ITC 1/53
Information Technology and Control
Vol. 53 / No. 1 / 2024
pp.128-145
DOI 10.5755/j01.itc.53.1.33456

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Received 2023/02/25 | Accepted after revision 2023/06/05

HOW TO CITE: Sun, B., Guo, N., Zhang, L., Li, Z. (2024). A Hybrid Strategy Guided Multi-Objective Artificial Physical Optimizer Algorithm. *Information Technology and Control,* 53(1), 128-145. https://doi.org/10.5755/j01.itc.53.1.33456

A Hybrid Strategy Guided Multi-Objective Artificial Physical Optimizer Algorithm

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Artificial physical optimizer (APO), as a new heuristic stochastic algorithm, is difficult to balance convergence and diversity when dealing with complex multi-objective problems. This paper introduces the advantages of R2 indicator and target space decomposition strategy, and constructs the candidate solution of external archive pruning technology selection based on APO algorithm. A hybrid strategy guided multi-objective artificial physical optimizer algorithm (HSGMOAPO) is proposed. Firstly, R2 indicator is used to select the candidate solutions that have great influence on the convergence of the whole algorithm. Secondly, the target space decomposition strategy is used to select the remaining solutions to improve the diversity of the algorithm. Finally, the restriction processing method is used to improve the ability to avoid local optimization. In order to verify the comprehensive ability of HSGMOAPO algorithm in solving multi-objective problems, five comparison algorithms were evaluated experimentally on standard test problems and practical problems. The results show that HSGMOAPO algorithm has good convergence and diversity in solving multi-objective problems, and has the potential to solve practical problems.

KEYWORDS: Multi-objective problem, artificial physical optimizer algorithm, R2 indicator, Target space decomposition strategy, Global optimization.

1. Introduction

In practical science and engineering applications, problems involving multiple conflicting objectives that need to be optimized at the same time are often referred to as multi-objective optimization problems. For example, urban bus route problem [13], shop scheduling problem [23], multi-agent system optimization problem [12] and engineering vehicle mechanical design [27]. The objective functions in the multi-objective optimization problem conflict with each other, and almost no single solution can satisfy all the objectives at the same time, so it is necessary to make trade-offs between different objectives. The set of all Pareto non-dominated solutions is called Pareto optimal solution set, and the mapping of Pareto optimal solution set in the object space is called Pareto frontier (PF). When solving multi-objective optimization problems, solutions obtained from Pareto optimal solution set should not only converge as much as possible, but also be distributed as evenly as possible on PF [2]. Therefore, how to achieve the balance between convergence and diversity is the key to solve the multi-objective optimization problem.

Based on the characteristics of meta-heuristics and population, the multi-objective evolutionary algorithm shows high efficiency in solving multi-objective problems. In the past 20 years, scholars at home and abroad have carried out a lot of research on the multi-objective optimization problem and proposed many multi-objective optimizer algorithms with good performance. A common multi-objective algorithm is an evolutionary algorithm based on Pareto advantages, which selects candidate solutions through non-dominated sorting. However, due to the lack of effective selection pressure, there are two alternative selection strategies based on decomposition method and performance indicator, respectively [10]. Evolutionary algorithms based on decomposition transform multi-objective problems into multiple single-objective subproblems through decomposition methods, and then optimize the subproblems simultaneously under the evolutionary framework [18]. Among them, MOEA/D [29] is the most typical algorithm of this kind. In MOEA/D, the multi-objective problem is decomposed into many optimization sub-problems. Different solutions of individuals are associated with different sub-problems, and the "diversity" among these sub-problems will naturally lead to the diversity of the population. When the decomposition method and weight vector are chosen correctly to make the optimal PF of the composite subproblems uniformly distributed, MOEA/D has a good chance to search for the uniform distribution of the Pareto solution if it can optimize all the subproblems well. Indicator-based evolutionary algorithm can simultaneously maintain the convergence and diversity of the population through only one strategy to calculate the indicator value in environmental selection, thus improving the performance of the population [11]. At present, the most common performance indicator is Hypervolume (HV) indicator, which combines convergence and diversity into a single indicator to select candidate solutions. However, the computational burden of HV indicator increases exponentially with the increase in the number of targets, which seriously prevents a large number of multi-objective optimization algorithms from using HV indicator [19]. It is found that R2 indicator achieves a comprehensive balance between convergence and diversity, and can replace HV indicator in solving multi-objective problems [25].

Zitzlerl in [31] defined R2 indicator in 2008 and provided the calculation formula of standard R2 indicator. Dimo in [3] made a comprehensive analysis of the attributes of R2 indicator and proposed a steady-state multi-objective evolutionary algorithm R2-EMOA based on R2 indicator. Phan in [15] proposed the R2-IBEA algorithm, which adaptively adjusts the position of reference points according to the degree of contemporary individuals in the target space to determine the relationship between the advantages and disadvantages of a given two individuals. The results show that the algorithm achieves excellent performance in terms of the optimality and diversity of solutions. Alan in [6] introduced R2 indicator on the basis of genetic algorithm and proposed MOGA algorithm. Coello in [7] proposed a multi-objective ant colony algorithm based on R2 indicator by integrating R2 indicator with ant colony algorithm, and verified the feasibility of this strategy.

Methodology for parallelization is an effective way to improve optimization algorithms. In [16], the author presents three propositions to improve the efficiency of classical methods. They are neighborhood search, partition solution space, and domain search combined with partition solution space. In [17], the author proposed a modification of the classic mechanism of federated learning and anew algorithm for data augmentation.

APO algorithm has been widely concerned by scholars due to its few parameters and simple operation. It has shown excellent competitiveness in single-objective optimization problems and has been well applied in various optimization tasks [26, 22]. However, as a random algorithm, APO algorithm faces the problems of low efficiency of non-dominant solution selection, convergence and diversity imbalance. In order to solve the problem of unbalanced convergence and diversity of APO algorithm, a hybrid strategy guided multi-objective mimicry physics optimizer algorithm, HSGMOAPO, is proposed in this paper. In this paper, we construct an external archive pruning technique based on R2 indicator selection and target space decomposition strategy to select candidate solutions. The two methods work together to select solutions with convergence and diversity equilibrium. In addition, in order to avoid the algorithm falling into local optimal, a limiting treatment method is proposed to improve the exploration and utilization ability of the population. Through numerical experiments on widely recognized benchmark problems, the good performance of the proposed algorithm is verified. In the design problems of double-bar truss, the proposed algorithm is proved to have high practical problem processing ability, and can be used as an effective means for multi-objective optimization problems.

2. Constrained Multi-objective Optimization Problem

Constrained multi-objective optimization problem consists of objective function and constraint conditions. Generally, the mathematical model of constrained multi-objective optimization problem is as follows:

$$\begin{aligned} &\min_{x\in\Omega} F(x) = \left(f_1(x),\ f_2(x), \cdots,\ f_m(x)\right) \\ &s.t.\ g_j(x) \le 0,\ j = 1, \cdots,\ p \\ &\quad h_j(x) = 0,\ j = p+1, \cdots,\ q \end{aligned}, \quad (1)$$

where, $x = (x_1, x_2, \cdots, x_d)^T$ is a D-dimensional decision vector; $\Omega \in R^D$ is decision space; $F(x)$ constitutes m conflicting objective functions, and $F(x) \in R^m$ represents the objective space. $g_j(x) \le 0$, $h_j(x) = 0$ represents inequality constraint function and equality constraint function, respectively.

3. Basic Principle of Artificial Physical Optimizer Algorithm

APO is an optimizer algorithm proposed by Professor Spear from the University of Wyoming inspired by Newton's Second Law. The solution of the optimization problem is regarded as a moving individual, and each individual adjusts its movement according to its own inertia and other individuals according to the rules of virtual force between particles. The inverse proportional relationship between individual quality and fitness value is established in the algorithm. Individuals with better fitness value attract individuals with poorer fitness value, individuals with worse fitness value repel individuals with better fitness value, and individuals with the best fitness value attract all other individuals. Each individual updates its quality according to its global fitness value and its own fitness value, so as to update its speed and position. Finally, population optimization is realized [21, 28].

Suppose the population size is *N*, the velocity of individual *i* in the *k*th dimensionis $v_{i,k}$, the position of individual *i* in the *k*th dimensionis $x_{i,k}$, and the mass function of individual *i* is:

$$m_i = e^{\frac{f(x_{best}) - f(x_i)}{f(x_{worst}) - f(x_{best})}}, \quad (2)$$

where, x_{best}, x_{worst} represents the position of optimal and worst adaptation values, respectively.

According to the mass of each individual and the formula of universal gravitation, the force exerted by each individual is calculated. Therefore, the virtual force on individual *i* by individual *j* in the *k*th dimension is:

$$F_{ij,k} = \begin{cases} Gm_i m_j (x_{j,k} - x_{i,k}), f(x_i) > f(x_j) \\ -Gm_i m_j (x_{j,k} - x_{i,k}), f(x_i) \le f(x_j) \\ \quad \forall i \ne j, i \ne best \end{cases}. \quad (3)$$

The resultant force on individual i in the kth dimension is:

$$F_{i,k} = \sum_{\substack{j=1 \\ i \neq j}}^{n} F_{ij,k}, \forall i \neq best, \quad (4)$$

where, G is the gravitational factor.

The formula for updating the velocity and position of individual i is:

$$v_{i,k}(t+1) = wv_{i,k}(t) + \lambda F_{i,k} / m_i, \ \forall i \neq best, \quad (5)$$

$$x_{i,k}(t+1) = x_{i,k}(t) + v_{i,k}(t+1), \quad \forall i \neq best, \quad (6)$$

where, $\omega \in (0,1)$ is the inertia weight; λ is a random number distributed in the interval $[0,1]$; $x_i \in [x_{\min}, x_{\max}]$, $v_i \in [v_{\min}, v_{\max}]$. Through Equations (5)-(6), the speed and position of the globally optimal individual are constantly updated to enter the next iteration.

The selection of inertia weight ω is very important to achieve the balance between population development and exploration. The linear inertia weight is used in iterative evolution, which reduces dynamically with iteration.

$$\omega = 0.9 - 0.5\frac{t-1}{\max t}, \quad (7)$$

where, t is the number of iterations; $\max t$ is the maximum number of iterations.

4. HSGMOAPO Algorithm

The HSGMOAPO algorithm proposed in this paper is based on the characteristics of R2 indicator and target space decomposition method, and adopts an external archive pruning technique to screen candidate solutions. In order to balance the convergence and diversity of the algorithm, HSGMOAPO algorithm is specially designed and mainly consists of three parts: (1) R2 indicator selection strategy. By calculating the contribution value of candidate solution R2 indicator, solutions with higher contribution value were selected to enter the external archive set. (2) Target space decomposition strategy. The solution position of the obtained non-dominated solution set in the target space is emphasized. (3) On the basis of the two candidate solution selection strategies, a limiting processing strategy is proposed to prevent the algorithm from falling into local optimization. Through the above strategies, the convergence and diversity of the algorithm are improved to achieve population optimization.

4.1. R2 Indicator Selection Strategy

R2 indicator is based on utility function to distinguish the pros and cons of candidate solutions, so as to select the candidate solutions with greater utility. It was initially used to evaluate the relative quality of two groups of individuals. Because R2 indicator can comprehensively evaluate the convergence and distribution of the population and obtain the uniform distribution solution set through fast calculation, it has been applied to solve the multi-objective optimization problems.

R2 indicator and R2 contribution values (CR2) were used to evaluate the relative quality of the two groups of individuals. Assuming that A standard weighted Chebyshev utility function has a specific reference point q^*, this indicator can be used to evaluate the quality of a single individual set (A) against q^*:

$$R2(A,W,q^*) = \frac{1}{|W|}\sum_{\mu \in W} \min_{x \in A}\left\{\max_{i=1,\cdots,m} \mu_i \left|f_i(x) - q^*\right|\right\}, \quad (8)$$

where, W is a group of uniform weight vectors, each weight vector $\mu = (\mu_1, \ \mu_2 \ \dots \ \ \mu_m) \in W$ is evenly distributed in the target space, and $1/|W|$ represents the probability of distribution on W.

The CR2 of the candidate solution is used to evaluate the quality of the solution. The lower the CR2, the smaller the selection probability of the newer population during evolution. The CR2 of the candidate solution $x \in A$ is:

$$CR2(x,A,W,q^*) = R2(A,W,q^*) - R2(A \setminus \{x\},W,q^*). \quad (9)$$

In order to better solve complex multi-objective problems. In this paper, the formula of R2 indicator is modified. In the algorithm design, the following formula is used to calculate the CR2.

$$R2(A,W,q^*) = \frac{1}{|W|}\sum_{\mu \in W} \min_{x \in A}\left\{\max_{i=1,\cdots,m} \frac{1}{\mu_i}\left|f_i(x) - q^*\right|\right\} \quad (10)$$

The selection strategy based on R2 indicator can be regarded as the mutual preference between a set of weight vectors and candidate solutions, and the calculation of the R2 contribution value of each candidate solution should consider the utility function value between each candidate solution and each weight vector. The weighted Chebyshev function is selected as the utility function. The R2 indicator was used to screen candidate solutions with great influence on the convergence of the whole algorithm. Firstly, the population solution set and the non-dominated solution in the archive set were combined, and the R2 indicator value of each candidate solution was calculated. Then, solutions with contribution value to R2 were found according to the CR2 value, and placed in the archive A_1, focusing on improving the convergence of the algorithm under the premise of maintaining diversity.

4.2. Target Space Decomposition Strategy

Objective space decomposition strategy is an effective means to maintain algorithm diversity. Firstly, the object space of the multi-objective problem is decomposed into a group of subregions according to a set of direction vectors. Assigning subregions to each subproblem, where each subproblem evolves in its own subregions, can effectively reduce repeated searches for subproblems. Then maintain the diversity of solutions by maximizing one solution per subarea.

For a given set of direction vectors $\gamma^1, \gamma^2, \cdots, \gamma^n$ and a population p, where n is the number of direction vectors. The decomposition and classification of the target space are defined as follows:

$$P^i = \left\{ x \middle| x \in P,\ \Delta(F(x),\gamma^i) = \max_{1\le j\le N}\left\{\Delta(F(x),\gamma^j)\right\}\right\}$$
$$\Delta(F(x),\gamma^i) = \frac{\gamma^i * \left(F(x)-q\right)^{\mathrm{T}}}{\gamma^i * \left(F(x)-q\right)}, i=1,2,\cdots,N \quad , \tag{11}$$

$$M_i = \left\{F(x)\in R^m, \Delta(F(x),\gamma^i) = \max_{1\le j\le N}\left\{\Delta(F(x),\gamma^j)\right\}\right\}, \tag{12}$$

where, $q = \left(q_1, q_2, \cdots, q_m\right)$ is the reference point; $\Delta(F(x),\gamma^i)$ is the cosine of the direction of $F(x)-q$ and γ^i; Population P is divided into $P^i, i=1,\cdots,n$ according to Equation (11), and target space M is divided into n subspaces $M_1, M_2, \cdots M_n$ according to Equation (12).

Since individuals are randomly generated, there is no guarantee that every subspace will be assigned to a suitable solution. Some subregions may have multiple individuals or no individuals. In this process, if the subspace is an empty set, the largest solution in $\Delta(F(x),\gamma^i)$ is selected as the candidate solution. If the candidate solution is unique in the subspace, in order to maintain the diversity of the solution set, the candidate solution is directly reserved. If the candidate solutions in the subspace are not unique, that is, when the number of candidate solutions is greater than 1, Chebyshev method (Tche) is adopted for calculation, and the solution with the minimum Tche value is selected as the candidate solution to enhance the diversity of the population under the condition of ensuring convergence. The calculation Formula of Tche is:

$$\min g_{Tche}(x|\lambda, z^*) = \max_{1\le i\le m}\left\{\lambda_i \left| f_i(x) - z_i^*\right|\right\},$$
$$\sum_{i=1}^{m}\lambda_i = 1,\ \lambda_i \ge 0, i=1,\cdots,m \tag{13}$$

where, $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_m)^{\mathrm{T}}$ is the weight vector, $z^* = \min\{f_i(x) | x\in\Omega\}, i\in\{1,2,\cdots,m\}$.

In order to maintain the diversity of the algorithm, the objective space decomposition strategy no longer considers other candidate solutions of the candidate solution subspace in archive A_1, and selects candidates from the remaining subspace to be liberated into archive A_2. It is worth noting that the PFS of all these subspaces form the PFS of the multi-objective optimization problem. Since the Pareto optimal set of each subspace is only a small part of the Pareto optimal set of the optimization problem, even if the entire Pareto optimal set of the optimization problem has nonlinear geometry, The Pareto optimal set of each subspace can also be approximately linear. Therefore, as far as the shape of Pareto optimal set is concerned, decomposition of object space makes the optimization easier. Secondly, this method only needs to select a set of direction vectors during decomposition, and Tche method is used when the number of candidate solutions of subspace is greater than 1. To some extent, it only needs less calculation.

4.3. Limiting Processing Policies

In order to avoid individuals falling into local optimality, the unfeasible individual HSGMOAPO algorithm uses a limiting processing strategy to adjust the position and velocity of particles to make them within the limit. If the position of an individual in dimension k exceeds the boundary, it is set back to the boundary according to equation, and the new position is defined as.

$$x'_{k,t+1} = \begin{cases} x_{k,\max} - (x_{k,t+1} - x_{k,\max}), & x_{k,t+1} > x_{k,\max} \\ x_{k,\min} + (x_{k,\min} - x_{k,t+1}), & x_{k,t+1} < x_{k,\min} \end{cases}, \quad (14)$$

where, $x_{k,\max}$, $x_{k,\min}$ are the upper and lower bounds of dimension k, respectively.

In any case, if only the position is modified, it is likely that the particle will leave the feasible region again in the next iteration based on the current velocity. For an individual whose position in dimension k exceeds the boundary, its velocity is defined as:

$$v'_{k,t+1} = -\lambda v_{k,t+1}, \quad (15)$$

where, λ is a uniformly distributed random number within [0,1]. HSGMOAPO algorithm explores the whole solution space, thus improving the ability to avoid local optimization.

4.4. HSGMOAPO Algorithm

Based on the above strategies, we use the HSGMOAPO algorithm to solve constrained multi-objective optimization problem. Figure 1 shows the design idea of the algorithm. The main process is described as follows:

We use the HSGMOAPO algorithm to solve MOPS. The following algorithm is described in pseudocode form.

算法 HSGMOAPO 算法

Input: population size N, External archiving A, Weight vector W, reference point q^*

Output: A

1: Initialize the population velocity and position, and the reference point q^*

2: Archives were generated following the R2 indicator strategy A_1

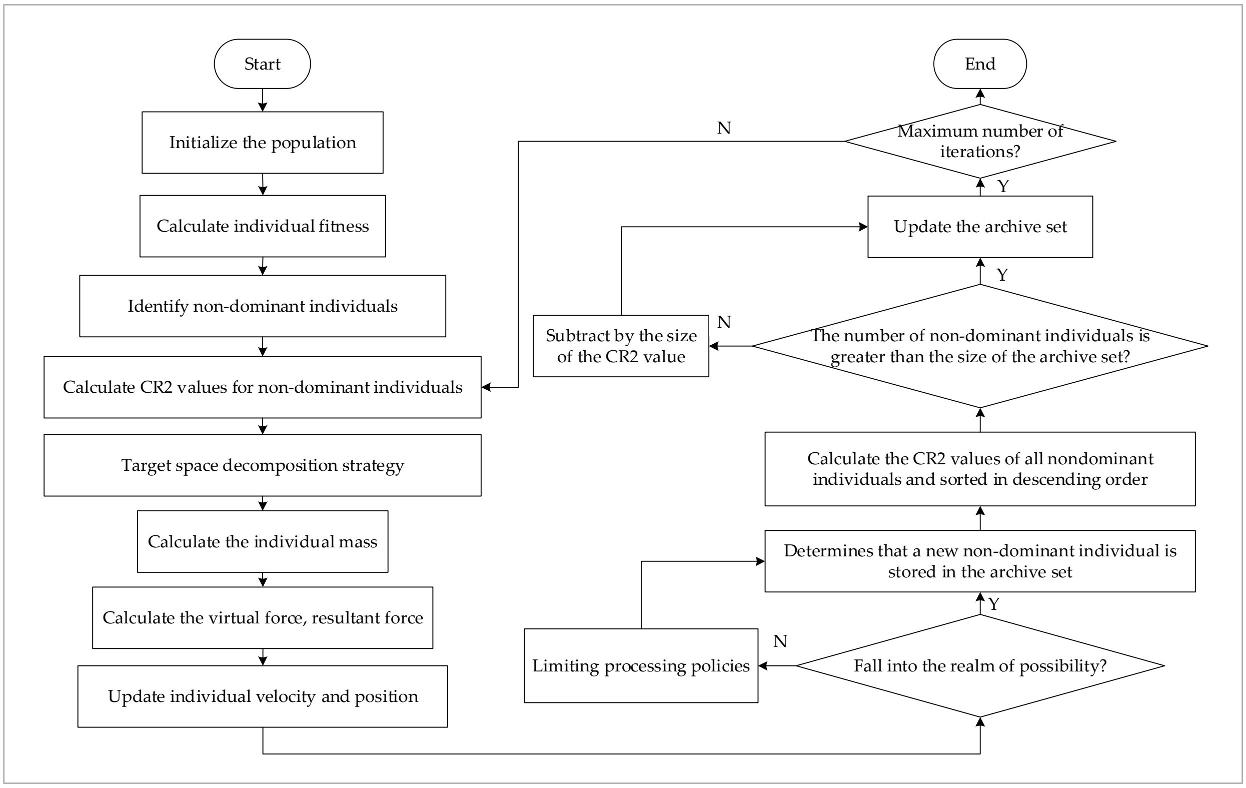
3: Generthe archive according to the target space decomposition strategy A_2

4: $A = A_1 \cup A_2$

5: When t less than the maximum number of iterations

Figure 1

The design idea of HSGMOAPO algorithm



6: for $i \in A$
7: Calculate individual quality
8: Calculate the individual virtual force
9: Calculate the individual to suffer the resultant force
10: Updates the individual speed and location
11: Determine whether the individual i is in the feasible area
12: if i belongs to a feasible area
13: Updates A
14: else restriction processing
15: End if
16: end for
17: $t = t + 1$
18: t greater than the maximum number of iterations
19: return A

5. Simulation Verification and Result Analysis

In order to test the comprehensive performance of HSGMOAPO algorithm in dealing with multi-objective problems, numerical experiments are carried out with five advanced multi-objective algorithms on benchmark problems.

5.1. Test Problems

MW series problems are standard constrained multiobjective testing problems, which are often used to evaluate the performance of constrained multiobjective optimization algorithms [28]. The MW series consists of 14 problems, covering different features extracted from constrained multi-objective problems in real situations, such as small feasible ratio, many constraints, and nonlinear constraints. In the experiment, the number of targets for MW4, MW8 and MW14 is set to 3, the number of targets for other problems is set to 2, the number of decision variables D is set to 15, and the maximum number of evaluation is 100,000. To ensure the fairness of the comparison algorithm, all experiments were conducted under the above unified parameter Settings.

5.2. Performance Specifications

Inverted Generation Distance (IGD) and Hypervolume (HV) are two indicators widely used to simultaneously measure the convergence and diversity of algorithms [9]. In this paper, these two indicators are used to evaluate the performance of the algorithm.

The IGD indicator represents the average distance from each reference point sampled on the real PF to the nearest solution.

$$IGD(P,Q) = \frac{\sum_{x \in P} d(x,Q)}{|P|}, \tag{16}$$

where, P is the set of uniformly sampled points on real PF, and Q is the set of approximate solutions obtained by the algorithm. $d(x,Q)$ represents the minimum Euclidean distance between individual x and population Q. A small IGD indicates that the resulting population is close to the true PF and has a good distribution.

The HV indicator measures the volume of the region in the target space surrounded by the solution set obtained by the algorithm and the predefined reference points.

$$HV(Q) = \lambda\left(\bigcup_{i=1}^{|S|} v_i\right), \tag{17}$$

where, Q is the set of approximate solutions obtained by the algorithm; λ represents the Lebesgue measure, which measures volume. $|S|$ represents the number of non-dominant solution sets, and v_i represents the supervolume formed by the reference point and the ith solution in the solution set. The larger the HV value, the better the performance of the algorithm.

5.3. Experimental Results and Analysis

In this paper, platemo platform was used to conduct a series of simulation experiments, and five advanced multi-objective optimization algorithms (MOEADD-AE [30], IDBEA [1], NSGA-III [5], CMOAPO [24] and R2-ICRMOAPO [20]) were selected for comparison. To study the performance of HSGMOAPO algorithm. The parameters of the comparison algorithm are consistent with those of the original paper. Tables 1-2 show the standard deviation and average value of IGD indicator and HV indicator obtained by 30 independent runs of six algorithms for MW test problem. The best results for each test question are highlighted in bold black, and it is worth noting that the best value for the IGD measure is the minimum, while the best value for the HV measure is the maximum. In addition, Wilcoxon rank sum test was performed at the significance level of 0.05 to show the significant difference between test results [4]. The symbols "+", "-" and "≈", respectively indicate that the results of other algorithms are stronger, weaker or approximate to

Table 1

IGD values of six algorithms run 30 times in test problem MW

Problem	MOEADDAE	IDBEA	NSGA-III	CMOAPO	R2-ICRMOAPO	HSGMOAPO
MW1	6.8024e-2(2.74e-2)-	3.2790e-1(1.16e-1)-	9.6776e-2(3.99e-2)-	2.5230e-1(1.55e-1)-	8.3417e-2(9.07e-2)-	**4.8247e-2(1.19e-2)**
MW2	8.9060e-2(3.45e-2)-	7.0846e-2(2.89e-2)-	5.4202e-2(1.47e-2)-	3.2250e-2(1.98e-2)-	2.4098e-2(9.94e-3)-	**1.8840e-2(8.06e-3)**
MW3	1.4960e-2(2.36e-3)-	9.3728e-2(1.69e-1)-	2.5194e-2(2.17e-3)-	9.9782e-2(1.74e-1)-	2.7758e-2(3.55e-2)-	**1.0226e-2(1.77e-3)**
MW4	1.0585e-1(1.04e-2)-	1.0308e-1(7.92e-2)-	9.5269e-2(1.11e-2)-	1.4538e-1(9.71e-2)-	**5.6394e-2(5.61e-3)+**	6.1618e-2(6.50e-3)
MW5	8.8398e-2(1.98e-2)-	1.3503e-1(1.70e-1)-	1.4253e-1(2.16e-2)-	2.2655e-1(2.84e-1)-	6.5572e-2(5.39e-2)-	**2.7673e-2(2.35e-2)**
MW6	4.9513e-1(1.92e-2)-	2.9726e-1(1.77e-1)-	2.4284e-2(5.82e-3)-	3.6232e-1(1.97e-1)-	2.1986e-1(2.17e-1)-	**1.9568e-2(1.22e-2)**
MW7	**1.2131e-2(3.21e-3)+**	2.1290e-2(3.25e-3)-	2.5448e-2(4.75e-3)-	5.5025e-2(1.11e-1)-	2.2866e-2(1.91e-2)=	1.3115e-2(3.28e-3)
MW8	1.2693e-1(3.08e-2)-	8.8035e-2(5.07e-2)-	6.3147e-2(4.63e-3)-	1.1749e-1(8.76e-2)-	6.5042e-2(3.36e-2)=	**4.9027e-2(4.71e-3)**
MW9	1.1994e-1(2.49e-1)-	2.2457e-1(2.77e-1)-	1.0918e-1(1.83e-1)=	6.0503e-1(2.54e-1)-	2.1337e-1(2.58e-1)-	**7.4851e-2(1.30e-1)**
MW10	5.0746e-1(1.36e-1)-	1.3595e-1(1.46e-1)-	4.5379e-2(6.15e-2)=	2.5263e-1(2.36e-1)-	8.5004e-2(1.15e-1)-	**4.3930e-2(4.24e-2)**
MW11	**1.8932e-2(3.40e-3)+**	6.8682e-1(9.66e-2)-	3.1836e-2(7.46e-3)-	7.6464e-1(5.82e-2)-	6.9412e-2(1.04e-2)=	2.9221e-2(9.74e-3)
MW12	7.0731e-2(7.14e-2)=	2.4188e-1(1.35e-1)-	1.6901e-1(3.16e-1)-	5.5162e-1(2.23e-1)-	3.3525e-1(3.23e-1)-	**6.1267e-2(1.38e-1)**
MW13	2.6698e-1(7.52e-2)-	3.8603e-1(2.21e-1)-	1.2580e-1(1.67e-2)-	8.1962e-1(4.41e-1)-	6.9690e-1(4.03e-1)-	**7.6077e-2(3.26e-2)**
MW14	1.4356e+0(1.36e-1)-	9.1440e-1(1.48e-1)-	5.2352e-1(1.30e-1)-	9.7152e-1(1.22e-1)-	4.7773e-1(1.43e-1)=	**2.3744e-1(1.11e-1)**
+/-/=	2/11/1	0/14/0	0/12/2	0/14/0	1/9/4	

Table 2

HV values of six algorithms run 30 times in test problem MW

Problem	MOEADDAE	IDBEA	NSGA-III	CMOAPO	R2-ICRMOAPO	HSGMOAPO
MW1	3.8835e-1(3.44e-2)-	1.7760e-1(8.46e-2)-	3.5812e-1(3.72e-2)-	2.5328e-1(1.32e-1)-	4.0510e-1(7.28e-2)-	**4.3630e-1(9.84e-2)**
MW2	4.5120e-1(4.61e-2)-	4.8285e-1(3.32e-2)-	4.9944e-1(2.14e-2)-	5.3807e-1(2.46e-2)-	5.4821e-1(1.44e-2)-	**5.5662e-1(1.33e-2)**
MW3	5.2733e-1(4.43e-3)-	4.5533e-1(1.08e-1)-	5.1385e-1(4.29e-3)-	4.6119e-1(1.07e-1)-	5.2242e-1(2.57e-2)=	**5.3611e-1(3.22e-3)**
MW4	7.3703e-1(1.73e-2)-	7.2389e-1(8.99e-2)-	7.5812e-1(2.11e-2)-	6.8488e-1(1.08e-1)-	**8.1491e-1(7.52e-3)=**	8.1055e-1(8.07e-3)
MW5	2.1332e-1(2.74e-2)-	2.1814e-1(4.37e-2)-	1.5216e-1(2.47e-2)-	2.2655e-1(8.39e-2)-	2.8390e-1(2.37e-2)-	3.0141e-1(1.97e-2)
MW6	1.2834e-1(1.82e-2)-	2.3578e-1(3.65e-2)-	3.0028e-1(9.09e-3)=	2.1010e-1(6.00e-2)-	2.4855e-1(5.76e-2)-	3.0322e-1(1.53e-2)
MW7	3.9458e-1(2.43e-3)-	4.0052e-1(4.96e-3)-	3.7866e-1(6.62e-3)-	3.8593e-1(4.20e-2)-	4.0192e-1(4.84e-3)-	4.0603e-1(1.68e-3)
MW8	3.6730e-1(5.28e-2)-	4.8950e-1(5.23e-2)-	5.0712e-1(9.89e-3)-	4.6242e-1(6.61e-2)-	5.1477e-1(3.22e-2)=	5.2376e-1(1.42e-2)
MW9	2.4717e-1(1.24e-1)-	**3.2691e-1(6.23e-2)=**	3.0336e-1(8.99e-2)-	6.1171e-2(1.21e-1)-	2.5307e-1(1.31e-1)-	3.2022e-1(7.37e-2)
MW10	1.8453e-1(5.57e-2)-	3.5644e-1(7.22e-2)-	4.0808e-1(3.61e-2)-	3.0046e-1(1.15e-1)-	3.8736e-1(5.97e-2)-	4.1465e-1(3.00e-2)
MW11	**4.4066e-1(1.33e-3)+**	2.7308e-1(1.49e-2)-	4.3569e-1(2.91e-3)=	2.6199e-1(7.23e-3)-	4.1887e-1(4.26e-2)=	4.3415e-1(7.17e-3)
MW12	5.2167e-1(7.13e-2)-	3.7797e-1(1.00e-1)-	**5.4452e-1(1.04e-1)+**	1.7439e-1(1.67e-1)-	3.4256e-1(2.43e-1)-	4.7549e-1(2.42e-1)
MW13	3.2358e-1(4.11e-2)-	3.4747e-1(3.69e-2)-	4.1845e-1(1.14e-2)-	2.7113e-1(8.93e-2)-	2.6227e-1(1.01e-1)-	4.4441e-1(1.62e-2)
MW14	4.6554e-2(1.51e-2)-	1.1799e-1(3.20e-2)-	2.9912e-1(6.61e-2)-	9.7813e-2(2.88e-2)-	3.1676e-1(6.61e-2)-	4.0181e-1(1.84e-2)
+/-/=	1/12/1	0/13/1	1/11/2	0/14/0	0/10/4	

R2-ICRMOAPO algorithm. Statistical results were recorded in the last two rows of the table.

Tables 1-2 show the detailed HV and IGD comparison results obtained by HSGMOAPO algorithm and five other comparison algorithms for MW problems. It can be seen from Table 1 that HSGMOAPO algorithm is optimized in 11 problems except MW4, MW7 and MW11. R2-ICRMOAPO is the best for MW4, and MOEADDAE is better than HSGMOAPO algorithm for MW7 and MW11. It can be seen from Table 2 that the HSGMOAPO algorithm is optimal for 10 problems except MW4, MW9, MW11 and MW12. R2-ICRMOAPO has the best effect on MW4, and MOEADDAE is superior to HSGMOAPO algorithm on MW11. For MW9 and MW12, IDBEA and NSGA-III get better results.

In addition, from the results in the last row of Tables 1-2, the HSGMOAPO algorithm achieves the best results in more than two-thirds of MW problems, verifying the good performance. Next came MOEADDAE, which performed best on three test questions. In order to show the optimizer results intuitively, we draw the approximate PF of HSGMOAPO algorithm for MW5, MW14 problem in Figures 2-3. In order to obtain better comparison effect, we also give the corresponding optimal solutions of other algorithms.

Figure 2 shows the comparison between the six algorithms and the real frontiers for MW5 problem. The constrained PF in the MW5 problem is part of the unconstrained PF because the constraints make part of the unconstrained PF impossible. It can be clearly seen from the figure that the PF obtained by HSGMOAPO algorithm achieves the best performance in terms of convergence and diversity. MOEADDAE and R2-ICRMOAPO obtain a well-distributed PF, but do not converge to a true PF. IDBEA and CMOAPO are not widely distributed enough to cover the entire PF because they converge on some parts of the true PF, but not all of it. The Pareto frontiers of each algorithm on MW14 problem are shown in Figure 3. The problem has a disconnected geometry caused by Pareto dominance, and its true PF is 4 disconnecting regions. It can be seen that MOEADDAE cannot cover the upper region completely, while other algorithms can find all the regions. Some of the Pareto solutions obtained by CMOAPO do not converge well to true PF. When IDBEA, NSGA-III, Rtwo-Icrmoapo and HSGMOAPO algorithms consider convergence and diversity at the same time, they show competitive performance. It can be intuitively seen that the distribution of HSGMOAPO is more regular.

In terms of algorithm convergence speed, the HV rising trend curve of each algorithm under some test problems is plotted in Figure 4, and the average ex-

Figure 2

PF scatter plots obtained by six algorithms on MW5 problem

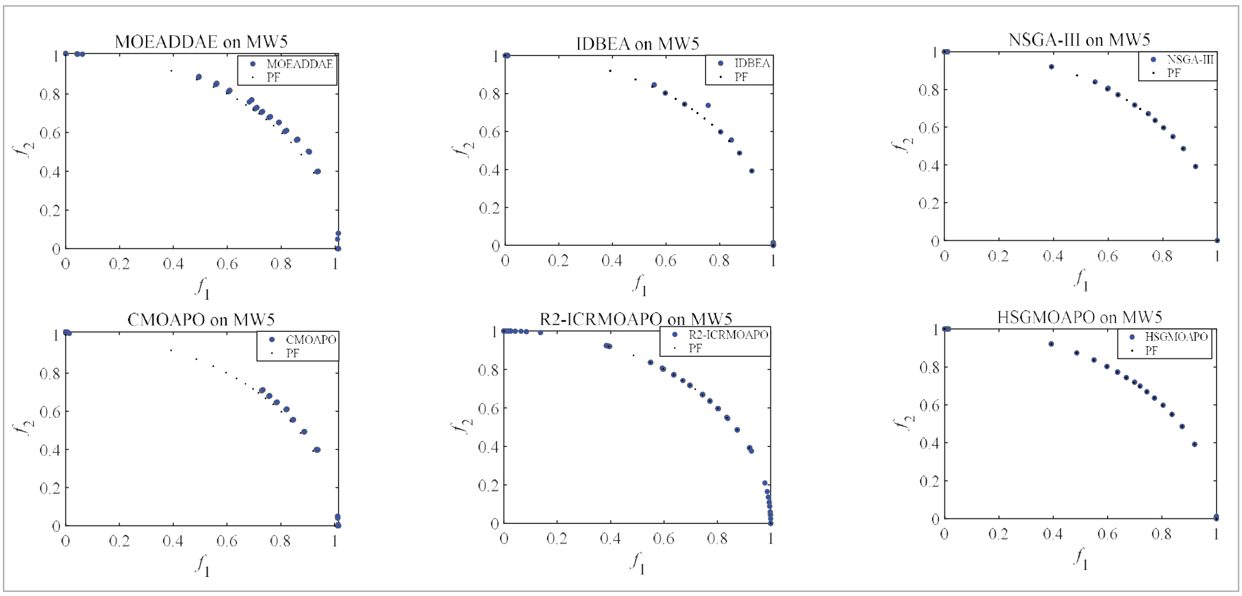


Figure 3

PF scatter plots obtained by six algorithms on MW14 problem

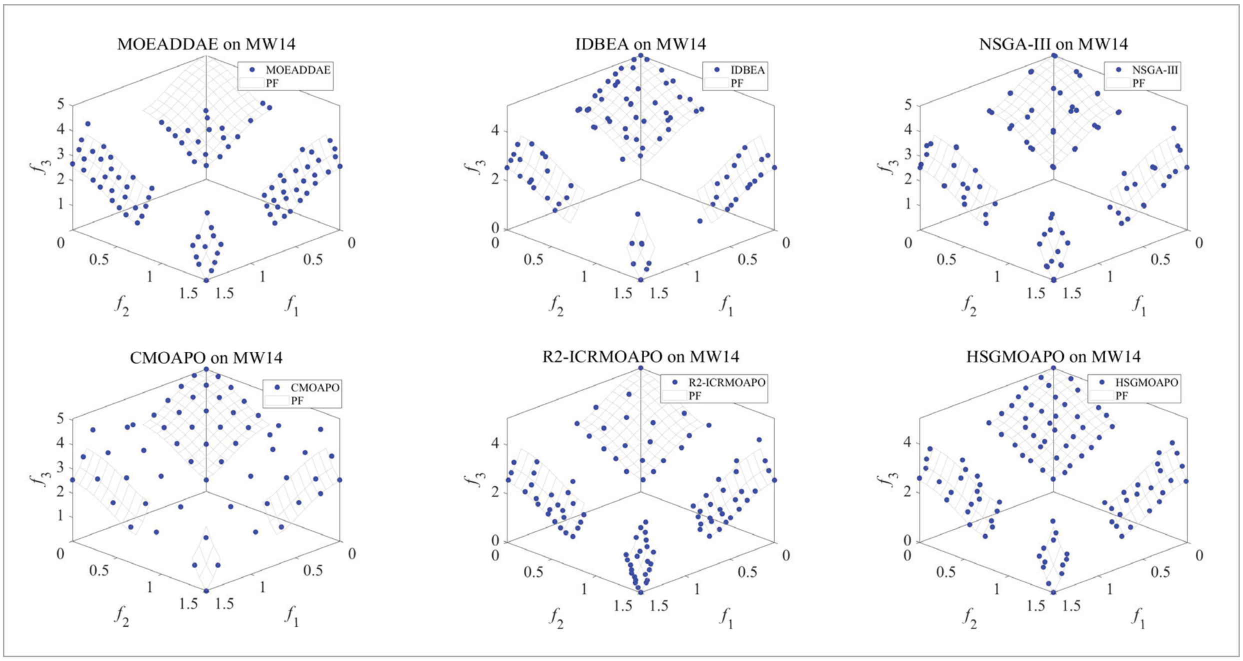
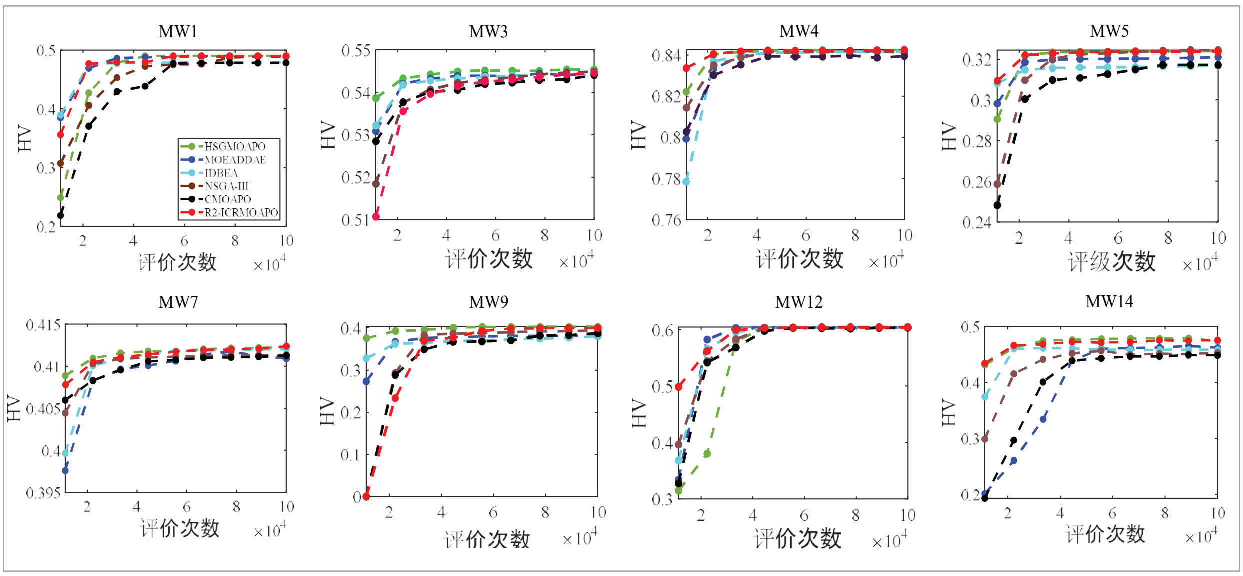


Figure 4

Convergence curve of HV indicator value

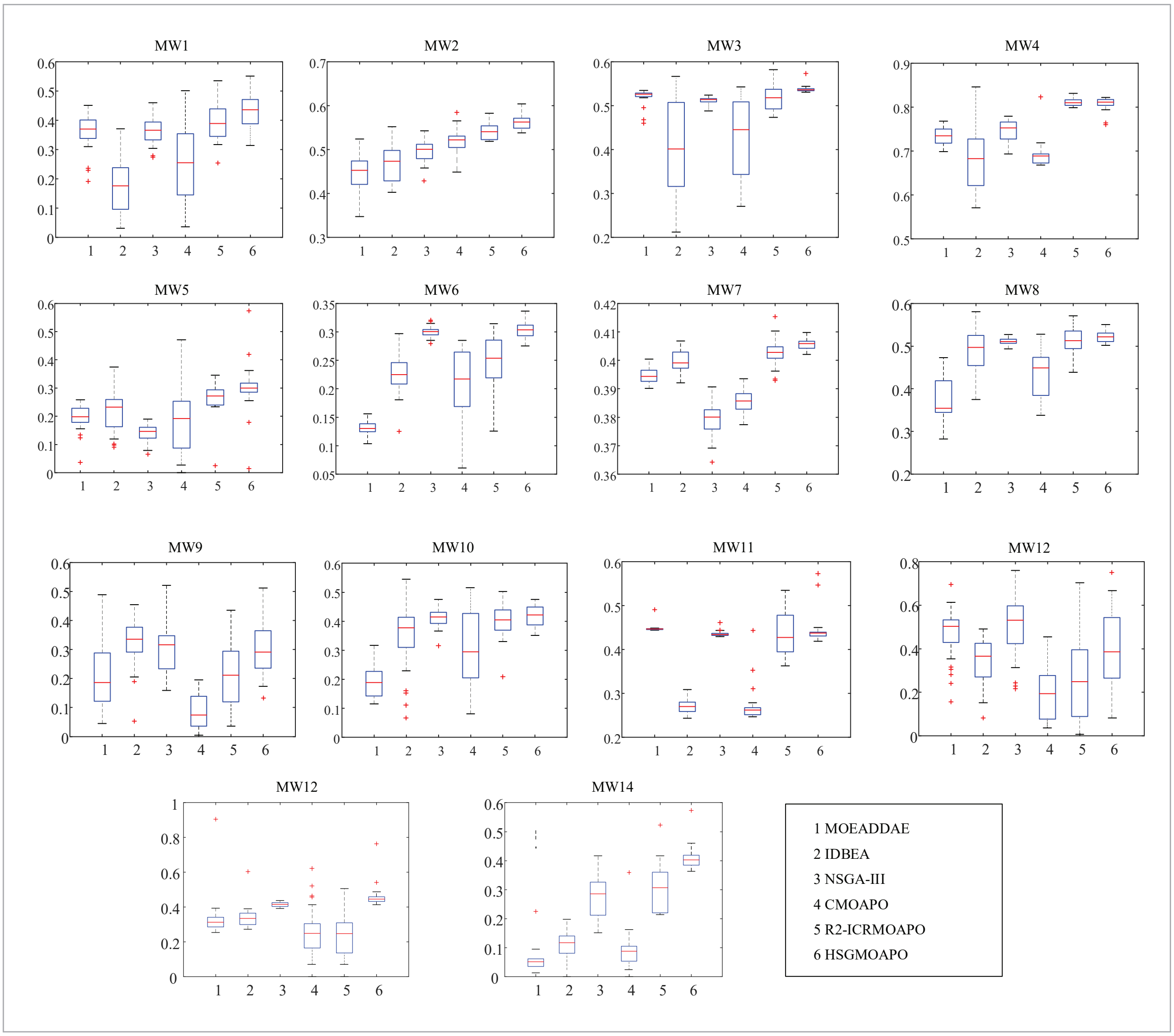


perimental data of 30 runs are recorded, respectively. The X-axis is the maximum number of evaluations per run, and the Y-axis is the HV indicator value for easy observation. As can be seen from Figure 3, HSG-MOAPO algorithm converges fastest on MW3, MW4, MW5, MW7, MW9 and MW14, and is also in a sub-optimal position when dealing with other problems. In addition, it can be seen that HSGMOAPO algorithm has the best convergence performance for MW1, MW3, MW5, MW7, MW9, and MW14 problems, which is consistent with the results obtained in Table 2. In summary, HSGMOAPO algorithm has good convergence and diversity on more than two thirds of test questions.

The boxplot can directly reflect the degree of data dispersion and overall distribution. Figure 5 shows a boxplot of HV values for MW test functions, where the horizontal labels 1, 2, 3, 4, 5 and 6 correspond to MOEADDAE, IDBEA, NSGA-III, CMOAPO, R2-ICRMOAPO and HSGMOAPO, respectively. As shown in Figure 5, the HSGMOAPO algorithm has an advantage over other comparison algorithms in HV statistics on test functions MW1, MW2, MW5, MW7, MW8, MW10, MW13, and MW14, it also shows that the improved algorithm has good convergence and distribution. At the same time, the effectiveness of the improved strategy is further illustrated by comparing the HV statistics of CMOAPO and R2-ICRMOAPO algorithms, the fusion of R2 indicator selection strategy, target spatial decomposition strategy and restric-

Figure 5

Boxplot of the statistical results of HV



tion processing strategy can significantly improve the performance of the improved APO algorithm. From the statistical results of HV indicator, there are some outliers in comparison algorithm. The outliers in HSGMOAPO algorithm are obviously reduced, but they still exist, which shows that the robustness of the improved algorithm needs to be further improved.

5.4. Calculation Efficiency Analysis

In order to verify the computational efficiency, the running time of all algorithms is recorded. All experiments were run on a computer with an Intel(R) Core(TM) i5-1035G1 CPU and 8 gigabytes of memory. Table 3 shows the average time of 30 runs of the six algorithms on the MW test problem. As can be seen from Table 3, the running time of the proposed HSGMOAPO is significantly reduced compared with MOEADDAE and IDBEA, and the running time is lower than that of CMOAPO and R2-icrmoapo algorithms, indicating that selecting candidate solutions using R2 indicator and target space decomposition strategy requires less computation than relying only on R2 indicator. Higher computational efficiency. However, in some test problems, the running time of NSGA-III is less than that of HSGMOAPO. However, according to the above experimental results, HSGMOAPO has the best balance performance in terms of convergence and diversity. Therefore, compared with the performance improvement, the above calculated cost is acceptable.

6. HSGMOAPO for Engineering Design Problems

In this section, we examine two real-world engineering application optimization problems with multiple objectives to show the efficiency and performance of the HSGMOAPO algorithm in finding non-dominated solutions with acceptable distribution and diffusion.

6.1. Welded Beam Design

The design goal of the welded beam design problem is to minimize the manufacturing cost and vertical shrinkage of the welded beam end. The schematic diagram of the problem is shown in Figure 6. This problem includes four optimization variables: weld thickness (h), clamping rod length (l), steel bar height

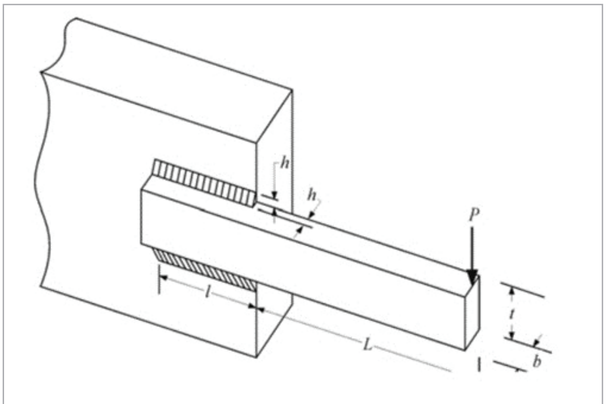
Table 3
Running time of six algorithms on MW test problem

problen	MOEADDAE	IDBEA	NSGA-III	CMOAPO	R2-ICRMOAPO	HSGMOAPO
MW1	28.83s	24.81s	2.79s	24.09s	10.39s	2.41s
MW2	31.43s	28.19s	2.02s	23.57s	11.68s	2.37s
MW3	36.46s	29.50s	5.11s	19.73s	9.40s	4.97s
MW4	34.19s	27.18s	2.89s	25.34s	12.21s	2.87s
MW5	30.85s	28.64s	2.93s	17.28s	9.55s	2.84s
MW6	33.81s	26.02s	3.66s	20.90s	11.39s	3.82s
MW7	35.94s	34.58s	2.91s	21.57s	11.77s	2.86s
MW8	40.08s	28.38s	3.35s	24.63s	13.26s	3.30s
MW9	46.69s	35.04s	2.60s	28.34s	15.31s	2.49s
MW10	38.37s	27.96s	4.56s	24.04s	12.05s	4.61s
MW11	32.93s	23.35s	2.78s	19.28s	9.83s	2.73s
MW12	48.02s	32.46s	4.73s	25.73s	14.27s	4.59s
MW13	45.26s	31.87s	4.03s	21.99s	12.38s	3.97s
MW14	47.95s	33.06s	4.52s	23.78s	13.01s	4.50s

(t), steel bar thickness (b). The constraint conditions include: shear stress (τ) and bending stress (θ) in the beam; The buckling load on the rod (Pc); The end of the beam is flat (δ) [8].

Figure 6

Schematic view of welded beam problem



The mathematical Formula for this problem is as follows:

$$\begin{aligned} \min\ & f_1(x) = 1.10471h^2 l + 0.04811tb(14.0 + l) \\ \min\ & f_2(x) = \frac{2.1952}{t^3 b} \\ s.t.\ & \tau(\vec{z}) - 13600 \le 0 \\ & \sigma(\vec{z}) - 30000 \le 0 \\ & h - b \le 0 \\ & 6000 - P_C(\vec{z}) \le 0 \end{aligned} \tag{18}$$

$$where\ \tau(\vec{z}) = \sqrt{(\tau')^2 + (\tau'')^2 + \frac{x_2 \tau' \tau''}{\sqrt{0.25\left(l^2 + (h+t)^2\right)}}}$$

$$\tau' = \frac{6000}{\sqrt{2}hl},$$

$$\tau'' = \frac{6000(14 + 0.5l)\sqrt{0.25\left(l^2 + (h+t)^2\right)}}{2\left[0.707x_1x_2\left(\frac{l^2}{12}\right) + 0.25\left(l^2 + (h+t)^2\right)\right]}$$

$$\sigma(x) = \frac{504000}{t^2 b}$$

$$P_C(x) = 64764.022(1 - 0.0282346t)tb^3$$

$$0.125 < h,\quad b < 5$$

$$0.1 < l,\ t < 10$$

Considering that the exact constraint PF of this engineering problem is unknown, this paper approximates the non-dominant feasible solutions found by six optimization algorithms, and then calculates the HV values of each algorithm on this engineering problem to evaluate the performance of each algorithm on this problem. Table 4 shows a comparison of the different algorithms for the best solution obtained. The results show that the performance of HSGMOAPO algorithm is better than other existing algorithms. For a more intuitive assessment, Figure 7 shows the Pareto frontier plots obtained by six comparison algorithms.

As shown in Figure 6, HSGMOAPO achieves the best performance in terms of convergence and distribution compared with other similar algorithms. CMOAPO and IDBEA do not converge well to the front edge, and the individual distribution of MOEADDAE is sparse in the front edge. R2-ICRMOAPO and NSGA-III have better competition.

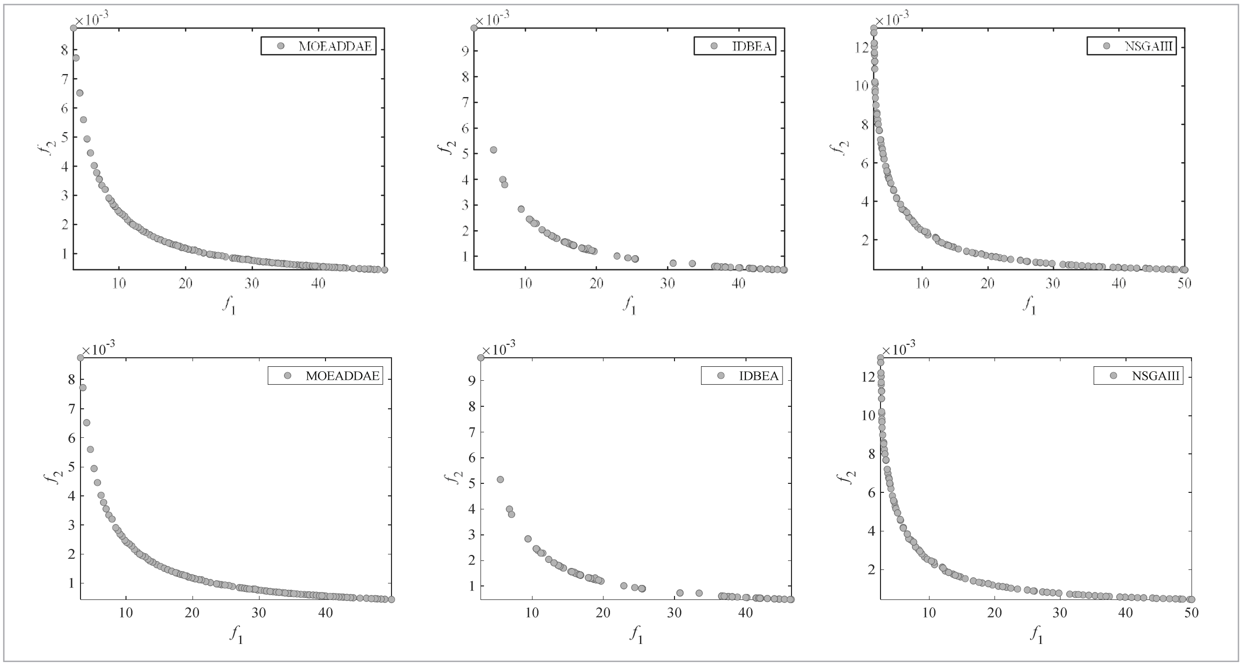
Table 4

Comparison of HV indicator results of six algorithms in welded beam design

HV	MOEADDAE	IDBEA	NSGA-III	CMOAPO	R2-ICRMOAPO	HSGMOAPO
Mean	9.8152e-1	9.8411e-1	9.8669e-1	9.5039e-1	9.8825e-1	9.9075e-1
SD	1.15e-2	1.39e-2	1.74e-2	2.23e-2	2.03e-2	1.20e-2
+/-/=	-	-	-	-	-	

Figure 7

Best Pareto optimal front obtained on welded beam design problem

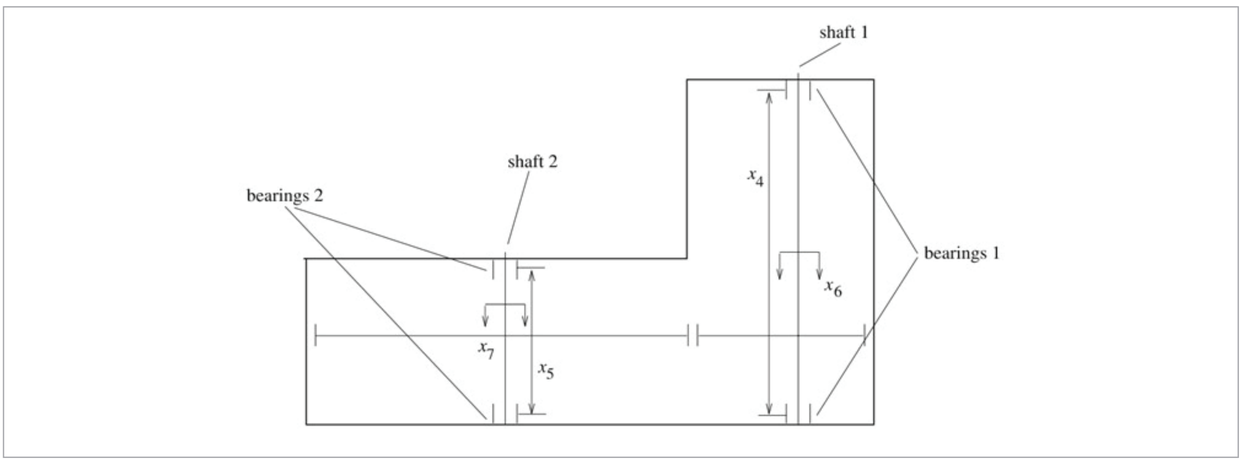


6.2. Speed Reducer Design Problem

Speed reducer design problem is a classic test problem, this problem takes reducer weight and reducer stress minimization as the objective function, and establishes a multi-objective optimization problem. There are 7 design variables (x_1-x_7) included in this problem, which are denoted as: face width (b); module of teeth (m); number of teeth in the pinion (z); length of the first shaft between bearings (l1); length of the second shaft between bearings (l2), diameter of first (d1) shafts; the diameter of second shafts (d2). Figure 8 is a schematic diagram of the design problem of the speed reducer [8].

Figure 8

Schematic view of speed reducer problem



The mathematical model of reducer optimization design problem is:

$$\min \quad f_1(x) = 0.7854x_1x_2^2\left(\frac{10x_3^2}{3} + 14.933x_3 - 43.0934\right) - 1.508x_1\left(x_6^2 + x_7^2\right) + 7.477\left(x_6^3 + x_7^3\right) + 0.7854\left(x_4x_6^2 + x_5x_7^2\right)$$

$$\min \quad f_2(x) = \sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 1.6910^7} \Big/ 0.1x_6^3$$

$$s.t. \quad \frac{1}{x_1x_2^2x_3} - \frac{1}{27} \le 0, \quad \frac{1}{x_1x_2^2x_3^2} - \frac{1}{397.5} \le 0, \quad \frac{x_4^3}{x_2x_3x_6^4} - \frac{1}{1.93} \le 0, \quad \frac{x_5^3}{x_2x_3x_7^4} - \frac{1}{1.93} \le 0, \tag{19}$$

$$x_2x_3 - 40 \le 0, \quad \frac{x_1}{x_2} - 12 \le 0, \quad 5 - \frac{x_1}{x_2} \le 0, \quad 1.9 - x_4 + 1.5x_6 \le 0, \quad 1.9 - x_5 + 1.1x_7 \le 0,$$

$$\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 1.6910^7} \Big/ 0.1x_6^3 \le 1300 \qquad \sqrt{\left(\frac{745x_5}{x_2x_3}\right)^2 + 1.57510^7} \Big/ 0.1x_7^3 \le 1100$$

$$2.6 \le x_1 \le 3.6, \quad 0.7 \le x_2 \le 0.8, \quad 17 \le x_3 \le 28, \quad 7.3 \le x_4 \le 8.3, \quad 7.3 \le x_5 \le 8.3, \quad 2.9 \le x_6 \le 3.9, \quad 5 \le x_7 \le 5.5$$

Table 5

Comparison of HV indicator results of six algorithms in Speed reducer design problem

HV	MOEADDAE	IDBEA	NSGA-III	CMOAPO	R2-ICRMOAPO	HSGMOAPO
Mean	8.7768e-1	7.8411e-1	7.1925e-1	6.9804e-1	7.9412e-1	9.4316e-1
SD	1.85e-2	2.02e-2	1.78e-2	2.00e-2	1.81e-2	1.74e-2
+/-/=	-	-	-	-	-	

Figure 9

Best Pareto optimal front obtained on speed reducer problem

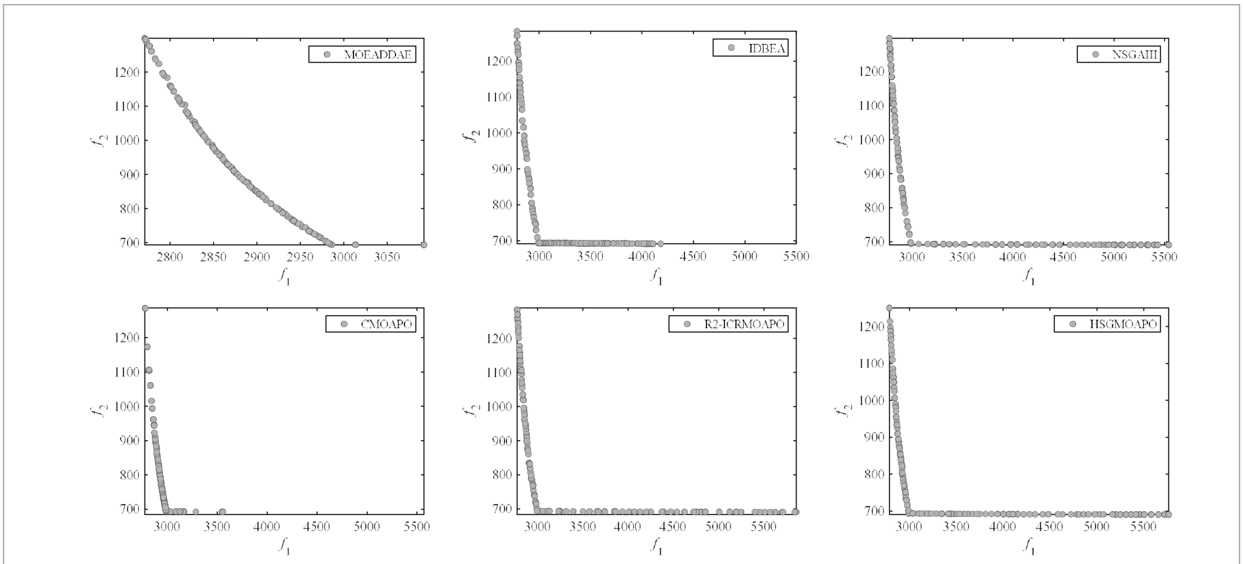


Table 5 shows the comparison between the five optimization algorithms and the proposed HSGMOAPO in HV indicator values. It is clear from Table 5 that the best solution produced by HSGMOAPO is a considerable improvement over the other algorithms. The HSGMOAPO algorithm can simultaneously find the design with the least weight and the least stress.

The best Pareto optimal solution obtained by six algorithms is shown in Figure 9. HSGMOAPO obtained the Pareto optimal solution of the reducer problem in the iterative process.

7. Conclusions and Future Studies

To solve the problem of unbalanced convergence and diversity of basic APO algorithm in complex multi-objective problems, a hybrid strategy-guided multi-objective mimicry physics optimizer algorithm, HSGMOAPO, is proposed in this paper. According to the characteristic of comprehensive evaluation solution set of R2 indicator, this paper takes it as an important strategy to improve the convergence. In the first selection, R2 indicator is used to select the candidate solutions that have a great influence on the convergence of the whole algorithm. Secondly, the diversity of population in decision space and target space is improved by the target space decomposition strategy. The selection based on R2 indicator and the target space decomposition strategy can complement each other in the process of evolution, thus improving the overall performance of the algorithm. Finally, considering the global convergence of the algorithm, a constraint processing method is proposed to adjust the position and velocity of the particles for the infeasible individuals, so as to improve the ability to avoid local optimization. Compared with the five advanced algorithms, HSGMOAPO algorithm is superior to other algorithms in the comprehensive performance of convergence, diversity, convergence speed and computational efficiency, so as to obtain better competitiveness. The algorithm is applied to two practical engineering problems, and the results show that the performance of HSGMOAPO algorithm is better than the other five algorithms in the design of double truss, and it has practical application potential. The improved strategy has achieved good results in the APO algorithm, but its idea is universal and can be flexibly introduced into other multi-objective optimization algorithms. Based on the characteristics of different strategies, the multi-strategy fusion method provides a choice for the improvement of the algorithm.

In the future, the HSGMOAPO algorithm will be used to solve a wider range of problems and update the strategy with the complexity of specific practical problems to enable the algorithm to deal with practical problems related to constraints. In addition, extending HSGMOAPO to higher-dimensional domains is the direction of further research.

Acknowledgement

This research is supported by National Natural Science Foundation of China (Grant No. 52272401), Basic Research Programs of Shanxi Province (Grant No. 202203021211185 and 202203021221150).

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