Design of Fractional Verhulst Model for Displacement Prediction of Landslide Based on the Optimization of Beetle Antennae Search Algorithm

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Landslides significantly impact economic development and public safety. Aiming at the problem of insufficient prediction accuracy of the displacement data series of the traditional grey Verhulst model, this paper proposes a fractional Verhulst model optimized using the beetle tentacle search algorithm. First, based on the grey Verhulst model, a fractional order operator is introduced to accurately adjust the magnitude between cumulative values, constructing a fractional order-based grey Verhulst model. Expanding the accumulative order range improves prediction performance. Second, the fractional operator is optimized. The beetle antennae search algorithm finds the optimal fractional order between 0 and 1 in the grey Verhulst model, minimizing average relative error. Finally, using Heifangtai landslide group displacement data from Gansu Province, simulation experiments verified that the model has higher fitting accuracy and prediction effect than the traditional grey Verhulst model, Huang’s improved Verhulst model, GM (1,1) model, cubic exponential smoothing model, and DGM (2,1) model. The average relative error is 2.949 %. Results show that the beetle antennae search algorithm optimized fractional order grey prediction model significantly improves fitting and prediction effect on data. The optimized fractional Verhulst model is more suitable for predicting landslide displacement deformation.

**KEYWORDS:** Landslide monitoring, Fractional Verhulst model, beetle antennae search algorithm, Heifangtai landslide.

### 1. Introduction

China is a country that has been seriously affected by natural geological disasters and suffered heavy losses in all aspects. According to statistics, nearly 300,000 potential geological hazards have been identified across China, threatening the safety of some 20 million people and their property. In recent years, catastrophic geological disaster events have frequently occurred in China. According to news reports, 7,840 geological disasters occurred nationwide in 2020, up 26.84 percentage points from 2019. Among them, landslides accounted for 4,810, or 61.35% of the total geological disasters, ranking first. The severe losses caused by landslide disasters are reflected not only in threats to the personal and property safety of the people but also in the resistance to the development of the country’s economy. In particular, in crucial transportation, industrial construction areas, and population activities, once a landslide occurs, it will be a considerable loss, sometimes more devastating, to industry, agriculture, and people’s lives and properties. Therefore, it is of tremendous research significance to monitor the deformation and displacement of landslides and to accurately predict their future deformation trends [26].

With the continuous development of landslide monitoring technology in China, landslide monitoring and early warning work has gradually transitioned from manual to intelligent. However, there are still problems, such as the high cost of monitoring systems, the inconvenience of maintenance, and the need for universality. The technology cannot be promoted nationwide, and monitoring systems can only be developed according to local geological characteristics. Each detection method targets different application environments and scenarios, and the results are not guaranteed optimal. Only by selecting suitable monitoring methods and prediction models can it provide accurate warning and prediction of mountain landslides. In the past fifty years, the GPS method of monitoring displacement has achieved comprehensive coverage in landslide monitoring, which monitors the three-dimensional continuous displacement of the landslide surface with high accuracy. Therefore, the time series of landslide deformation displacement collected by GPS provides the basic data for landslide warning, establishing landslide prediction models, and eventually predicting the time of landslide occurrence [2]. At present, the methods for predicting landslide displacement mainly include machine learning methods and traditional mathematical modeling methods. For example, Han et al. [4] used variational modal decomposition and deep confidence neural network model to predict the displacement of the Bajijabao landslide in the Three Gorges Reservoir area; Jiang et al. [6] used an optimized differential evolutionary-support vector machine model to predict landslide displacement, which provided a new idea for non-linear methods; Jiang et al. [7] combined LSTM neural network and
SVR algorithm to the prediction of the periodic term of landslide displacements. To address the uncertainty in the process of landslide displacement prediction, Wang et al. [21] used exponential smoothing (DES) to predict the linear part of landslide displacement change and Bayesian deep neural network (BDNN) to predict the non-linear part of residuals, constructing a new framework for landslide displacement probability prediction: DES-BDNN. Although machine learning and artificial intelligence methods have better non-linear fitting effects, they require more data and optimization. Traditional mathematical modeling methods, such as the grey Verhulst model, the GM (1,1) model, DGM (2,1) model, and other grey prediction models [16]. Grey prediction theory deals with uncertain systems with partly known and partly unknown information. Only need a small sample of data to model landslide displacement [30]. It makes them more advantageous for areas with little information and potential unexplored landslides. Qiu et al. [18] used the GM (1,1) model in grey theory to predict ancient landslides. The DGM (2,1) grayscale model does not need to accumulate generated sequences and directly uses the original sequences [1]. The Verhulst model harshly selects early warning indicators, requiring many trial calculations to select appropriate ones. The grey Verhulst model obtains event change laws through analyzing and processing known data, adapting well to landslide complexity and monitoring limitations. Li et al. [11] first combined the twin support vector regression and the Hausdorff derivative operator, then proposed a new grey prediction model, achieving excellent results in predicting the displacement of the Bazimen landslide in China’s Three Gorges Reservoir area. Huang et al. used the reciprocal sequence (RS) of the accumulative generation operation (AGO) to construct identification parameters for the grey Verhulst model [5]. This method effectively solves problems of initial value optimization and parameter misplacement replacement in the model.

In practical problems, there is a problem of inequivalence and randomness between the original data. Fractional calculus can better explore the object’s essence and flexibly solve complex problems than integer order. The fractional grey system model combines both advantages, revealing the grey system's development law more deeply. With the rapid development of artificial intelligence algorithms, optimizing the order of the fractional Verhulst model is essentially an optimization process. Relevant scholars have proposed using intelligent algorithms to optimize the parameters of the fractional grey forecasting model. Wang et al. [22] proposed a particle swarm optimization (PSO) algorithm to determine the optimal parameters of the FGRM (1,1) model. Shalaby et al. [19] used the GWO algorithm to adjust FOPID parameters. Li et al. [11] improved the standard Salp swarm algorithm (SSA) by introducing Levy’s flight (LF) strategy and chaotic local search (CLS) strategy and then solved the parameters of the new gray prediction model. Li et al. [12] used the PSO algorithm to solve the parameters of the grey multivariate prediction model. Zhu et al. [33] used the Marine predator algorithm to solve the hyperparameters of the CFNGBM (r, N) model. Wang et al. [23] designed a new surrogate-assisted evolutionary algorithm (RESAPSO) to optimize the hyperparameters of the LSTM network based on the surrogate model. The Beetle Antennae Search (BAS) algorithm is a bionic intelligent algorithm proposed by scholars inspired by the foraging of longicorn beetles. The algorithm has two advantages: first, it does not need to know the specific form of the objective function and gradient information; second, it can reduce the amount of computation and shorten optimization time. These two characteristics make it very suitable for Verhulst model order rectification. Compared with the PSO algorithm, the BAS algorithm has more advantageous in terms of jumping out of the local extremes and convergence speed. Compared with the genetic algorithm (GA), the BAS algorithm does not require binary encoding, and the calculation speed is faster. Compared with the bat algorithm (BA) and artificial bee colony (ABC) algorithm, the BAS algorithm has higher efficient and less complex. In addition, due to the lower in terms of time and space complexity, the efficiency of the BAS algorithm is higher than that of most swarm intelligence algorithms. Zhang et al. [31] used the BAS optimization algorithm with low computational effort and fast computational speed to optimize the parameters of the random forest regression (RF) algorithm, thereby improving the accuracy of the dam horizontal displacement prediction model. Zhang et al. [32] effectively realized the parameter tuning of the PID controller and fractional order PID controller by using the BAS algorithm.
In summary, this paper takes the Hefangtai landslide group in Gansu Province as an example, analyzes its displacement variation law, and adds the fractional order operator based on the Grey Verhulst model to precisely adjust the order of magnitude between the cumulative numbers, to construct the Grey Verhulst model based on fractional order. The beetle antennae search algorithm precisely adjusts the order to optimize model accuracy. Comparative analysis with the traditional grey Verhulst model, GM (1,1) model, Cubic Exponential Smoothing algorithm model, DGM (2,1) model, and the Huang optimized grey Verhulst model \[5\] shows the optimized model has a high fitting and prediction effects on landslide data. This paper provides a reference for early warning and prevention of landslide disasters.

2. Experimental Details

2.1. Landslide Prediction Forecasting Models

The study of landslide hazard warning and prediction is a cutting-edge topic worldwide and a problem humanity has constantly been exploring but has yet to solve perfectly. Due to the sudden, random, and non-linear nature of landslide formation, landslides are highly complex non-linear dynamical systems. Despite rapid scientific and technological development, landslide prediction remains challenging. Most consider early research, the beginning of the work on landslide prediction. After decades of research by scholars from various countries, landslide prediction theory has developed considerably. The main types are deterministic forecasting models, non-linear forecasting models, statistical forecasting models, and macroscopic forecasting models.

2.1.1. Model Selection

Deterministic models have strict mathematical expressions and deterministic mathematical relationships. Historical monitoring data curves of previous landslides are fitted to the models to derive the model parameters. The mathematical relationship between the historical data and the models determines prediction reliability. Representative models include the Saito model and the limit equilibrium method. Non-linear models apply non-linear scientific theories to complex landslide forecasting and prediction problems. Representative models include non-linear dynamics models and neural network models. To build prediction models, statistical models use statistical and mathematical theories to analyze the relationship between landslide deformation parameters and time. Typical methods include the grey model and the Verhulst model. Macroscopic forecasting models blend deterministic and statistical models, and they are hybrid models \[17\].

The traditional Saito method is based on creep theory and can, therefore, only be used to predict unobstructed earth slides on the leading edge, which is limited in scope. The non-linear kinetic model is based on non-linear kinetic theory and has some physical significance. Nevertheless, it also needs the following shortcomings: the specific expressions of anonymous functions, initial conditions, and process state variables cannot be specified. The neural network model has a powerful non-linear fitting ability and can be trained repeatedly by the computer to learn available landslide data to make predictions. However, its disadvantage is that it requires a large amount of data for training. At the same time, the actual situation can only collect a little landslide data, with the problem of the insufficient sample size of the original data. The design goal of this paper is to develop a landslide monitoring system in the direction of low cost, high precision, and intelligence. While ensuring the prediction effect, we should also consider the cost. Unlike the large number of computing resources required by artificial intelligence, the prediction model proposed in this paper is a lightweight model design. Therefore, this paper selects the grey Verhulst model, a statistical model, to predict landslide displacement. Fractional order theory will apply in the analysis example.

2.1.2. Grey Verhulst Model

Professor Deng Julong first proposed grey systems theory in the early 1980s, a discipline created by Chinese scholars to address the uncertainty and complexity of nature. The theory mainly extracts effective information through the transforming of partial known information. It is used to solve the problems of “small sample” and “information-poor,” which are difficult to solve by some methods. It mainly uses less data for modeling \[17\]. The grey prediction model is a part of the grey system theory and has an important position. The grey Verhulst model is a unique form of the GM (1,1) model, which has good prediction accuracy and applicability for problems with an
“S” shaped data series [10]. For example, Duan et al. [3] used a grey Verhulst model to predict CO2 emissions from coal combustion, which coincides with the concept of “carbon neutrality” proposed by China. A system with information between explicit and ambiguous is called a grey system, and a grey system is a practical problem that can be solved by knowing the least amount of information [9]. Grey systems theory refers to processes with any set of random variables that vary over time as grey processes. Most systems are complex with messy data, but correlations exist. The original data sequence needs to be processed and transformed to discover the relationship within it. This process is called grey series generation [29]. Differential equation models then predict future change patterns. The data series should have been sampled at equal intervals when modeling the grey Verhulst model. The data series should first be processed for first-order accumulative generation operation (1-AGO), which is to highlight the regularity of the original data series, and then modeled into the calculation. The simulated values after processing will be reduced by first-order accumulative reduction to finally arrive at the predicted values [20].

Suppose the original displacement data sequence is $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}$, calculate the 1-AGO sequence to obtain $X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\}$

where
\[
x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, \ldots, n
\]

Calculate the sequence of the generated mean value of consecutive neighbors to obtain $Z^{(1)} = \{z^{(1)}(1), z^{(1)}(2), \ldots, z^{(1)}(n)\}$ where
\[
z^{(1)}(k) = 0.5\left(x^{(1)}(k) + x^{(1)}(k-1)\right), k = 2, 3, \ldots, n
\]

The basic grey Verhulst model is:
\[
x^{(0)}(k) + az^{(0)}(k) = b\left(z^{(0)}(k)\right)^2, \quad k = 1, 2, \ldots, n
\]

From the above equation, it is obtained that
\[
x^{(0)}(k) = -az^{(0)}(k) + b\left(z^{(0)}(k)\right)^2 \Rightarrow x^{(0)}(k) - x^{(0)}(k-1) = -az^{(0)}(k) + b\left(z^{(0)}(k)\right)^2.
\]

According to the Newton-Leibniz formula, there are:
\[
x^{(1)}(k) - x^{(1)}(k-1) = \int_{k-1}^{k} \frac{dx^{(1)}(t)}{dt} dt.
\]

According to the geometric meaning of a definite integral, there are:
\[
z^{(1)}(k) = \frac{x^{(1)}(k) + x^{(1)}(k-1)}{2} \approx \int_{k-1}^{k} x^{(1)}(t) dt.
\]

The geometric meaning of the two definite integrals is shown in Figure 1.

**Figure 1**
Geometric curve of definite integrals

According to the above formula:
\[
\begin{pmatrix}
-\varepsilon^{(2)}(2) \\
-\varepsilon^{(3)}(3) \\
\vdots \\
-\varepsilon^{(n)}(n)
\end{pmatrix}
\begin{pmatrix}
\varepsilon^{(0)}(2) \\
\varepsilon^{(0)}(3) \\
\vdots \\
\varepsilon^{(0)}(n)
\end{pmatrix}
= \begin{pmatrix}
a \\
b
\end{pmatrix} = \begin{pmatrix}
x^{(0)}(2) \\
x^{(0)}(3) \\
\vdots \\
x^{(0)}(n)
\end{pmatrix}
\]

Get matrix $B$ and $Y$. $\hat{\alpha} = [a, b]$ as parameter estimators. Then obtain estimated parameter estimators through the least squares estimation method:
\[
\hat{\alpha} = \left(B^T B\right)^{-1} B^T Y.
\]

From Equations (4), (5), and (6) obtain the shadow equation of the Verhulst model:
\[
\frac{dx^{(i)}}{dt} + ax^{(i)} = b\left(x^{(i)}\right)^2.
\] (9)

Solve the shadow equation and discretize it to obtain the time response sequence:

\[
x^{(i)}(t) = \frac{ax^{(i)}(1)}{bx^{(i)}(1) + (a - bx^{(i)}(1))e^{at}}.
\] (10)

\[
\hat{x}^{(i)}(k+1) = \frac{ax^{(i)}(1)}{bx^{(i)}(1) + (a - bx^{(i)}(1))e^{ak}},
\] (11)

where \(\hat{x}^{(i)}(1) = x^{(i)}(1) = x(0)\)

Find the sequence \(\hat{x}^{(i)}\) according to \(\hat{x}^{(i)} = (k+1)\). Then use the first-order inverse accumulating generation operation (1-IAGO) to restore the predicted sequence \(\hat{X}^{(0)}\). The 1-IAGO is:

\[
\hat{x}^{(i)}(k+1) = \hat{x}^{(i)}(k+1) - \hat{x}^{(i)}(k), k = 1, 2, \ldots, n.
\] (12)

Finally, conduct an error test, where the residuals are:

\[
\varepsilon(k) = x^{(0)}(k) - \hat{x}^{(0)}, k = 2, 3, \ldots, n.
\] (13)

The relative error or Absolute Percentage Error (APE) is:

\[
\Delta_t = \frac{\left|\varepsilon(k)\right|}{x^{(0)}(k)}, k = 2, 3, \ldots, n.
\] (14)

The grey Verhulst model effectively describes and predicts saturated (S-shaped) state processes under minor sample conditions. It is commonly used to predict populations, biological reproduction, and product longevity. In actual problems, since much data tends to be close to the “S” shape, the grey Verhulst model can model it directly [13].

2.2. Fractional Verhulst Model

As humans continue exploring and studying the objective world, it is found that most dynamic systems are not of integer order. Therefore, using the theory of fractional calculus theory to model objects in the objective world is more accurate than using integer order. The calculation of fractional theory is more complicated to understand, so most research remains theoretical without breakthroughs. In recent years, with the progress of science and technology in various countries, fractional order has also moved from theory to practical engineering applications. Therefore, fractional theory modeling has unique advantages and great significance [34].

2.2.1. Fractional Accumulation Operator

**Definition 1.** Suppose \(X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}\) is the original data sequence, \(Y\) is the sequence operator, then \(X^{(0)}Y = \{x^{(0)}(1)y, x^{(0)}(2)y, \ldots, x^{(0)}(n)y\}\) is the first-order cumulative generating sequence, where

\[
x^{(0)}(k)y = \sum_{i=1}^{k}x^{(0)}(i), k = 1, 2, \ldots, n.
\] (15)

Then call \(Y\) the first-order accumulative generating operator of \(X^{(0)}\), denoted as 1-AGO.

**Theorem 1.** Suppose \(X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}\) is the original data sequence, \(\in \mathbb{R}\), then \(X^{(0)}rYr = \{x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n)\}\) is the cumulative generating sequence of order \(r\), where

\[
x^{(r)}(k) = \sum_{i=1}^{k}x^{(r-1)}(i), k = 1, 2, \ldots, n.
\] (16)

Then call \(Yr\) the accumulative generating operator of order \(r\) of \(X^{(0)}\), denoted as \(r\)-AGO.

1. When \(r\) is a positive integer,

\[
x^{(r)}(k) = \sum_{i=1}^{k} \frac{(k-i+1)(k-i+2)\cdots(k-i+r-1)}{(r-1)!}x^{(0)}(i), k = 1, 2, \ldots, n.
\] (17)

Integer order cumulative generating operator called \(X^{(r)} = X^{(0)}Yr = \{x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n)\}\) as \(X^{(0)}, r \in \mathbb{Z}^{+}\).

2. When \(r\) is a positive real number, according to the infinite integral definition of the Gamma function,

\[
\Gamma(z) = \int_{0}^{\infty} e^{-t}t^{z-1}dt.
\] (18)

Through the subsection integral method, there are
Therefore, the factorial formula is:

\[ \Gamma(z + 1) = \frac{1}{z} \Gamma(z) \]

The Gamma function generalizes the factorial to non-integer \( z \). Extending the accumulation operator (Equation (17)) to a positive real order, Equation (17) becomes:

\[ x^{(r)}(k) = \sum_{i=1}^{k} \frac{\Gamma(r + k - i)}{\Gamma(k - i + 1) \Gamma(r)} x^{(0)}(i), \quad k = 1, 2, \ldots, n. \]  (21)

This extends the integer order to the fractional order, called the fractional order accumulative generating operator \([28]\), \( r \in \mathbb{R}^+ \). Source coefficient of \( x^{(0)}(i) \) with \( x^{(0)}(i-1) \) in Equation (21) to verify the weight ratio of the new and old information. Among them

\[
\begin{align*}
 a_{ik} &= \frac{\Gamma(r + k - i)}{\Gamma(k - i + 1) \Gamma(r)}, \quad k = 1, 2, \ldots, n \\
 a_{i(k-1)} &= \frac{\Gamma(r + k - i + 1)}{\Gamma(k - i + 2) \Gamma(r)} \\
 a_{ik} &= \frac{\Gamma(r + k - i)}{\Gamma(k - i + 1) \Gamma(r)} = \frac{r + k - i}{k - i + 1} \frac{\Gamma(r + k - i)}{\Gamma(r) k - i + 1} \\
 &= \frac{1 - i + k}{r - i + k} = 1 + \frac{1 - r}{r - i + k}
\end{align*}
\]

\( a_{i(k-1)} \) is the original data sequence, \( x^{(0)}(i) \) is called the fractional Verhulst model, where \( X^{(r)} = \{ x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n) \} \) is the original data sequence, \( r \in \mathbb{R}^+ \), the order \( r \) cumulative generating sequence of \( X^{(0)} \) is \( X^{(r)} = \{ x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n) \} \), the sequence of generating the mean value immediately adjacent to \( X^{(r)} \) is called \( Z^{(r)} = \{ z^{(r)}(2), z^{(r)}(3), \ldots, z^{(r)}(n) \} \) then it is called

\[ x^{(r)}(k) + az^{(r)}(k) = b(z^{(r)}(k))^2 \]  (22)

the fractional Verhulst model, where

\[
\begin{align*}
 x^{(r-1)}(k) &= x^{(r)}(k) - x^{(r)}(k - 1) \\
 &= \sum_{i=r}^{k} \frac{\Gamma(r + k - i)}{\Gamma(k - i + 1) \Gamma(r)} x^{(0)}(i) - \sum_{i=r}^{k} \frac{\Gamma(r + k - i - 1)}{\Gamma(k - i) \Gamma(r)} x^{(0)}(i) \\
 z^{(r)}(k) &= \frac{x^{(r)}(k) + x^{(r)}(k - 1)}{2}, \quad k = 2, 3, \ldots, n
\end{align*}
\]  (23)

Here,

\[ \Gamma(r + k - i) = (r + k - i - 1), \quad \Gamma(k - i + 1) = (k - i)!, \quad \Gamma(r) = (r - 1)! \]

Using the least square method, estimate the model parameter column \( \hat{a} = [a, b]^T \).

\[
\begin{pmatrix}
 -z^{(r)}(2) \\
 -z^{(r)}(3) \\
 \vdots \\
 -z^{(r)}(n)
\end{pmatrix} \begin{pmatrix}
 -z^{(r)}(2) \\
 -z^{(r)}(3) \\
 \vdots \\
 -z^{(r)}(n)
\end{pmatrix}^2 = \begin{pmatrix}
 a \\
 b
\end{pmatrix} = \begin{pmatrix}
 x^{(r-1)}(2) \\
 x^{(r-1)}(3) \\
 \vdots \\
 x^{(r-1)}(n)
\end{pmatrix}
\]
where \( z^{(r)}(k), k = 2, 3, \ldots, n \) is the sequence of immediately adjacent mean generation; \( a \) and \( b \) are the parameters to be sought; 
\[
\hat{a} = \left[ a, b \right]
\]
Then the least square estimation of the parameter column \( \hat{a} \) of the fractional Verhulst model is: 
\[
\hat{a} = \left( B^T B \right)^{-1} B^T Y.
\]

**Definition 3.** Define
\[
\frac{dx^{(r)}(t)}{dt} + ax^{(r)}(t) = b\left[x^{(r)}(t)\right]^2
\]  
(24)
As the whitening equation of the fractional Verhulst model.

**Theorem 2:**
1. The time sequence of the model obtained by solving the whitening equation is
\[
\hat{x}^{(r)}(k + 1) = \frac{ax^{(r)}(1)}{bx^{(r)}(1) + (a - bx^{(r)}(1))e^{\eta k}}, \quad k = 1, 2, \ldots, n
\]  
(25)
2. Then, the prediction sequence of the original sequence is obtained by the fractional reduction operator:
\[
\hat{x}^{(0)}(k) = \left( \hat{x}^{(r)} \right)^{(r-r)}(k)
\]  
(26)
\[
= \sum_{i=0}^{k-1} (-1)^i \frac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)} \hat{x}^{(r)}(k-i)
\]  
(26)

**2.3. Order Optimization Based on the Beetle Antennae Search Algorithm**

When the fractional Verhulst model is used for prediction, it is necessary to optimize the order \( r \). The optimal order \( r \) is selected to minimize the average relative error. This paper uses the Beetle Antennae Search (BAS) algorithm for parameter optimization. BAS algorithm is an intelligent optimization algorithm proposed by Jiang et al. in 2017, a meta-heuristic algorithm [8]. As an effective tool for solving complex optimization problems, heuristic algorithms have been widely used in many research fields and engineering applications. For example, In machine learning and data mining, it is used for feature selection, parameter optimization and hyperparameter optimization. In the fields of UAV path planning and robot motion control, it is used to find feasible solutions quickly. In image processing and pattern recognition, it is used for clustering center optimization, sensor array layout optimization, etc. In the fields of power system planning and control, supply chain management, and engineering design optimization, it is used to solve complex optimization problems with high dimensionality. Compared with intelligent optimization algorithms such as the PSO algorithm, the perceptual ability of antennae can effectively reduce the probability of purposeless search and improve the convergence rate. Moreover, the BAS algorithm only needs a single individual to avoid the process of group optimization and significantly reduce the calculation. The algorithm’s main idea is that its tentacles have a perception function during foraging. Its tentacles can judge the next step direction based on odor strength when seeking a mate or food and effectively find the food position. The algorithm has simple steps, fewer setting parameters, and less computation than other optimization algorithms [24]. The BAS algorithm has significant low-dimensional target optimization advantages [14]. In this paper, the range of order is the real number between 0 and 1, which belongs to the one-dimensional optimization problem. Therefore, the BAS algorithm effectively solves the optimal solution of order. The optimization flow chart of the BAS algorithm is shown in Figure 2.

The pseudo-code for the fractional order based on BAS optimization is as follows:

**Algorithm 1.** Beetle Antennae Search optimizes fractional order algorithm

**Input:** Objective function \( f(X^{(r)}) \), where variable \( X^{(r)} = \{x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n)\} \), the maximum number of iterations \( N_{max} \)

**Output:** \( pBest, f(pBest) \)

1. Initialize algorithm-related parameters: landslide original data sequence \( X^{(0)} \), step length \( s \), step coefficient \( \eta \), tentacle length \( d \);
2. \( i = 1; \)
3. \( r = 0; \)
4. **while** \( i < N_{max} \) **do**
5. Generation of the random search direction vector \( \hat{b} \) according to Equation (29);
6. Search the left and right beetles of the beetle in the variable space according to Equation (30);
7. Updating the beetle state variable $x_g$ using Equation (30);
8. Calculate the r-order accumulation of the original data sequence $X^{(0)}$ according to Equation (16), to generate the sequence $X^{(r)}$;
9. Do the generated mean value of consecutive neighbors according to Equation (23) to get $Z^{(r)}$;
10. According to Equation (16), do first-order accumulation processing to get the 1-AGO operator;
11. Solve parameter column $\hat{a} = [a, b]^T$;
12. Solve the time response equation $\hat{x}^{(r)}(k)$ according to Equation (24);
13. Calculate the simulated value of $X^{(r)}$;
14. Reduce to obtain the simulated value of $\hat{X}^{(0)}$;
15. Calculate the average relative errors $f(r)$ and $f(gBest)$;
16. if $f(r) < f(gBest)$ then do
17. Update the optimal solution and its fitness value $x = pBest, f = f(pBest)$;
18. Update the beetle search step $s(g)$ using Equation (31);
19. end if
20. $i = i + 1$;
21. end while
22. return $pBest, f(pBest)$

Solving the optimal order $r$ of the fractional Verhulst model is equivalent to solving the problem of minimum average relative error:

$$\min f(r) = \frac{1}{n-1} \sum_{k=2}^{n} \frac{|x^{(0)}(k) - \hat{x}^{(0)}(k)|}{x^{(0)}(k)}, r \in \mathbb{R}^r$$

Set $\min f(r)$ as objective function and apply the BAS algorithm to determine the optimal value $r_0$, $r \in (0, 1)$.

In this paper, the initial parameters of the BAS algorithm are set as follows: initial the step size of searching $s(1) = 1$, step length coefficient $\eta = 0.95$, the fixed constant $c = 5$ represents the ratio between the step length $s$ and the distance $d$ between the two antennae of a longhorn beetle, the target space dimension is $k = 1$, the number of iterations is 50. The calculation steps of the optimal order based on the BAS algorithm are as follows:

1. Randomly initialize the position of the beetle population and the orientation of the beetle antennae, and then normalize them:

$$gBest = rand(k,1)$$

$$b = \frac{rand(k,1)}{\|rand(k,1)\|}$$

where $gBest$ is the initial optimal position, $rand()$ represents the random number of randomly generated 0-1, $k$ represents the size of the spatial dimension.
2 Create a spatial coordinate position equation based on the left and right tentacles of the longhorn beetle:

\[
\begin{align*}
\begin{cases}
x_g &= x_g + \frac{b}{2} \rightarrow \\
x_{rg} &= x_g - \frac{b}{2}
\end{cases}
\end{align*}
\tag{30}
\]

where, \(x_g\) represents the spatial position of the left antennae of the longhorn beetle at the \(g\) iteration. \(x_{rg}\) represents the spatial position of the left antennae of the longhorn beetle at the \(g\) iteration. \(x_g\) represents the spatial position of the mass center of the beetle in the \(g\) iteration. \(d\) represents the induction length between the beetle antenna.

3 According to the objective function, calculate the function values of the left and right antenna of the beetle, and move to the direction of the antenna with the smaller function value.

4 The equation for updating the spatial position of the longicorn is:

\[
x_{g+1} = x_g - s(g) \rightarrow \frac{b}{2} \text{ sign} \left[ f(x_g) - f(x_{rg}) \right]
\tag{31}
\]

\[
s(g + 1) = \eta s(g)
\tag{32}
\]

\[pBest = f(x_{g+1}).\]
\tag{33}

where \(s(g)\) represents the step length of the \(g\) iteration, \(f(x_g)\) and \(f(x_{rg})\) represent the function values corresponding to the left and right antenna of longicorn at the \(g\) iteration, \(\text{sign}(\ )\) represents the sign function, \(\eta\) is the step coefficient, \(pBest\) is the current location.

5 The average relative error of the fractional Verhulst model is calculated when \(r = pBest\).

a Calculate the \(r\) order cumulative generating sequence \(X^{(r)}\) of the original data sequence \(X^{(0)}\).

b Make adjacent mean generation processing obtains \(Z^{(r)}\).

c After the first-order cumulative reduction, \(X^{(r-1)}\) is obtained.

d Solving parameter column \(\hat{\alpha} = [a, b]^\top\).

e After solving, get the time response equation \(\dot{x}^{(r)}(k)\).

f Calculate the simulated value of \(x^{(r)}\).

g Restore the simulated value \(\hat{x}^{(0)}\).

h Whether the average relative error \(f(pBest)\) is less than \(f(gBest)\) is calculated, if satisfied, \(x = pBest, f = f(pBest)\) Otherwise, the first step is returned for normalization.

6 Iteration completed, output \(f(pBest)\) and \(pBest\). That is when the order \(r = pBest\) is the optimal order, the average relative error of the fractional Verhulst model is the smallest.

2.4. Evaluation Criteria

To intuitively measure and evaluate the model’s prediction performance, this paper uses the following:

Root Mean Squared Error (RMSE) to evaluate model interpretability. Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) to evaluate model prediction accuracy. The coefficient of determination (R-squared, R2) to evaluate model goodness of fit. The smaller the MAE, RMSE, and MAPE, the better the model performance and accuracy. The closer R2 is to 1, the better the model fit.

The calculation formula is as follows:

\[
\text{RMSE} = \sqrt{\frac{1}{n-1} \sum_{i=2}^{n} (x^{(0)}(k) - \hat{x}^{(0)}(k))^2}, k = 2, 3, \ldots, n
\tag{34}
\]

\[
\text{MAE} = \frac{1}{n} \sum_{i=2}^{n} |x^{(0)}(k) - \hat{x}^{(0)}(k)|
\tag{35}
\]

\[
\text{MAPE} = \frac{100}{n} \sum_{i=2}^{n} \frac{|x^{(0)}(k) - \hat{x}^{(0)}(k)|}{x^{(0)}(k)}
\tag{36}
\]

\[
R^2 = \frac{\sum_{i=1}^{n} (x^{(0)}(k) - \hat{x}^{(0)}(k))^2}{\sum_{i=1}^{n} (\bar{x}^{(0)}(k) - x^{(0)}(k))^2},
\tag{37}
\]

where \(\bar{x}^{(0)}(k) = \frac{1}{n} \sum_{i=1}^{n} x^{(0)}(k), k = 1, 2, \ldots, n\).
3. Results

There are many types of geological disasters in China, among which landslide disasters occupy the first place. In 2018, Lu Hao, Minister of the Ministry of Natural Resources, emphasized that current protection against geological disasters is mainly divided into two issues: “Where is the hidden danger?” and “What time may happen?” \cite{25}. Therefore, in order to protect the safety of the people, it is urgent to accurately monitor the deformation of landslides, excavate and analyze existing data, and scientifically predict when the landslide may occur, it is imperative.

Professor Yan \cite{27} believed that the evolution of landslides had a process of breeding, development, occurrence, and extinction. The Verhulst model predicted organisms’ reproduction, growth, and death. The two had a high similarity. Therefore, the Verhulst model was applied to predict landslide displacement time, laying the foundation for landslide disaster prediction. This paper introduces the fractional operator into the grey Verhulst model, and the beetle antennae search algorithm optimizes the order to minimize the average relative error.

3.1. Performance Analysis of the BAS Algorithm in Benchmark Test Function

In order to verify the effectiveness and superiority of the performance of the BAS algorithm, a variety of benchmark functions from the CEC2013 benchmark suite \cite{15} (see Table 1) were used to test the BAS algorithm, the PSO algorithm, the GOA algorithm, the WOA algorithm and the GA algorithm. The population size of each algorithm was set to 100, the maximum number of iterations was 1000, the upper and lower bounds were set to [-100, 100], the search times were 100 times; the Inertia factor \( w = 0.6 \); the learning factors \( c_1, c_2, c_3 \) are 2. To avoid uncertainty in the operation of the algorithms, the five algorithms were tested independently 100 times. The average and standard deviation of the results are shown in Table 2.

Table 2 shows that the average value (AVG) and standard (STD) deviation of the BAS algorithm are significantly better than those of the GA, WOA, PSO, and GOA optimization algorithms. Other algorithms such as GOA and WOA have more stable results, but the optimal values are not optimal. The performance of PSO algorithm is more balanced. The GA performed relatively poorly. For the Rosenbrock function, the BAS algorithm converged to the global optimal solution, while the GOA algorithm fell into a local optimal solution. For the multimodal benchmark functions of Rastrigin’s function, Ackley’s function, and Griewank’s function, the BAS algorithm was able to find the global optimal region by jumping out of most local optima, although it was more difficult. Overall, the BAS algorithm has a stronger global search capability than other algorithms. Therefore, the BAS algorithm

<table>
<thead>
<tr>
<th>Function name</th>
<th>Expression</th>
<th>Peak type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>( f_1(x) = \sum_{i=1}^{n} x_i^2 )</td>
<td>unimodal</td>
</tr>
<tr>
<td>Rosenbrock</td>
<td>( f_2(x) = \sum_{i=1}^{n} \left( 100 \left( x_{i+1} - x_i^2 \right)^2 - \left( x_i - 1 \right)^2 \right) )</td>
<td></td>
</tr>
<tr>
<td>Rastrigin</td>
<td>( f_3(x) = \sum_{i=1}^{n} \left( x_i^2 - \cos(2\pi x_i) + 10 \right) )</td>
<td></td>
</tr>
<tr>
<td>Ackley</td>
<td>( f_4(x) = -20 \exp \left( -0.2 \sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2} \right) - \exp \left( \frac{1}{n} \sum_{i=1}^{n} \cos(2\pi x_i) \right) + 20 + e )</td>
<td>multimodal</td>
</tr>
<tr>
<td>Griewank</td>
<td>( f_5(x) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos \left( \frac{x_i}{\sqrt{i}} \right) + 1 )</td>
<td></td>
</tr>
</tbody>
</table>
Table 2
Test Benchmark function optimization results

<table>
<thead>
<tr>
<th>Fun</th>
<th>GA</th>
<th>PSO</th>
<th>WOA</th>
<th>GOA</th>
<th>BAS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AVG</td>
<td>STD</td>
<td>AVG</td>
<td>STD</td>
<td>AVG</td>
</tr>
<tr>
<td>$f_1$</td>
<td>2.62E+03</td>
<td>1.67E+02</td>
<td>2.77E+03</td>
<td>6.02E+03</td>
<td>-5.78E+02</td>
</tr>
<tr>
<td>$f_2$</td>
<td>1.34E+02</td>
<td>5.48E+04</td>
<td>2.38E+04</td>
<td>1.37E+03</td>
<td>2.22E+02</td>
</tr>
<tr>
<td>$f_3$</td>
<td>1.72E+06</td>
<td>1.09E+06</td>
<td>4.68E+05</td>
<td>4.20E+05</td>
<td>1.70E+04</td>
</tr>
<tr>
<td>$f_4$</td>
<td>2.29E+03</td>
<td>3.91E+01</td>
<td>2.55E+03</td>
<td>2.15E+00</td>
<td>2.31E+03</td>
</tr>
<tr>
<td>$f_5$</td>
<td>2.76E+03</td>
<td>2.86E+01</td>
<td>2.64E+03</td>
<td>1.63E+00</td>
<td>2.63E+03</td>
</tr>
</tbody>
</table>

is chosen to optimize the fractional order operator in this paper.

Figure 3
BAS algorithm iterative process

The BAS algorithm solves the optimal fractional Verhulst model. Figure 3 is the process of an iterative algorithm to obtain the optimal solution, and the number of iterations is 50. The graph shows that when the number of iterations is about 30, the algorithm has basically converged, and the algorithm converges faster. Finally, the optimal order $r = 0.064672$, parameter $a = -0.2206$, $b = -0.0005$. The time response is:

$$\dot{x}^{(0.064672)}(k+1) = \frac{-10.7478526}{-0.0243605 - 0.1962395^{0.22064}}$$

3.2. Fitting Results of Each Model

In order to verify the prediction effect of the optimization model, this paper establishes the model according to the cumulative displacement data of the Gansu Heifangtai landslide group from January to December 2016, monitored by GPS. The optimization model of this paper is compared with the Huang-optimized grey Verhulst model, grey Verhulst model, GM (1,1) model, cubic exponential smoothing model, and DGM (2,1) model. The residual and relative errors of the six models are compared by analyzing the original and predicted values of the six models. MATLAB2018b software is used to program the six models, respectively, and the accuracy of each model is shown in Table 3.

In order to express the fitting of each model more intuitively, the fitting curves and error bar graphs of each model are plotted with the origin software according to Table 3, as shown in Figure 4. It was evident from the figure that the fractional order operator Verhulst model proposed in this paper has a minor error value compared with the other five models and can eliminate the extreme effect to a certain extent. The fitting curve also fits better with the original curve. The fitting curve and error bar chart is presented as the best, showing that the prediction model proposed in this paper has high accuracy and reliability.
Table 3
Accuracy checklist of six models of the Heifangtai landslide group in Gansu

<table>
<thead>
<tr>
<th>No.</th>
<th>$x^{(0)}$ /mm</th>
<th>Grey Verhulst model</th>
<th>Cubic Exponential Smoothing model</th>
<th>DGM(2,1) model</th>
<th>GM(1,1) model</th>
<th>Huang optimized Verhulst model</th>
<th>Fractional operator Verhulst model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\hat{x}^{(0)}$ /mm</td>
<td>$\Delta_1$ (%)</td>
<td>$\hat{x}^{(0)}$ /mm</td>
<td>$\Delta_1$ (%)</td>
<td>$\hat{x}^{(0)}$ /mm</td>
<td>$\Delta_1$ (%)</td>
</tr>
<tr>
<td>1</td>
<td>48.721</td>
<td>48.721</td>
<td>0</td>
<td>48.721</td>
<td>0</td>
<td>48.721</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>55.093</td>
<td>17.151</td>
<td>68.869</td>
<td>49.365</td>
<td>10.397</td>
<td>53.510</td>
<td>2.873</td>
</tr>
<tr>
<td>3</td>
<td>70.554</td>
<td>22.989</td>
<td>67.417</td>
<td>51.886</td>
<td>26.459</td>
<td>63.600</td>
<td>9.856</td>
</tr>
<tr>
<td>4</td>
<td>80.436</td>
<td>30.653</td>
<td>61.892</td>
<td>67.833</td>
<td>15.688</td>
<td>74.520</td>
<td>7.355</td>
</tr>
<tr>
<td>5</td>
<td>92.059</td>
<td>40.587</td>
<td>55.912</td>
<td>83.574</td>
<td>9.217</td>
<td>86.340</td>
<td>6.212</td>
</tr>
<tr>
<td>6</td>
<td>95.561</td>
<td>53.248</td>
<td>44.279</td>
<td>99.783</td>
<td>4.418</td>
<td>105.389</td>
<td>3.724</td>
</tr>
<tr>
<td>7</td>
<td>122.434</td>
<td>69.017</td>
<td>43.629</td>
<td>107.702</td>
<td>12.033</td>
<td>120.589</td>
<td>7.754</td>
</tr>
<tr>
<td>8</td>
<td>146.305</td>
<td>88.066</td>
<td>39.806</td>
<td>133.360</td>
<td>8.848</td>
<td>127.900</td>
<td>12.580</td>
</tr>
<tr>
<td>9</td>
<td>160.634</td>
<td>110.156</td>
<td>31.424</td>
<td>163.000</td>
<td>1.473</td>
<td>144.070</td>
<td>10.312</td>
</tr>
<tr>
<td>10</td>
<td>186.818</td>
<td>134.402</td>
<td>28.057</td>
<td>184.064</td>
<td>1.473</td>
<td>161.570</td>
<td>13.514</td>
</tr>
<tr>
<td>11</td>
<td>201.363</td>
<td>159.095</td>
<td>20.991</td>
<td>211.528</td>
<td>5.048</td>
<td>200.510</td>
<td>10.356</td>
</tr>
</tbody>
</table>

Note: No. represents the Serial number, $x^{(0)}$ represents Original value, $\Delta k$ represents Residual error.

3.3. Comparison of Prediction Accuracies

The visual comparison analysis of the fitting accuracy of each model is shown in Figures 5-7. The APE histogram of Figure 5 shows that the APE of the proposed model is significantly lower than that of other predictive models. The prediction error is significantly lower. Figure 6 shows that the fitting curve is more consistent with the original curve. Figure 7 shows that the residual curve fluctuates less. The fractional grey Verhulst model can complement each other’s advantages and improve the efficiency and prediction accuracy of the traditional grey Verhulst model. The fractional grey Verhulst model complements the traditional model’s advantages, improving efficiency and prediction accuracy.

The accuracy comparison results of the fitted values and the actual values of the six models are shown in Table 4. In the fitting stage, the traditional Verhulst model has the largest MAPE (43.669%), MAE (47.820 mm), and RMSE (48.202 mm). Its most minor $R^2$ (0.557) indicates that the model’s accuracy is low. The model proposed in this paper has the smallest RMSE (4.385 mm), the lowest MAE (3.085 mm) and MAPE (2.949%), and the largest $R^2$ (0.993). Its evaluation standard is lower than other models, with the best fitting effect. Optimizing the order dramatically reduces the grey Verhulst model’s average relative error from 43.669% to 2.94%, indicating that the optimized model has higher accuracy and better suits landslide displacement and deformation prediction. Huang’s improved Verhulst model has a better fitting effect. In contrast, the fractional Verhulst model reduces the average relative error by 1.093 percentage points greater. The residual value fluctuation is slightly smaller and generally more stable with higher prediction accuracy.
Figure 4
Fitting plots and error bars of predicted data for each model. Letters (a-f) represent each model's fitting plot; Numbers (1-6) represent each model’s error bars of simulated and actual values.
4. Discussion

Landslides significantly impact economic development and public safety. With artificial intelligence and IoT advancing monitoring systems, landslide monitoring has gradually become intelligent. The manual monitoring of landslide monitoring systems is complex, traditional prediction methods are high, and it is not easy to reason for the complex situation. In order to predict and warn of the occurrence of...
landsides in advance and ensure the safety of people and property, it is of tremendous research significance to monitor the deformation and displacement of landslides and accurately predict the future deformation trend.

In general, the displacement information of landslides is partly known, so the landslide system is usually regarded as uncertain. The original data obtained in the study of its displacement characteristics are usually white, but the distribution type is grey, called grey data. The characteristic of grey prediction theory is to deal with uncertain systems with partly known and partly unknown information. The basic theory is to convert the original non-negative grey data into the sequence transformation of approximate exponential law generated by accumulation and then to establish the internal rules of the grey model mining system. At the same time, the dynamic fitting modeling of landslide deformation displacement is more accurate. This paper introduces fractional calculus theory into the modeling process. The fractional operator adjusts the grey Verhulst model cumulative series variation accuracy, selecting operators based on different scene data sequences to optimize fitting and prediction accuracy.

Aiming at the problem that the traditional grey Verhulst model has insufficient fitting accuracy in predicting displacement data series, this paper proposes a fractional Verhulst model optimized based on the beetle antennae search algorithm. It uses the BAS algorithm to find the optimal fractional order between 0 and 1 in the grey Verhulst model, minimizing average relative error. This paper improves landslide displacement prediction.

The fractional Verhulst model specifically applies to landslide disaster prediction, enabling intelligent landslide warning. Results show that the proposed method provides ideas and guidance for intelligent landslide warning, which has a certain guiding significance. The main idea is to establish a fractional Verhulst model by adding a fractional operator to the grey Verhulst model. Optimizing the order via search algorithms and accurately adjusting the order optimizes model accuracy. This prediction method provides a specific basis for landslide disaster prediction.

5. Conclusion

In this paper, taking the Heifangtai landslide group in Gansu Province as an example, a prediction method based on the fractional Verhulst model is proposed for its displacement time series. The fractional operator accumulates the original sequence, and the beetle antennae search algorithm is selected to obtain the original optimal order between 0 and 1. The order of magnitude between the accumulation operators is accurately adjusted, and the average relative fitting error is reduced to a minimum to improve the model's accuracy. The proposed optimization model is compared with the traditional grey Verhulst model, and the Huang optimized grey Verhulst model, GM (1,1) model, cubic exponential smoothing model, and DGM (2,1) model on the displacement value of mountain landslide. The accuracy of the traditional grey Verhulst model is low, and the accuracy of the Huang-optimized grey Verhulst model is slightly lower than that of this model. The average relative error of the fractional Verhulst model proposed in this paper can reach 2.949%, and the accuracy is the highest among the six models. The results show that compared with the traditional grey prediction model, this model has a better fitting effect and prediction accuracy and can effectively summarize the displacement variation law of mountain landslides, which has a certain application value for the prediction of mountain landslide disasters.

This paper uses the beetle antennae search algorithm that optimizes the fractional order Verhulst model, an improved swarm intelligent optimization algorithm prediction model for landslide displacement time series. For the fractional-order Verhulst landslide prediction problem, the landslide formation mechanism and the time-period trend term are analyzed and combined with the system dynamics to form a combined prediction model, which makes the prediction accuracy further improved. However, the natural factors affecting the instability of landslide movement are more complex. Future research should consider different landslide evolution mechanisms and factor impacts, establishing a reasonable regularization parameter determination method. Exploring landslide formation mechanisms and evolution processes from monitoring data is needed. With intelligent optimization/machine learning algorithms, remote sensing
satellites, surface monitoring equipment, drone field data, AI data centers, and landslide prediction/early warning databases, developing landslide geological, geophysical, and hydrological survey data is enriching. Combining landslide system dynamics, high-precision landslide displacement observation, comprehensive impact factors, and considering landslide physical properties to construct a comprehensive prediction model will be essential.

Data Sharing Agreement
The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declaration of Conflicting Interests
The author(s) declared no potential conflicts of interest with respect to the research, author-ship, and/or publication of this article.

References


