Research on the Prediction Model of Chinese Tax Revenue Based on GM (1,1) and LSSVM

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In view of the complex influencing factors of tax revenue, the highly non-linear relationship among the influencing factors and the difficulty in predicting tax revenue, this paper proposes to use GM (1,1) Combined with LSSVM, and it calculates the tax forecasting of China. This paper selects the proportion of the first industry, the ratio of import and export trade to GDP, GDP, the number of urban employment population, the proportion of residents’ disposable income and tax revenue in fiscal revenue as the influencing factors, and uses GM (1, 1) and LSSVM, respectively, to predict the tax revenue of our country, establishes the quadratic programming model to determine the optimal combination weight for the formation of the combination predicting model of tax revenue in our country, make an empirical analysis with the tax revenue of our country from 2000 to 2018 as the research object, and compare the prediction results with LSSVM model, GM (1,1) model and improved GM (1,1) model. The results show that the prediction model of China’s tax revenue based on GM (1,1) and LSSVM has a high fitting accuracy with the test set, which can reflect the complex non-linear relationship between various factors. It is of great significance for the development of prediction on Chinese tax revenue and the formulation of a scientific and effective national financial budget.

KEYWORDS: GM (1,1), LSSVM, influencing factors, secondary planning, tax revenue forecasting.
Along with the high-speed development of economy in our country since the reform and opening up, China's tax revenue has increased steadily, and it has become the main source of the financial revenue in our country recently, our revenue also will be taken from the people, and is used in the interests of the people, which not only has direct impact on the income level and life quality of Chinese residents, but also affects the financial condition and development potential of enterprises. As an important means of product redistribution and macro-control, tax revenue is becoming more and more important in economic and social development. Tax forecasting analysis is a kind of scientific management work through a trend analysis and future judgment on the tax revenue prospect by using the combination of historical data on tax revenue and mathematical statistics method on the basis of the analysis of the factors affecting tax revenue.

Because of the influence of economic society and history, the taxation system in our country is a complex system influenced by many factors, with all influencing factors interacting and influencing with each other, and many influencing factors have a large number of factors of uncertainty, which lead to a greatly increased complexity of tax revenue forecasting in our country [6]. Tax revenue prediction is the basis for the scientific formulation of tax policies, it’s also an essential condition for formulating on fiscal budgets and carrying out macro regulation in the country, and it will contributes to the healthy development of the economy and society [11]. Therefore, scholars have done a lot of research on tax forecasting. Shemyakina, Murzina and Yalyalieva [8] established a an research method of tax revenue prediction of the partial least squares with supporting vector’s system for particle swarm optimization by taking the added value, GDP and per capita GDP of the primary, secondary and tertiary industries as input parameters and the tax revenue will be used as a prediction parameter. Sang [7] established the forecasting method of tax revenue in Sichuan Province based on BP neural network model, and analyzed the influence degree of different influencing factors, she believed that the secondary and tertiary industries and the total import and export volume were the key influencing factors for tax revenue. Liu [10] established the relationship between tax revenue and various factors of industrial tax sources by using a measurement regression model of the system GMM, he also predicted the tax revenue in the 13th Five-Year Plan period based on the model results, and put forward relevant suggestions to improve the accuracy of tax revenue prediction.

Grey system theory replaces uncertain random variables with grey quantities and it is simulated in the random processes within a certain range. And the GM (1,1) model is the most widely used one, which accumulates the original data, weakens the influence of random disturbance factors, and describes the increasing growth rules by differential equation. It has a good effect on the prediction of non-negative smooth data. With constant progress of China’s tax reform process, the relationship between tax and economic has changed largely, the results which are predicted by the gray theory have some defects. Therefore, this paper will improve the GM (1,1) method, and introduces a method of the combination on support vector machine (SVM) model, which aiming at the problem that a single method cannot meet the requirement of the precision. SVM model has characteristics of fast learning speed, simple calculation structure and refined algorithm. Based on the improved GM (1,1) and SVM methods to predict tax revenue, with an optimal way to determine the combination weight of them, so the prediction results can be more conformed to the reality, and the prediction accuracy of the model will be improved.

1. An Introduction of the Algorithm
1.1. An Improved Model GM (1,1)

GM (1,1) is the most common grey model, it cumulatively generates a sequence with the rule of index according to the history data, accumulative sequence of differential equation model was constructed to calculate time-responded function, and then forecast data can be made by an accumulative reduction, and there has a good effect in dealing with a non-negative smooth data [9], but for the data with characteristics of fluctuation usually cannot reach the predicted accuracy [3]. In order to overcome this defect, this paper will improve the sequence generation method, and weaken the volatility of data, so as to form an improved GM (1,1) model. In this model, the exponential transformation and geometric average transformation are added to the original GM (1,1), and then the
accumulation is carried out. Finally, the exponential transformation and geometric average transformation are added to the original GM (1,1) so as to restore the predicted value. Specific steps are as follows:

1. Make exponential generation transformation on the original data series \( X(0) = \{x(1), x(2), \cdots, x(n)\}, x(k) > 0, k = 1, 2, \cdots n \) and \( x(k) > 0, k = 1, 2, \cdots n \), here is the formula of index transform:

\[
x(k) e_1 = x(k) T^{k-1}, k = 1, 2, \cdots n.
\]

In this formula, \( T = M / m \), and \( M \) is the maximum value of the original data sequence and \( m \) is the minimum value of the original data sequence.

2. A transformation of Geometric average generation is performed on the sequence after exponential transformation, and we can obtain:

\[
X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\}, \text{here is an expression formula of the geometric generation:}
\]

\[
x^{(0)}(k) = \left[ \prod_{j=1}^{k} x(k) e_1 \right]^{\frac{1}{k}}, k = 1, 2, \cdots n.
\]

3. The sequence generated in step (2) is accumulated once to generate an accumulated sequence \( X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\}:

\[
x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(k), k = 1, 2, \cdots n.
\]

4. Establish a whitening background value sequence according to that accumulate sequence \( Z^{(0)} = \{z^{(0)}(2), z^{(0)}(3), \cdots, z^{(0)}(n)\}:

\[
Z^{(0)} = \{z^{(0)}(2), z^{(0)}(3), z^{(0)}(n)\}:
\]

\[
z^{(0)}(k) = a x^{(0)}(k-1) + (1-a) x^{(0)}(k), a = 0.5, k = 2, 3, \cdots n.
\]

5. Constructing grey differential equation on the basis of accumulation sequence:

\[
x^{(0)}(k) + c\xi^{(1)}(k) = u.
\]

Representative development coefficient \( u \) represents the amount of gray action. The least square method is used to solve the equation:

\[
\begin{bmatrix} a \\ u \end{bmatrix} = (A^T A)^{-1} A^T Y.
\]

In this formula:

\[
A=\begin{bmatrix} -z^{(0)}(2) & 1 \\ -z^{(0)}(3) & 1 \\ \vdots & \vdots \\ -z^{(0)}(n) & 1 \end{bmatrix}, \quad Y=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}.
\]

6. Build a whitening equation on that basis of the accumulate sequence:

\[
\frac{d\xi^{(1)}}{dt} + A\xi^{(1)} = u.
\]

The time function is expressed as:

\[
\xi^{(0)}(t) = \left( x^{(0)}(1) - \frac{u}{a} \right) e^{-at} + \frac{b}{a}.
\]

7. Time series can be presented as:

\[
\xi^{(0)}(k+1) = \left( x^{(0)}(1) - \frac{u}{a} \right) e^{-ak} + \frac{b}{a}.
\]

It can be restored through a cumulative reduction:

\[
\xi^{(0)}(k+1) = \xi^{(1)}(k+1) - \xi^{(1)}(k) = (1-e^{-a}) \left( x^{(0)}(1) - \frac{u}{a} \right) e^{-ak}, k = 1, 2, \cdots.
\]

8. Finally, an exponential transformation and a geometric average generating transformation are performed to restore the predicted value.

1.2. LSSVM Algorithm

Based on the nonlinear feature of tax forecasting, the least squares support vector machine (SVM) method will be used to forecast the tax revenue, the least squares support vector machine (LSSVM) will make a change from the standard support vector machine (SVM) of equality constraint improvement into equality constraints, LSSVM owns a quickly operating speed, the advantage of the relatively simple operation. It can be widely used in the prediction, evaluation and classification, its linear regression function can be expressed as follows:

\[
y(x) = \omega \varphi(x) + b.
\]
In this formula, \( y \) presents the input variable; \( w \) presents the weight tensor; \( \varphi(x) \) presents mapping function; \( b \) represents the bias tensor.

In order to reduce the structural risk, LSSVM adopts the optimal way to solve the related parameters.

\[
\min_{w,b,e} J(w,e) = \frac{1}{2} w^T w + \frac{1}{2} \gamma \sum_{k=1}^{N} e_k^2
\]  

(13)

\[
y_k = w^T \varphi(x_k) + b + e_k, k = 1, \ldots, N.
\]  

(14)

In this formula, \( y \) represents punishing tensor; \( e_k \) represents error of fitting; \( b \) represents threshold value.

To solve this problem, LSSVM introduces Lagrange multiplier \( \alpha_k \) (non-negative number), and then constructs a Lagrange function to transform the optimization problem into:

\[
L(w, b, e, \alpha) = J(w,e) - \sum_{k=1}^{N} \alpha_k \left[ w^T \varphi(x_k) + b + e_k - y_k \right].
\]  

(15)

\( L \) will be taken derivatives into \( w, b, e \) and \( \alpha \), and we can get:

\[
\frac{\partial L}{\partial w} = 0 \rightarrow \omega = \sum_{k=1}^{N} \alpha_k \varphi(x_k)
\]

\[
\frac{\partial L}{\partial b} = 0 \rightarrow \sum_{k=1}^{N} \alpha_k = 0
\]

\[
\frac{\partial L}{\partial e_k} = 0 \rightarrow \alpha_k = \gamma e_k
\]  

(16)

\[
\frac{\partial L}{\partial \alpha_k} = 0 \rightarrow \omega^T \varphi(x_k) + b + e_k - y_k
\]

\[
k = 1, \ldots, N.
\]

\( W \) and \( e_k \) can be eliminated after we introduced a kernel function \( K(x_m, x_n) = \varphi(x_m)^T \varphi(x_n), m,n = 1, \ldots, N, \) and we can get:

\[
\begin{bmatrix} 0 \\ 1 \end{bmatrix} T_{\Omega + \gamma^{-1} I} \begin{bmatrix} b \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix}.
\]  

(17)

In this formula, \( I^T = [1 \cdots 1]; \quad \alpha = [a_1 \cdots a_N]^T. \)

LSSVM prediction model can be expressed as:

\[
y(x) = \sum_{k=1}^{N} \alpha_k K(x, x_k) + b.
\]  

(18)

2. Establishment on Tax Forecasting Model

**Step One:** Determine the influencing factors of tax revenue, collect the historical data of each indicator, and establish the test set and sample set.

**Step Two:** Use the modified GM (1,1) model and LSSVM algorithm to forecast the tax revenue.

**Step Three:** According to the prediction results of the two methods in the second step, the quadratic programming is used to establish the combined prediction model, and the weights need to be determined when establishing this model. Firstly, the error functions of the two methods will be calculated, and the optimization equation is established with the minimum sum of squared errors as the constraint condition, so as to determine the optimal weight. \( x_i(t) \) is the prediction value for GM (1,1) and SVM method, respectively, its corresponding error can be described as:

\[
e_{it} = x(t) - x_i(t)
\]  

(19)

\( W = [w_1, w_2]^T \) is the weight corresponding to the two prediction methods, and \( \sum w_i = 1. \) Thus, the error sum of squares of this prediction model is:

\[
S = \sum_{t=1}^{m} e_i^2 = \sum_{t=1}^{m} \left( \sum_{t=1}^{m} w_i e_{it} \right)^2.
\]  

(20)

The quadratic programming model is established with \( S \) minimum as the objective constraint condition, and the optimal weight is obtained by solving.

**Step Four:** Make an analysis of error for the forecasting method. If the error meets the requirements, the calculation can be terminated and the model will be used for the tax forecasting; otherwise, we should go back to the first step.

The detailed steps are shown in Figure 1.
3. Case Analysis

3.1. Index Selection of Tax Influencing Factors

National tax revenue is determined by the number of tax sources and tax system and other factors. In this paper, the influencing factors and indicators are determined as follows:

1. Proportion of the primary industry. On the one hand, the output value of the primary industry is mainly agricultural products, and there is generally no standard accounting record of agricultural products in the transaction process, which brings great inconvenience to taxation [14]. On the other hand, the primary industry is in a weak position in the development of the national economy, but it is also crucial. In order to promote a better development, the country often adopts supportive policies, especially abolished the collection of agricultural tax, butchery tax, pastoral tax and agricultural and forestry special tax after 2006. Through these two points, we can find that the tax in our country mainly comes from the second and third industries [1]. When the proportion of primary industry is high, the tax in our country is relatively less.

2. Degree of economic openness. Imports and exports are easier to monitor and collect than other taxes, and taxes on imports and exports account for a significant proportion of taxes in developing countries [12]. The ratio of import and export volume to GDP is used to reflect the degree of economic openness.

3. Level of economic development. GDP is an important indicator reflecting the level of economic development. The higher the level of economic development is, the more tax sources and tax revenue will increase [5].

4. The number of employed urban population. With the development of China's urbanization, more people has gathered in Chinese town, which created more employment opportunities and consumption demand, and promoted the development of the second and third industry, as the important engine of economic and social development, urbanization has increased the number of tax sources, reduced the tax collection cost [15], promoted a constant optimization of tax structure, and the number of our country tax revenue increased.

5. Household disposable income. The more disposable income of residents are, the higher the income level of residents will be, and the more personal income tax will be [13]. At the same time, the disposable income of residents can also reflect a country's ability of creating wealth to a certain extent. The more wealth created, the more tax sources it will have, it also means it can have more taxes sources, such as, more corporate income tax and value-added tax [4].

6. The proportion of tax revenue in fiscal revenue. Non-tax revenue includes common income, such as property gains, and capital income, such as sales of investment assets. The lower the share of tax revenue in fiscal revenue is, the easier other income is to gain, the less effort the state makes to collect taxes, and the lower the revenue it generates [2].

3.2. Data Sources

This paper takes China’s tax revenue from 2000 to 2018 as the research object, and the data comes from China Statistical Yearbook (2001-2019), China Tax
Yearbook (2001-2019) and the relevant data of National Taian Research Service Center. Table 1 is the data table of relevant factors.

Both prediction models select the data from 2000 to 2015 as the training set and the data from 2016 to 2018 as the test set. Among them, the tax value is the total annual national tax revenue, the proportion of primary industry is the total proportion of primary industry in GDP, the ratio of import and export trade to GDP in that year, the number of employed people in cities and towns, the per capita disposable income and the proportion of tax revenue in fiscal revenue. These data are all disclosed by the National Bureau of Statistics and the Tax Bureau.

### 3.3. Verification Method of Prediction Results

The sample mean square error was used to verify the prediction results MSE, Mean Absolute Error MAD and Mean Absolute Percentage Error MAPE, as for the data \( X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(k)\} \), we can predict \( x^{(0)}(k+1) \), and three expressions of error are, respectively:

#### Table 1

<table>
<thead>
<tr>
<th>Year</th>
<th>Tax Value(a hundred million)</th>
<th>The Proportion of Primary Industry (%)</th>
<th>The Ratio of Import and Export Volume to GDP(%)</th>
<th>GDP(a hundred million)</th>
<th>Population of Urban Employment(ten thousand of people)</th>
<th>Urban Disposable Income(yuan)</th>
<th>The Proportion of Tax Revenue in Fiscal Revenue(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>12581.5</td>
<td>14.7</td>
<td>39.2</td>
<td>100280.1</td>
<td>23151</td>
<td>3721.3</td>
<td>93.9</td>
</tr>
<tr>
<td>2001</td>
<td>15301.4</td>
<td>14.0</td>
<td>38.1</td>
<td>110863.1</td>
<td>23940</td>
<td>4070.4</td>
<td>93.4</td>
</tr>
<tr>
<td>2002</td>
<td>17636.5</td>
<td>13.3</td>
<td>42.2</td>
<td>121717.4</td>
<td>24780</td>
<td>4531.6</td>
<td>93.3</td>
</tr>
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<td>2003</td>
<td>20017.3</td>
<td>12.3</td>
<td>51.3</td>
<td>137422.0</td>
<td>25639</td>
<td>5006.7</td>
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<td>2004</td>
<td>24165.7</td>
<td>12.9</td>
<td>59.0</td>
<td>161840.2</td>
<td>26476</td>
<td>5660.9</td>
<td>91.5</td>
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<td>28778.5</td>
<td>11.6</td>
<td>62.4</td>
<td>187318.9</td>
<td>27331</td>
<td>6384.7</td>
<td>90.9</td>
</tr>
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<td>34804.4</td>
<td>10.6</td>
<td>64.2</td>
<td>219438.5</td>
<td>28310</td>
<td>7228.8</td>
<td>89.8</td>
</tr>
<tr>
<td>2007</td>
<td>45622.0</td>
<td>10.3</td>
<td>61.8</td>
<td>270232.3</td>
<td>29350</td>
<td>8583.5</td>
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<td>2008</td>
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<td>10.3</td>
<td>56.3</td>
<td>319515.5</td>
<td>30210</td>
<td>9856.5</td>
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<tr>
<td>2009</td>
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<td>9.8</td>
<td>43.2</td>
<td>349081.4</td>
<td>31120</td>
<td>10977.5</td>
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<td>2010</td>
<td>73210.8</td>
<td>9.5</td>
<td>48.8</td>
<td>413030.3</td>
<td>34687</td>
<td>12519.5</td>
<td>88.1</td>
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<td>2011</td>
<td>89738.4</td>
<td>9.4</td>
<td>48.3</td>
<td>489300.6</td>
<td>35914</td>
<td>14550.7</td>
<td>86.4</td>
</tr>
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<td>2012</td>
<td>100614.3</td>
<td>9.4</td>
<td>45.2</td>
<td>540367.4</td>
<td>37102</td>
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<td>2013</td>
<td>110530.7</td>
<td>9.3</td>
<td>43.4</td>
<td>595244.4</td>
<td>38240</td>
<td>18310.8</td>
<td>85.5</td>
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<tr>
<td>2014</td>
<td>119175.3</td>
<td>9.1</td>
<td>41.0</td>
<td>643974.0</td>
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</tr>
<tr>
<td>2015</td>
<td>124922.2</td>
<td>8.8</td>
<td>35.6</td>
<td>689052.1</td>
<td>40410</td>
<td>21966.2</td>
<td>82.0</td>
</tr>
<tr>
<td>2016</td>
<td>130360.7</td>
<td>8.6</td>
<td>32.7</td>
<td>743585.5</td>
<td>41428</td>
<td>23821.0</td>
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</tr>
<tr>
<td>2017</td>
<td>144369.9</td>
<td>7.9</td>
<td>33.6</td>
<td>8271217</td>
<td>42462</td>
<td>25973.8</td>
<td>83.6</td>
</tr>
<tr>
<td>2018</td>
<td>156402.9</td>
<td>7.9</td>
<td>33.9</td>
<td>900309.5</td>
<td>43419</td>
<td>28228.0</td>
<td>85.3</td>
</tr>
<tr>
<td>2019</td>
<td>158000.46</td>
<td>7.1</td>
<td>32</td>
<td>986515.2</td>
<td>45249</td>
<td>30732.8</td>
<td>83.0</td>
</tr>
<tr>
<td>2020</td>
<td>154312.29</td>
<td>7.7</td>
<td>31.6</td>
<td>1015986.2</td>
<td>46271</td>
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<tr>
<td>2021</td>
<td>172731</td>
<td>7.3</td>
<td>34.2</td>
<td>1143670</td>
<td>46773</td>
<td>35128</td>
<td>85.3</td>
</tr>
</tbody>
</table>
In order to further verify the rationality of the model, the tax revenue from 2016 to 2018 is taken as a test set, and the calculation results show that the model has a certain generalization ability and high prediction accuracy, and also shows that the selection of the influencing factors of forecasting indicators for tax revenue has applicability and rationality.

In order to further verify the rationality of the model, the tax revenue from 2016 to 2018 is taken as a test set, and the prediction results of the combined model are compared with those of LSSVM model, GM (1,1) model and improved GM (1,1) model. The comparison results are shown in Table 2.

### Table 2
Comparison of test sample output results of three different models

<table>
<thead>
<tr>
<th></th>
<th>sample</th>
<th>MSE</th>
<th>MAD</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Composite pattern</td>
<td>217.87</td>
<td>128.01</td>
<td>8.42</td>
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<tr>
<td>LSSVM model</td>
<td>225.88</td>
<td>157.45</td>
<td>10.48</td>
<td></td>
</tr>
<tr>
<td>GM(1,1) model</td>
<td>264.17</td>
<td>204.26</td>
<td>13.52</td>
<td></td>
</tr>
<tr>
<td>Improve GM(1,1) model</td>
<td>245.70</td>
<td>194.01</td>
<td>12.30</td>
<td></td>
</tr>
</tbody>
</table>

As can be seen from Table 5, the MSE of LSSVM model, GM (1,1) model and improved GM (1,1) model calculated on test samples were 225.88, 264.17 and 245.70, respectively, and the MAD were 157.45, 204.26 and 194.01, respectively. MAPE were 10.48, 13.52 and 12.30, respectively, which were all larger than the prediction errors of the combined model. Therefore, we can judge the combined forecasting results proposed in this paper have a high precision, and the validity of the model is verified, which can be used to forecast the tax revenue in China.

### 4. Conclusion

Tax revenue plays a great role in the development of national economy, and the improvement on the accuracy of tax revenue forecast is of great and practical significance for formulating reasonable economic policies. The tax revenue is affected by many factors, which are nonlinear and coupled. It is difficult
to achieve the ideal forecast result by using a single method.
In this paper, the evolution of China’s tax system structure, the total amount and structure of tax revenue, and the industrial structure of tax revenue are analyzed. At the same time, the tax revenue forecasting model based on GM (1,1) and LSSVM is established. Taking the tax revenue from 2000 to 2018 as an example, the error comparison shows that the combined forecasting results proposed in this paper have high accuracy, which verifies the effectiveness of the model, and can be used to forecast China’s tax revenue.

Data Sharing Agreement
The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

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Conflict of Interest
The authors have no conflicts of interest to declare.

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