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STATISTIC CHARACTERISTICS OF M-ARY FSK SIGNAL IN THE PRESENCE OF GAUSSIAN NOISE, IMPULSE NOISE AND VARIABLE SIGNAL AMPLITUDE

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Abstract. In this paper the receiver for the demodulation of M-FSK signals in the presence of Gaussian noise, impulse noise and variable signal amplitude is considered. The communication systems are subject to Gaussian noise, impulse noise and Rayleigh signal amplitude that can seriously degrade their performance.

1. Introduction

In this paper we consider a system for coherent demodulation of *M*-ary FSK signals in the presence of Gaussian noise, impulse noise and variable signal amplitude. These annoyances can seriously degrade the performance of communication systems [1]-[3]. In the paper [4], the performance evaluation of several types of FSK and CPFSK receivers was investigated in detail using the modified moment's method. Also, the error probability of the cross-correlator receiver for binary digital frequency modulation detection is studied using theoretical analysis and computer simulations [5]. In [6], average bit-error probability performance for optimum diversity combining of non-coherent FSK over Rayleigh channels is determined. Performance analysis of wide-band *M*-ary FSK systems in Rayleigh fading channels is given in [7]. The influence of impulsive noise in noncoherent *M*-ary digital systems is observed in the paper [8].

In order to view an influence of the Gaussian noise, impulse noise and variable signal amplitude on the performances of an *M*-ary FSK system, we derived the probability density function of an *M*-ary FSK receiver output signal, the joint probability density function of output signal and its derivative, and the joint probability density function of output signal at two time instants. The bit error probability, the signal error probability and an outage probability can be determined by the probability density function of an output signal. Also, the moment generating function, the cumulative distribution of output signals and the moment and variance of output signals can be derived by probability density function of output signals. An average level crossing rate and an average fade derivation of an output signal process can be calculated by the joint probability density of the output signals and its derivative. An expression for calculation autocorrelation function can be derived by a joint probability density function of the output signals at two time instants. By using Winner-Hinchine theorem, the spectral power density function of an output *M*-ary FSK signal can be obtain. Based on this, the results obtained in this paper have a great significance.

This paper is organized as follows: the first section is the introduction. In the second section the model of the *M*-ary FSK system is defined. The expressions for the probability density function of the output signal and the joint probability density function of the output signal and its derivative at one time instant are obtained in the third section. In the next, fourth section, the characteristics of the signal on the output of an *M*-ary FSK receiver at two time instants is given. The fifth section presents the numerical results in the case *M*=2. The last section is the conclusion.

2. Model of the M-ary FSK System

The model of an M-ary FSK system, which we consider in this paper, is shown at Fig. 1. This system has *M* branches. Each branch consists of the bandpass filter and correlator. The correlator is consisting of multiplier and lowpass filter. That system can be used for the transmission of signals in fading indoor power line environment.

The signal at the input of the receiver is digital frequently modulated signal corrupted by additive Gaussian noise, impulse noise and variable signal amplitude.

The transmitted signal for the hypothesis H_i is:

$$s(t) = A\cos\omega_i t \quad (1)$$

where A denotes the amplitude of the modulated signal and has Rayleigh distribution.

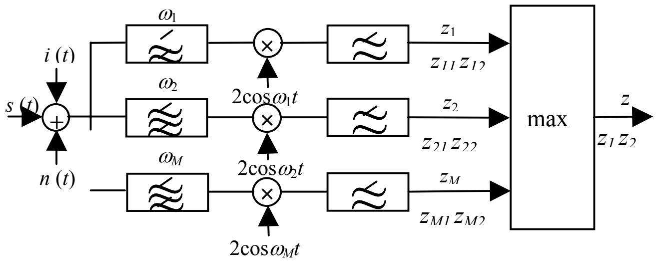

Figure 1. Block diagram of the system for coherent demodulation of M-ary FSK signal

Gaussian noise at the input of the receiver is given as follows:

$$n(t)=\sum_{i=1}^{M} x_i\cos\omega_i t+y_i\sin\omega_i t\ ,\quad i=1,2,...M\ , \tag{2}$$

where x_i and y_i are the components of Gaussian noise, with zero means and variances σ^2.

The pulse interference $i(t)$ can be written as:

$$i(t)=\sum_{i=1}^{M}\left(A_i+cn\right)\cos\left(\omega_i t+\theta_i\right), \tag{3}$$

where n has Poisson's distribution:

$$p(n)=\frac{\lambda^n}{n!}e^{-\lambda}\ , \tag{4}$$

λ is the intensity of the impulse processes. Phases θ_i, i=1,2,...,M, have uniform probability density function.

These signals pass first through bandpass filters whose central frequencies ω_1, ω_2,..., ω_M correspond to hypotheses H_1,H_2,...,H_M. Then, after multiplying with signal from the local oscillator, they pass through lowpass filter which cut all spectral components which frequency is greater than the border frequency of the filter.

If z_1,z_2, ...z_M are the output signals of the branch of the receiver, then the M-FSK receiver output signal is:

$$z=\max\{z_1,z_2...z_M\} \tag{5}$$

The probability density of the output signal is

$$p_z(z)=\sum_{i=1}^{M}p_{z_i}(z)\cdot\prod_{\substack{j=1\\ j\neq i}}^{M}F_{z_j}(z). \tag{6}$$

The joint probability density function of the output signal z and its derivative is:

$$p_{z\dot z}(z,\dot z)=\sum_{i=1}^{M}p_{z_i\dot z_i}(z,\dot z)\cdot\prod_{\substack{j=1\\ j\neq i}}^{M}F_{z_j}(z). \tag{7}$$

3. Characteristics of the Signal at One Time Instant

In the case of the hypothesis H_1, the transmitted signal is:

$$s(t)=A\cos\omega_1 t\ , \tag{8}$$

while the output branch signals of the receiver are:

$$z_1=A+x_1+A_1\cos\theta_1 \tag{9}$$

$$z_k=x_k+A_k\cos\theta_k\ ,\quad k=2,3,\dots,M. \tag{10}$$

It is necessary to define the probability density functions on the output of branches and the cumulative density of these signals to obtain the output probability density function of M-ary FSK receiver.

The conditional probability density functions for the signals z_1, z_2,..., z_M are:

$$p_{z_1/A,\theta_1}(z_1)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{\left(z_1-A-(A_1+cn)\cos\theta_1\right)^2}{2\sigma^2}}\ , \tag{11}$$

$$p_{z_k/A,\theta_k}(z_k)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{\left(z_k-(A_k+cn)\cos\theta_k\right)^2}{2\sigma^2}}\ ,\quad k=2,3,\dots,M. \tag{12}$$

By averaging (11) and (12), we obtain the probability density functions of the signals on the output of the branches:

$$p_{z_1}(z_1)=\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{\left(z_1-A-(A_1+cn)\cos\theta_1\right)^2}{2\sigma^2}}\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{A}{\sigma^2}e^{-\frac{A^2}{2\sigma^2}}dA\frac{1}{2\pi}d\theta_1 \tag{13}$$

$$p_{z_k}(z_k)=\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{\left(z_k-(A_k+cn)\cos\theta_k\right)^2}{2\sigma^2}}\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{A}{\sigma^2}e^{-\frac{A^2}{2\sigma^2}}dA\frac{1}{2\pi}d\theta_k\ . \tag{14}$$

The cumulative distributions of the signals z_1, z_2,..., z_M are:

$$F_{z_1}(z_1)=\int_{-\infty}^{z_1}\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{\left(z_1-A-(A_1+cn)\cos\theta_1\right)^2}{2\sigma^2}}\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{A}{\sigma^2}e^{-\frac{A^2}{2\sigma^2}}dA\frac{1}{2\pi}d\theta_1 dz_1\ , \tag{15}$$

$$F_{z_k}(z_k)=\int_{-\infty}^{z_k}\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{\left(z_k-(A_k+cn)\cos\theta_k\right)^2}{2\sigma^2}}\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{A}{\sigma^2}e^{-\frac{A^2}{2\sigma^2}}dA\frac{1}{2\pi}d\theta_k dz_k\ . \tag{16}$$

The probability density function of the M-ary FSK receiver output signal in the case of the hypothesis H_1 can be obtained from:

$$p_{z_1}(z_1) = \sum_{i=1}^{M} p_{z_{1i}}(z_1) \cdot \prod_{\substack{j=1 \\ j\neq i}}^{M} F_{z_{1j}}(z_1). \tag{17}$$

4. Characteristics of the Signals at Two Time Instants

The M-ary FSK receiver output signals at instant t_1 are z_{11}, z_{21},..., z_{M1} and at the instant t_2, they are z_{12}, z_{22},..., z_{M2}. The phases of the interference in each branch remain constant at both time instants.

The joint probability density of the signals z_{11} and z_{12} is:

$$p_{z_{11}z_{12}}(z_{11}, z_{12}) = \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{2\pi\sigma^2\sqrt{1-\gamma^2}} \cdot$$
$$\cdot e^{-\frac{(z_{11}-A-(A_1+cn)\cos\theta_1)^2}{2\sigma^2(1-\gamma^2)}} \cdot$$
$$\cdot e^{-\frac{-2\gamma(z_{11}-A-(A_1+cn)\cos\theta_1)(z_{12}-A-(A_1+cn)\cos\theta_1)}{2\sigma^2(1-\gamma^2)}} \cdot$$
$$\cdot e^{-\frac{(z_{12}-A-(A_1+cn)\cos\theta_1)^2}{2\sigma^2(1-\gamma^2)}} \cdot$$
$$\cdot \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda} \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_1 \tag{18}$$

where γ denotes the correlation coefficient of the noise.

Similarly,

$$p_{z_{k1}z_{k2}}(z_{k1}, z_{k2}) = \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{2\pi\sigma^2\sqrt{1-\gamma^2}} \cdot$$
$$\cdot e^{-\frac{(z_{k1}-(A_k+cn)\cos\theta_k)^2}{2\sigma^2(1-\gamma^2)}} \cdot$$
$$\cdot e^{-\frac{-2\gamma(z_{k1}-(A_k+cn)\cos\theta_k)(z_{k2}-(A_k+cn)\cos\theta_k)}{2\sigma^2(1-\gamma^2)}} \cdot$$
$$\cdot e^{-\frac{(z_{k2}-(A_k+cn)\cos\theta_k)^2}{2\sigma^2(1-\gamma^2)}} d\theta_k \cdot$$
$$\cdot \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda} \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_k \tag{19}$$

The M-ary FSK receiver output signals at two time instants, z_1 and z_2, are equal:

$$z_1 = \max\{z_{11}, z_{21}, ..., z_{M1}\}, \tag{20}$$
$$z_2 = \max\{z_{12}, z_{22}, ..., z_{M2}\}. \tag{21}$$

The joint probability density of the output signals z_1 and z_2 is:

$$p_{z_1z_2}(z_1, z_2) = \sum_{i=1}^{M}\sum_{j=1}^{M} p_{z_{i1}z_{j2}}(z_1, z_2) \cdot$$
$$\cdot \prod_{\substack{k=1 \\ k\neq i}}^{M} \prod_{\substack{l=1 \\ l\neq i}}^{M} F_{z_{k1}z_{l2}}(z_1, z_2). \tag{22}$$

The joint probability density of signals z_{11} and z_{12} and their derivatives $\dot{z}_{11}$ and $\dot{z}_{12}$ is:

$$p_{z_{11}z_{12}\dot{z}_{11}\dot{z}_{12}}(z_{11}, z_{12}, \dot{z}_{11}, \dot{z}_{12}) = \frac{1}{2\pi\sigma^2\sqrt{1-\gamma^2}} \cdot$$
$$\int_0^\infty \int_{-\pi}^{\pi} e^{-\frac{(z_{11}-A-(A_1+cn)\cos\theta_1)^2 - 2\gamma(z_{11}-A-(A_1+cn)\cos\theta_1)(z_{12}-A-(A_1+cn)\cos\theta_1)}{2\sigma^2(1-\gamma^2)}} \cdot$$
$$\cdot e^{-\frac{(z_{12}-A-(A_1+cn)\cos\theta_1)^2}{2\sigma^2(1-\gamma^2)}} \cdot \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{\dot{z}_{11}^2+\dot{z}_{12}^2}{2\sigma_1^2}} \cdot$$
$$\cdot \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda} \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_1. \tag{23}$$

Similarly,

$$p_{z_{k1}z_{k2}\dot{z}_{k1}\dot{z}_{k2}}(z_{k1}, z_{k2}, \dot{z}_{k1}, \dot{z}_{k2}) = \frac{1}{2\pi\sigma^2\sqrt{1-\gamma^2}} \cdot$$
$$\cdot \int_0^\infty \int_{-\pi}^{\pi} e^{-\frac{(z_{k1}-(A_k+cn)\cos\theta_k)^2 - 2\gamma(z_{k1}-(A_k+cn)\cos\theta_k)(z_{k2}-(A_k+cn)\cos\theta_k)}{2\sigma^2(1-\gamma^2)}} \cdot$$
$$\cdot e^{-\frac{(z_{k2}-(A_k+cn)\cos\theta_k)^2}{2\sigma^2(1-\gamma^2)}} \cdot \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{\dot{z}_{k1}^2+\dot{z}_{k2}^2}{2\sigma_{21}^2}} \cdot$$
$$\cdot \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda} \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_k. \tag{24}$$

The joint probability density of the signals z_1 and z_2 and their derivatives $\dot{z}_1$ and $\dot{z}_2$ is:

$$p_{z_1z_2\dot{z}_1\dot{z}_2}(z_1, z_2, \dot{z}_1, \dot{z}_2) =$$
$$= \sum_{i=1}^{M}\sum_{j=1}^{M} p_{z_{i1}z_{j2}\dot{z}_{11}\dot{z}_{12}}(z_1, z_2, \dot{z}_1, \dot{z}_2) \cdot$$
$$\cdot \prod_{\substack{k=1 \\ k\neq i}}^{M} \prod_{\substack{l=1 \\ l\neq i}}^{M} F_{z_{k1}z_{l2}}(z_1, z_2). \tag{25}$$

5. Numerical Results

Now, we consider the dual branch FSK receiver because of its easy implementation and very good performances. It is employed in many practical telecommunication systems.

The probability density function, in the case of dual branch, has the following form:

$$p_z(z_1) = p_{z_{11}}(z_1) \cdot F_{z_{12}}(z_1) + p_{z_{12}}(z_1) \cdot F_{z_{11}}(z_1). \tag{26}$$

The probability density functions $p(z)$ for various values of the parameters c, σ, λ and interference amplitude are given in Figures 2. to 10.

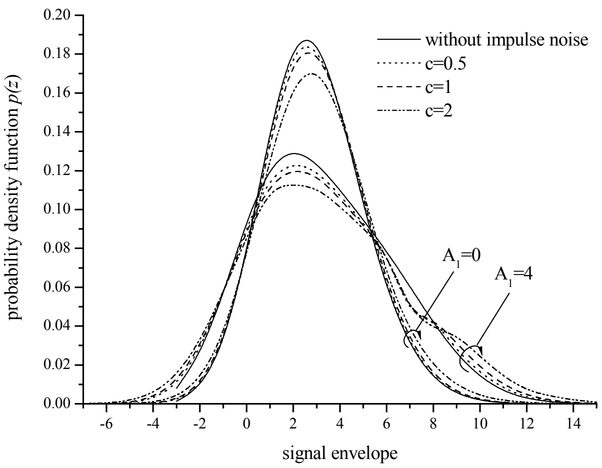

Figure 2. The probability density functions $p(z)$ for the parameters σ=2, λ=0.5

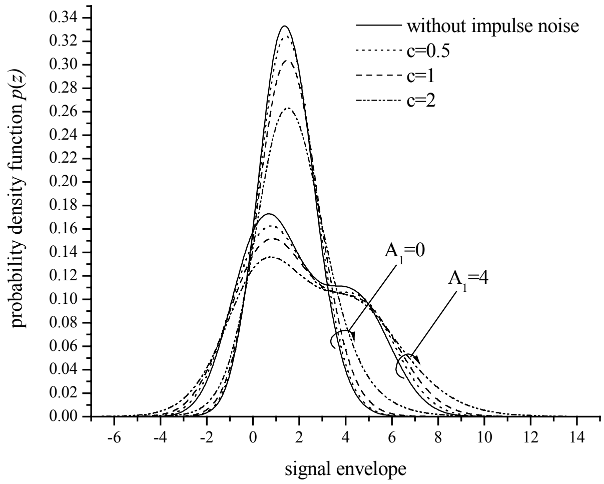

Figure 5. The probability density functions $p(z)$ for the parameters σ=1, λ=0.5

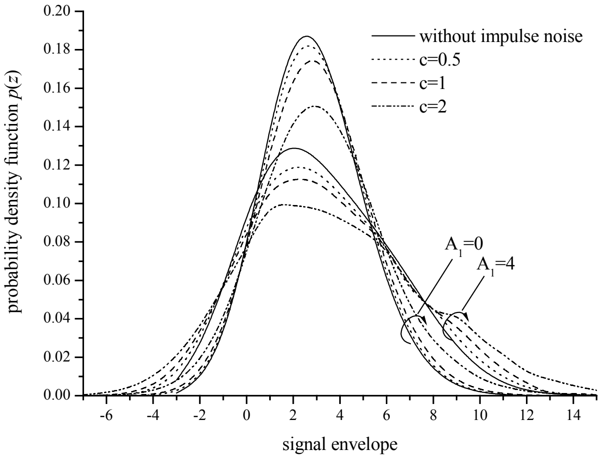

Figure 3. The probability density functions $p(z)$ for the parameters σ=2, λ=1

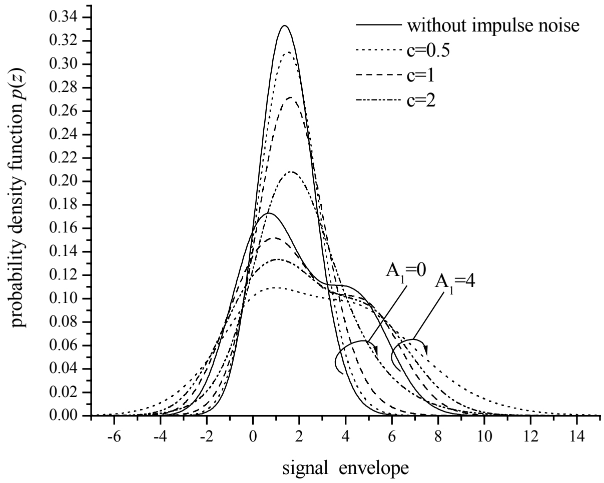

Figure 6. The probability density functions $p(z)$ for the parameters σ=1, λ=1

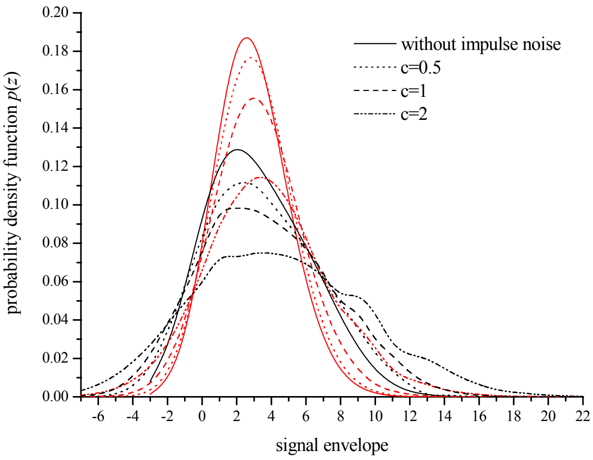

Figure 4. The probability density functions $p(z)$ for the parameters σ=2, λ=2

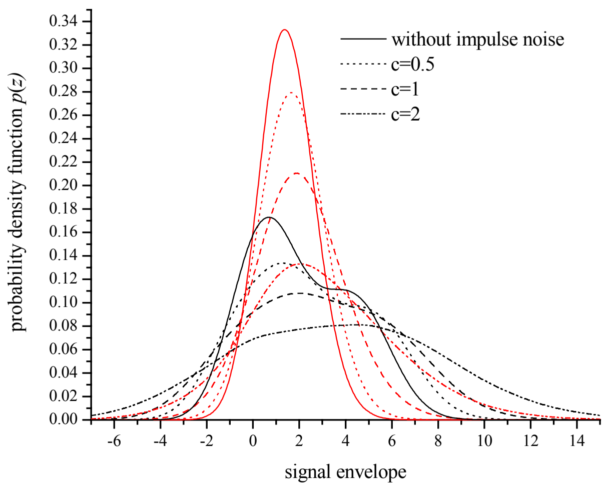

Figure 7. The probability density functions $p(z)$ for the parameters σ=1, λ=2

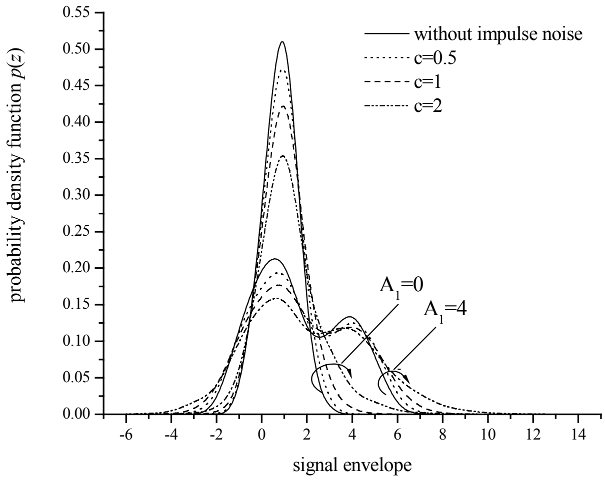

Figure 8. The probability density functions $p(z)$ for the parameters σ=0.5,λ=0.5

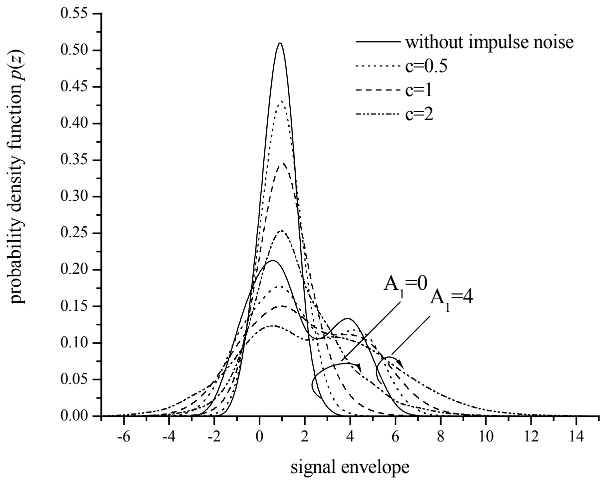

Figure 9. The probability density functions $p(z)$ for the parameters σ=0.5, λ=1

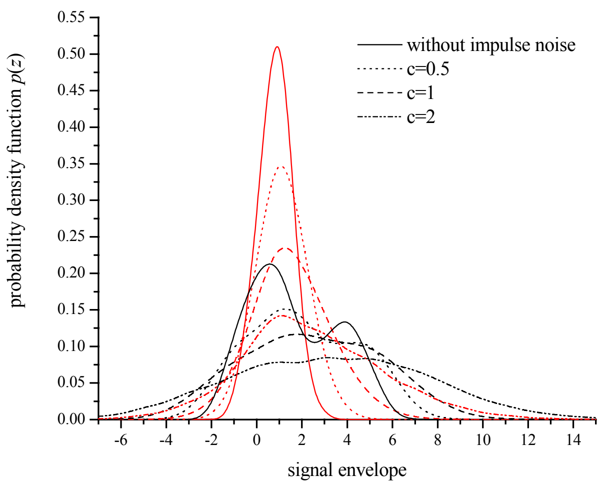

Figure 10. The probability density functions $p(z)$ for the parameters σ=0.5, λ=2

6. Conclusion

In this paper, the statistical characteristics of the signal at the output of the receiver for coherent FSK demodulation are derived. The input signal of the receiver is digital frequently modulated signal corrupted by additive Gaussian noise, impulse noise and variable signal amplitude. The interference appears in each receiver branch. In this paper the probability density function of an *M*-ary FSK receiver output signal, the joint probability density function of output signal and its derivative, and the joint probability density function of output signal of two time instants are derived.

The joint probability density function of the output signal at two times instants is important for the case where the noise is correlated. The bit error probability and the outage probability can be determined by the probability density function of an output signal. The level crossing rate and an average fade duration of an output signal process can be calculated by the joint probability density function of an output signal and its derivative. An expression for calculation autocorrelation function of the output signal can be obtained by a joint probability density function of the output signals at two time instants. Also, by this function a likelihood function of an *M*-FSK system, when the decision is done by two samples, can be calculated.

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