described in the form

$$e_l(m) = A_l(z^{-1})y_l(m) - B_l(z^{-1})x_l(m) \tag{11}$$

where

$$A_l(z^{-1}) = 1 + a_1(l)z^{-1} + ... + a_M(l)z^{-M},$$

$$B_l(z^{-1}) = b_0(l) + b_1(l)z^{-1} + ... + b_N(l)z^{-N}.$$

The input and the output distributors reduce the sampling rate of the input sequence $x(k)$ and of the output sequence $y(k)$. The sampling rate reduction is achieved by forming the subsequences $x_{M+i}(m)$ and $y_{M+i}(m)$, i = 0, 1, …, L–1 and by saving every Lth sample of the sequences $x(k)$ and $y(k)$. The sampling rate reduction is equal to the periodicity L of the LPTV system. The subsequencies $x_{M+i}(m)$ and $y_{M+i}(m)$ are used for the parameter estimation. We begin the parameter estimation at the time moment M. The estimation procedure can be accomplished in a parallel manner for l = M, …, M+L–1. The parameters are estimated independently at time moments M, …, M+L–1. In case the LPTV filter periodicity L is unknown, we can repeat the parameter estimation process with different periodicity values L^* = 2, 3, and so on. If $L^* \neq L$, we obtain large values of loss functions (8) and bad parameter estimates. By repeating the estimation process with different values of the periodicity L^*, calculating loss functions (8), and comparing these values, we can estimate true value of the LPTV system periodicity L.

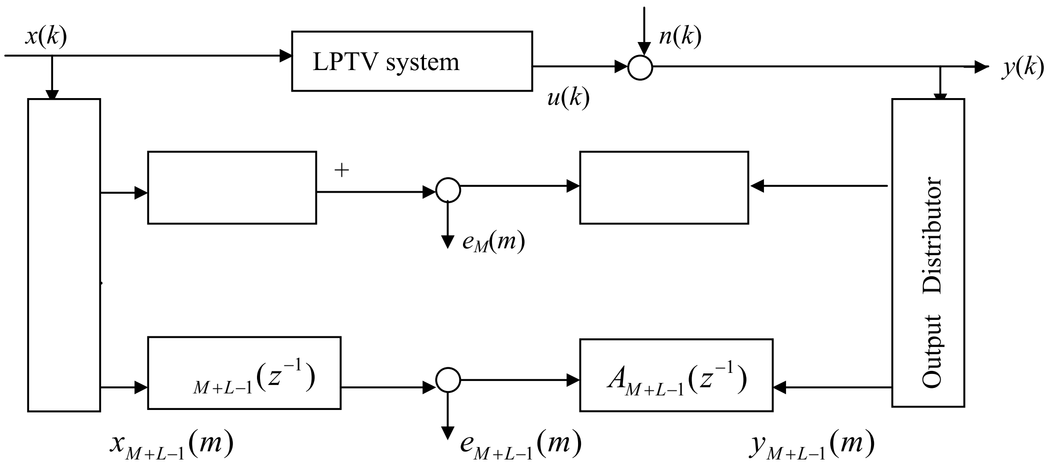

Figure 1. Parallel structure for estimation of the LPTV system parameters

4. Experimental results

Assume that the LPTV system with periodicity $L = 3$ is described by the equation

$$y(k) = \sum_{i=0}^{2} \beta_i(k) x(k-i) - \sum_{j=1}^{2} \alpha_j(k) y(k-j) + \lambda n(k), \qquad (12)$$

where $\beta_0(0) = 0.9$, $\beta_0(1) = 1.1$, $\beta_0(2) = 1.3$, $\beta_1(0) = 1.9$, $\beta_1(1) = 2.1$, $\beta_1(2) = 2.3$, $\beta_2(0) = 2.9$, $\beta_2(1) = 3.1$, $\beta_2(2) = 3.3$, $\alpha_1(0) = -1.3734$, $\alpha_1(1) = -1.6514$, $\alpha_1(2) = -1.1731$, $\alpha_2(0) = 0.4871$, $\alpha_2(1) = 0.6933$, $\alpha_2(2) = 0.1364$.

The noise $n(k)$ is generated as a zero-mean, unit-variance white noise sequence with a Gaussian distribution. Coefficient λ determines the intensity of additive noise $n(k)$. The performance of the estimated model is summarized in Table 1, in which signal to noise ratio SNR $= 20\log_{10}\frac{\sigma_u}{\sigma_n}$, where σ_u – standard deviation (std) of the LPTV system output $u(k)$, σ_n – standard deviation of the noise $n(k)$. In Table 1, α and β are true parameters of the LPTV system and $\hat{a}$ and $\hat{b}$ are averaged estimated parameters of the model. For calculation of averaged estimated parameters, we repeated estimation procedure 100 times, obtained 100 sets of estimated values, then calculated mean and standard deviation of the estimated values for SNR = 42, 22, and 12 dB.

Table 1. LPTV system (12) parameter estimates via SNR

True parameters	$\beta_0(0)$=0.9	$\beta_1(0)$=1.9	$\beta_2(0)$=2.9	$\alpha_1(0)$=−1.3734	$\alpha_2(0)$= 0.4871	SNR(dB)
Estimates	$\hat{b}_0(0)$	$\hat{b}_1(0)$	$\hat{b}_2(0)$	$\hat{a}_1(0)$	$\hat{a}_2(0)$	
mean std	0.9 0.0121	1.9013 0.0118	2.8991 0.0108	−1.3706 0.0023	0.4863 0.0026	42
mean std	0.8954 0.1016	1.9063 0.1103	3.0164 0.1215	−1.3221 0.0237	0.4305 0.0205	22
mean std	0.9301 0.3101	2.313 0.3311	3.5617 0.3689	−0.9621 0.1505	−0.1421 0.0451	12
True parameters	$\beta_0(1)$=1.1	$\beta_1(1)$=2.1	$\beta_2(1)$=3.1	$\alpha_1(1)$=−1.6514	$\alpha_2(1)$=0.6933	
Estimates	$\hat{b}_0(1)$	$\hat{b}_1(1)$	$\hat{b}_2(1)$	$\hat{a}_1(1)$	$\hat{a}_2(1)$	
mean std	1.0893 0.0131	2.1020 0.0134	3.1046 0.0152	−1.6483 0.0032	0.6912 0.0032	42
mean std	1.1413 0.1439	2.2130 0.1405	3.3871 0.1671	−1.5641 0.0324	0.5719 0.0307	22
mean std	1.0031 0.3275	2.6715 0.3691	4.4896 0.3897	−0.9872 0.0817	0.1528 0.0798	12
True parameters	$\beta_0(2)$=1.3	$\beta_1(2)$=2.3	$\beta_2(2)$=3.3	$\alpha_1(2)$=−1.1731	$\alpha_2(2)$=0.1364	
Estimates	$\hat{b}_0(2)$	$\hat{b}_1(2)$	$\hat{b}_2(2)$	$\hat{a}_1(2)$	$\hat{a}_2(2)$	
mean std	1.3008 0.0103	2.3007 0.0093	3.3282 0.0102	−1.1730 0.0028	0.1353 0.0161	42
mean std	1.3097 0.1034	2.3352 0.1151	3.4512 0.1107	−1.0713 0.0151	0.1289 0.0162	22
mean std	1.4576 0.3457	2.6732 0.3457	3.8715 0.2915	−0.8423 0.0465	0.0390 0.0427	12

In Figure 2 it is shown the LPTV system parameter recursive estimation (10) quality via the number T of processed observations for SNR = 22 dB.

A measure of the parameter estimation quality is:

$$B0 = \frac{\left|\beta_0(0) - \hat{b}_0(0)\right|}{\left|\beta_0(0)\right|} \cdot 100\%,$$

$$A1 = \frac{\left|\alpha_1(0) - \hat{a}_1(0)\right|}{\left|\alpha_1(0)\right|} \cdot 100\% , \qquad (13)$$

and so on, where $\hat{b}_0(0)$, $\hat{a}_1(0)$ are averaged estimated parameters and $\beta_0(0)$, $\alpha_1(0)$ are true parameters of the LPTV system at time moment $k = 0$.

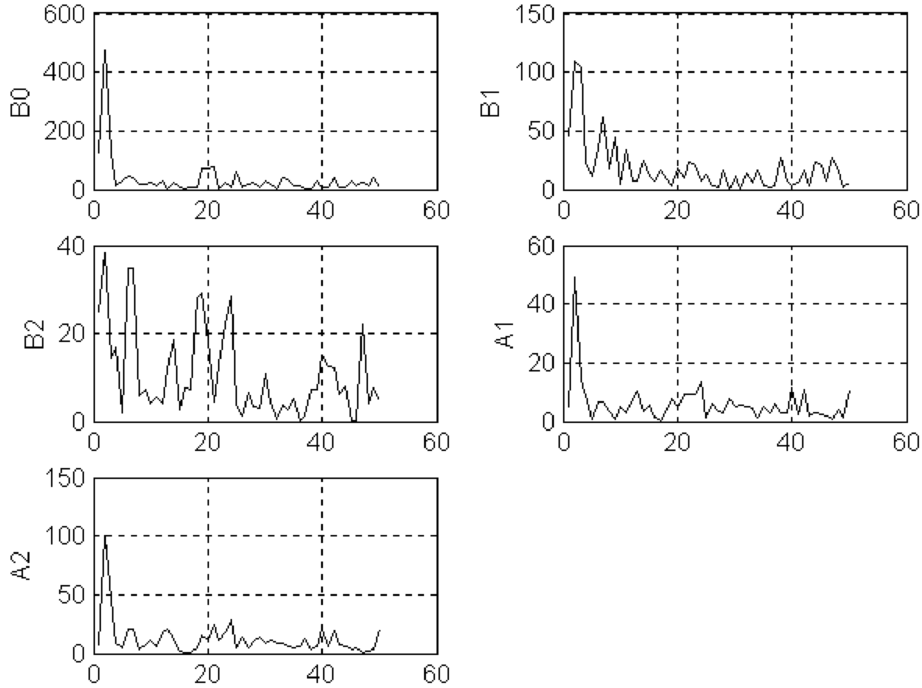

Figure 2. The parameter estimation quality (13) of the LPTV system recursive parameter estimation (10) at time $k = 0$ vs the number of observations T; SNR = 22 dB

4. Conclusions

We have proposed a nonrecursive and recursive parallel parameter estimation algorithms for LPTV system. The algorithms provide a computational tool based on a theoretical foundation for parameter estimation of LTI system. The estimated values are close to the true parameter values at high SNR. This property can be confirmed by the analysis of the simulation results of Table 1. The parallel parameter estimation structure is proposed. If periodicity $L = 1$, the parallel algorithm reduces to the LTI system parameter estimation algorithm. The algorithm can be extended to estimate LPTV system parameters from colored noise corrupted measurements.

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Received May 2007.