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IMAGE PROCESSING ALGORITHM FOR PLOTTING ISOCLINICS IN PHOTOELASTIC ANALYSIS

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Abstract. The calculation of the precise stress field from the experimental photoelastic images requires the determination of the isoclines for various directions of the vector of polarisation. For solution of this problem an image processing optimisation algorithm utilising the concept of radial neighbours of a pixel is proposed.

1. Indroduction

The photoelastic method utilises the phenomenon of stress (load) induced birefringence [1, 2]. The fringe order is determined by placing the birefringent model in a polariscope. The standard plane polariscope consists of the light source and a pair of polarisers, termed as polariser and analyser, with crossed polarisation axes on either side of the model (Figure 1) [3, 4].

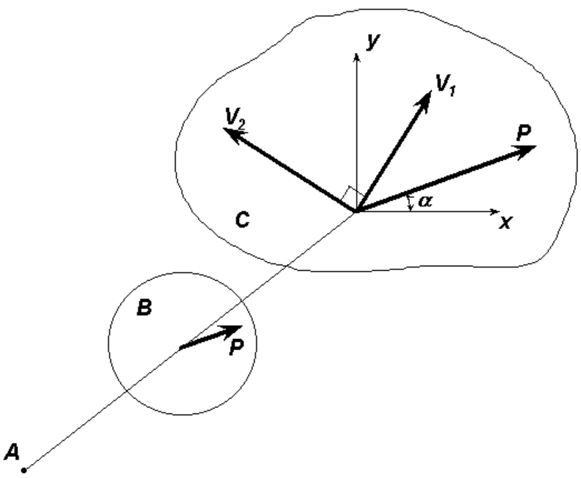

Figure 1. The schematic structure of a plane polariscope: A – the light source; B – polarizer (analyser is not shown); C – stressed specimen; *x, y* – axes of the orthogonal Cartesian frame; $\{P\}$ - the vector of polarisation; $\{V_1\}$, $\{V_2\}$ - the directions of the principle stresses.

The produced isochromatics can be classified to isoclinic lines and isochromatic fringes. Isoclinic lines provide information on the direction of principal stresses throughout the model. Isochromatic fringes provide the information on the constant difference of principal stresses. The standard plane polariscope shows both the isoclinics and isochromatics intertwined. Visualisation of isochromatics alone can be achieved by using a circular polariscope.

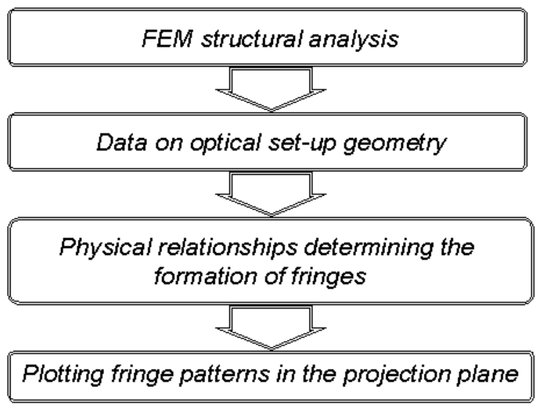

Figure 2. Schematic diagram representing the formation of fringe patterns in the projection plane.

It can be noted, that the hybrid experimental – numerical techniques [5, 6] iterate the scheme described in Figure 2 until a satisfactory correlation between the reconstructed pattern of fringes and the experimental data is achieved.

The object of this paper is to advance the fringe pattern plotting procedures. Specifically, photoelastic analysis requires identification of the centers of isoclinics what is necessary for subsequent reconstruction of stresses. It is clear that higher accuracy of the localisation of the fringe centers will result in better reconstruction of the whole field of stresses in the analysed specimen.

2. Construction of the digital photoelastic images

The eigenmodes for the structure in the state of plane stress are calculated by using the displacement formulation common in the finite element analysis [7, 8].

It is assumed that the structure performs high frequency vibrations according to the eigenmode (the frequency of excitation is about equal to the eigenfrequency of the corresponding eigenmode and the eigenmodes are not multiple). The vibrations of the structure are registered stroboscopically when the structure is in the state of extreme deflections according to the eigenmode. In this case the nodal stresses for the eigenmode (the eigenmode of stresses) are obtained, which are further used for the calculation of the photoelastic images.

If σ_x, σ_y, τ_{xy} are denoted as the components of the stresses in the problem of plain stress, then the principal stresses σ_1, σ_2 are calculated as the eigenvalues of the following matrix:

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}, \tag{1}$$

and the normalised eigenvectors of this matrix $\{V_1\}$, $\{V_2\}$ are assumed to represent the directions of the principal stresses.

The vector of polarisation is assumed to be given as:

$$\{P\} = \begin{Bmatrix} \cos\alpha \\ \sin\alpha \end{Bmatrix}, \tag{2}$$

where α is the angle of the vector of polarisation with the x axis shown in Figure 1.

Then the intensity in the photoelastic image of the plane polariscope (isoclinics and isochromatics intertwined) is calculated as:

$$I = \left(\left(\{V_1\}\cdot\{P\}\right)\left(\{V_2\}\cdot\{P\}\right)\sin C\left(\sigma_1 - \sigma_2\right)\right)^2, \tag{3}$$

where C - the constant dependent on the thickness of the analysed structure in the state of plane stress and on the material from which it is produced [1, 9].

The intensity of the photoelastic image for the circular polariscope (isochromatics) is calculated as:

$$I = \left(\sin C\left(\sigma_1 - \sigma_2\right)\right)^2, \tag{4}$$

and the intensity of the isoclinics pattern is calculated as:

$$I = \left(\left(\{V_1\}\cdot\{P\}\right)\left(\{V_2\}\cdot\{P\}\right)\right)^2. \tag{5}$$

The latter image can not be obtained directly from the experimental investigations but is required for the determination of the stress field. It can be obtained from the two experimental images (of the plane polariscope and of the circular one) as a result of image processing.

3. Algorithm for the construction of the digital image of isoclines

For the determination of the stress field the directions of principal stresses determined by (5) are necessary. For this purpose the lines corresponding to the darkest points of those images for a number of values of α are to be plotted on a single image. For the solution of this problem an image processing algorithm is developed.

The concepts of 8-neighbours, 4-neighbours and diagonal neighbours of a pixel are common in computer graphics and image. Here this concept is on a circle of a given radius surrounding the analysed pixel. Points $p_2, \ldots, p_9$ are radial 8-neighbours of the pixel p_1. Those pixels are denoted as a set:

$$N_8(p_1) = \{p_2, \ldots, p_9\}. \tag{6}$$

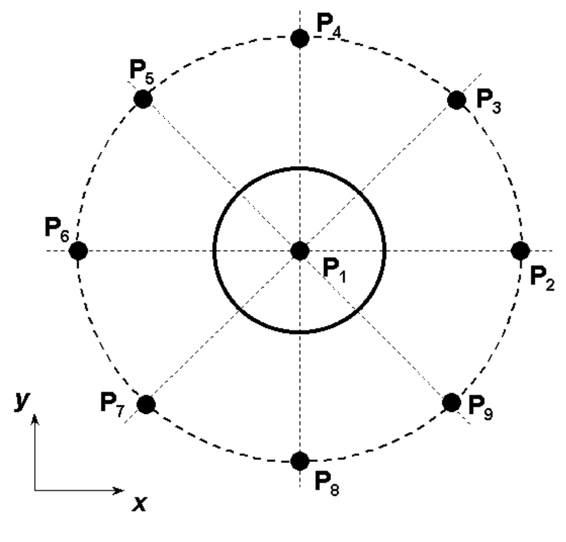

Figure 3. The pixel and its radial neighbours

Analogously, the points p_2, p_4, p_6 and p_8 are radial 4-neighbours of the pixel p_1:

$$N_4(p_1) = \{p_2, p_4, p_6, p_8\}; \tag{7}$$

and points p_3, p_5, p_7 and p_9 are radial diagonal neighbours of the pixel p_1:

$$N_D(p_1) = \{p_3, p_5, p_7, p_9\}. \tag{8}$$

The values of intensities given by eq.(5) are calculated for a given pixel and its radial 8-neighbours.

The calculations are performed for a sequential number of values of the local coordinates inside the currently analysed finite element. The spatial orthogonal Cartesian coordinates of these points can be calculated using the shape functions of the analysed finite element. Afterwards, the procedure of perspective projection is applied as a result of which the coordinates of the point in the projection plane X and Y are obtained. The reconstructed digital image consists of the matrix of pixels where the columns are indexed from 0 to m_x and the rows – from 0 to m_y . Thus the point (X, Y) is mapped to the pixel (i_x, i_y):

$$i_x = \mathrm{round}\left(\frac{X - X_{\min}}{X_{\max} - X_{\min}} m_x\right),$$
$$i_y = \mathrm{round}\left(m_y - \frac{Y - Y_{\min}}{Y_{\max} - Y_{\min}} m_y\right), \tag{9}$$

where the subscripts min and max indicate the minimum and maximum values of the corresponding quantities.

The rounding operation in eq. (9) can distort the quality of the reconstructed image. Therefore, the shift to the coordinates of the center of the corresponding pixel (i_x, i_y) is introduced:

$$\overline{X} = i_x \frac{X_{\max} - X_{\min}}{m_x} + X_{\min},$$
$$\overline{Y} = Y_{\min} - \left(i_y - m_y\right)\frac{Y_{\max} - Y_{\min}}{m_y}. \tag{10}$$

The spatial co-ordinates of the center of the pixel are found from the values of $\left(\overline{X}, \overline{Y}\right)$ and the data related with the perspective projection. Further, the usual calculations of the plotting procedure are performed.

In our problem previously described calculations are to be performed not only for a given point, but also for its radial 8-neighbours, that is for the points:

$$\left(i_x + r, i_y\right),\ \left(i_x + \frac{r}{\sqrt{2}}, i_y - \frac{r}{\sqrt{2}}\right),\ \left(i_x, i_y - r\right),$$
$$\left(i_x - \frac{r}{\sqrt{2}}, i_y - \frac{r}{\sqrt{2}}\right),\ \left(i_x - r, i_y\right),$$
$$\left(i_x - \frac{r}{\sqrt{2}}, i_y + \frac{r}{\sqrt{2}}\right),\ \left(i_x, i_y + r\right),$$
$$\left(i_x + \frac{r}{\sqrt{2}}, i_y + \frac{r}{\sqrt{2}}\right).$$

The co-ordinates of those points are used in eq. (10) instead of (i_x, i_y) and further calculations of the plotting procedure are performed.

Let I_i denote the intensity of point p_i. If the intensity of the pixel is not greater than the intensities of the radial 4-neighbours, that is, if:

$$\begin{cases} I_1 \le I_2 \\ I_1 \le I_4 \\ I_1 \le I_6 \\ I_1 \le I_8 \end{cases}; \tag{11}$$

or if the intensity of the pixel is not greater than the intensities of the radial diagonal neighbours, that is, if:

$$\begin{cases} I_1 \le I_3 \\ I_1 \le I_5 \\ I_1 \le I_7 \\ I_1 \le I_9 \end{cases}, \tag{12}$$

then it is considered that the point p_1 belongs to the isocline. The radius of the radial neighbours is chosen on the basis of numerical experiments (it is to be a small quantity usually equal to about several inter-pixel distances).

4. Numerical results

The rectangular cantilever plate with fixed edge in the state of plane stress is analysed.

The reconstructed image for the circular polariscope (isochromatics) is shown in Figure 4.

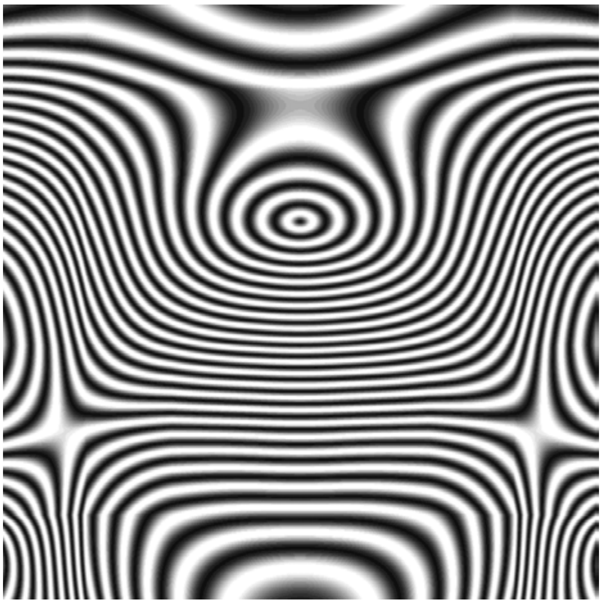

Figure 4. Isochromatics (the image produced by the circular polariscope) for the third eigenmode

The images of the isoclines when $\alpha = 0$ and the corresponding image for the plane polariscope (isoclinics and isochromatics intertwined) are presented in Figure 5 and Figure 6.

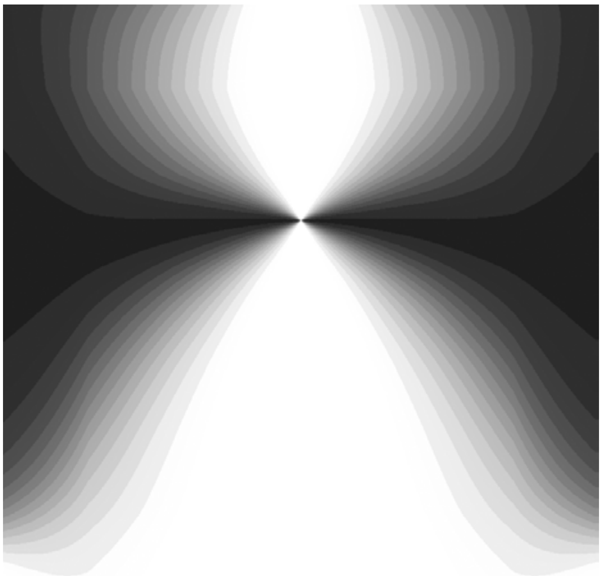

Figure 5. The image of the isoclines of the third eigenmode at $\alpha = 0$

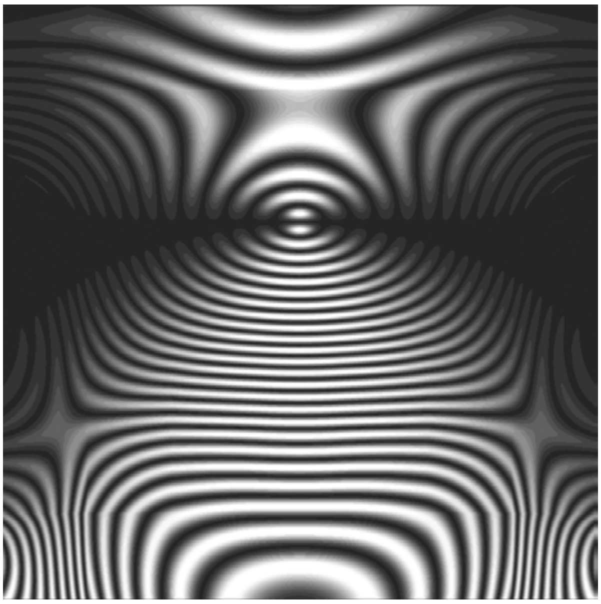

Figure 6. Isoclinics and isochromatics intertwined for the third eigenmode at $\alpha = 0$

The corresponding images at $\alpha = \pi/8$ are shown in Figure 7 and Figure 8.

The lines of the principal directions of the stresses corresponding to the darkest parts from the images of the isoclines (pattern of isoclinics) are shown in Figure 9. They were obtained by plotting the points of the images which are darker than a given threshold value.

The same lines obtained as a result of the proposed optimisation algorithm of image processing type are shown in Figure 10.

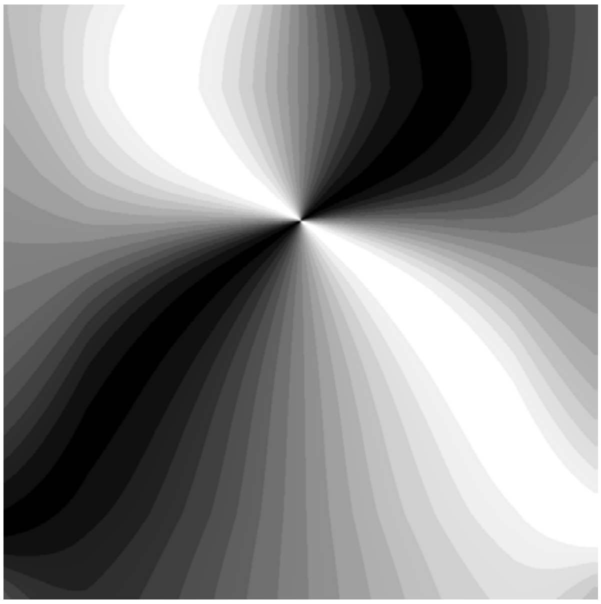

Figure 7. The image of the isoclines of the third eigenmode at $\alpha = \pi/8$

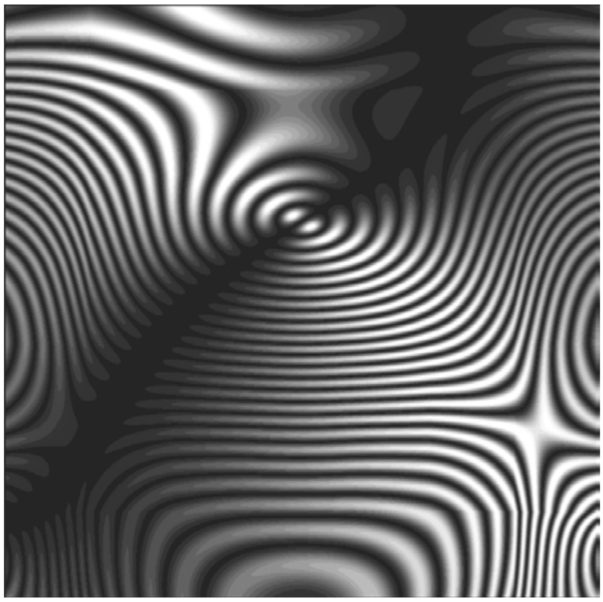

Figure 8. Isoclinics and isochromatics intertwined for the third eigenmode at $\alpha = \pi/8$

5. Concluding remarks

The calculation of the precise stress field from the experimental photoelastic images requires the determination of the isoclines for various directions of the vector of polarisation. Image processing algorithm utilising the introduced concept of radial neighbours of a pixel is proposed for the solution of this problem.

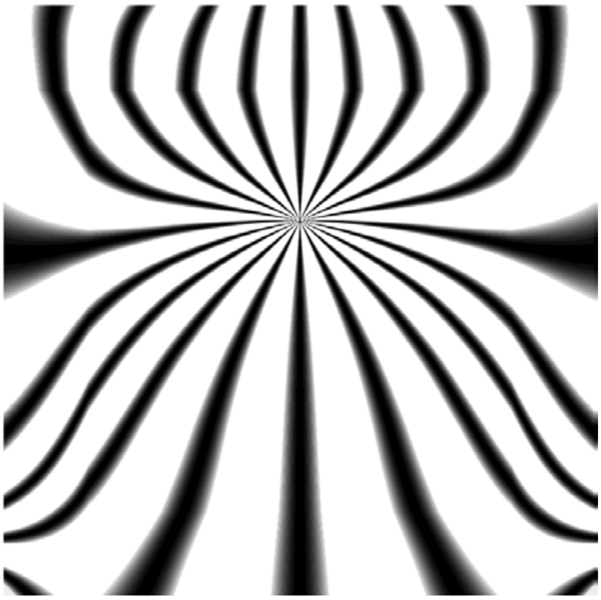

Figure 9. The lines of the principal direction of the stresses for the third eigenmode at $\alpha = (i-1)\pi / 20$, $i = 1, \ldots, 10$

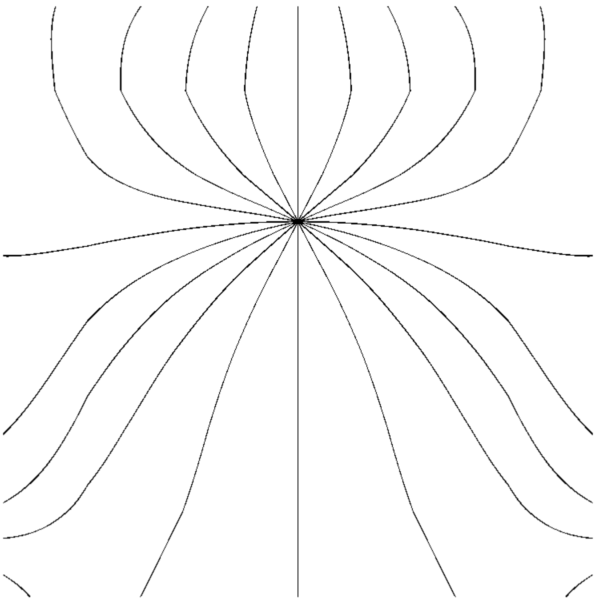

Figure 10. The reconstruction of the centers of isoclinics as a result of application of the proposed algorithm

6. References

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