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THE HEAT BALANCE MODEL OF RESIDENTIAL HOUSE

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Abstract. This paper deals with the heat balance model of residential house. It is very important to maintain desired comfort in building and reduce heating costs. Heating costs depend on building heat loss factor, air infiltration rate and heat loss in the heat supply system. Values of these parameters may be reduced by improving construction of the building and developing optimal control of heating system. The heat balance model was used for development of a control system, which enables to regulate the heating installation of a building with continuous occupation. The system consists of autonomous water flow controllers in radiators and water temperature controller in boiler. It uses simple control algorithm. The system was developed and tested by simulation. The results obtained showed that the system allows providing high-energy conservation and reliable comfort.

1. Introduction

A heating system for building compensates the heat loss and maintains comfort. Heating and cooling loads are thermal energy that must be supplied to or removed from interior of a building in order to maintain desired comfort conditions. Once the loads have been established, one can proceed to the supply side and determine the performance of the required heating and cooling equipment.

A load calculation consists of a careful accounting of all the thermal energy terms of building. While the basic principle is very simple, a serious complication can arise from storage of heat in the mass of the building. For annual energy consumption the effect of heat capacity depends on the control of thermostat: it is negligible if indoor temperature is constant but can be quite significant with thermostat set back or up.

The heat is lost by heat transfer through the building surfaces and by exchange of air between the heated space and the building's surroundings. Fresh air in buildings is essential for comfort and health, and energy for conditioning this air is an important term. Not enough air, and risks sick building syndrome; too much air, and one wastes energy. There are two mechanisms that contribute to the total air exchange:

- Infiltration – uncontrolled airflow through all the little cracks and openings in a real building,
- Ventilation – natural ventilation through open windows or doors and mechanical ventilation by fans.

In the past, not much attention was paid to air tight construction and older buildings tend to have rather high infiltration rates in the range of 1 or 2air changes per hour. With current conventional construction one finds lower values, around 0.3 to 0.7. These values are seasonal averages; instantaneous values vary with wind and indoor-outdoor temperature difference. When infiltration is insufficient to guarantee adequate indoor air quality, forced ventilation becomes necessary. The required air exchange rates depend on the density of occupants. In residential buildings the density is relatively low and infiltration is likely to be sufficient.

The heat loss is mainly a function of outdoor air temperature. By taking the outdoor temperature as a primary influencing factor for the weather, the heat loss is [1]:

$$Q_{loss} = k_l (T_i - T_o) \tag{1}$$

where: Q_{loss} – building heat loss (W),

k_l – building heat loss factor (W/˚C),

T_i – indoor temperature (˚C),

T_o – outdoor temperature (˚C).

When the heat supplied by heating system is equal to the heat loss, constant indoor temperature is maintained. A thermal control system for buildings is intended both to maintain comfort and to minimize the power consumption.

There are three mechanisms that contribute to the total heat consumption [2]:

- building heat loss factor – k_l,
- air infiltration rate,
- heat loss in the heat supply system.

Building heat loss factor and air infiltration rate may be improved by changing construction of the house. Heat loss in the heat supply system may be reduced by developing the optimal control of heating systems [2-6]. In order to evaluate heat loss in the heat supply system the heat balance model must include model of heating system. Several studies regarding the optimal control of the heating systems have been undertaken [7-13]. The work was mainly focused on the use of optimal control methods based on minimum principle. However, the disadvantage of such methods lies in the fact that they require optimization at each time step and they cannot be implemented on computers other than ones with high calculating capacity power. In [14], thermal control system is based on quadratic optimization principle. The heating system is considered to be a multidimensional linear system, which may be represented by a discrete state model. This model may be subjected to any control or observation. This model has the following formula:

$$\begin{aligned}\mathbf{x}(i+1) &= \mathbf{A}\mathbf{x}(i) + \mathbf{B}\mathbf{u}(i) + \mathbf{F}\mathbf{P}(i)\\ \mathbf{y}(i) &= \mathbf{C}\mathbf{x}(i)\end{aligned} \tag{2}$$

where: $\mathbf{x}(i)$ – state vector (dimension n),
$\mathbf{y}(i)$ – output vector (dimension m),
$\mathbf{u}(i)$ – input vector (dimension r),
$\mathbf{P}(i)$ – disturbance vector (dimension rl),
$\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{F}$ – constant matrices.

If $\mathbf{e}(i)$ represents the difference between the index value of controlled variable Z and the output $\mathbf{y}(i)$, the optimal control desired, which is the sequence of vectors $\mathbf{u}(i)$, shall minimize a criterion called J as follows:

$$J = \sum_{i=1}^{N} \left[\mathbf{e}(i)^{\mathrm{T}} \mathbf{Q}\mathbf{e}(i) + \mathbf{u}(i)^{\mathrm{T}} \mathbf{R}\mathbf{u}(i)\right]. \tag{3}$$

Penalization matrices **Q** and **R** are symmetric and defined as positive.

2. Macroscopic heating model

We need building heating model in order to define optimal sequence of control vectors. Basically, a building model is composed of such components as a building energy storage component, a heat loss component, a radiator component, a flow controller component, and the main fuel boiler. A block diagram for heating model is shown in Figure 1.

The building energy storage element describes the building thermal storage effect, which is the reaction of indoor temperature to the heat flow into a building. The heat loss element describes the heat lost to the surroundings as a function of the weather and indoor temperature. The radiator element describes how heat is transferred to the building as a function of water mass flow, building water supply temperature and indoor temperature. The heat transferred from the water is written as:

$$Q_{\mathrm{supp}} = mc_p\left(T_s - T_r\right) \tag{4}$$

where: Q_{supp} – heat supplied (W);
m – water flow (kg/°C);
c_p – water heat capacity;
T_s – supply water temperature (°C);
T_r – return water temperature (°C).

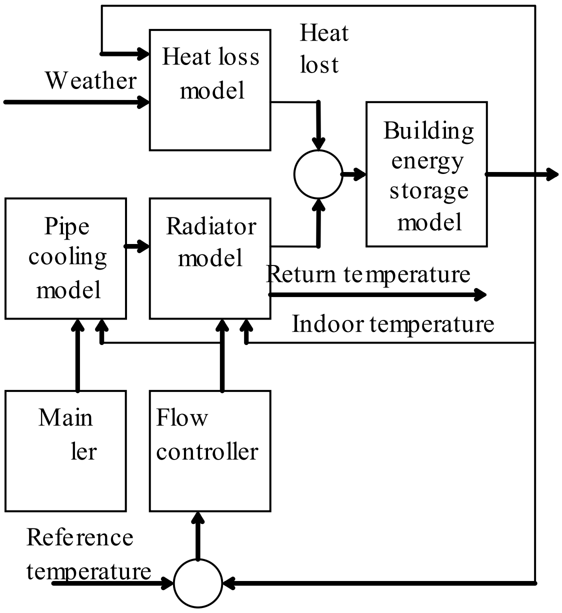

Figure 1. Block diagram of macroscopic heating model

By assuming that all heated parts of the building are heated at a uniform indoor temperature at all times, the building can be modeled as a single heat capacity element. A differential equation is then written relating the heat flow to the building to time derivative of the indoor temperature and the building heat capacity:

$$\frac{dT_i}{dt} = \frac{1}{C}\left(Q_{supp} - Q_{loss}\right), \tag{5}$$

where: C – building heat capacity.

Some heat is lost in the pipes connecting boiler and radiators. The amount of the heat loss can be calculated by using the heating pipe transmission parameter. The transmission effectiveness τ is defined as:

$$\tau = \frac{T_o - T_e}{T_i - T_e} = e^{-\frac{U_p}{mC_p}}, \tag{6}$$

where: T_i – pipe inlet temperature (˚C);
T_o – pipe outlet temperature (˚C);
T_e – pipe environment temperature (˚C);
U_p – pipe heat loss factor (W/˚C).

The supply temperature to the radiator can be calculated as [1]:

$$T_o = T_e + \left(T_i - T_e\right)\tau = T_e + \left(T_i - T_e\right)\tau_0^{\frac{m_0}{m}}. \tag{7}$$

The reference value of τ can be calculated from reference flow conditions:

$$t = \frac{T_{o0} - T_e}{T_{i0} - T_e} = e^{-\frac{U_p}{m_0 C_p}}, \tag{8}$$

where: T_{i0} – pipe inlet temperature at reference condition (°C);

T_{o0} – pipe outlet temperature at reference condition (°C);

m_0 – water flaw at reference condition (kg/s).

Buildings have different regulating systems. Let us consider a PI (Proportional and Integral) controller. It uses the proportional part of error and the integral of temperature error:

$$m = k_p\left(T_{i\,ref} - T_i\right) + k_i \int_0^t \left(T_{i\,ref} - T_i\right)dt, \tag{9}$$

where: k_p – flow controller gain ((kg/s)/°C);

$T_{i\,ref}$ – indoor temperature set point (°C);

k_i – flow controller integration factor (kgC);

t – time.

The classical state form for heating system PI regulator is:

$$\begin{bmatrix} \frac{dT_i}{dt} \\ \frac{dm}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{k_l}{C} & \frac{c_p\left(T_s - T_r\right)}{C} \\ \frac{k_p k_l}{C} - k_i & -\frac{k_p c_p\left(T_s - T_r\right)}{C} \end{bmatrix} \begin{bmatrix} T_i \\ m \end{bmatrix}$$
$$+ \begin{bmatrix} \frac{k_i}{C} & 0 \\ -\frac{k_p k_l}{C} & k_i T_{i\,ref} \end{bmatrix} \begin{bmatrix} T_o \\ 1 \end{bmatrix} \tag{10}$$
$$m = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} T_i \\ m \end{bmatrix}, \quad T_i = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} T_i \\ m \end{bmatrix}.$$

This equation can easily be solved by the discrete method. A common expression of differential matrix equation is:

$$\frac{d\mathbf{x}}{dt} = \mathbf{Ax} + \mathbf{Bu}. \tag{11}$$

The discrete version of the equation can be written as:

$$\mathbf{x}(i+1) = \mathbf{F}(i)\mathbf{x}(i) + \mathbf{G}(i)\mathbf{u}(i), \tag{12}$$

where

$$\mathbf{F} = e^{\mathbf{A}\Delta\mathbf{t}}. \tag{13}$$

By using Taylor equation, this can be written as:

$$\mathbf{F} = e^{\mathbf{A}\Delta\mathbf{t}} = \mathbf{I} + \Delta t\mathbf{A} + \frac{\left(\Delta t\mathbf{A}\right)^2}{2} + \ldots = \sum_{j=0}^{\infty} \frac{\left(\Delta t\mathbf{A}\right)^j}{j!}. \tag{14}$$

When the inverse of **A** exists, **G** can be solved as:

$$\mathbf{G} = \mathbf{A}^{-1}\left(\mathbf{F} - \mathbf{I}\right)\mathbf{B}. \tag{15}$$

The matrix **A** in (2) corresponds to matrix **F** in (12). The entries of the matrix **A** depend on the radiator return water temperature T_r. An iteration loop is used to find out T_r [15]:

$$T_r[n+1] = \left(T_s - T_i\right)e^{-y} + T_i, \tag{16}$$

where

$$y = \frac{\ln\left(\frac{T_{s0} - T_{i0}}{T_{r0} - T_{i0}}\right)}{\left(\frac{T_s - T_r[n]}{T_{s0} - T_{r0}}\right)^{\left(\frac{3}{4}-1\right)} \left(\frac{m}{m_0}\right)^{\frac{3}{4}}}. \tag{17}$$

Radiator return water temperature T_r is not constant, it depends on mass flow *m*. In this case, the matrix **A** in (2) is not constant, and minimization of criterion *J* is required at each time step. The amount of calculations increases and a computer with high calculating power capacity is needed in heating system control based on quadratic optimization principle.

3. Energy conservation

There are buildings with discontinuous occupation and buildings with continuous occupation. In the case of heated buildings with discontinuous occupation, energy is saved by precise forecasting preheating time. The optimal control provides energy conservation. In the case of heated buildings with continuous occupation, we must supply the same amount energy Q_{supp} as building loss energy Q_{loss}. The only way to save energy is to choose heating system operating conditions reducing heat loss in pipes connecting heating boiler and radiators.

Simulations of the heating system operating conditions were carried out. *Simulink* was used. A block diagram for building heating system model is shown in Figure 2.

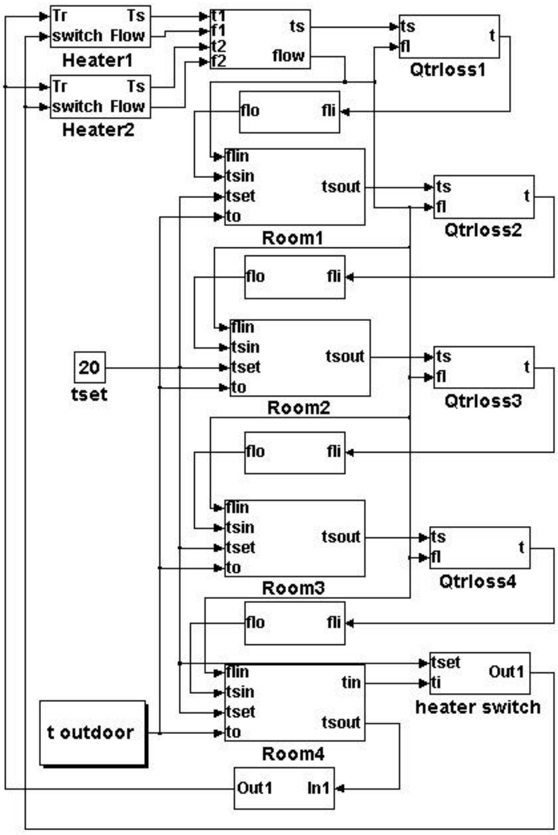

Figure 2. Building heating system model

Here the heat balance equation is written for each zone in the form of equation (5) for the heat flow between zones *j* and *k*. However, even when the entire building is kept at the same temperature, multizone analysis becomes necessary if the spatial distribution of heat gains is too non-uniform. Consider, for example, a building with large windows on the north and south sides, during a sunny winter day when the gains just balance the total heat loss. Then neither heating nor cooling would be required, according to a one-zone analysis. But how can the heat from the south get to the north?

The heat flow is the product of the heat transfer coefficient and the temperature difference, as in Equation (4). Temperature differences between occupied zones are small, usually not more than a few °C; otherwise there would be complaints about comfort. The heat transfer coefficients between zones are often not sufficiently large for effective redistribution of heat, especially if there are walls or partitions.

The model of room is shown in Figure 3.

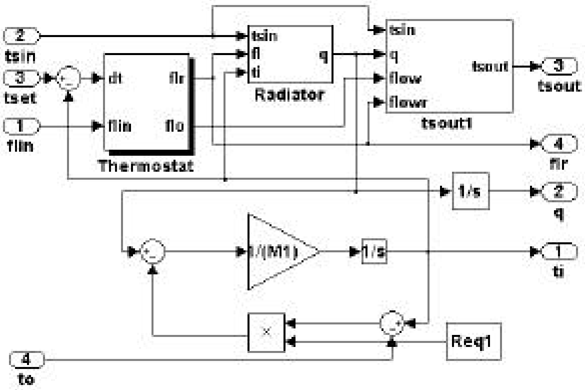

Figure 3. Room heating model

There are four rooms in this building. Water flow in radiator is controlled by separate controller in each room. The heat transferred to the room by radiator depends on the water mass flow, building water supply temperature, indoor and outdoor temperatures. The same amount of heat may be transferred by increasing water supply temperature and decreasing water mass flow. Consumed energy dependence on supply water temperature is shown in Figure 4.

The lower supply water temperature the less energy is used for heating because heat loss in pipes connecting main heating boiler and radiators is reduced. In this case, the minimum water supply temperature is restricted by weather (outdoor temperature, humidity, strength and direction of wind). The radiator performance depends on supply water temperature and may be insufficient to maintain desirable indoor temperature in the case where water supply temperature is reduced. As shown in Figure 4, minimum water supply temperature is 50°C when outdoor temperature is +5°C, 55°C when outdoor temperature is -5°C and 60°C when outdoor temperature is -15°C.

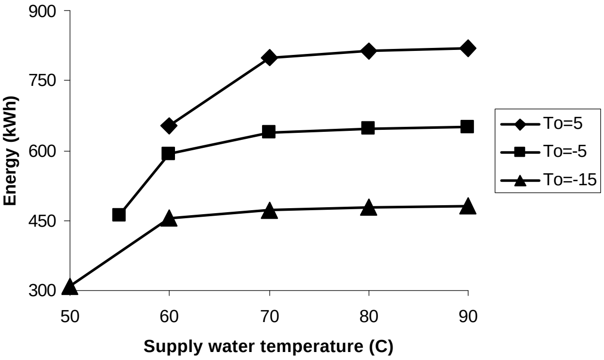

Figure 4. Dependence of heating energy on the temperature on supplied water

In order to minimize power consumption for heating it is necessary to choose water supply temperature for maintaining desirable indoor temperature. Maximum water flow corresponds to optimal water supply temperature. Temperature control in the main boiler is based on the following feature:

$$\begin{aligned} T_b[n+1] &= T_b[n] + \\ &\operatorname{sgn}(m_{t1} - m_{\max})\frac{1+\operatorname{sgn}(m_{t1} - m_{\max})}{2}\Delta T_b + \\ &\operatorname{sgn}(m_{\max} - m_{t2})\frac{1-\operatorname{sgn}(m_{\max} - m_{t2})}{2}\Delta T_b. \end{aligned} \qquad (18)$$

where: T_b water temperature in boiler;

$m_{\max}$ – water flow in radiator with maximum flow;

m_{t1}, m_{t2} – thresholds.

Simulation of the application of such control system was performed. Variation of outdoor temperature is shown in Figure 5.

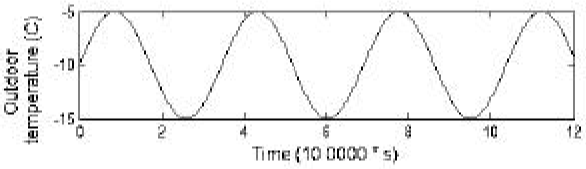

Figure 5. Outdoor temperature

Such diurnal change of outdoor temperature corresponds to the most severe conditions and occurs infrequently. The changes of indoor temperature are shown in Figure 6.

Indoor temperature $T_i = 18°C$ at the start of simulation. Indoor temperature set point $T_{reff} = 20°C$.

The water flow m_i in radiators *i* are shown in Figure 7.

The maximum water flow (the fourth room) sometimes differs from its maximum value due to inertness of heating system. In this case, heat energy accumulated in water is used. The fuel for water heating in boiler is not used in these time periods and water temperature in boiler and the temperature of water supplied to the radiators decreases. Variation of supplied water temperature is shown in Figure 8.

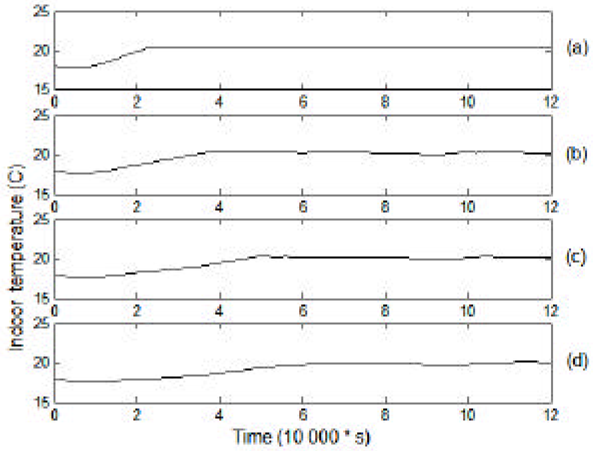

Figure 6. Indoor temperature: the first room (a), the second room (b), the third room (c), the fourth room (d)

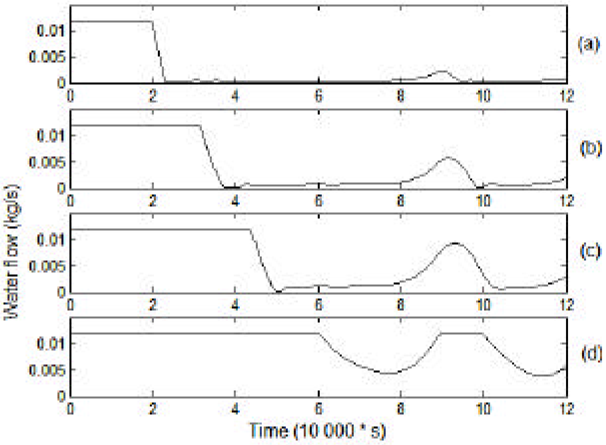

Figure 7. Water flow in the first room radiator (a), in the second room radiator (b), in the third room radiator (c), in the fourth room radiator (d)

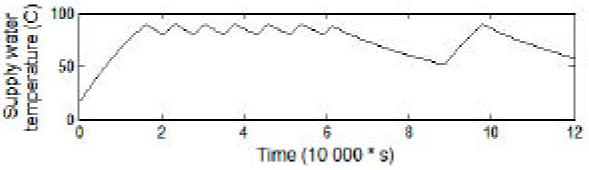

Figure 8. Supply water temperature

Supply water temperature is decreased when water flow threshold in radiator with maximum water flow value is exceeded. Influence of the heating system inertness is more evident when outdoor temperature changes faster as shown in Figure 9.

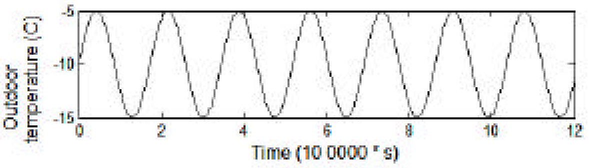

Figure 9. Outdoor temperature

The water flow m_i in radiators i in this case is shown in Figure 10.

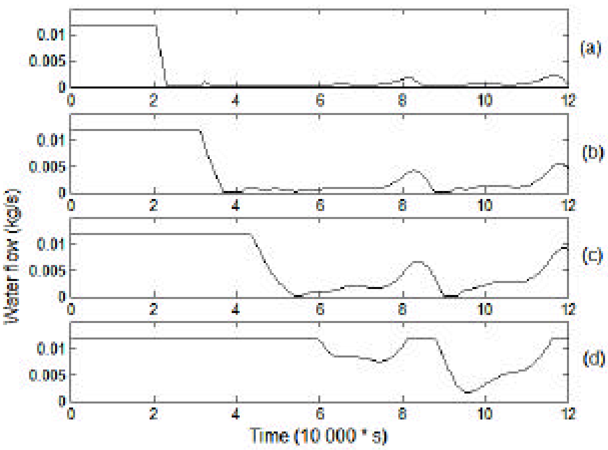

Figure 10. Water flow in the first room radiator (a), in the second room radiator (b), in the third room radiator (c), in the fourth room radiator (d)

Time periods, when the maximum water flow level (the fourth room) is less than threshold m_{t1} are longer in this case and time periods when fuel is not used increase. Due to this energy consumption in heating system does not increase.

Energy consumption in the heating system is shown in Figure 11. In both cases simulation starts when indoor temperature $T_i = 18^\circ C$ and indoor temperature set point $T_{reff} = 20^\circ C$.

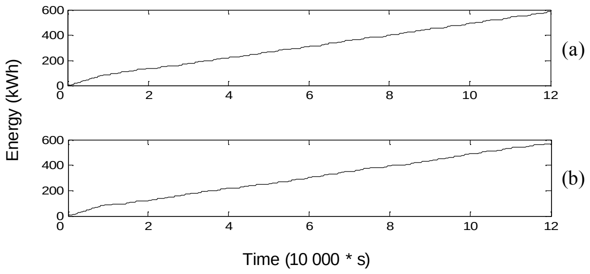

Figure 11. Consumed energy: outdoor temperature changes once a day (a), twice a day (b)

Heating system consumes 586.7 kWh energy, when outdoor temperature changes once a day (Figure 5) and 582.3 kWh, when outdoor temperature changes twice a day (Figure 9). In the case when supply water temperature does not change and outdoor temperature changes once a day, the heating system consumes 638.2 kWh energy.

4. Conclusions

The heat loss of building depends on building heat loss factor, air infiltration rate and heat loss in the heat supply system. These parameters are calculated using heating balance model. Such a model was developed and used for heating system design.

A control system which enables to regulate the heating installation of a building with continuous occupation is suggested. The system consists of autonomous water flow controllers in radiators and water temperature controller in boiler. Water temperature in the boiler is changed when maximum water flow in radiator exceeds some boundaries.

The system is simple. It does not require a computer with high calculating power and can be implemented on computers that concurrently solve other tasks.

Due to autonomous flow controllers the system measures only indoor temperatures in the rooms. All other disturbances are compensated automatically.

Due to autonomous flow controllers the system can be easily expanded.

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