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An Optimization-Simulation Approach to Chance-Constraint Programming

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This paper considers a stochastic programming problem with a number of random parameters in the set of constraints. The method used for solving the problem is the iterative optimization-simulation approach. It consists of two phases: optimization phase, which includes solving a deterministic counterpart of the original chance-constrained problem, and a simulation phase in which the original constraints are checked using Monte Carlo simulation. One iteration corresponds to one scenario. If the decision maker is dissatisfied with the results, a new scenario is generated in which the deterministic values of stochastic parameters are changed in the direction that will provide a more robust solution. The deterministic counterpart in the new scenario is formulated depending on the result of the previous iteration. To that end, different heuristics are considered. The main goal is to provide a good insight on the optimization problem under uncertainty by performing a relatively small number of iterations. The general approach and results of the proposed framework are illustrated on an example of advertisement placement.

KEYWORDS: Stochastic programming, Simulation, Chance-constraints, Heuristics, Scenario generation.

1. Introduction

Chance-constrained programming (CCP) is a well-known approach to solving stochastic problems. It was first introduced in 1959 [8], where the optimization problem is formulated so that the probability of meeting a constraint is above a certain acceptable level. This changes the nature of the stochastic problem from one

of uncertainty to one of risk, meaning that the decision maker is willing to accept a certain risk that constraints will not be satisfied. Even though this problem was introduced back in the 1950's it is still widely considered to be a very difficult problem and often intractable [5, 27, 22].

Therefore, a novel optimization-simulation approach to solving CCP problem is proposed. Optimization-simulation approaches are not new to optimization, and their aim is to integrate optimization methods with Monte Carlo simulation experiments for solving stochastic problems [33, 18, 19, 45, 16, 7, 1, 32, 31]. Their number has increased significantly only recently, due to increased computational abilities of computers.

Simulation is generally used as input for the other part of the framework – optimization, to create deterministic models and solve them.

The novelty in the approach proposed in this paper is that the power of simulation is used to find the probability of satisfying chance-constraints. Simulation is used to solve the hardest part of the CCP problem, by finding the probability of satisfying the constraints, which is often why CCP problems are intractable.

The first phase of the approach – the optimization phase, relies on generating appropriate deterministic counterparts of the original problem to be optimized. The efficient generation of such scenarios is the central problem in stochastic programming in general [9, 17, 41], so some simple heuristics are proposed. The proposed approach is illustrated using the problem of advertisement placement.

The paper is organized as follows: Section 2 formulates the advertisement placement model, i.e. the mathematical model, notation, and a gives a short explanation of the optimization task; Section 3 is reserved for a detailed description of the proposed optimization-simulation approach; Section 4 shows the heuristics used to generate scenarios. Research results are presented in Section 5. Concise results are given in tables and diagrams for every scenario used. Section 6 summarizes the conclusions.

2. Advertisement Placement Problem

Operational research (OR) techniques were first applied in marketing with the emergence of the famous theorem for marketing mix optimization in 1954 [15]. Linear programming and goal programming became first choices when talking about OR in marketing, and Markov models and various simulation tech-

niques, including game theory, were applied in the 1960's and 1970's [29,34]. Even in these early days of interaction between OR and marketing, researchers observed the stochastic nature of marketing problems through utilizing the theory of decision making under uncertainty [38]. Not much was done in the upcoming years in the field. But recently, due to substantially increased computational capacities of computers, modeling uncertainty through various estimation techniques has found its place in OR applications in marketing [39, 26, 3].

Consequently, stochastic programming is not new to marketing, but generally was not the main focus of researchers in the beginning. Nevertheless, it has sparked a new interest among researchers in various fields of marketing problems.

As part of the marketing mix, advertising was particularly researched. However, researchers deemed its effects on sales and market share to be doubtful. This particular opinion was formed due to concerns that advertising might have no effect on sales. But, since [10, 30, 43, 14, 23] confirmed that advertising has a positive influence on sales, all suspicions on the effectiveness of these models and its application in decision making were dismissed. Naturally, significant effort was put into developing models that could optimize the allocation of an advertising budget, which is also the task of this paper.

The problem considered in this paper is the selection of the optimal number of advertisements to be placed in different newspapers. Apart from the obvious task of determining the number of advertisements, their positions and size for every newspaper have to be determined as well, while reaching the needed views and scores both in total population, and within target groups. The goal is to achieve all the aforementioned with the lowest cost possible. Daily, weekly and monthly newspapers, their total views and ratings for the targeted groups are considered. Ratings are data with a stochastic nature and are assumed to have a Gaussian distribution. The problem is formulated as a mixed-integer stochastic programming model.

Notation

I - Set of newspapers (daily, weekly and monthly)

J - Set of possible positions in all newspapers

P_i - Set of possible positions for i -th single newspaper

T - Set of target groups

S - Minimum number of unique advertisement views

c_{ij} - Price for j -th position in i -th newspaper

v_i - Number of i -th newspaper views

r_{ik} - i -th newspaper rating for k -th target group

p_j - Visibility percentage for j -th advertisement position

t_k - Wanted rating for k -th target group

m_i - Minimum number of advertisement in i -th newspaper

h_i - Maximum number of advertisement in i -th newspaper

g_j - Minimum number of advertisement on j -th position

d_j - Maximum number of advertisement on j -th position

x_{ij} - Number of j -th advertisement in i -th newspaper

Mathematical model

With reference to the introduced notation, the advertisement placement model follows:

Minimize

$$f(x) = \sum_{i \in I} \sum_{j \in P_i} c_{ij} x_{ij} \quad (1)$$

subject to constraints

$$\sum_{i \in I} \sum_{j \in P_i} v_i p_j x_{ij} \geq S, \quad (2)$$

$$\Pr \left[\sum_{i \in I} \sum_{j \in P_i} r_{ik} x_{ij} \geq t_k \right] \geq p_k, k \in T, \quad (3)$$

$$\sum_{j \in P_i} x_{ij} \geq m_i, i \in I, \quad (4)$$

$$\sum_{j \in P_i} x_{ij} \leq h_i, i \in I, \quad (5)$$

$$\sum_{i \in I | j \in P_i} x_{ij} \geq g_j, j \in J, \quad (6)$$

$$\sum_{i \in I | j \in P_i} x_{ij} \leq d_j, j \in J. \quad (7)$$

Constraint (2) ensures the minimum required advertisement views; constraint (3) is a stochastic constraint that sets the wanted rating for every target group; constraints (4) and (5) set the minimum and maximum number of possible advertisement for ev-

ery newspaper, respectively; and constraints (6) and (7) set the minimum and maximum total number of advertisement that can be placed on the assigned positions in newspapers, respectively.

The stochastic nature of the model is reflected in equation (3), and in its set of constraints. The obvious reason is the stochastic parameter r_{ik} , which represents the newspaper rating for target groups. These ratings are calculated based on the number of copies printed, newspaper reputation and tradition, and the position of advertisement placed in those newspapers. The method used to calculate these parameters is not the focus of this paper. For the purpose of the model, we considered a case where uncertain parameters have a Gaussian distribution with expected value - μ and variance - σ .

There are 14 different target groups, which gives us a set of 14 stochastic constraints to be considered. Target groups, and consequently the set of stochastic constraints, can be classified in four groups:

- Gender: the first two constraints define the minimum wanted rating achieved within male and female population,
- Age: six different age categories are considered, and consequently the same number of constraints that define minimum wanted ratings within every age group,
- Geographic location: the model differentiates between three geographic locations where newspapers are sold, hence the minimum ratings that need to be achieved within the assigned geographic region are set,
- Education: three education levels are considered, and minimum ratings are assigned for them.

Decision makers usually require a high percentage of stochastic constraint satisfaction. Therefore, stochastic constraints are usually divided into two groups. The first group is the so-called "hard" constraints group, and the decision maker demands that these constraints are always satisfied, which means their p_k levels are 1. The second group is the "soft" constraints group and the decision maker accepts a certain risk that these constraints will not be satisfied. Usually, p_k levels for these sets of constraints are above 0.9 - 0.95. The idea of dividing constraints in an LP model to "hard" and "soft" ones was first introduced in [28] and is also used in this paper. One can

make further groups of constraints, but these are the general guidelines on how this specific problem was modeled in this paper.

3. Optimization-Simulation Approach

The approach consists of two phases – the optimization phase, where scenarios are generated and the deterministic model is solved to optimality using GLPK (GNU Linear Programming Kit - <https://www.gnu.org/software/glpk/>), and a simulation phase, where the validity of the scenario is tested by checking the chance-constraints through simulation.

The main idea is to replace the original, stochastic problem with a new, deterministic model that will provide the best solution under the condition that constraints are satisfied with a predefined probability. Generating a new deterministic counterpart of the original model corresponds to one scenario. Obviously, this approach will require defining a considerable number of scenarios that need to be checked in order to find a scenario that will achieve the wanted probability of satisfying the constraints. The upside is that every iteration provides further understanding of the original problem.

The second phase of the method uses the power of simulation to check the generated scenario.

Upon solving the deterministic counterpart of the original problem in the first phase, an optimal solution is obtained. This is needed to determine the probability of satisfying the chance-constraints. Next, a random deterministic value for every stochastic parameter is set. The values are determined from each stochastic parameter’s predefined set with assigned distribution. This way a matrix of random deterministic values from stochastic parameters is generated,

or in other words, the randomness of stochastic data is simulated. Whether the constraints are satisfied or not is checked by taking into account the deterministic values of stochastic parameters and optimal solutions of the scenario generated in the first phase.

These simulations are repeated N times, generating N of these matrices, inserting the numbers into the constraints and then calculating if they are satisfied or not. The probability for satisfying stochastic constraints can be calculated by counting how many times constraints were satisfied, using the following formula:

$$\frac{\text{Number of cases when constraint was satisfied}}{\text{Total number of cases } N} * 100$$

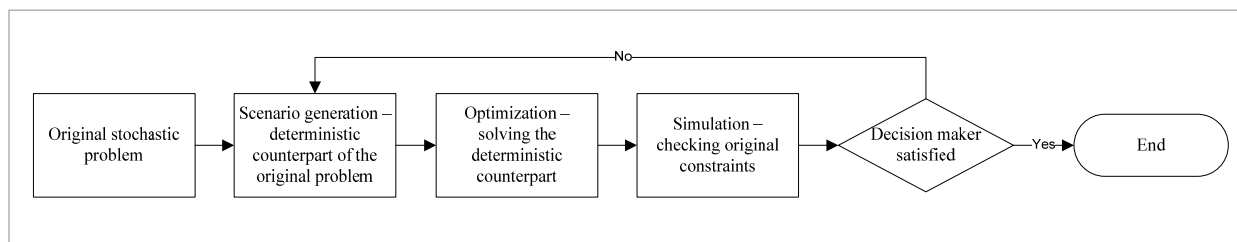
This is done for every constraint in model.

This approach is valid under the presumption that the decision maker is willing to accept a certain risk of constraints not being satisfied. In return, the value of the criterion function would be better than if the decision maker removed all uncertainty and accepted a robust approach. If the decision maker is dissatisfied with the results, a new scenario is created, i.e. a new deterministic counterpart of the original problem, and the whole procedure is repeated. The new scenario is formulated depending on the results of the previous one, and the goal is to find a scenario that meets the minimum criteria set by the decision maker.

The whole approach is summarized in Figure 1.

The main issue of the procedure is scenario generation, namely, how to define a new deterministic counterpart of the original problem. There is no strict rule in scenario generation, and it is strongly suggested to use different heuristics. The particular heuristics

Figure 1
Optimization-simulation approach



used will depend on the nature of the problem, the assumed distribution of the random data, and the experience of the decision maker. The heuristics used in this paper are explained in the following section.

4. Heuristics Used for Scenario Generation

Effectiveness and efficiency of the optimization-simulation method relies heavily on heuristics. The reason for this is the infinite number of possible scenarios to be considered. Heuristics are the cornerstone of the whole method.

Heuristic strategies are simple rules of thumb that solve complex uncertain situations [12]; they are “efficient cognitive processes that ignore information” [20] and “cognitive shortcuts that emerge when information, time and processing capacity are limited” [36].

Heuristics play a key role when facing complex problems under conditions of high uncertainty [35, 24]. Simple-rules strategy of few heuristics are desirable in predictable environments, but are indispensable in unpredictable environments [13, 21, 4, 6, 11].

Nevertheless, heuristics are not shortcuts to solving complex problems at the expense of reduced processing time, but rely heavily on environmental information [2, 25, 37]. By acknowledging these facts, heuristics can be ecologically rational. Ecological rationality is determined by two key factors, structure of the environment and lack of computing speed and power to solve the problem exactly due to a bounded rationality [42].

Ecological rationality lies in the foundation of the fast-and-frugal paradigm that provides a positive view on the heuristics, which is also the standpoint in this paper.

Fast and frugal heuristics will enable finding the right scenario by generating a relatively small number of scenarios, which is key to efficiency and effectiveness of the whole method. Considering the observed problem of advertisement placement, the distribution of the stochastic parameters and the desired p_k levels, we present heuristics used to efficiently search through the scenarios.

4.1. Heuristic 1

The first two scenarios to be generated correspond to deterministic doubles whose stochastic parameters are set to μ and $\mu-3\sigma$ respectively.

The $\mu-3\sigma$ scenario represents the robust scenario and, consequently, p_k levels expected for this scenario are 1. Constraints are satisfied regardless of the realizations of the stochastic parameters. The reason this scenario is first checked is that the decision maker expects high p_k levels (above 0.9 and 0.95) for the acceptable scenario, which means that the wanted scenario is expected to be somewhere near the robust one.

The scenario with expected values μ gives the first insight into the nature of the observed problem. After checking the scenario using simulation, p_k levels are calculated for all of the 14 chance-constraints. It is to be expected that all active constraints will have p_k levels at around 0.5, while inactive ones will be at around 0.9 or above. There will be a third group of constraints between active and inactive ones.

This heuristic indicates the existing constraints: active - stochastic parameters in these constraints need to be moved in the subsequent scenarios towards the robust solution to get the desired p_i ; inactive - constraints which already meet the set p_i and whose stochastic parameters should not be changed; constraints between the previous two - whose stochastic parameters need to be moved to some extent towards the robust solution, depending on the results of the simulation for the scenarios.

4.2. Heuristic 2

Exploit the positive correlation between the constraints within the groups when generating new scenarios.

Positive correlation between the constraints within groups is hypothesized, but to a different extent among groups. Therefore, three heuristics are derived from this general one:

Heuristic 2.1:

There is a strong positive correlation within the constraints in the age group.

These correlations are stronger between the age groups that are closer to each other than for those that are not. For example, there is a strong positive correlation between the age groups 30-39 and 40-49, and

little to no positive correlation between age groups 12-19 and 66+. The positive correlation between constraints within an age group increases or decreases correspondingly.

Heuristic 2.2:

There is no positive or negative correlation within the gender group of constraints.

Since the male and female population have different interests in newspapers and magazines, there is no significant correlation to be expected to be of use when generating scenarios.

Heuristic 2.3:

For the remaining two groups of constraints, geographic region and education, there is a moderate positive correlation to be expected and taken into account when generating new scenarios.

The most important knowledge obtained through this second group of heuristics is that it gives further insight into the problem and indicates the relations that exist between the stochastic constraints.

This means that it will not be necessary to create scenarios that will be closer to a robust one for all the parameters in every constraint in consecutive iterations, but only for some constraints. In exchange, one can still expect an increase in p_k levels for all constraints within groups. By applying this heuristic, both of the prerequisites for a good scenario are achieved: high probability levels for constraint satisfaction, and a better value of criterion function.

4.3. Heuristic 3

Determine the "simulation step".

The simulation step is defined as the value for which the stochastic parameters are adjusted when generating a new deterministic equivalent of the original problem. In this paper, the simulation step is fixed to 0.1σ , meaning that when generating a new scenario, deterministic values of the stochastic parameters are adjusted by 0.1σ towards the robust parameters, relative to the previous scenario.

The first heuristic indicated the constraints whose parameters need to be changed and to what extent; the second heuristic provided an insight into the relations of constraints within groups, further reducing the need to linearly change all of the parameters in

the designated constraints; and the final heuristic determined the concrete value for which the stochastic parameters are adjusted.

It provides consistency when generating new scenarios and enables the decision maker to get an insight into the sensitivity of the constraints when changing stochastic parameters through consecutive scenarios.

Simulation step can be defined as declining, meaning that it decreases as scenarios get closer to a robust scenario, or it can be mixed, which is the combination of the fixed and declining step. Since heuristics are idiosyncratic by its very nature, [4], decision makers can use simulation steps best suited for their problem.

This concludes the list of heuristics used and is followed by the results.

5. Computational Testing and Results

The set of 14 stochastic constraints, defined by equation (3) in the model formulation, are chance-constraints with a predefined probability level of satisfaction p_k .

The decision maker sets the p_k level for every constraint. The decision maker owns a chain of supermarkets and therefore wants to target a specific group through advertising. His target group are women aged between 40 and 49, from the geographical region G1, with a university degree. This means that second, sixth, ninth and fourteenth constraint need to have the highest p_k level. The decision maker is willing to accept less than 0.005 risk that these constraints will not be satisfied, which means that p_k levels for these four constraints need to be higher than 0.995. For all other constraints, the decision maker is willing to accept a 0.05 risk, namely, p_k levels are equal or higher than 0.95. This is presented in Table 1.

In return, the decision maker demands at least 15% in currency less spent than if the robust approach was to be used on this model. So the tradeoff is described by accepting certain levels of risk in return for a better value of criterion function.

The results of computational testing of the proposed scenarios are presented in Table 2, for every group of constraints.

Table 1

Probability levels for soft constraints

Constraint	Gender		Age						Geographic region			Education		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
p_k	≥ 0.95	>0.995	≥ 0.95	≥ 0.95	≥ 0.95	>0.995	≥ 0.95	≥ 0.95	>0.995	≥ 0.95	≥ 0.95	≥ 0.95	≥ 0.95	>0.995

The columns denote 14 target groups, TG_{i_p} , $k=1, \dots, 14$ as presented in the model formulation in Section 3. The last column represents the value of the criterion function. Changes in its value can be tracked through different scenarios. Rows of the table represent the scenario, with allocated deterministic values of the stochastic parameters, and for each scenario there is a p_k level indication that shows the probability of satisfying the constraints.

Scenario with expected value (S_{exp}) and robust scenario (S_{rob}) are two marginal cases designated by the heuristic 1, and scenarios are generated between these two cases. For each scenario there is an indication what the new deterministic equivalent is set to, and the following row presents the probability of satisfying chance-constraints of the scenario. Each scenario is run through 10,000 simulations and appropriate probabilities are calculated and presented in the table. Shaded cells indicate that changes were made to the scenario for the assigned constraint, relative to the previous scenario. The desired scenario is S_{10} as it meets the wanted p_k levels and the value of criterion function set by the decision maker.

The value of the criterion function is the worst for the robust scenario S_{rob} , which is expected, since it removes all uncertainty from the model. The lowest and the best value of the criterion function is for the scenario with the expected values, but chances that constraints are satisfied are the lowest. The value of the criterion function increases slowly as we move towards the desired scenario S_{10} that gives us the desirable probabilities for chance-constraints, keeping them within the risk accepted by the decision maker. By comparing the values of the criterion functions between the robust scenario and the desirable S_{10} , there is an 18,4% decrease in the value of the function, which meets the limit set by the decision maker.

We draw your attention to the fact that only ten scenarios are presented in the results section due to the limited space of the paper, and with the aim of a clearer presentation of results.

Note that the number of possible scenarios (iterations) in the observed stochastic problem is infinite. Even a simple discretization of the probability distributions for stochastic parameters leads to an exponential growth of the number of iterations and scenarios to be checked. The main goal of the optimization-simulation approach is to reduce the number of scenarios to a reasonable level.

We would like to refer our readers to the following book [40], especially Chapter 6, which considers the issue of generating and checking scenarios of stochastic programming problems with Monte-Carlo simulation to full extent.

As for our problem, scenarios are generated and checked until minimal requirements of the decision maker are met. In our case, it meant generating exactly 112 different scenarios before reaching the desired probabilities for constraints and the wanted value of the criterion function. Having in mind that the CPU time for the optimization part is 10.5 seconds and for the simulation part 2 seconds (RAM: 4.00GB; Processor: Intel (R) Core (TM) i3-2310M 2.10GHz), reaching the desired scenario for such a complex problem was not time consuming.

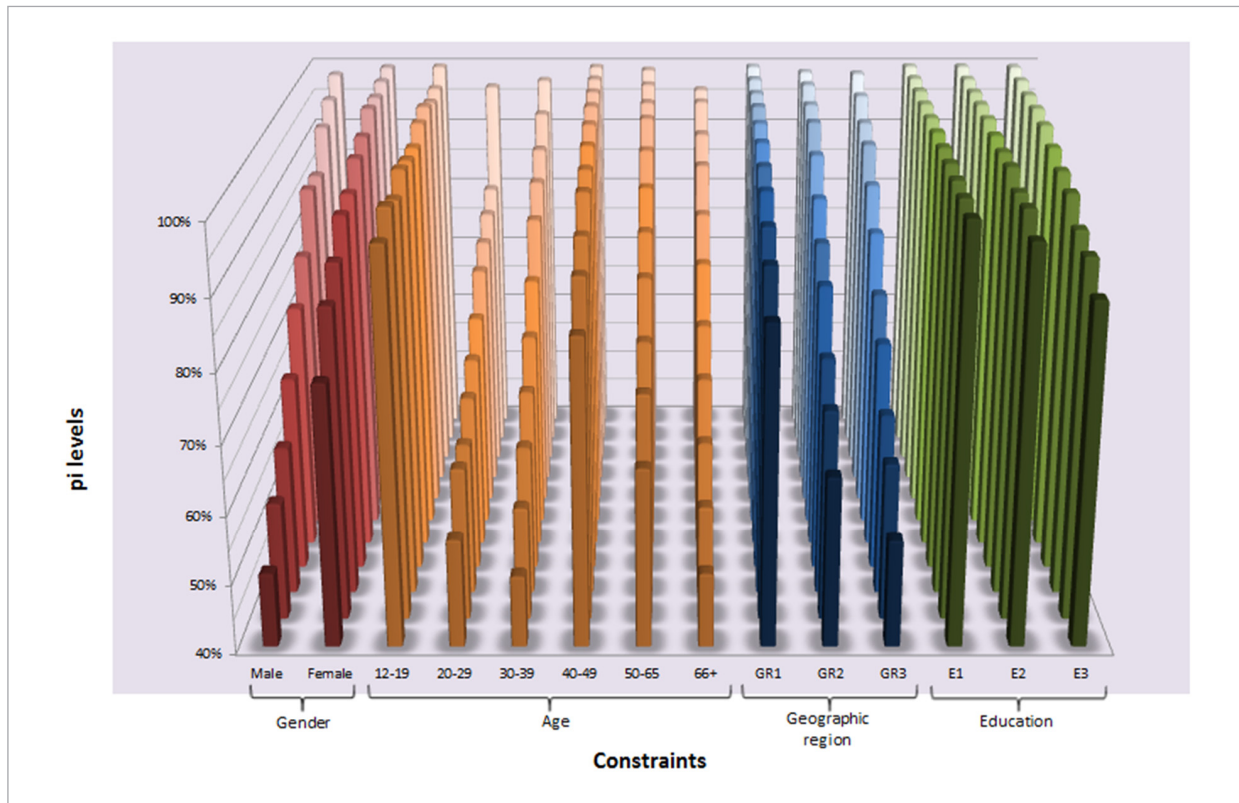
For the sake of clarity, only ten key scenarios are presented. The results presented show the general approach, how the heuristics were checked, and new heuristic rules acquired and applied when solving this problem.

Figure 2 graphically summarizes the results of the computational testing of scenarios:

Each group of constraints has its own color, and a to-

Figure 2

Graphical representation of computational results



tal of 11 scenarios are presented here, all but the robust one. This chart shows how p_k levels increase with every new scenario.

The optimization simulation approach proposed in this paper is absolutely dependent on heuristics. Heuristics are what drive this method from the beginning until the end. On the other hand, the approach provides a perfect platform for the validation of the proposed heuristics. Hogarth and Karelaia show that simulation with computer-generated data is an excellent way to evaluate a heuristic [25], meaning that whilst checking the generated scenarios, heuristics are being validated simultaneously. Heuristics also capitalize on learning processes [44]. This way the decision maker gets an insight into the problem and learns about the nature of the problem itself. This win-win situation provides a perpetual checking and learning process for both ends of the method – scenario checking and heuristic validation.

Short computation time and memory, limited number of simulations considered and the speed with which the method is run makes it ideal for generating and testing new heuristics on similar problems.

6. Conclusion

In this paper, an original approach for tackling uncertainty in chance-constraint programming was put forward and general guidelines on how to use an optimization-simulation approach were given. Advertisement placement model, which is by nature a stochastic problem, was used to show how the optimization-simulation model could be applied in practice. The approach relies on heuristics when generating new scenarios. Heuristics are obtained through scenario testing, and the goal of the paper was to confirm them so they could be used in solving similar problems.

Computational testing shows that the approach is valid under the assumptions given in this paper. It also shows that we can obtain valid approximate solution to chance-constraint programming problem of

advertisement placement by following the guidelines of the method. Knowledge obtained in solving this particular problem can be used as a basis for solving similar problems.

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