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MODELLING, SIMULATION AND OPTIMISATION OF INTERBANK SETTLEMENTS

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Abstract. Interbank payment and settlement systems establish conditions for the circulation of financial funds in the market and guarantee the distribution of assets. Non-cash payments are intensively growing in the payment and settlement market. Modern electronic systems of interbank payments are introduced to satisfy this need. Interbank payment and settlement sector is very sensitive to changes in the market. This calls the demand to foresee adaptation of the payment and settlement system in the dynamic environment. The technological renewal of the payment and settlement system was aimed at increasing fund turnover as well as complying with the requirements applied with regard to payment systems. Such challenges make the subject of modelling and analysis of financial flows topical in interbank systems. The article presents a stochastic model for the interbank payment and settlement system and analyses possibility for optimisation of system costs. The results of application of the model developed to the analysis of the real flow of payments in the payment system are given.

Keywords: Interbank payments, settlements, modelling of interbank settlements, optimisation of settlement costs.

1. Introduction

Interbank payment and settlement sector is very sensitive to changes. Interbank payment and settlement systems establish conditions for the circulation of financial funds in the market and guarantee the distribution of assets. The main purpose of such systems is to warrant a fast and rational turnover of settlements to balance payments and to reduce the movement of money supply. Active introductions of the means of electronic data transfer in banking and concentration of a great part of settlements at the centres of interbank payments were related with the creation of an automated system of clearing. Any change in such a system can have a great influence on the finance and capital markets. A change in the system can influence the development of national economy. Therefore by the interbank payment and settlement systems and their participants special requests are presented. These systems should provide the principles of stability, efficiency, and security. Participants of the system must satisfy the requirements of liquidity and capital adequacy measures. It invokes a requirement for the increased supervision and control of parameters of the system and its participants. The owner, operator, and supervisor of such a system by default are the central bank. It installs a request for the participants of the system, conducts supervision over their performance and takes measures that guarantee a stable system operation.

The systems of payments can be divided into that of discrete clearing and real-time systems. In the systems of discrete clearing, payments are made in the set intervals of time. In the real-time systems, payments are made continuously. Non-cash payments are growing in the market of payments. Recently non-cash payments have been growing in the market of payments of Lithuania. In 2005, the Clearinghouse of the Bank of Lithuania processed 17.3 per cent of all non-cash domestic payments in the country. The Clearinghouse processed 18.46 million of payment calls at the cost of 228 billion litas. Compared to 2004, the volume of payment transactions has grown by 18.7 per cent and their value went up by 19.4 per cent. The share of very small value payment transactions was growing gradually and in 2005 it made up 88.3 per cent of total payments. The Bank of Lithuania has designed and implemented a new real-time payment system LITAS which replaced the discrete clearing payment system TARPBANK that has been operating since 1993. A substantial renewal of the payment system in Lithuania was prompted by the implementation of the new banking technologies and was aimed at increasing funds turnover and complying with the requirements applied with regard to payment systems in the European Union. After the implementation of the new real-time payment system, the value of processed payment calls in this system was gradually growing.

The above-mentioned conditions demand not only to supervise and administer the system operation as well as to control the operations of the participants, but also to foresee changes in the system due to parameter change of the system. Interbank payments and settlements sector is very sensitive to changes in the market. A foul-up in the settlements can have a negative influence on the economic and social environment. Practical experiments in the active system are very risky. These require the demand for modelling the payment and settlement system.

Possibilities of modelling various situations of the market of payments are widely discussed with a view to avoid similar situations in the world practice (i.e., simulation of a system by applying BoF-PSS (Bank of Finland Payment and Settlement Simulator)). We could not manage to find a similar research, analysing the situation in the market of payments of Lithuania. Therefore, in this article, we present a model of the payment and settlement system by the example of the Clearinghouse of the Bank of Lithuania.

The topicality. Sensitiveness of the sector of interbank payments and changes in the market of settlements require to foresee the adaptation of the payment and settlement system in the dynamic environment. The scarcity of research on modelling Lithuanian interbank payments makes this subject of investigation topical both in theory and in practice.

The object of investigation of this article is modelling of the payment and settlement system.

The objective of the article is to present a model of the payment and settlement system and survey the possibility of statistical optimization of settlements costs.

The methods of the article are a systematic analysis of literature, practical analysis of the payment and settlement system, graphic and monographic analysis, analysis of real flow of payments in the payment system LITAS, and modelling of the interbank payment and settlement system.

2. Description of Settlement system and modelling data

2.1. Description of Settlement system

The payment and settlement systems consist of the system operator and participants of the system. In an establishment the participants keep the correspondent accounts for making payments.

In Lithuania, the Bank of Lithuania is the owner and operator of the system.

The payment system is designed to process payment instructions in real time and the set time.

The system operator must grant a possibility for a payment participant to submit payment instructions of another system participants to the system on the basis of a document presented by the payment participant, confirming right to submit payment instructions of other participants.

The payment system processes all kinds of interbank and customer payments regardless of their value. The system also settles payments arising from the Bank of Lithuania's own transactions with participants. Additionally, the payment system provides cash leg of securities transactions in real time following the delivery versus payment principle. It also establishes a possibility for other payment systems to perform settlements through the accounts of their participants with the Bank of Lithuania and provides a possibility to perform not only credit transfers, but also debit transfers.

The payment system is regulated by the Rules of Operation of the Payment System and bilateral bank account agreements between the Bank of Lithuania and system participants.

In addition to the Bank of Lithuania, the system is open to banks that have a banking licence issued by the Bank of Lithuania and foreign bank branches that have a permission of the Bank of Lithuania to operate in the country as well as Central Securities Depository of Lithuania, brokerage companies, Central Credit Union of Lithuania and clearing houses registered in the Republic of Lithuania. A credit institution of a state located in the European economic area and by the decision of the Board of the Bank of Lithuania, a financial or clearing institution of such a state may also join the system. A detailed list of participants of the payment system is presented in the Official List of Systems [7].

The system is designed for the following goals [7]:

- to prepare payment instructions of system participants by performing real time gross settlements (RTGS) and the set time settlements (DTS) through their accounts held with the Bank of Lithuania by ensuring safe, final and irrevocable settlement of system participants;
- to warrant the processing of payment instructions for securities transactions according to the delivery versus payment principle jointly with the Securities Settlement System;
- to fulfil the functions of information exchange among system participants required for the operation of the system and the Securities Settlement System.

A system participant has the following rights [7]:

- to submit payment instructions to the system for performing RTGS and/or DTS;
- to submit instructions to the system to change the settlement conditions of its application submitted earlier and not yet entered into the system;
- to change the priority of an application for making credit transfer and to revoke an application;
- to receive real-time information on the settlement of applications submitted to the system by other participants where he is indicated as a beneficiary;
- to monitor the latest information on its payment instructions submitted to the system and its settlement account (accounts);
- to receive the end-of-day reports: information on the settlement results of its applications, the settlement of applications that act on the change of the balance in the settlement account and the final balance of its settlement account;
- to receive information from the system operator on the fee calculated for the settlement of applications in the system. On ascertaining an error in the calculation of the amount of the fee, to request its correction.

A system participant has the following responsibilities [7]:

- to comply with legal acts of the Bank of Lithuania regulating settlements, organisation of the internal control of the bank and electronic certification;
- to manage the operational risk of its information systems related to the system;
- to ensure the functioning of organisational, hardware and software facilities of the interface to the system;
- a payment participant must submit a document to the system operator in advance that confirms his right to submit applications for transferring the funds of other participants and the list of system participants that granted this right.

The system is open each business day established in the Republic of Lithuania. Information exchange between the system participants and the system is performed by means of electronic messages signed with a digital signature. An application submitted to the system is entered into the payment queue and remains in it until the moment of entry into the system. The moment of entry into the system is the beginning of the settlement of an application. A system participant, the operator, or a third party may not revoke the application entered into the system. The application is treated as settled, when the funds are credited into the settlement account of the beneficiary.

An urgent payment instruction is processed in real time if the participant has sufficient funds for settling the application. If the participant is scant of funds, urgent applications are entered into the queue.

Every 20 minutes the system performs the optimisation procedure during which these applications are selected from the queue the netting of which shows that there are sufficient funds for settling them.

Unexceptional payment instructions are processed four times per day: at 9:00 a.m., 12:00 a.m., 3:00 p.m. and 4:00 p.m.

Information is exchanged between the system participants and the system by means of electronic messages. A digital signature is used to sign:

- application messages and that including the instructions to change the settlement conditions;
- messages including the processing results of payment instructions and that including the processing results of instructions to change the settlement conditions of payment instructions;
- messages of the end-of-day reports.

System participants who submit applications to the system assign their priority according to priority values described in the technical documentation of the system.

A payment instruction submitted to the system is validated by the system, if its structure conforms to the message structure requirements described in the technical documentation of the system. Otherwise, the application is rejected and the system participant who submits the application is notified about the reason of rejection.

An application validated by the system is placed into the payment queue, if its settlement conditions (if indicated by the system participant submitting the application) are fulfilled.

An application remains in the queue until the moment of its entry into the system, i.e., the moment when the system establishes that available funds are sufficient for settling the application and the processing of the application is started immediately by transferring the funds indicated in it from the settlement account of the payer to the settlement account of the beneficiary.

Neither a system participant, nor its operator or a third party may revoke an application entered into the system.

During the processing of applications the settlement account of a system participant is credited with the funds indicated in the application of another system participant or the balance of this account is debited with the amount of funds indicated in the application to be transferred to another system participant.

The application is recognised as settled, when the funds are credited into the settlement account of the beneficiary. After settling the application, the system notifies the payer, the beneficiary and the payment participant about it, if the application was submitted by him.

Processing payment instructions in real time and immediately transferring their settlement results to the Securities Settlement System ensure the processing of applications for settling securities transactions based on the delivery versus payment principle.

The processing of applications is completed by the end-of-day procedure during which the following actions are performed in sequence:

- the optimisation procedure is applied to all the applications in the payment queue;
- after the optimisation procedure all the remaining applications, where available funds are sufficient for settlement, are settled in sequence, and the those where available funds are insufficient are omitted.
- applications that remain not executed due to the lack of available funds are removed from the payment queue and the system participant that submitted the application is notified about it;
- the end-of-day reports are prepared and submitted to system participants.

In order to manage the operational risk of the system, the system operator sets system security requirements, plans and implements respective security measures, assesses the system security situation and determines the residual risk.

The payment system payment costs are based on the principles of full cost coverage, transparency and equal rights of system participants.

2.2. Modelling data

We simulate payments flow of the interbank payment and settlement system. The system consists of J agents, who execute payments between themselves. We call by agents the participants of a system: banks, foreign banks branches, credit unions, and other financial or clearing institution members of the payment and settlement system. The participants send applications to the payment and settlement system. Each application is described in the system by the name of a sender, name of the addressee, moment of delivery of the application, and the volume of the transaction.

The receipt of real data is bound up with a problem of confidentiality. Usually the institutions which take part in interbanking operations avoid to reveal the data of transactions. Exceptionally, it is possible to receive encoded data.

We consider the anonymous data of the interbank settlement session of a typical labour day presented by the Bank of Lithuania. These data consist of 74637 applications of 11 participants of the Payment and Settlement System. The data include the code name (number) of a participant of the payment and settlement system, time of delivery of the applications, volume of the applications and the flow of applications. Further we use the term "payment" instead of "payment order" for simplicity.

3. Modelling and simulation

For modelling of Interbank payments we apply the Poisson-lognormal model [1]. The frequency of delivery and volume of payments are random. According to this model we consider the flow of applications of the i^{th} agent to the j^{th} one following from the Poisson distribution with intensities λ_{ij}, and the volumes of transactions assumed to be lognormal with parameters μ, σ. The Poisson distribution of application flow was tested according to the Shapiro-Wilk criterion [10]. The assumption on log normality of transactions volume was tested according to asymmetry criterion [3].

Using this model, a system for simulating settlements has been developed. The system generates flows of moments of bilateral payments by the Poisson distribution and the corresponding flow of payment volumes according to lognormal distribution.

The real data of the payment and settlement systems were used during the simulation:

- the number of participants of the payment and settlement system;
- time of delivery of the applications;
- volume of the applications;
- flow of applications.

The primary data were obtained from these data by the imitative model of generation of payments flow:

- frequency of submission of applications;
- average of the value of one transfer;
- standard deviation of one transfer.

The flow of payments was analysed on the line of time divided by equal time intervals and grouped to periods of length T (Figure 1). Since the costs of settlements are calculated for the periods of 30 days, typically a time interval is assumed to be one day, while period T is 30 days.

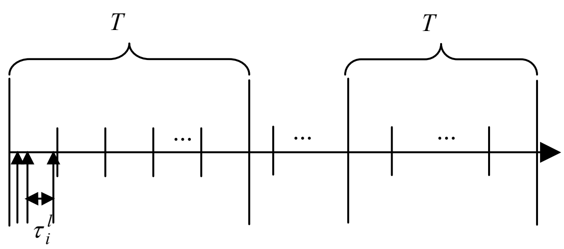

Figure 1. The line of time of submitting the payment applications

Let us consider the flow of a settlement period. Thus, every agent generates a flow of payments which are delivered to other participants. For $i, j = 1,...,J$, let z_{ij}^{l} be the number of payments from bank i to bank j per day l, $1 \le l \le T$. The time $t_{ij}^{k,l}$ of applications of each participant generated according to $t_{ij}^{k,l} = t_{ij}^{k-1,l} + \tau_{ij}^{k,l}$, $\tau_{ij}^{k,l} = \frac{-\ln(\zeta)}{\lambda_{ij}}$, ζ is uniformly random [0,1], $1 \le k \le z_{ij}^{l}$, $i \ne j$.

The value of the amount of applications was generated by the lognormal law:

$$p_{ij}^{k,l} = \exp\left(\mu + \sigma \cdot \eta_{ij}^{k,l}\right). \quad (1)$$

Where the average μ and standard deviation σ were estimated according to the real data, $\eta_{ij}^{k,l}$ is a standard normal variable.

4. Analysis of Settlement balance

4.1. Balance of the correspondent account

Let us have a flow of payments. Then the value of payments made by the i^th agent is:

$$\varphi_i^l = \sum_{j=1}^{J} \left(\sum_{k=1}^{z_{ij}^{l}} p_{ij}^{k,l} \right). \quad (2)$$

We introduce a variable $C_{ij}^{k,l} \in \{0,1\}$, $i, j = 1,...,J$, $k = 1,...,z_{ij}^{l}$. Here $C_{ij}^{k,l} = 1$ denotes that the k^{th} payment from bank i to bank j is included into the set of settled payments. Respectively, $C_{ij}^{k,l} = 0$ means that the payment is not included into the set.

The day net balance between two agents i and j is the difference of the amount of assets sent from bank i to j and back:

$$\xi_{ij}^{l} = \sum_{k=1}^{z_{ij}^{l}} p_{ij}^{k,l} C_{ij}^{k,l} - \sum_{k=1}^{z_{ji}^{l}} p_{ji}^{k,l} C_{ji}^{k,l} \quad (3)$$

The day net balance of bank i is the total sum of money that other banks sent to bank i minus the total sum of money that bank i sends to other banks [9].

$$\delta_i^l = \sum_{j=1}^{J} \xi_{ij}^{l} = \sum_{j=1}^{J} \left(\sum_{k=1}^{z_{ij}^{l}} p_{ij}^{k,l} C_{ij}^{k,l} - \sum_{k=1}^{z_{ji}^{l}} p_{ji}^{k,l} C_{ji}^{k,l} \right). \quad (4)$$

For interbank payments the bank uses correspondent accounts deposited in the Central bank or a Clearinghouse. Denote the amount of assets at the end of the l^{th} day in the correspondent account of the i^{th} agent by K_i^l, $i = 1, J$, $1 \le l \le T$.

We study the bank policy for control of their correspondent account by deposit or withdrawal of a certain fixed amount of assets from this account. According to the clearing rules, agents cannot have a negative correspondent account balance. If the correspondent account balance is positive the participants may withdraw a certain amount from the correspondent account. On the other hand, participants can deposit a certain amount of asset to their correspondent account in order to equilibrate the liquidity and payment costs. If the amount in the correspondent account is insufficient for settlements, an agent can take loans.

The correspondent account of the l^{th} day consists of the correspondent account of the previous day, net balance and deposited (or withdrawn) amount of asset of the participant himself. Thus, the amount on the correspondent account may be computed as follows:

$$K_i^l = \max(0, K_i^{l-1} + \delta_i^l + G_i^l), \quad (5)$$

where K_i^l is the correspondent account residue of the bank i for day l, G_i^l is the deposited or withdrawn sum of the bank i.

4.2. Costs of settlements

The payment and settlement system is characterized by operational, credit and liquidity risk. For simplicity, we assume that all applications of payments are executed without adjournment, i.e., an application is executed at once: $C_{ij}^{k,l} = 1$. A successful performance of the payment system is guaranteed by keeping sufficient sums in the corresponding accounts. Insufficient sums of the clearing accounts cannot satisfy the credit obligations, because this fact destabilizes interbank payments and sets gridlocks in the payment and settlement system. The Central bank allows borrowing overnight loans and installs reserve requirements to the settlement system participants in order to prevent the illiquidity in the payment system. Therefore the Central bank establishes reserve requirements RR_i for the participants of settlement system. The reserve requirements depend on liabilities of a participant.

In order to study the policies of credit and liquidity risk control, we consider a probability of exceeding the correspondent account and operational costs of settlements.

The total cost of settlements of the i^{th} agent during one period consists of several parts:

$$D_i = RE_i + F_i + B_i + TT_i + AC_i, \quad (6)$$

where RE_i is the premium for deposit, F_i is the pay of nonconformity of reserve requirements, B_i is the cost of short-term loans, TT_i is the bank indirect losses due to the freeze of the deposited amount of assets (or possible profit of withdrawal) in the correspondent account, and AC_i is the operation cost.

The i^{th} participant gets the premium RE_i at the positive balance of the corresponding account which does not exceed reserve requirements, and pay penalty F_i in the case of nonconformity of reserve requirements. Thus, the amount of premium RE_i is represented by the formula:

$$RE_i = \frac{\sum_{l=1}^{T} \max\left(RR_i, K_i^l\right) \cdot r}{100 \cdot 360}, \quad r = \sum_{l=1}^{T} \frac{LR^l}{T}, \qquad (7)$$

where LR^l is the interest rate of refinancing by the Central bank of operations, RR_i are reserve requirements for the settlement system participants.

The amount of penalty F_i of the i^{th} agent, is represented by the formula:

$$F_i = \frac{\left(\sum_{l=1}^{T} (\max\left(0, RR_i - K_i^l\right)\right) \cdot \left(r + p\right)}{360 \cdot 100}, \qquad (8)$$

where p is an added penalty percentage item (in the Lithuanian Central bank $p = 2.5$).

If the participant of the system lacks of assets to execute settlements, the Central bank can confer a short-term loan under the certain interest rate.

Income of a participant of the system may be computed taking into account the expressions of penalty and premium by the following formula:

$$B_i = -STL \cdot \sum_{l=1}^{T} \min\left(0, K_i^{l-1} + \delta_i^l + G_i^l\right), \qquad (9)$$

where STL is the interest rate of a short-term loan.

Let us analyze how banks can manage settlements costs by depositing (or withdrawing) assets on the correspondent account. We consider the policy when banks deposit or withdraw certain fixed sums X_i. When computing operational costs we have to take into consideration that a bank cannot withdraw sum larger than that present in the corresponding account. Thus, after simple considerations, we have that the deposited or withdrawn amount is as follows:

$$G_i^l = \max\left(X_i, -\max\left(K_i^{l-1} + \delta_i^{l-1}, 0\right)\right), \qquad (10)$$

The frequency of liquidity loss is computed as follows:

$$P_{likv} = \frac{\sum_{l=1}^{T} \sum_{i=1}^{J} H\left(\min\left(0, K_i^{l-1} + \delta_i^l + G_i^l\right)\right)}{T}, \qquad (11)$$

where $H(\cdot)$ is Heaviside function.

The indirect losses due to the freeze of deposit (or possible profit of withdrawal) on the correspondent account are represented by the formula:

$$TT_i = IBR \cdot \sum_{t=0}^{T} G_i^l, \qquad (12)$$

where IBR is the interest rate of the interbank loan market.

The operating costs of the i^{th} agent are computed assuming that the cost of one operation is fixed at ϕ:

$$AC_i = \phi \cdot \sum_{t=1}^{T} \sum_{j=1}^{J} z_{i,j}^l, \qquad (13)$$

The payment and settlement system is characterized by a probability of losses of liquidity P_{likv} given in (10) and total settlements costs

$$D = \sum_{i=1}^{J} D_i. \qquad (14)$$

5. Statistical simulation of settlements costs

We calculate the average costs of service and evaluate the probability of losses of liquidity by simulating a few periods of settlements.

Denote the cost of transactions during one period by $D_i = D_i(X_i, \delta_i)$, which is a random function in general, depending on the deposit X_i and the vector of balances of the correspondent account $\delta_i = (\delta_i^1, \delta_i^2, \ldots, \delta_i^T)$, here $\delta_i = \sum_{l=1}^{T} \delta_i^l$, $1 \le i \le J$.

Denote the expected cost during one period by

$$L_i\left(X_i\right) = ED_i\left(X_i, \delta_i\right). \qquad (15)$$

In order to estimate the influence of the parameter X_i on the cost, it is necessary to find a derivative of the cost function on parameter X_i. Note, the function $D_i(X_i, \delta_i)$ is a piecewise function in general, which is not differentiable in usual sense. Therefore we introduce a generalized gradient of this function, using the following expressions for computing subgradients [4].

Let

$$g(x) = \min(g_1(x), g_2(x)), \qquad (16)$$

where $g_1(x)$ and $g_2(x)$ are generalized differentiable functions. Then the subgradient is:

$$\partial_x g(x) = \begin{cases} \partial g_1(x), & if\ g_1(x) \le g_2(x) \\ \partial g_2(x), & if\ g_1(x) > g_2(x) \end{cases}. \quad (17)$$

Note that a subgradient is coincidental with the gradient of the function which is differentiable in the usual sense.

Using this approach, we find subgradients of the functions $K_i^l(x)$ and $G_i^l(x)$. Hence we have:

$$\partial_x K_i^l(x) = \begin{cases} \partial_x K_i^{l-1}(x) + \partial_x G_i^l(x), & if\ K_i^{l-1}(x) + \delta_i^l + G_i^l(x) \ge 0 \\ 0, & otherwise \end{cases}, \quad (18)$$

$$\partial_x G_i^l(x) = \begin{cases} 0, & if\ x \le 0, K_i^l + \delta_i^l \le 0 \\ \partial_x K_i^l(x), & if\ 0 < K_i^l + \delta_i^l < -x_i \\ 1, & otherwise \end{cases}. \quad (19)$$

Using these formulas, we can compute the subgradients $\partial_x D_i(X_i, \delta_i)$. It is easy to make sure that expectation of the subgradient of the cost function yields the gives us gradient of expected costs [4]:

$$\frac{dL_i(X_i)}{dX_i} = E\partial_x D_i(X_i, \delta_i). \quad (20)$$

Now we consider the statistical simulation procedure of settlement costs (15) and their derivatives (20). Let N periods of settlement performance be simulated and random vectors of incomes and outcomes $\delta_{i,n}$, $1 \le n \le N$, $1 \le i \le J$, be generated. Thus, the statistical estimate of settlement costs is the average cost:

$$\tilde{L}_i(X_i) = \frac{1}{N}\sum_{n=1}^{N} D_i(X_i, \delta_{i,n}). \quad (21)$$

The Monte-Carlo estimator of gradient (20) is obtained by virtue of:

$$Q_i(X_i) = \frac{1}{N}\sum_{n=1}^{N} \partial_x D_i(X_i, \delta_{i,n}). \quad (22)$$

Denote the vector of agent impact on its correspondent account as $X = (X_1, \ldots, X_J)$. The quality of settlement system can be defined by general expected cost

$$\tilde{L}(X) = \sum_{i=1}^{J} \tilde{L}_i(X_i).$$

During the simulation the sampling variance can be computed:

$$d_N^2(X) = \frac{1}{N}\sum_{n=1}^{N}\left(D^n - \tilde{L}(X)\right)^2, \quad (23)$$

where $D^n = \sum_{i=1}^{J} D_i(X_i, \delta_{i,n})$, $1 \le n \le N$, as well as the $J \times J$ sampling covariance matrix:

$$A(X) = \frac{1}{N}\sum_{n=1}^{N}\left(\eta^n - \overline{\eta}\right)' \cdot \left(\eta^n - \overline{\eta}\right), \quad (24)$$

where η^n is a vector with the components

$$\eta_i^n = \partial_x D_i(X_i, \delta_{i,n}),$$

and $\overline{\eta}$ is a vector with the components

$$\overline{\eta}_i = Q_i(X_i),\ 1 \le n \le N,\ 1 \le i \le J.$$

6. Statistical optimisation of settlements costs

We develop the statistical optimization procedure for minimizing the costs using the approach of stochastic nonlinear programming by the Monte-Carlo estimators [8]. Let some initial vector of agents deposits $X^0 = \left(X_1^0, X_2^0, \ldots, X_J^0\right)$ be given, and a random sample of income and outcome vectors be generated. Let the initial sample size be N^0. Now, the Monte-Carlo estimators of the gradient of expected costs are computed according to (22). Next, the iterative stochastic procedure of gradient search could be introduced:

$$X^{t+1} = X^t - \rho \cdot Q_i(X_i), \quad (25)$$

where $\rho > 0$ is a certain step-length multiplier.

Let us consider a choice of the sample size during iterations. Note, that there is no great necessity to compute estimators with a high accuracy on starting the optimization, because then it suffices only to approximately evaluate the direction leading to the optimum. Therefore, one can obtain not so large samples at the beginning of the optimum search and later on increase the size of samples so as to obtain the estimate of the objective function with a desired accuracy only at the time of decision making on finding the solution of the optimization problem. We pursue this purpose by choosing the sample size at every next iteration inversely proportional to the square of the gradient estimator from the current iteration.

$$N^{t+1} = \frac{n \cdot Fish(\gamma, n, N^t - n)}{\rho \cdot Q(X^t) \cdot (A(X^t))^{-1} \cdot (Q(X^t))'}, \quad (26)$$

where $Fish(\gamma, n, N^t - n)$ is the γ-quintile of the Fisher distribution with $(n, N^t - n)$ degrees of freedom.

The step length ρ could be chosen experimentally. We introduce minimal and maximal values N_{min} (usually ~20-50) and N_{max} (usually ~ 1000-2000) to avoid great fluctuations of sample size in iterations. Note that N_{max} may also be chosen from the conditions on the permissible confidence interval of estimates of the objective function (see the next section).

A possible decision should be examined at each iteration of the optimization process on optimal solution finding. The proposed procedure guarantees the global convergence to some stationary point of the objective function [8]. Since we know only the Monte-Carlo estimates of the objective function and that of its gradient, we can test only the statistical optimality hypothesis. Since the stochastic error of these estimates essentially depends on the Monte-Carlo sample size, a possible optimal decision could be made, if, first, there is no reason to reject the hypothesis of equality of the gradient to zero, and, second, the sample size is sufficient to estimate the objective function with the desired accuracy.

Note that the distribution of sampling averages $\widetilde{L}_i$ and Q_i can be approximated by the one- and multidimensional Gaussian law [2], [5]. Therefore it is convenient to test the hypothesis of equality to zero of the gradient by means of the well-known multidimensional Hotelling T^2-statistics [6]. Hence, the optimality hypothesis could be accepted for some point X^t with significance $1-\mu$, if the following condition is satisfied:

$$\begin{aligned}&(N^t-n)\cdot(Q(X^t))\cdot(A(X^t))^{-1}\cdot(Q(X^t))\\&/n\le Fish(\mu,n,N^t-n)\end{aligned}, \quad (27)$$

Next, we can use the asymptotic normality again and decide that the objective function is estimated with a permissible accuracy ε, if its confidence bound does not exceed this value:

$$\eta_\beta\cdot d_{N^t}(X^t)/\sqrt{N^t}\le\varepsilon, \quad (28)$$

where η_β is the β -quintile of the standard normal distribution and standard deviation d_{N^t} is defined by (23). Thus, the procedure (25) is iterated adjusting the sample size according to (26) and testing conditions (27) and (28) at each iteration. If the latter conditions are met at some iteration, then there are no reasons to reject the hypothesis on the optimality of the current solution. Therefore, there is a basis to stop the optimization and make a decision on the optimum finding with a permissible accuracy. If at least one condition out of (27), (28) is violated, then the next sample is generated and the optimization is continued. As follows from the previous section, the optimization should stop after generating a finite number of Monte-Carlo samples.

2. Results of simulation and optimization

In this section, we present some Monte-Carlo simulation results which were calculated using the proposed model calibrated with respect to real data. In Table 1, the matrix of intensities generation per minute is presented. The parameters of the lognormal transaction volume (1) are as follows: $\mu = 7{,}813$, $\sigma = 2{,}189$.

Table 1. Matrix of intensities generations (number of applications / minute)

j/i	0	1	2	3	4	5	6	7	8	9	10
0	0.0000	0.0555	0.0000	0.0049	0.0205	0.0014	0.0000	0.0014	0.0021	0.0014	0.0410
1	0.0000	0.0000	0.0000	0.2053	2.1398	0.1565	0.0000	0.1473	0.3943	0.1395	3.9189
2	0.0000	0.6161	0.0000	0.0099	0.2798	0.0240	0.0000	0.0272	0.0353	0.0148	0.5783
3	0.0000	0.2063	0.0000	0.0000	0.0572	0.0039	0.0000	0.0039	0.0113	0.0025	0.1226
4	0.0000	1.3340	0.0000	0.0325	0.0000	0.0297	0.0000	0.0420	0.0834	0.0272	1.1111
5	0.0000	0.0544	0.0000	0.0011	0.0177	0.0000	0.0000	0.0004	0.0035	0.0004	0.0339
6	0.0000	0.3300	0.0000	0.0078	0.1561	0.0099	0.0000	0.0148	0.0173	0.0078	0.2063
7	0.0000	0.2445	0.0000	0.0035	0.1491	0.0053	0.0000	0.0000	0.0127	0.0046	0.3437
8	0.0000	0.3681	0.0000	0.0131	0.0703	0.0067	0.0000	0.0071	0.0000	0.0071	0.2240
9	0.0000	0.1346	0.0000	0.0035	0.0343	0.0028	0.0000	0.0021	0.0071	0.0000	0.0714
10	0.0000	5.2183	0.0000	0.1724	2.5009	0.1604	0.0000	0.1915	0.3282	0.1208	0.0000

In Figures 2 and 4, we present dependences of costs of settlements $\widetilde{L}_i$ on the deposited amount X_i for the first and last agents (i=0,10), estimated by the Monte-Carlo method, which illustrate the existence of the minimum point (N=5000). In Figures 3 and 5, we give dependences of derivatives of cost function with respect to the variable X_i. In these figures we see that change of derivatives is concerted with an increase and decrease of cost functions in Figures 2 and 4. Analogous dependences are similar for other agents.

Thus, the minimum exists and we search it by the iterative method (25), (27) and (28). In Figures 6-10, we present results of settlement costs optimisation by the approach described above. The optimisation required 62 iterations and 144 459 Monte-Carlo trials in total. In Figure 6 dynamics of general settlement costs is presented which illustrates the decrease of costs during optimisation (from 76999 LTL to 74083 LTL). In Figure 7, the dependence of the general sum of deposits is presented, which also illustrates the convergence of the optimisation process. Figures 8-11 also illustrate the dependencies of settlement costs and the deposited amount for the first and last agent. Figure 12 shows dynamics of the Monte-Carlo sample size during the optimisation which illustrates the adjustment of this size according to (26).

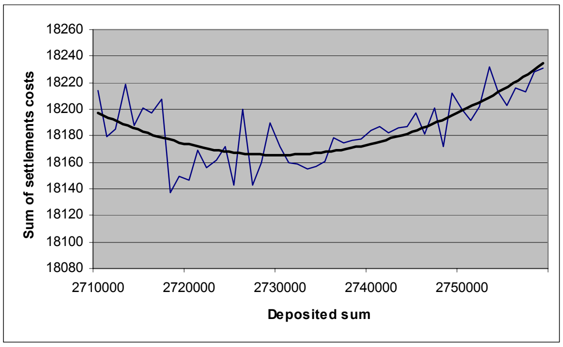

Figure 2. Dependence of settlement costs $\widetilde{L}_1$ on the sum of deposit X_1, N=5000

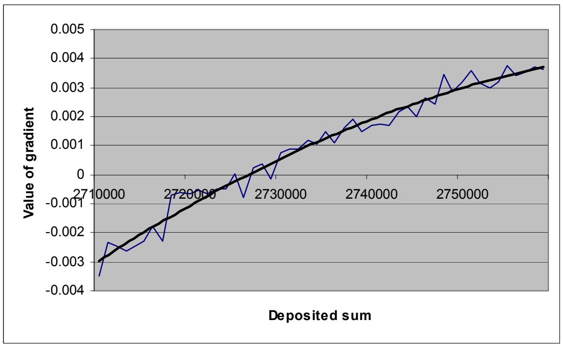

Figure 3. Dependence of the derivative $\widetilde{Q}_1(X_1)$ on the deposit X_1, N=5000

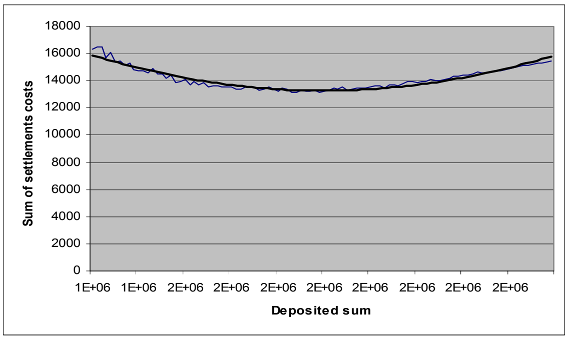

Figure 4. Dependence of settlement costs $\widetilde{L}_{10}$ on the sum of deposit X_{10}, N=5000

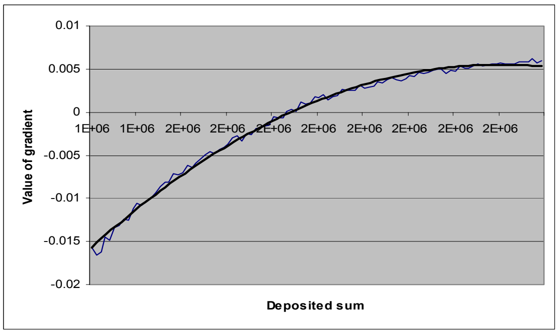

Figure 5. Dependence of the derivative $\widetilde{Q}_{10}(X_{10})$ on the deposit X_{10}, N=5000

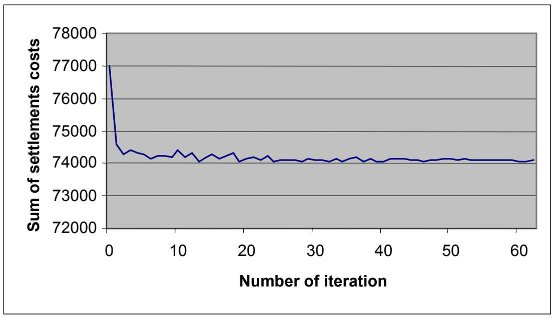

Figure 6. Dependence of the general settlement costs $\widetilde{L}^t$ on the number of iterations

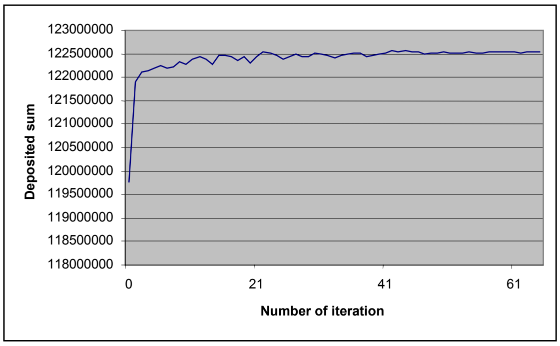

Figure 7. Dependence of the sum of deposits on the number of iterations

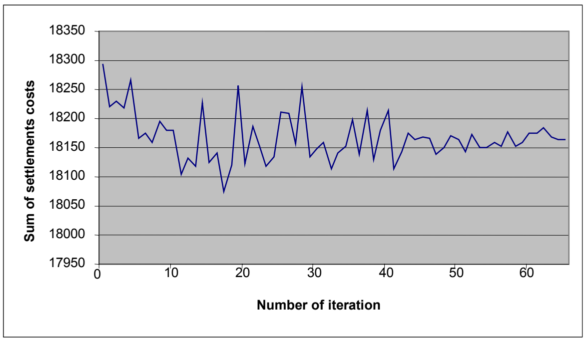

Figure 8. Dependence of the settlement costs $\widetilde{L}_1^t$ on the number of iterations

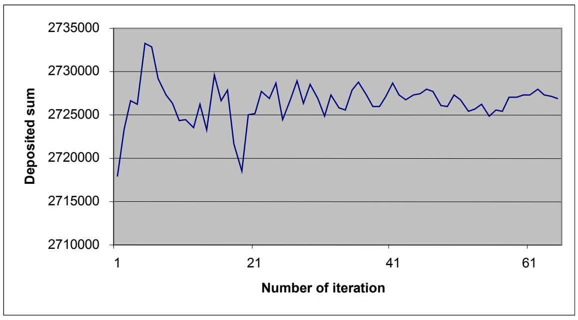

Figure 9. Dependence of the deposit X_1^t on the number of iterations

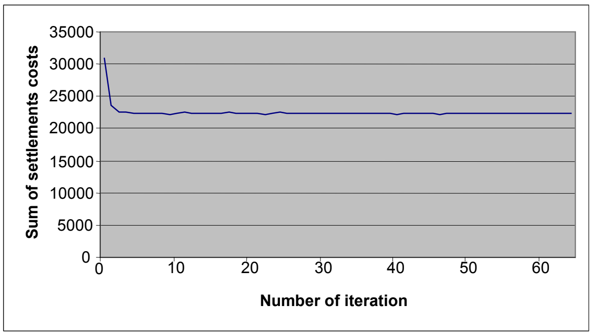

Figure 10. Dependence of the settlement costs $\widetilde{L}_{10}^t$ on the number of iterations

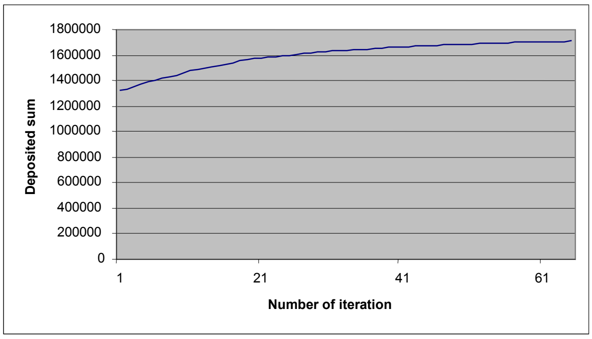

Figure 11. Dependence of the deposit X_{10}^t on the number of iteration

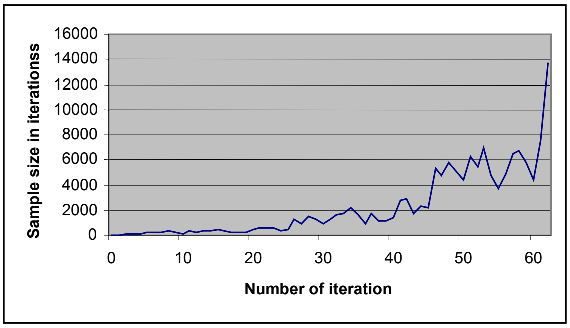

Figure 12. Dependence of the sample size N^t on the number of iteration.

8. Conclusion

The growth of non-cash payments, and the need to execute real-time payments invokes new challenges to electronic systems of the interbank clearing. Simulation and optimisation of transaction costs illustrate an opportunity for banks to maximize the future profit. In this situation it is especially important to study the strategies of management by banks of their correspondent accounts in Clearing house. In this paper, we analyze how banks can manage settlement costs by depositing (or withdrawing) assets on the correspondent account. We consider the policy when banks deposit or withdraw certain fixed sums. The stochastic optimisation method to regulate the correspondent agent account has been developed by Monte-Carlo estimators and investigated by computer simulation.

The outcome of the performed simulation shows that applying the given model of the income of a Clearinghouse as well as information technologies it is possible to optimise the parameters for management of risks of the credit, liquidity, and operational costs.

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