ISSN 1392 – 124X INFORMATION TECHNOLOGY AND CONTROL, 2006, Vol.35, No.1

A STUDY OF STABLE MODELS OF STOCK MARKETS

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Abstract. Since the middle of the last century, financial engineering has become very popular among mathematicians and analysts. Stochastic methods were widely applied in financial engineering. Gaussian models were the first to be applied, but it has been noticed out that they inadequately describe the behavior of financial series. Since the classical Gaussian models were taken with more and more criticism and eventually have lost their positions, new models were proposed. Stable models attracted special attention; however their adequacy in real market should be justified. Nowadays, they have become an extremely powerful and versatile tool in financial modeling. Stock market modeling problems are considered in this paper. Adequacy and efficiency of the chosen model are demonstrated. The parameters of stable laws are estimated by the maximal likelihood method. Multifractality and self-similarity hypotheses are tested and the Hurst analysis is made as well.

Keywords: Stable distributions, financial modeling, self-similarity, multifractal, infinite variance, Hurst exponent, Anderson – Darling, Kolmogorov – Smirnov criteria.

1. Introduction

Modeling of financial processes and their analysis is a very fast developing branch of applied mathematics. For a long time processes in economics and finance have been described by Gaussian distribution (Brownian motion). At present, normal models are taken with more criticism [43]. Real data are often characterized by skewness, kurtosis and heavy tails [22], [33], [35] and because of that reasons they are odds with requirements of the classical models. There are two essential reasons why the models with a stable paradigm [23], [24] are applied to model financial processes. The first one is that stable random variables (r.vs.) justify the generalized central limit theorem (CLT), which states that stable distributions are the only asymptotic distributions for adequately scaled and centered sums of independent identically distributed random variables (i.i.d.r.vs.) [20]. The second one is that they are leptokurtotic and asymmetric [9]. This property is illustrated in Figure 1, where (a) and (c) are graphs of stable probability density functions (with additional parameters) and (b) is the graph of the Gaussian probability density function, which is also a special case of stable law.

The paper is structured as follows. Overviews of related problems are given in Section 2, description and overview of stable r.vs. are introduced in Section 3, research object and analysis of its characteristics are presented in Section 4. Section 5 is devoted to the analysis of stability and self–similarity by Hurst exponent estimation. The results and conclusions are presented in Section 6.

2. Problems

Long ago in empirical studies [26], [27] it was noticed that returns of stocks (indexes, funds) are badly fitted by Gaussian distribution, because of heavy tails and strong asymmetry. Stable laws were one of the solutions in creating mathematical models of stock returns. There arises a question – why are stable laws, but not any others chosen in financial models? The answer is: because the sum of n independent stable random variables has a stable and only stable distribution, which is similar to the CLT for distributions with a finite second moment (Gaussian). If we are speaking about hyperbolic distributions, so, in general, the Generalized Hyperbolic distribution does not have this property, whereas the Normal-inverse Gaussian (NIG) [3] has it. In particular, if Y_1 and Y_2 are independent normal inverse Gaussian random variables with common parameters α and β but having different scale and location parameters $\delta_{1,2}$ and $\mu_{1,2}$, respectively, then $Y = Y_1 + Y_2$ is $\mathrm{NIG}(\alpha, \beta, \delta_1 + \delta_2, \mu_1 + \mu_2)$. So NIG fails against a stable random variable, because, in the stable case, only the stability parameter α must be fixed and the others may be different, i.e., stable ones are more flexible for portfolio construction with different asymmetry.

The other reason why stable distributions are selected from the list of other laws is that they have heavier tails than the NIG (its tail behavior is often classified as "semi-heavy").

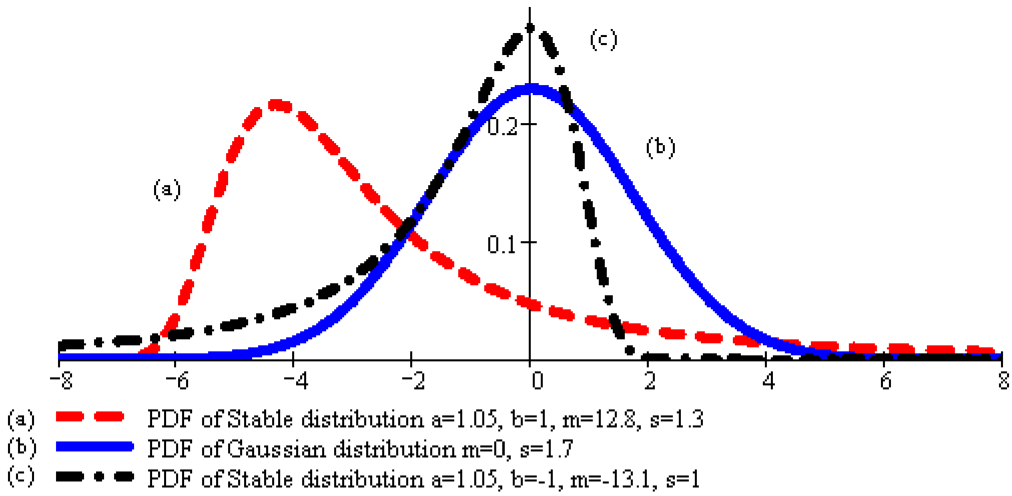

Figure 1. Stable distributions[1] are leptokurtotic and asymmetric

As it has been noticed before, stable distributions justify the generalized CLT, so from the point of view of financial engineering, they should be applied in modeling of financial portfolio. Why? Let us have n stocks with the returns r.vs. X_i from the class of stable distributions, here $i=1,\ldots,n$. Then the portfolio with the weights w_i will also be a r.v.

$$Y=\sum_{i=1}^{n} w_i X_i$$

from the class of stable distributions. But here arises a fundamental problem: whether our data are really stable and how to determine that. This work offers some approaches to the problem.

3. The stable distributions and an overview of their properties

We start with a definition of stable random variable.

We say that a r.v. X is distributed by the stable law and denote

$$X \overset{d}{=} S_\alpha(\sigma,\beta,\mu)$$

where S_α is the probability density function, if a r.v. has the characteristic function:

$$\phi(t)=$$

$$=\begin{cases} \exp\left\{-\sigma^\alpha\cdot|t|^\alpha\cdot\left(1-i\beta sign(t)\tan(\frac{\pi\alpha}{2})\right)+i\mu t\right\}, & \text{if } \alpha\neq 1 \\ \exp\left\{-\sigma\cdot|t|\cdot\left(1+i\beta sign(t)\frac{2}{\pi}\cdot\log|t|\right)+i\mu t\right\}, & \text{if } \alpha=1 \end{cases}$$

Each stable distribution is described by 4 parameters: first and most important is the stability index $\alpha\in(0;2]$, which is essential when characterizing financial data. The others respectively are: $\beta\in[-1,1]$ is skewness, $\mu\in\boldsymbol{R}$ is a position, σ is the parameter of scale, $\sigma>0$.

The probability density function is

$$p(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\phi(t)\cdot exp(-ixt)dt\,.$$

In the general case, this function cannot be expressed as elementary functions. The infinite polynomial expressions of the density function are well known, but it is not very useful for Maximal Likelihood estimation because of infinite summation of its members, for error estimation in the tails, and so on. We use an integral expression of the PDF in standard parameterization

$$p(x,\alpha,\beta,\mu,\sigma)=\frac{1}{\pi\sigma}\int_0^\infty e^{-t^\alpha}\cdot$$

$$\cdot\cos\left(t\cdot\left(\frac{x-\mu}{\sigma}\right)-\beta t^\alpha\tan\left(\frac{\pi\alpha}{2}\right)\right)dt.$$

It is important to notice that Fourier integrals are not always practical to calculate PDF because the integrated function oscillates. That is why a new formula is proposed which does not have this problem:

[1] Here a is a stability parameter, b - asymmetry parameter, m – location parameter and s is a scale parameter

$$p(x,\alpha,\beta,\mu,\sigma)=$$
$$=\begin{cases}\dfrac{\alpha\left|\dfrac{x-\mu}{\sigma}\right|^{\frac{1}{\alpha-1}}}{2\sigma\cdot|\alpha-1|}\displaystyle\int_{-\theta}^{1}U_\alpha(\varphi,\theta)\exp\left\{-\left|\frac{x-\mu}{\sigma}\right|^{\frac{\alpha}{\alpha-1}}U_\alpha(\varphi,\theta)\right\}d\varphi, \text{if } x\neq\mu\\ \dfrac{1}{\pi\sigma}\cdot\Gamma\left(1+\dfrac{1}{\alpha}\right)\cdot\cos\left(\dfrac{1}{\alpha}\arctan\left(\beta\cdot\tan\left(\dfrac{\pi\alpha}{2}\right)\right)\right), \text{if } x=\mu\end{cases}$$

$$U_\alpha(\varphi,\vartheta)=$$
$$=\left(\frac{\sin\left(\frac{\pi}{2}\alpha(\varphi+\vartheta)\right)}{\cos\left(\frac{\pi\varphi}{2}\right)}\right)^{\frac{\alpha}{1-\alpha}}\cdot\left(\frac{\cos\left(\frac{\pi}{2}((\alpha-1)\varphi+\alpha\vartheta)\right)}{\cos\left(\frac{\pi\varphi}{2}\right)}\right),$$

where $\theta=\arctan\left(\beta\tan\frac{\pi\alpha}{2}\right)\frac{2}{\alpha\pi}\cdot sign(x-\mu)$.

If μ=0 and σ=1, then $p(x,\alpha,\beta)=p(-x,\alpha,-\beta)$.

A stable r.v. has a property, which may be stated in two equivalent forms:

- If $X_1, X_2, \ldots, X_n$ are independent r.vs. distributed by $S_\alpha(\sigma,\beta,\mu)$, then $\sum_{i=1}^{n}X_i$ will be distributed by $S_\alpha(\sigma\cdot n^{1/a},\beta,\mu\cdot n)$.
- If $X_1, X_2, \ldots, X_n$ are independent r.vs. distributed by $S_\alpha(\sigma,\beta,\mu)$, then [35]

$$\sum_{i=1}^{n}X_i\overset{d}{=}\begin{cases}n^{1/\alpha}\cdot X_1+\mu\cdot(n-n^{1/\alpha}), \text{ if } \alpha\neq 1\\ n\cdot X_1+\frac{2}{\pi}\cdot\sigma\cdot\beta\cdot n\ln n, \text{ if } \alpha=1\end{cases}.$$

One of the most fundamental stable law statements [20] is as follows.

Let $X_1, X_2,\ldots,X_n$ be independent identically distributed random variables and

$$\eta_n=\frac{1}{B_n}\sum_{k=1}^{n}X_k+A_n,$$

where $B_n>0$ and A_n are constants of scaling and centering. If $F_n(x)$ is a cumulative distribution function of r.v. η_n, then the asymptotic distribution of functions $F_n(x)$, as $n\rightarrow\infty$, may be stable and only stable. And vice versa: for any stable distribution $F(x)$, there exists a series of random variables, such that $F_n(x)$ converges to $F(x)$, as $n\rightarrow\infty$.

Let X have distribution $S_\alpha(\sigma,\beta,0)$ with $\alpha<2$. Then there exist two i.i.d. random variables Y_1 and Y_2 with the common distribution $S_\alpha(\sigma,\beta,0)$ such that

$$X=\left(\frac{1+\beta}{2}\right)^{\frac{1}{\alpha}}Y_1-\left(\frac{1-\beta}{2}\right)^{\frac{1}{\alpha}}Y_2, \text{ if } \alpha\neq 1.$$

Let X_1 and X_2 be independent random variables with $X_i\sim S_\alpha(\sigma_i,\beta_i,\mu_i)$, i=1,2. Then $X_1+X_2\sim S_\alpha(\sigma,\beta,\mu)$, with

$$\sigma=\left(\sigma_1^\alpha+\sigma_2^\alpha\right)^{1/\alpha},\ \beta=\frac{\beta_1\sigma_1^\alpha+\beta_2\sigma_2^\alpha}{\sigma_1^\alpha+\sigma_2^\alpha},\ \mu=\mu_1+\mu_2.$$

The pth moment $E|X|^p=\int_0^\infty P(|X|^p>y)dy$ of random variable X exists and is finite only if $0<p<\alpha$. Otherwise, it does not exist.

Stable processes. A stochastic process $\{X(t),t\in T\}$ is stable if all its finite dimensional distributions are stable.

Let $\{X(t),t\in T\}$ be a stochastic process. $\{X(t),t\in T\}$ is α-stable if and only if all linear combinations $\sum_{k=1}^{d}b_kX(t_k)$ (here $d\geq1$ $t_1,t_2,\ldots,t_d\in T$, $b_1, b_2,\ldots,b_d$ – real) are α-stable. A stochastic process $\{X(t),t\in T\}$ is called the (standard) α-stable Levy motion if:

(1) $X(0)=0$ (almost surely);

(2) $\{X(t)$: $t\geq0\}$ has independent increments;

(3) $X(i)-X(s)\sim S_\alpha((t-s)^{1/\alpha}, \beta,0)$, for any $0\leq s<t<\infty$ and $0<\alpha\leq2$, $-1\leq\beta\leq1$.

Note that the α-stable Levy motion has stationary increments. As α=2, we have the Brownian motion.

4. Research object

In this paper, we pay our attention only to international indexes, because only they have long enough series to analyze. As we will see further, most of the statistical methods require long or very long sets. The Baltic and other Central and Eastern Europe countries have "young" financial markets and they are still developing, financial instruments are badly realizable and therefore they are often non-stationary. Stagnation effects are often observed in such markets, expressed by an extremely strong passivity: at some time periods, stock prices do not change because there are no transactions at all. In such a case, the number of zero returns can reach 89 % and the stability parameter α as well as the scale parameter σ extremely decrease and tend to 0. A new kind of model should be developed and analyzed, i.e., we have to include one more additional condition into the model – the daily stock return is equal to zero with a certain (rather high) probability p, while it is not so with the probability 1-p it changes. Models of this kind require special research in the future. In this paper, we apply stable models if the number of zeros does not exceed 16%.

Nevertheless, the studied series represent a wide spectrum of stock market. Information that is typically (finance.yahoo.com, www.omxgroup.com etc.) included into a financial database is:

- Unique trade session number and date of trade;
- Stock issuer;
- Par value;
- Stock price of last trade;
- Opening price;
- High - low price of trade;
- Average price;
- Closure price;
- Price change %;

- Supply – Demand;
- Number of Central Market (CM) transactions;
- Volume;
- Maximal – Minimal price in 4 weeks;
- Maximal – Minimal price in 52 weeks;
- Other related market information.

We use here only the closure price, because we will not analyze data as a time series and its dependence.

We focused on 26 international companies, firms, indexes and funds (Table 1), and we analyzed the following r.vs.

$$X_i = ln\frac{P_{i+1}}{P_i} = ln\,P_{i+1} - ln\,P_i$$

where P is a set of stock prices. While calculating such a variable, we transform (Figure 2) from price to log price changes ("return").

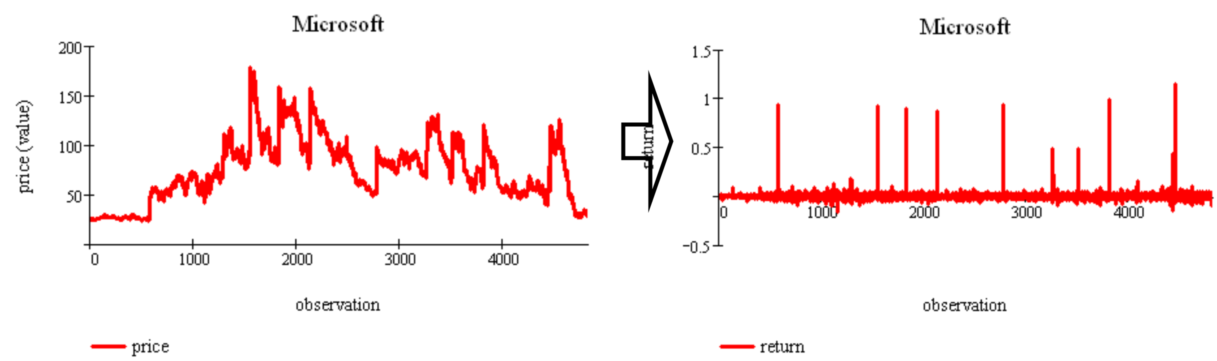

Figure 2. Data transformation

Table 1. Name of the company, index or fund

Full name	Index	Time period	Series N	Market
AIM S&P 500 INDEX INV (^ISPIX)	ISPIX	10-08-98 – 27-05-05	1712	Fund
AMEX COMPUTER TECHNOLOGY (^XCI)	AMEX	26-08-83 – 27-05-05	5486	Technology Index
AT&T CORP (T)	AT&T	02-01-62 – 27-05-05	10928	Telecom
BP PLC(BP)	BP	03-01-77 – 27-05-05	7171	Oil & Gas
CAC 40 (^FCHI)	FCHI	01-03-90 – 30-05-05	3838	Index
CAMDEN NATIONAL CORP (CAC)	CAC	08-10-97 – 27-05-05	1922	Finance
COCA-COLA CO (COKE) (KO)	COCA	02-01-62 – 27-05-05	10928	Consumer Goods
DAX IND (^GDAXI)	GDAXI	26-11-90 – 30-05-05	3652	Index
DOW JONES AIG COMMODITY INDEX (^DJC)	DJC	03-01-91 – 27-05-05	3634	Index
DOW JONES COMPANY INC (DJ)	DJ	01-07-85 – 27-05-05	5019	Services
DOW JONES INDUSTRIAL AVERAGE	DJIA	26-05-1896 – 16-01-04	26958	Industry Index
DOW JONES TRANSPORTATION AVERAGE	DJTA	26-10-1896 – 26-08-03	29296	Transportation
FIAT SPA (FIA)	FIAT	30-06-89 – 27-05-05	4014	Automobile
GENERAL ELECTRIC CO (GE)	GE	02-01-62 – 27-05-05	10928	Conglomerates
GENERAL MOTORS CORP (GM)	GM	02-01-62 – 27-05-05	10928	Consumer Goods
INTERNATIONAL BUSINESS MACHINES (IBM)	IBM	02-01-62 – 27-05-05	10928	Technology
LOCKHEED MARTIN CORP (LMT)	LMT	03-01-77 – 27-05-05	7172	Industrial Goods
MCDONALD'S CORP (MCD)	MCD	02-01-70 – 27-05-05	8935	Services
MERRILL LYNCH & CO INC (MER)	MER	03-01-77 – 27-05-05	7166	Finance
MICROSOFT CORP (MSFT)	MSFT	13-03-86 – 27-05-05	4849	Technology
NASDAQ 100 TRUST SERIES 1 (QQQQ)	NASDAQ	10-03-99 – 27-05-05	1566	Index
NIKE INC (NKE)	NIKE	19-08-87 – 27-05-05	4480	Consumer Goods
NIKKEI 225 INDEX (^N225)	NIKKEI	04-01-84 – 30-05-05	5267	Index
KONINKLIJKE PHILIPS ELECTRONICS (PHG)	PHILE	30-12-87 – 27-05-05	4393	Technology
S&P 500 INDEX (^SPX)	S&P	03-01-50 – 27-05-05	13941	Index
SONY CORP (SNE)	SONY	06-04-83– 27-05-05	5585	Technology

One can see that the length of series is very different starting from 1566 (6 years, NASDAQ) to 29296 (107 years, DJTA). Very different industries are chosen also, to represent the whole market. Their empirical characteristics are calculated and given in Table 2.

5. Analysis of stability

Examples of stability analysis can be found in the works of Rachev [4], [15] and Weron [44]. In the latter paper, Weron analyzed the DJIA index (from 1985-01-02 to 1992-11-30, 2000 data points in all). The stability analysis was based on the Anderson – Darling criterion and by the weighted Kolmogorov criterion (D'Agostino [6]), the parameters of stable distribution were estimated by the regression method proposed by Koutrouvelis [21] and fully described in [16]. The author states that DJIA characteristics perfectly correspond to stable distribution.

The problem of estimating the parameters of stable distribution is usually severely hampered by the lack of known closed form density functions for almost all stable distributions. Most of the methods in mathematical statistics cannot be used in this case, since these methods depend on an explicit form of the PDF. However, there are numerical methods that have been found useful in practice and are described below. Given a sample $x_1,\ldots,x_n$ from the stable law, they provide estimates $\hat{\alpha}$, $\hat{\beta}$, $\hat{\mu}$, and $\hat{\sigma}$ of α, *β*, *μ*, and *σ*, respectively. Stable parameters usually are estimated by these methods: maximal likelihood, regression, the method of moments, etc. All the methods are decent, but the maximal likelihood estimator yields the best results. From the practical point of view, the MLM is the worst method, because it is very time-consuming [16]. Anyway in this paper, all the 4 parameters are estimated by the MLM since it is most precise.

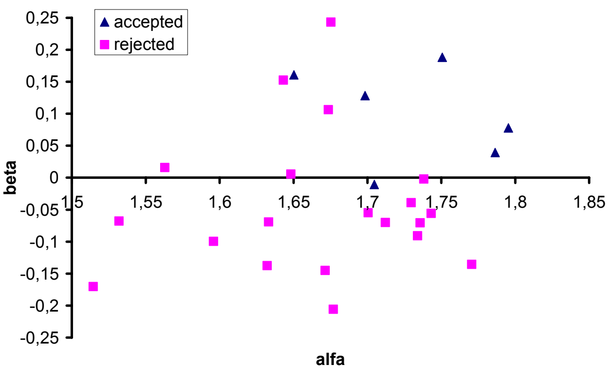

Figure 3. Distribution of *α* and *β*

Almost all data series are strongly asymmetric ($\hat{\gamma}_1$), and the empirical kurtosis ($\hat{\gamma}_2$) shows that density functions of series are more peaked than Gaussian. That is why we make an assumption that Gaussian models are not applicable to these financial series. The distribution (Figure 3) of α and *β* estimates shows that usually α is over 1.5 and for sure less than 2 (this case 1.8) for financial data.

Now we will check two hypotheses: the first one – H_0^1 is our sample (with empirical mean $\hat{\mu}$ and empirical variance $\hat{\sigma}$) distributed by the Gaussian distribution. The second – H_0^2 is our sample (with parameters *α, β,μ* and *σ*) distributed by the stable distribution. Both hypotheses are examined by two criteria: Anderson – Darling (A-D) method [19] and Kolmogorov – Smirnov (K-S) method [19]. The first criterion is more sensitive to the difference between empirical and theoretical distribution functions in far quantiles (tails), in contrast to the K-S criterion, which is more sensitive to the difference in the central part of distribution.

These two methods were very nicely applied in the works of Rachev et al. [15], [4], Weron [44], and in [16], too (to test the distribution of stock portfolio).

In Table 2, one can see the results of statistical analysis by A-D and K-S criteria.

In the marked cells, the values of A-D and K-S criteria are given which are acceptable with the confidence level of 5%. The A-D criterion rejects the hypothesis of Gaussianity in all cases with the confidence level of 5%. Hypotheses of stability were rejected only in 15 cases out of 27, but the values of criteria even, in the rejected cases, are better than for the Gaussian distributions. Referring to the K-S criterion, we give only such values that are acceptable with the 5% confidence level. One can see that there are only four acceptable cases which means that our samples are better fitted in tails than in the central part of distribution.

Then a new question arises – which kind of models could be expedient? There is only one answer – non–Gaussian models, and because of high kurtosis it would be useful to choose models with Pareto properties.

Following Rachev [4, 15] – "the α-stable distribution offers a reasonable improvement if not the best choice among the alternative distributions that have been proposed in the literature over the past four decades". But before applying stable models to financial data, it is *necessary* to demonstrate that data sets are really stable.

To prove the stability hypothesis, other researchers applied the method of infinite variance, because non–Gaussian stable r.vs. have infinite variance. The method of converging variance was proposed [14], [31] to test the hypothesis of stability. The set of empirical variances S_n^2 of random variable *X* with infinite variance diverges [14].

Let $x_1,\ldots\ x_n$ be a series of i.i.d.r.vs. *X*. Let $n \le N < \infty$ and $\bar{x}_n$ be the mean of the first *n* observations, $S_n^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x}_n)^2$, $1 \le n \le N$. If a distribution has finite variance, then there exists a finite constant $c<\infty$ such, that $\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x}_n)^2 \to c$ (almost surely), as $n\to\infty$. And vice versa, if the series is simulated by the non–Gaussian stable law, then the series S_n^2 diverges. Fofack [10] has applied this assumption to a series with finite variance (standard normal, Gamma) and with infinite variance (Cauchy and totally skewed stable). In the first case, the series of variances converged very fast and, in the second case, the series of variances oscillated with a high frequency, as $n\to\infty$. Fofack and Nolan [11] applied this method in the analysis of distribution of Kenyan shilling and Morocco dirham exchange rates in the black market. Their results allow us to affirm that the exchange rates of those currencies in the black market change with infinite variance, and even worse – the authors state that distributions of parallel exchange rates of some other countries do not have the mean (α<1 in the stable case). We present, as an example, the graphical analysis of the variance process of Microsoft corporation stock prices returns (Figure 4).

Table 2. Empirical characteristics of data sets and criterion probabilities of Anderson – Darling and Kolmogorov – Smirnov statistics

Index	Empirical characteristics					Parameters estimates of stable model					
	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	Value of A-D crit.	α	β	μ	σ	Value of A-D crit.	Value of K-S crit.
ISPIX	0	0.0002	-0.0203	2.1077	0.99970	1.7864	0.0393	0.0001	0.0078	0.63744	0.02584
AMEX	-0.0004	0.0003	0.4237	11.725	0.99999	1.6984	0.1283	-0.0001	0.0099	0.83053	0.01824
AT&T	0.0002	0.0005	18.403	1190.7	0.99999	1.5319	-0.0679	-0.0001	0.0075	0.99999	-
BP	0	0.0006	14.415	426.03	0.99999	1.7356	-0.0706	-0.0004	0.0101	0.98932	-
FCHI	-0.0002	0.0002	0.1002	2.7032	0.99999	1.7506	0.1881	-0.0000	0.0081	0.28359	0.01198
CAC	0.0002	0.001	20.754	739.51	0.99999	1.5145	-0.1701	-0.0006	0.0088	0.99866	-
COCA	0.0001	0.0006	19.445	660.44	0.99999	1.7121	-0.0699	-0.0004	0.0088	0.98818	-
GDAXI	-0.0003	0.0002	0.1928	3.5719	0.99999	1.6502	0.1607	-0.0001	0.0079	0.84363	-
DJC	-0.0001	0.0001	0.5157	7.4797	0.99999	1.7954	0.0778	-0.0001	0.0046	0.87824	-
DJ	0	0.0004	1.9248	41.859	0.99999	1.7046	-0.0107	-0.0000	0.0103	0.91486	-
DJIA	0.0002	0.0001	-0.9114	26.040	0.99999	1.5958	-0.0995	0.0002	0.0056	0.99834	-
DJTA	0.0001	0.0001	-0.1545	15.259	0.99999	1.5629	0.01586	0.0002	0.0056	0.99970	-
FIAT	0.0004	0.0007	-3.7374	126.16	0.99999	1.6331	-0.0692	0.0006	0.0127	0.99999	-
GE	0.0001	0.0006	20.253	749.55	0.99999	1.7431	-0.0558	-0.0003	0.0090	0.97881	-
GM	0.0001	0.0003	5.3537	204.43	0.99999	1.7339	-0.0908	-0.0000	0.0098	0.99769	-
IBM	0.0002	0.0006	23.209	1114.1	0.99999	1.7005	-0.0548	-0.0002	0.0091	0.91800	-
LMT	-0.0003	0.0008	12.869	476.76	0.99999	1.6322	-0.1375	-0.0007	0.0115	0.97905	-
MCD	0	0.0007	11.471	284.85	0.99999	1.7296	-0.039	-0.0005	0.0106	0.88734	-
MER	-0.0002	0.0009	7.6567	184.51	0.99999	1.7705	-0.1355	-0.0005	0.0148	0.89021	-
MSFT	0	0.0013	9.9985	180.47	0.99999	1.7381	-0.0021	-0.0010	0.0141	0.83503	-
NASDAQ	0.0006	0.001	7.9646	188.53	0.99999	1.6753	0.2431	0.0013	0.0145	0.82352	0.02928
NIKE	-0.0003	0.0009	9.2816	223.60	0.99999	1.6714	-0.1450	-0.0010	0.0130	0.93361	-
NIKKEI	0	0.0002	0.1113	7.6903	0.99999	1.6431	0.1526	0.0003	0.0076	0.98046	-
PHILE	-0.0001	0.001	16.206	663.69	0.99999	1.6482	0.0058	-0.0004	0.0138	0.98355	-
S&P	-0.0003	0.0001	1.3313	35.117	0.99999	1.6735	0.1064	-0.0002	0.0049	0.99913	-
SONY	-0.0002	0.0005	4.8083	150.51	0.99999	1.6769	-0.2057	-0.0005	0.0115	0.98979	-

The columns in this graph show the variance at different time intervals, the solid line shows the series of variances S_n^2. One can see that, as n increases, i.e. $n\to\infty$, the series of empirical variance S_n^2 not only diverges, but also oscillates with a high frequency. The same situation is for mostly all our data sets presented.

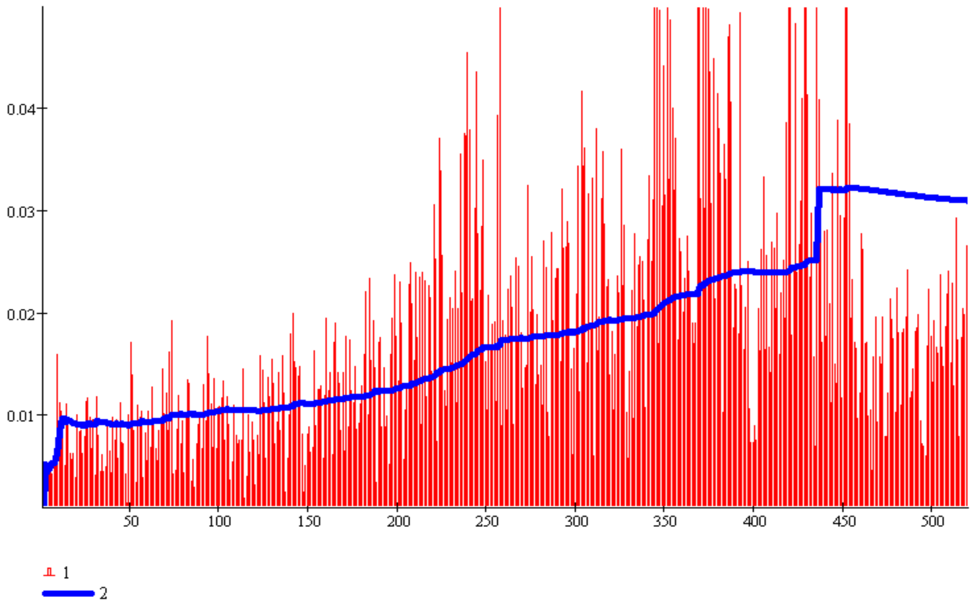

Figure 4. Series of empirical variance of the MICROSOFT company (13-03-86 – 27-05-05)

5.1. Stability by homogeneity

The third method to verify the stability hypothesis is based on the fundamental statement. Suppose we have an original financial series (returns or subtraction of logarithms of stock prices) $X_1, X_2, \ldots, X_n$. Let us calculate the partial sums $Y_1, Y_2, \ldots, Y_{[n/d]}$, where $Y_k = \sum_{i=(k-1)\cdot d+1}^{k\cdot d} X_i$, $k=1\ldots[n/d]$, and d is the number of sum components (freely chosen). The fundamental statement implies that original and derivative series must be homogeneous. Homogeneity of original and derivative (aggregated) sums was tested by the Smirnov and Anderson criteria (ω^2) [19].

The accuracy of both methods was tested with generated sets, which were distributed by the uniform R(-1,1), Gaussian $N(0,1/\sqrt{3})$, Cauchy C(0,1) and stable $S_{1.75}(1,0.25,0)$ distributions. Partial sums were scaled, respectively, by $\sqrt{d}$, $\sqrt{d}$, d, $d^{1/1.75}$. The test was repeated for a 100 times. The results of this modeling show that the Anderson criterion (with confidence levels 0.01, 0.05 and 0.1) is more precise than that of Smirnov with the additional confidence level (for details see Appendix A).

It should be noted that these criteria require large samples (of size no less than 200), that is why the original sample must be large enough. The best choice would be if one could satisfy the condition $n/d>200$.

The same test was performed with real data from Table 1, but homogeneity was tested only by the Anderson criterion. Partial series were calculated by summing d=10 and 15 elements and scaling with $d^{1/\alpha}$. The parameter α was taken from Table 2. The results of this test are presented in Table 3.

Table 3. Test of homogeneity of the series of partial sums and original series by the Anderson criterion (significance level 5%)

Index	$m = 10$	$m = 15$
ISPIX	+	+
AMEX	+	+
AT&T	+	-
BP	+	+
FCHI	+	+
CAC	-	-
COCA	+	+
GDAXI	+	+
DJC	+	+
DJ	+	+
DJIA	-	-
DJTA	+	+
FIAT	-	-
GE	+	+
GM	+	+
IBM	+	+
LMT	+	+
MCD	+	+
MER	+	+
MSFT	+	+
NASDAQ	-	+
NIKE	+	+
NIKKEI	-	-
PHILE	+	+
S&P	+	+
SONY	+	+

The value "+" means that the hypothesis of homogeneity of the original and derivative samples is acceptable, with the confidence level 5% and, vice versa, the value "-" means that it is unacceptable with the 5% confidence level.

One may draw a conclusion from the fundamental statement that for international indexes ISPIX, AMEX, BP, FCHI, COCA, GDAXI, DJC, DJ, DJTA, GE, GM, IBM, LMT, MCD, MER, MSFT, NIKE, PHILE, S&P, SONY the hypothesis on stability is acceptable.

5.2 Self – similarity and multifractality

As mentioned before, for a long time it has been known that normal models do not properly describe financial series. Due to that, there arises a hypothesis of fractionallity or self–similarity. The Hurst indicator (or exponent) is used to characterize fractionallity. The process with the Hurst index H=½ corresponds to the Brownian motion, when variance increases at the rate of $\sqrt{t}$, where t is the amount of time. Indeed, in real data this growth rate (Hurst exponent) is longer [5]. As $0.5<H\leq 1$, the Hurst exponent implies a persistent time series characterized by long memory effects, and when $0\leq H<0.5$, it implies an anti-persistent time series that covers less distance than a random process. Such behavior is observed in mean – reverting processes.

There are a number of different, not equivalent definitions of self-similarity [37]. The standard one states that a continuous time process $Y=\{Y(t),t\in T\}$ is self-similar, with the self-similarity parameter H (Hurst index), if it satisfies the condition:

$$Y(t)\overset{d}{=}a^{-H}Y(at),\quad \forall t\in T,\forall a>0,0\leq H<1, \quad (1)$$

where the equality is in the sense of finite-dimensional distributions. The canonical example of such a process is Fractional Brownian Motion (H=½). Since the process Y satisfying (1) can never be stationary, it is typically assumed to have stationary increments [5].

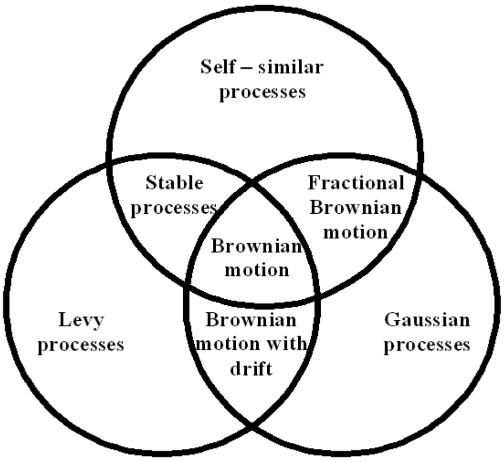

Figure 5. Self-similar processes and their relation to Levy and Gaussian processes

Figure 5 shows that stable processes are the product of a class of self–similar processes and that of Levy processes. Suppose a Levy process $X=\{X(t), t\geq 0\}$. Then X is self-similar if and only if each $X(t)$ is strictly stable [7]. The index α of stability and the exponent H of self-similarity satisfy $\alpha=1/H$.

Consider the aggregated series $X^{(m)}$, obtained by dividing a given series of length N into blocks of length m, and averaging the series over each block.

$$X^{(m)}(k)=\frac{1}{m}\sum_{i=(k-1)m+1}^{km}X_i \text{ , here } k=1, 2\ldots[N/m].$$

Self-similarity is often investigated not through the equality of finite-dimensional distributions, but through the behavior of the absolute moments. Thus, consider

$$AM^{(m)}(q)=E\left|\frac{1}{m}\sum_{i=1}^{m}X(i)\right|^q=\frac{1}{m}\sum_{k=1}^{m}\left|X^{(m)}(k)-\overline{X}\right|^q$$

If X is self-similar, then $AM^{(m)}(q)$ is proportional to $m^{\beta(q)}$, it means that $ln\,AM^{(m)}(q)$ is linear in $ln\,m$ for a fixed q:

$$ln\,AM^{(m)}(q)=\beta(q)ln\,m+C(q)\,. \quad (2)$$

In addition, the exponent $\beta(q)$ is linear with respect to q. In fact, since $X^{(m)}(i)\overset{d}{=}m^{1-H}X(i)$, we have

$$\beta(q)=q(H-1) \quad (3)$$

Thus, the definition of self-similarity is simply that the moments must be proportional as in (2) and that $\beta(q)$ satisfies (3).

This definition of a self-similar process given above can be generalized to that of multifractal processes. A non-negative process $X(t)$ is called multifractal if the logarithms of the absolute moments scale linearly with the logarithm of the aggregation level m. Multifractals are commonly constructed through multiplicative cascades [8]. If the multifractal can take positive and negative values, then it is referred to as a signed multifractal (the term "multiaffine" is sometimes used instead of "signed multifractal"). The key point is that, unlike self-similar processes, the scaling exponent $\beta(q)$ in (2) is not required to be linear in q. Thus, signed multifractal processes are a generalization of self-similar processes. To discover whether a process is (signed) multifractal or self-similar, it is not enough to examine the second moment properties. One must analyze higher moments as well.

As one can see from Appendix B and Table 4, all the considered series are multifractal, since $ln\,AM^{(m)}(q)$ is linear on $ln\,m$ (2), and most of them are also self-similar, because $\hat{H}(q)=1+\frac{\beta(q)}{q}$ (3) is linear on q. However, visually one can see that only few have nice linear dependence. That is because we estimate only from 10 points, which is really too little and, thus, the reliability of the correlation coefficient is very doubtful.

Table 4. Correlation coefficient between $H(q)$ and q

Index	Corelation Coef.	Visualy linear?	Self-similar or Multifractal
ISPIX	97.57	yes	Self-similar
AMEX	99.01	yes	Self-similar
AT&T	92.572	no	Multifractal
BP	92.445	no	Multifractal
FCHI	98.533	yes	Self-similar
CAC	90.77	no	Multifractal
COCA	92.48	no	Multifractal
GDAXI	99.174	yes	Self-similar
DJC	99.597	yes	Self-similar
DJ	97.828	yes	Self-similar
DJIA	98.103	yes	Multifractal
DJTA	98.501	yes	Self-similar
FIAT	96.829	Yes-no	Multifractal
GE	92.435	no	Multifractal
GM	96.915	Yes-no	Multifractal
IBM	92.989	no	Multifractal
LMT	95.257	no	Multifractal
MCD	93.778	no	Multifractal
MER	95.25	no	Multifractal
MSFT	92.567	no	Multifractal
NASDAQ	94.513	no	Multifractal
NIKE	94.367	no	Multifractal
NIKKEI	98.305	yes	Self-similar
PHILE	93.651	no	Multifractal
S&P	98.581	yes	Self-similar
SONY	96.918	Yes-no	Multifractal

Finally only 9 indexes are self–similar: ISPX, Amex, FCHI, gdaxi, djc, dj, djta, Nikkei, s&p.

5.2.1. Hurst exponent estimation

There are many methods to evaluate this index, but in literature the following are usually used [17]:

a. Time-domain estimators,

b. Frequency-domain/wavelet-domain estimators,

The methods:

- *Absolute Value method(*Absolute Moments*)* [36, 37, 39];
- *Variance method (*Aggregate Variance*)* [36, 39, 40];
- *R/S method*. R/S is one of the better known methods. It has been discussed in detail since 1969. The author of this idea was Mandelbrot [28, 29, 30, 36, 39]. This method also has some robust modifications, the best known of them is Lo – *R/S* [25, 42].
- *Variance of Residuals*. [32, 39];

are known as time domain estimators. Estimators of this type are based on investigating the power law relationship between a specific statistic of the series and the so-called aggregation block of size *m*.

The following three methods and their modifications are usually presented as time-domain estimators:

- *Periodogram method* [13, 36, 39]. Also one can find some modifications of this method [38].
- *Whittle* [12, 41]. Some robust methods, such as the Aggregated Whittle Method [18] or Local Whittle Method [34] were developed;
- *Abry-Veitch* (AV) [2, 17].

The methods of this type are based on the frequency properties of wavelets.

All Hurst exponent estimates were calculated with SELFIS software (Table 5), which is freeware and can be found on the web page [45].

For estimating by the method of Aggregate Variance, R/S, Periodogram, Absolute Moments and Variance of Residuals, the correlation coefficients are found as well. Estimates of Abry-Veitch and Whittle confidence intervals are given, too.

The correlation coefficient (Table 5 and Figure 6) for the Hurst exponent illustrates the adequacy of estimation. Since we need a high adequacy, we take the Hurst estimates only with the correlation over 0.9 (Figure 7). Over 60% of the cases fall into this area (only the methods of Aggregate Variance, R/S, and Variance of Residuals). One can see that the methods of periodogram and absolute moments are not very good applicable, to estimate the Hurst exponent.

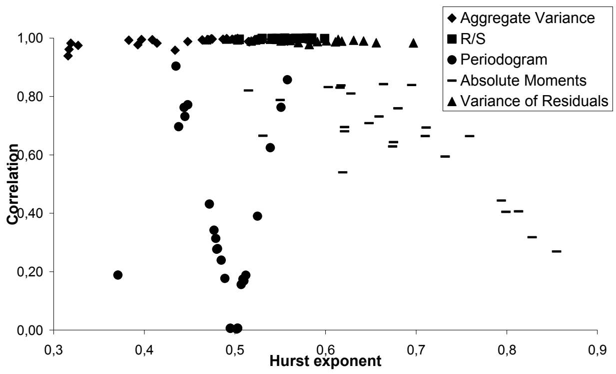

Figure 6. Correlation of the Hurst exponent estimate, by the methods of Aggregate Variance, R/S, Periodogram, Absolute Moments, and Variance of Residuals

If one looks again at Figure 7, one can see that in almost 30% (mostly estimated by the method of Aggregate Variance) of the cases the Hurst exponent is less than 0.5, so we have to reject the hypothesis of stability (in that case) or not to use that method.

Figure 8 shows us overall distribution of the Hurst exponent of all series, but it gives not much information since all the methods (either with correlation less than 0.9) are included.

Table 5. Hurst index estimates and their correlation coefficient or confidence interval

INDEX	Parameter	Aggregate Variance	R/S	Periodo-gram	Absolute Moments	Variance of Residuals	Abry-Veitch Estimator	Whittle Estimator
ISPIX	Hurst e. e. Correlation coef. or confidence interval	0.516 98.70%	0.561 99.66%	0.438 6.967%	0.855 26.90%	0.601 99.03%	0.540 [0.483-0.598]	0.500 [0.461-0.538]
AMEX	Hurst e. e. CC or CI	0.470 99.52%	0.573 99.92%	0.448 7.720%	0.675 64.35%	0.584 99.40%	0.527 [0.503-0.552]	0.500 [0.480-0.519]
AT&T	Hurst e. e. CC or CI	0.393 97.70%	0.555 99.96%	0.507 1.561%	0.616 83.01%	0.656 98.40%	0.452 [0.435-0.468]	0.500 [0.486-0.513]
BP	Hurst e. e. CC or CI	0.522 99.48%	0.541 99.77%	0.481 2.790%	0.759 66.42%	0.611 98.71%	0.494 [0.469-0.518]	0.500 [0.481-0.519]
FCHI	Hurst e. e. CC or CI	0.558 98.92%	0.568 99.92%	0.435 9.037%	0.813 40.65%	0.570 98.41%	0.543 [0.506-0.580]	0.500 [0.472-0.527]
CAC	Hurst e. e. CC or CI	0.317 96.10%	0.563 99.82%	0.371 18.87%	0.619 54.03%	0.469 99.45%	0.541 [0.483-0.598]	0.500 [0.461-0.538]
COKE	Hurst e. e. CC or CI	0.397 99.48%	0.505 99.59%	0.503 0.590%	0.617 83.76%	0.591 98.77%	0.514 [0.497-0.531]	0.500 [0.486-0.513]
GDAXI	Hurst e. e. CC or CI	0.549 99.37%	0.585 99.97%	0.495 0.578%	0.794 44.38%	0.551 99.00%	0.529 [0.492-0.566]	0.500 [0.472-0.527]
DJC	Hurst e. e. CC or CI	0.540 99.58%	0.599 99.84%	0.485 2.394%	0.828 31.82%	0.561 99.11%	0.544 [0.507-0.581]	0.504 [0.477-0.531]
DJ	Hurst e. e. CC or CI	0.409 99.45%	0.545 99.82%	0.444 7.625%	0.621 68.04%	0.491 99.60%	0.523 [0.498-0.548]	0.500 [0.480-0.519]
DJIA	Hurst e. e. CC or CI	0.471 99.41%	0.580 99.94%	0.551 7.628%	0.648 70.84%	0.539 99.59%	0.564 [0.553-0.576]	0.511 [0.501-0.520]
DJTA	Hurst e. e. CC or CI	0.434 95.85%	0.579 99.92%	0.525 3.902%	0.621 69.54%	0.567 99.71%	0.597 [0.586-0.609]	0.544 [0.534-0.554]
FIAT	Hurst e. e. CC or CI	0.503 99.85%	0.549 99.62%	0.445 7.318%	0.732 59.42%	0.518 99.00%	0.513 [0.476-0.551]	0.500 [0.472-0.527]
GE	Hurst e. e. CC or CI	0.414 98.24%	0.530 99.88%	0.489 1.772%	0.628 80.99%	0.631 99.22%	0.605 [0.588-0.622]	0.500 [0.486-0.513]
GM	Hurst e. e. CC or CI	0.474 99.75%	0.547 99.89%	0.495 0.669%	0.659 73.12%	0.523 99.33%	0.558 [0.542-0.575]	0.500 [0.486-0.513]
IBM	Hurst e. e. CC or CI	0.448 98.82%	0.543 99.73%	0.509 1.748%	0.664 84.22%	0.614 99.55%	0.443 [0.426-0.460]	0.500 [0.486-0.513]
LMT	Hurst e. e. CC or CI	0.487 99.61%	0.568 99.93%	0.558 8.571%	0.711 69.35%	0.642 98.97%	0.458 [0.433-0.483]	0.500 [0.480-0.519]
MCD	Hurst e. e. CC or CI	0.319 98.21%	0.541 99.88%	0.479 3.137%	0.515 82.05	0.560 99.31%	0.548 [0.531-0.564]	0.500 [0.486-0.513]
MER	Hurst e. e. CC or CI	0.475 99.66%	0.549 99.72%	0.480 2.766%	0.680 75.91%	0.582 99.77%	0.531 [0.506-0.556]	0.500 [0.480-0.519]
MSFT	Hurst e. e. CC or CI	0.383 99.26%	0.503 99.35%	0.472 4.317%	0.603 83.19%	0.586 99.44%	0.493 [0.468-0.518]	0.500 [0.480-0.519]
NASDAQ	Hurst e. e. CC or CI	0.515 98.82%	0.567 99.89%	0.502 0.392%	0.799 40.49%	0.582 97.81%	0.497 [0.440-0.554]	0.500 [0.461-0.538]
NIKE	Hurst e. e. CC or CI	0.327 97.48%	0.528 99.49%	0.510 1.688%	0.550 78.79%	0.546 99.79%	0.611 [0.587-0.636]	0.503 [0.484-0.522]
NIKKEI	Hurst e. e. CC or CI	0.316 93.90%	0.572 99.92%	0.512 1.880%	0.531 66.54%	0.505 99.53%	0.528 [0.504-0.553]	0.500 [0.480-0.519]
PHILE	Hurst e. e. CC or CI	0.491 99.81%	0.570 99.93%	0.503 0.708%	0.695 83.92%	0.697 98.32%	0.569 [0.544-0.594]	0.500 [0.480-0.519]
S&P	Hurst e. e. CC or CI	0.464 99.37%	0.578 99.94%	0.477 3.421%	0.674 62.89%	0.546 99.52%	0.570 [0.554-0.587]	0.520 [0.506-0.534]
SONY	Hurst e. e. CC or CI	0.497 99.60%	0.579 99.95%	0.539 6.246%	0.710 66.43%	0.618 98.95%	0.642 [0.617-0.666]	0.522 [0.503-0.541]

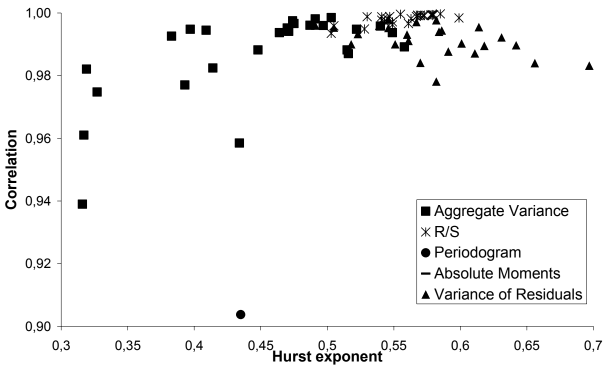

Figure 7. Correlation of the Hurst exponent estimate, by the methods of Aggregate Variance, R/S, and Variance of Residuals (correlation ≥0,9)

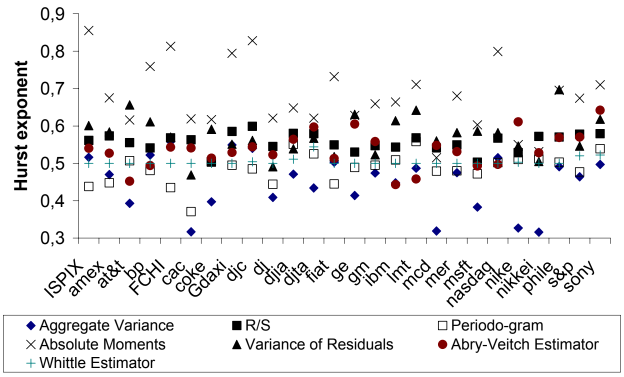

Figure 8. Hurst exponent estimates by all the methods for all series

Hurst exponent in 75% of the cases is over the level H=0.5, which means that α is less than 2 (non–Gaussian) and 68% are in the interval (0.5 ; 0.666], when α∈[1.5 ; 2). In fact, none is over 0.9 (α<1.1). Finally, we can conclude that only the methods of R/S and Variance of Residuals are good enough to estimate the Hurts exponent and stability parameter α.

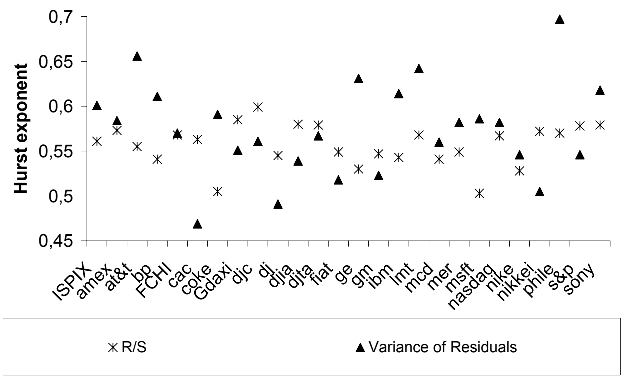

Figure 9. Hurst exponent estimates by the methods of R/S, and Variance of Residuals for all series

In these cases (Figure 9), correlation is in the interval [0.97 ; 1] and the Hurst exponent H∈(0.5 ; 0.7), which means that α∈(1.42 ; 2).

Finally, we can find an empirical dependence between stability index and Hurst exponent (Figure 10 and Table 6).

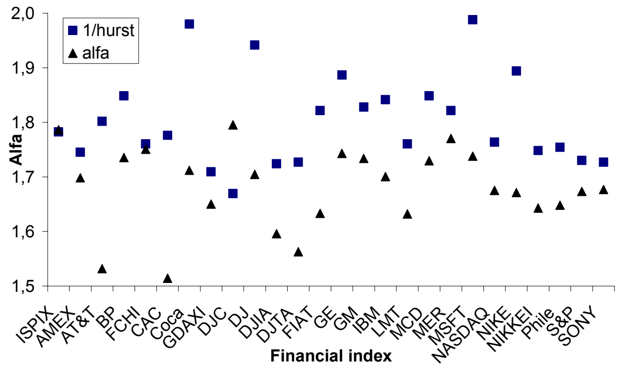

Figure 10. Dependence between Hurst exponent and stability index alfa

Hurts exponent is estimated by R/S method, whereas the stability parameter *α* by maximal likelihood method.

Table 6. Hurst exponent and stability index alfa

	R/S Hurst	1/Hurts	alfa
ISPIX	0.561	1.783	1.786
AMEX	0.573	1.745	1.698
AT&T	0.555	1.802	1.532
BP	0.541	1.848	1.736
FCHI	0.568	1.761	1.751
CAC	0.563	1.776	1.515
COCA	0.505	1.980	1.712
GDAXI	0.585	1.709	1.650
DJC	0.599	1.669	1.795
DJ	0.515	1.942	1.705
DJIA	0.58	1.724	1.596
DJTA	0.579	1.727	1.563
FIAT	0.549	1.821	1.633
GE	0.53	1.887	1.743
GM	0.547	1.828	1.734
IBM	0.543	1.842	1.701
LMT	0.568	1.761	1.632
MCD	0.541	1.848	1.730
MER	0.549	1.821	1.771
MSFT	0.503	1.988	1.738
NASDAQ	0.567	1.764	1.675
NIKE	0.528	1.894	1.671
NIKKEI	0.572	1.748	1.643
PHILE	0.57	1.754	1.648
S&P	0.578	1.730	1.674
SONY	0.579	1.727	1.677
ISPIX	0.561	1.783	1.786

One can see that indexes in third and fourth rows are similar (theoretically they should be equal). The average absolute difference is equal to 0.132 (min 0.004 and max 0.27).

6. Conclusions

For a long time Gaussian models were applied to model stock price return. Empirically, it has been shown that some stock price returns are not distributed by Gaussian distribution, therefore a stable (max-stable, geometric stable, α-stable, symmetric stable, and others) approach was proposed. Stable random variables satisfy the generalized CLT. Since fat tails and asymmetry are typical for them, they fit the empirical data distribution better (than Gaussian). Besides, they are leptocurtotic. But small question arise: are the data distributed by the stable law? This work offers some approaches to the problem.

The adequacy of the mathematical model for financial modeling was tested in this paper by two methods: that of Anderson – Darling and Kolmogorov – Smirnov. The first criterion is more sensitive to the difference between empirical and theoretical distribution functions in far quantiles (tails), in contrast to the second criterion, which is more sensitive to the difference in the central part of the distribution. Since the stable law is heavy-tailed, the A-D criterion was chosen as principal one.

Another approach to the stability hypothesis is the homogeneity test of partial sums and original series. It has been proved that the Anderson criterion is more precise than the Smirnov criterion. The Anderson criterion was chosen as the principal one.

We have investigated 26 international financial series focusing on the issues of stability, multifractality, and self-similarity. It has been established that the hypothesis of stability was ultimately rejected in 14.81% cases, definitely stable in 22.22%, and the rest are doubtful (see Appendix C). It is important to note that, even in the case of rejection, the value of the A-D criterion for stability testing was much better than for the test of Gaussianity. No series was found with the Gaussian distribution. For more information on the dependence between the stability tests see Appendix C.

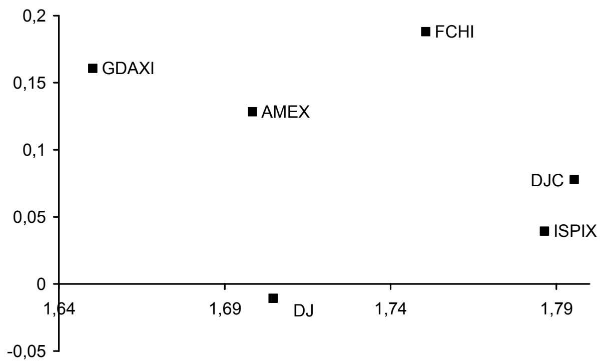

Figure 11. Distribution of α and β for stable series

Stable model parameters were estimated by the maximal likelihood method. As one can see in Figure 11, the stability indexes of stable series are concentrated between 1.65 and 1.8, which confirms the results of other authors that the stability parameter of financial data is over 1.5. Asymmetry parameters are scattered in the area between -0.017 and 0.2.

The investigation of self-similarity has concluded that 66.67% of the series are only multifractal and the other 33.33% concurrently are self-similar.

The Hurst analysis has showed that the methods of R/S and Variance of Residuals are significant in the stability analysis. Following these two methods, Hurst exponent estimates are in the interval $H \in (0.5;0.7)$, which means that the stability index $\alpha \in (1.42\ ;\ 2)$. When the Hurst exponent is calculated by R/S method, $H \in (0.5;\ 0.6)$, then $\alpha \in (1.666\ ;\ 2)$.

The stable models are suitable for financial engineering, however the analysis has shown that not all (only 22% in our case) the series are stable, so the model adequacy and other stability tests are necessary before model application. The studied series represent a wide spectrum of stock market, but it must be stressed that the research requires a further continuation: to extend the models.

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Received January 2006.

Appendix A

Appendix A deals with testing of reliability of two criteria (Anderson and Smirnov) for homogeneity of two samples. They are described in detail in Section 5.2.

Notations in Tables:

P – confidence level;
h – number of components in sums;
T – sample size (of original – simulated series);
Max – maximum value in that table;
Min – minimum value in that table;
Average – average homogeneity of all tests

The number in a marked cell shows how many times (out of 100 tries) original and derivate series were homogenous.

The significance level in all the tables is P=0.05.

A.1. Uniform distribution $R(-1,1)$

Table A.1. Results of testing of the Anderson criterion

$h\backslash T$	300	400	500	600	700	800	900	1000		
5	62	52	28	9	5	2	1	0		
10			1	0	0	0	0	0	*min*	0
15						0	0	0	*max*	62
20								0	*average*	8,89

Table A.2. Results of testing of the Smirnov criterion

$h\backslash T$	300	400	500	600	700	800	900	1000		
5	0	0	0	0	0	0	0	0		
10			0	0	0	0	0	0	*min*	0
15						0	0	0	*max*	0
20								0	*average*	0

A.2. Cauchy distribution $C(0,1)$

Table A.3. Results of testing of the Anderson criterion

$h\backslash T$	300	400	500	600	700	800	900	1000		
5	100	100	100	99	100	99	100	100		
10			99	96	97	95	98	100	*min*	95
15						100	97	98	*max*	100
20								95	*average*	98,5

Table A.4. Results of testing of the Smirnov criterion

$h\backslash T$	300	400	500	600	700	800	900	1000		
5	99	97	100	98	99	93	96	97		
10			96	97	95	99	96	98	*min*	93
15						96	95	96	*max*	100
20								97	*average*	96,89

A.3. Gaussian distribution $N(0,1/\sqrt{3})$

Table A.5. Results of testing of the Anderson criterion

$h\backslash T$	300	400	500	600	700	800	900	1000		
5	100	100	100	100	100	99	100	100		
10			100	100	100	98	100	100	*min*	97
15						100	100	97	*max*	100
20								99	*average*	99,61

Table A.6. Results of testing of the Smirnov criterion

$h\backslash T$	300	400	500	600	700	800	900	1000		
5	100	100	100	99	100	100	100	100		
10			97	98	95	98	99	98	*min*	93
15						98	100	96	*max*	100
20								93	*average*	98,4

A.4. Stable distribution $S_{1.25}(1,0.5,0)$

Table A.7. Results of testing of the Anderson criterion

$h\backslash T$	300	400	500	600	700	800	900	1000		
5	100	100	100	100	100	100	100	100		
10			100	96	97	97	98	97	*min*	96
15						96	98	100	*max*	100
20								99	*average*	98,78

Table A.8. Results of testing of the Smirnov criterion

$h\backslash T$	300	400	500	600	700	800	900	1000		
5	100	95	99	99	100	98	99	98		
10			95	99	100	99	100	97	*min*	95
15						96	98	96	*max*	100
20								96	*average*	98

Some cells in the tables are empty, because the methods require that the sample size of each sample would be no less than 50.

In the Uniform distribution case, random variables X (original series) and Y (series of sums) are distributed differently, i.e., they are not homogeneous. Obviously (Tables A.1, A.2), the Smirnov method indicates non-homogeneity better than that of Anderson. In other cases, the random variables X and Y must be distributed by the same distribution function, i.e., they are homogenous and, as one can see (Tables A.3. – A.8.), in the average, the criterion of Anderson distinguishes homogeneity better than that of Smirnov.

Appendix B

<table>
<tr><th>Column A</th><th>Column B</th></tr>
<tr><td>Plots of $ln\,AM^{(m)}(q)$ versus $ln\,m$ for the respective financial series with mean subtracted (q=1,…,5 from top to bottom).</td><td>Plots of $\hat{H}(q)$ versus q for the respective financial series.</td></tr>
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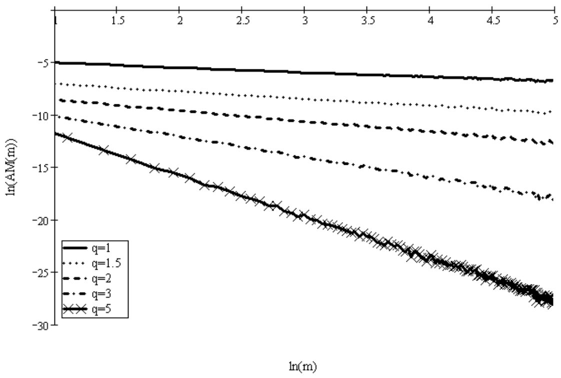
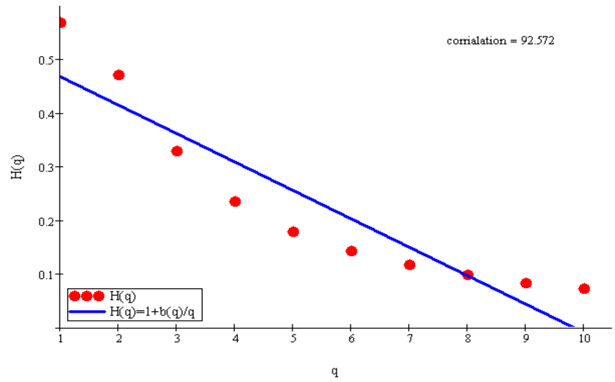

</td><td>
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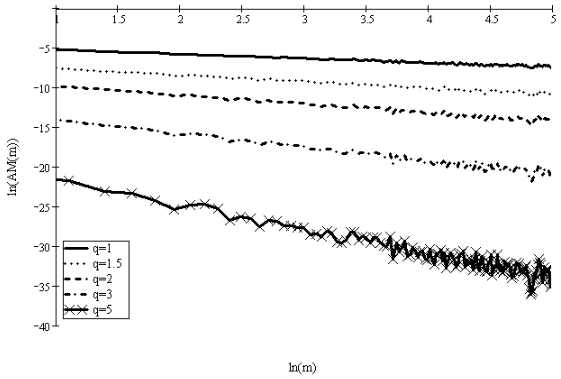
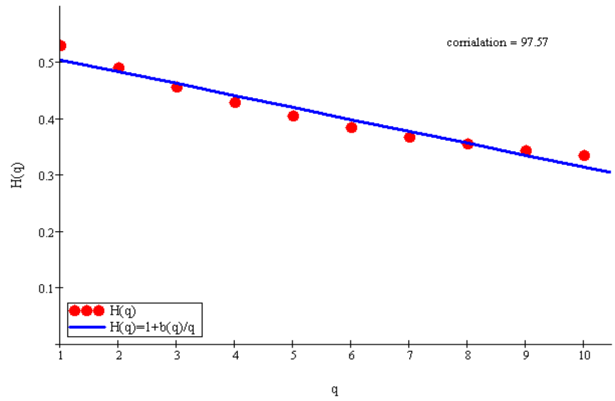

</td><td>
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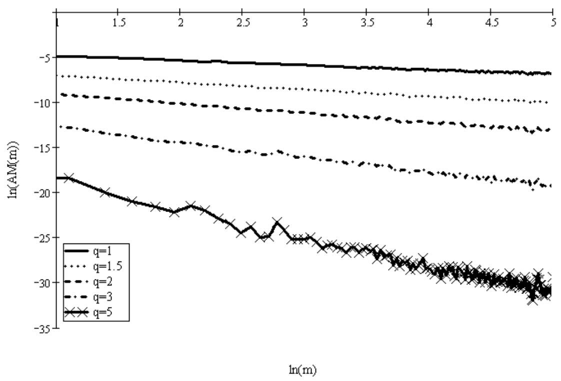
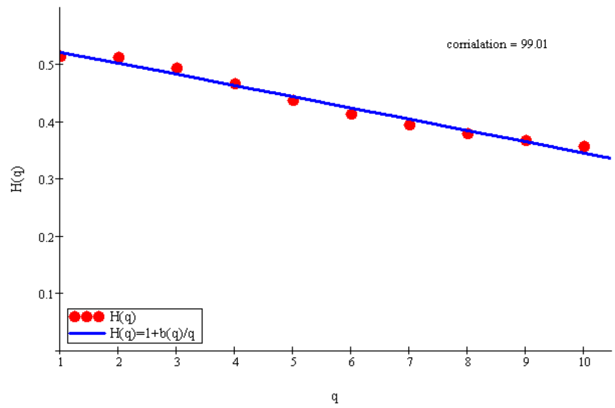

</td><td>
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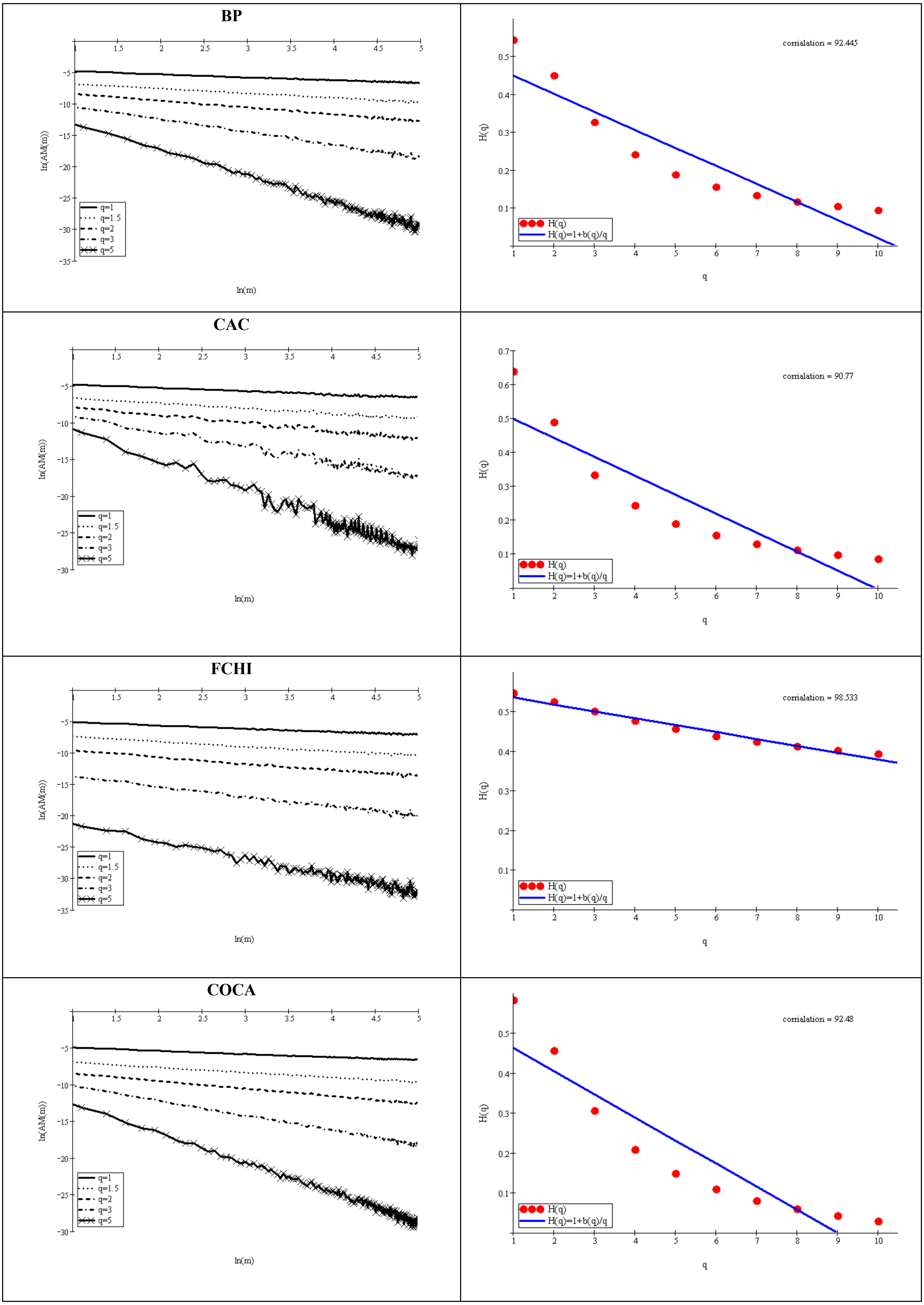
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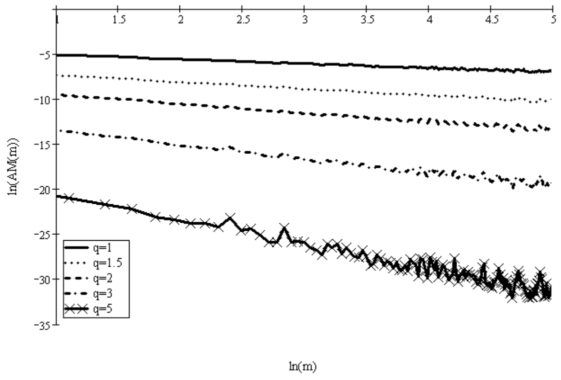
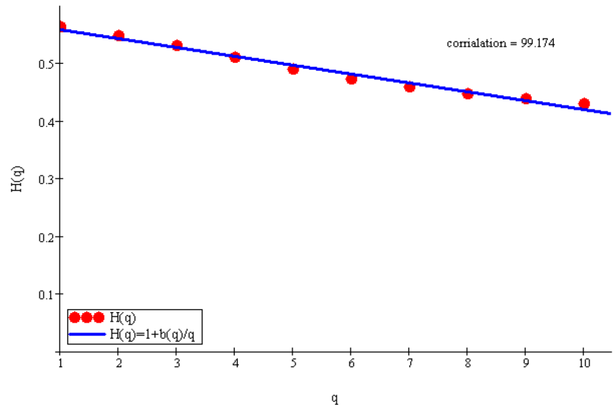
ln(AM(m))
ln(m)
q=1
q=1.5
q=2
q=3
q=5
corrialation = 92.445
H(q)
H(q)=1+b(q)/q
q
CAC
ln(AM(m))
ln(m)
q=1
q=1.5
q=2
q=3
q=5
corrialation = 90.77
H(q)
H(q)=1+b(q)/q
q
FCHI
ln(AM(m))
ln(m)
q=1
q=1.5
q=2
q=3
q=5
corrialation = 98.533
H(q)
H(q)=1+b(q)/q
q
COCA
ln(AM(m))
ln(m)
q=1
q=1.5
q=2
q=3
q=5
corrialation = 92.48
H(q)
H(q)=1+b(q)/q
q

GDAXI





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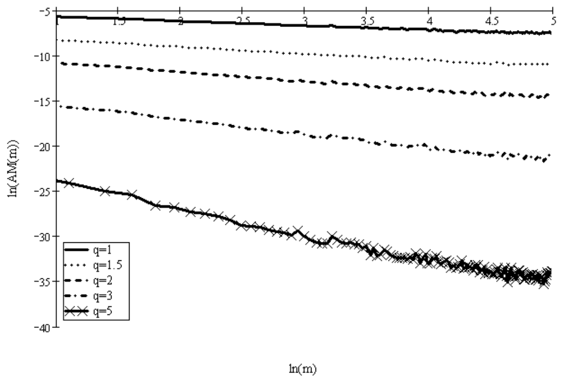

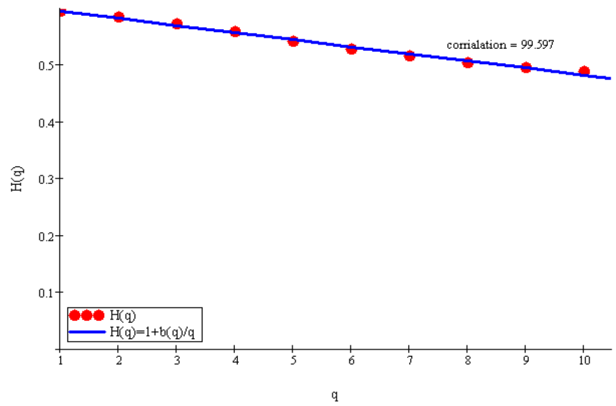

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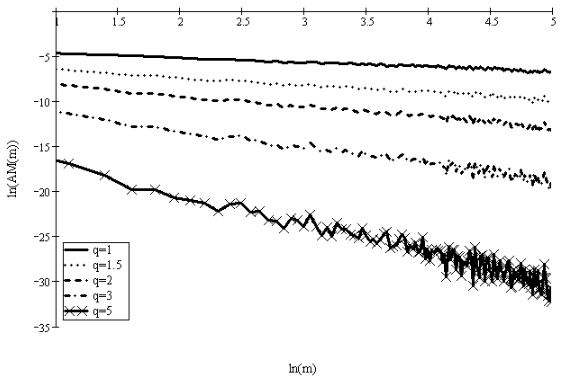

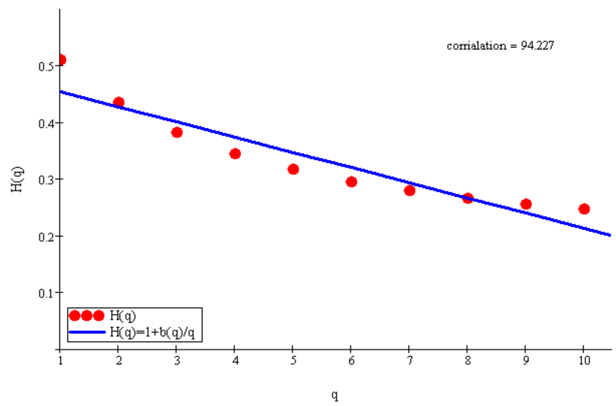

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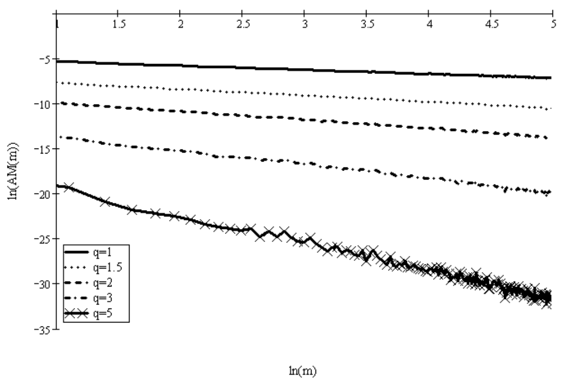

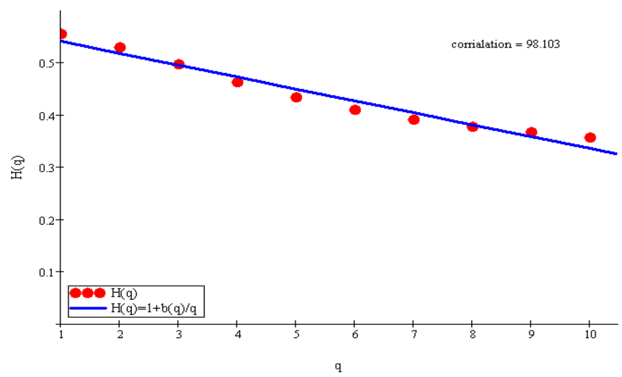

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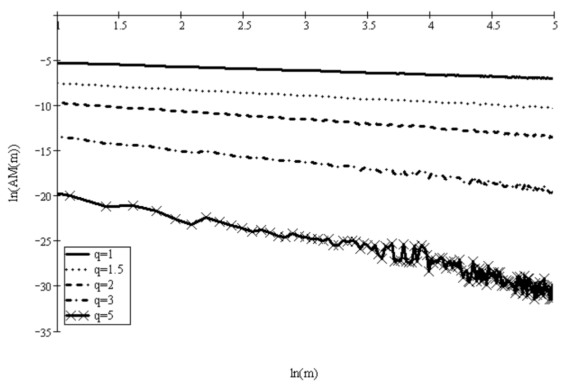

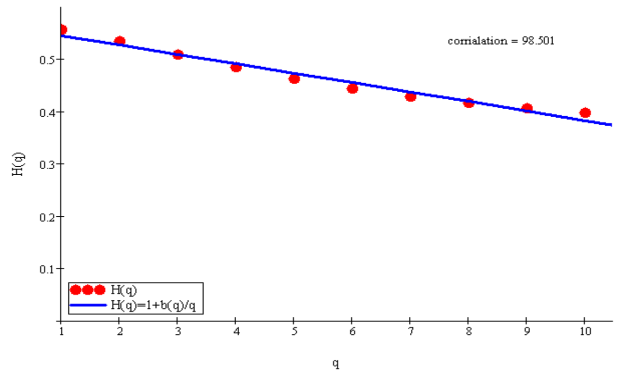

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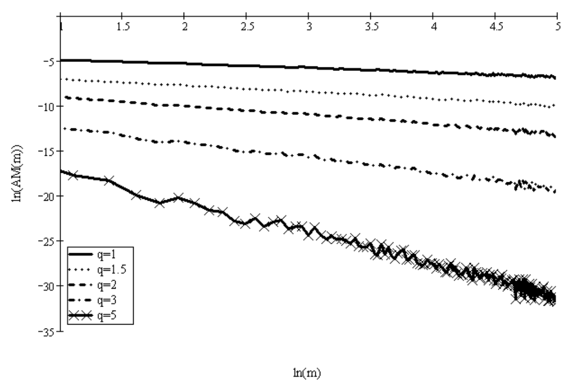

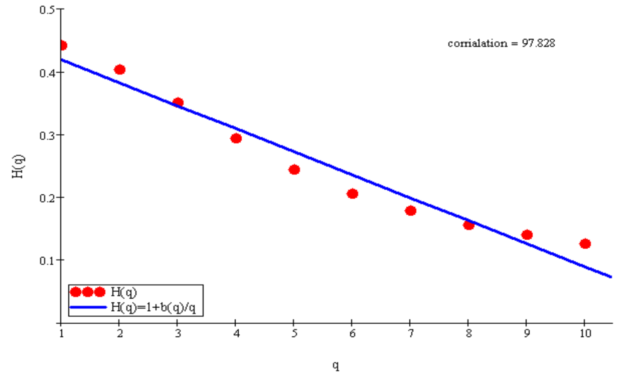

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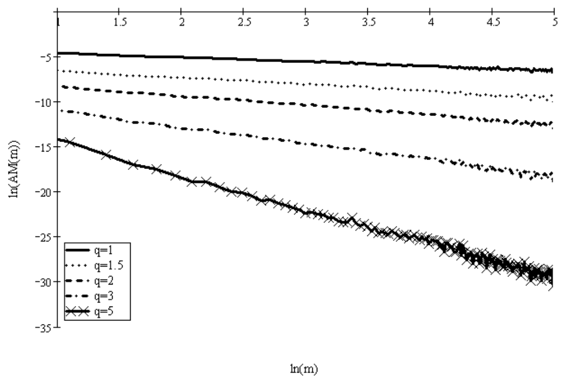

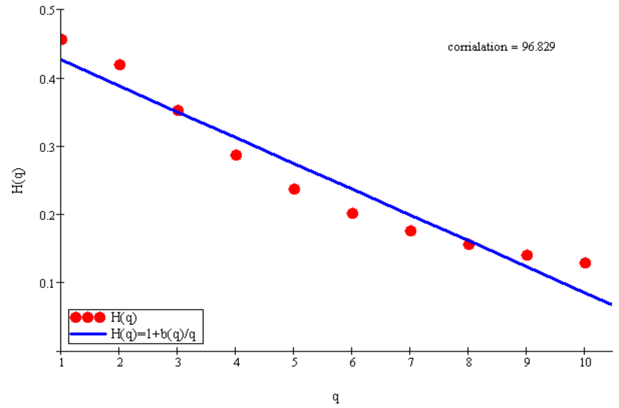

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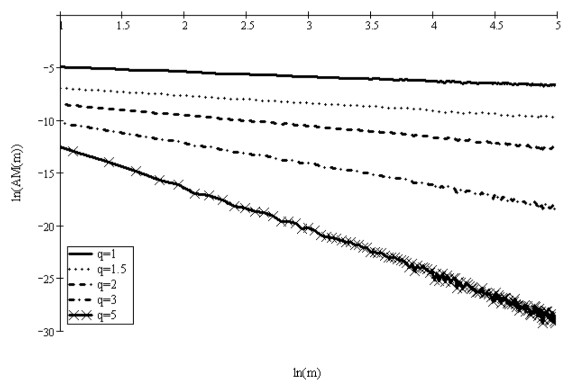
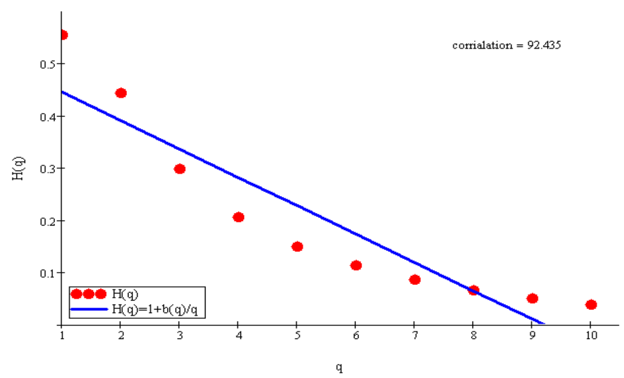
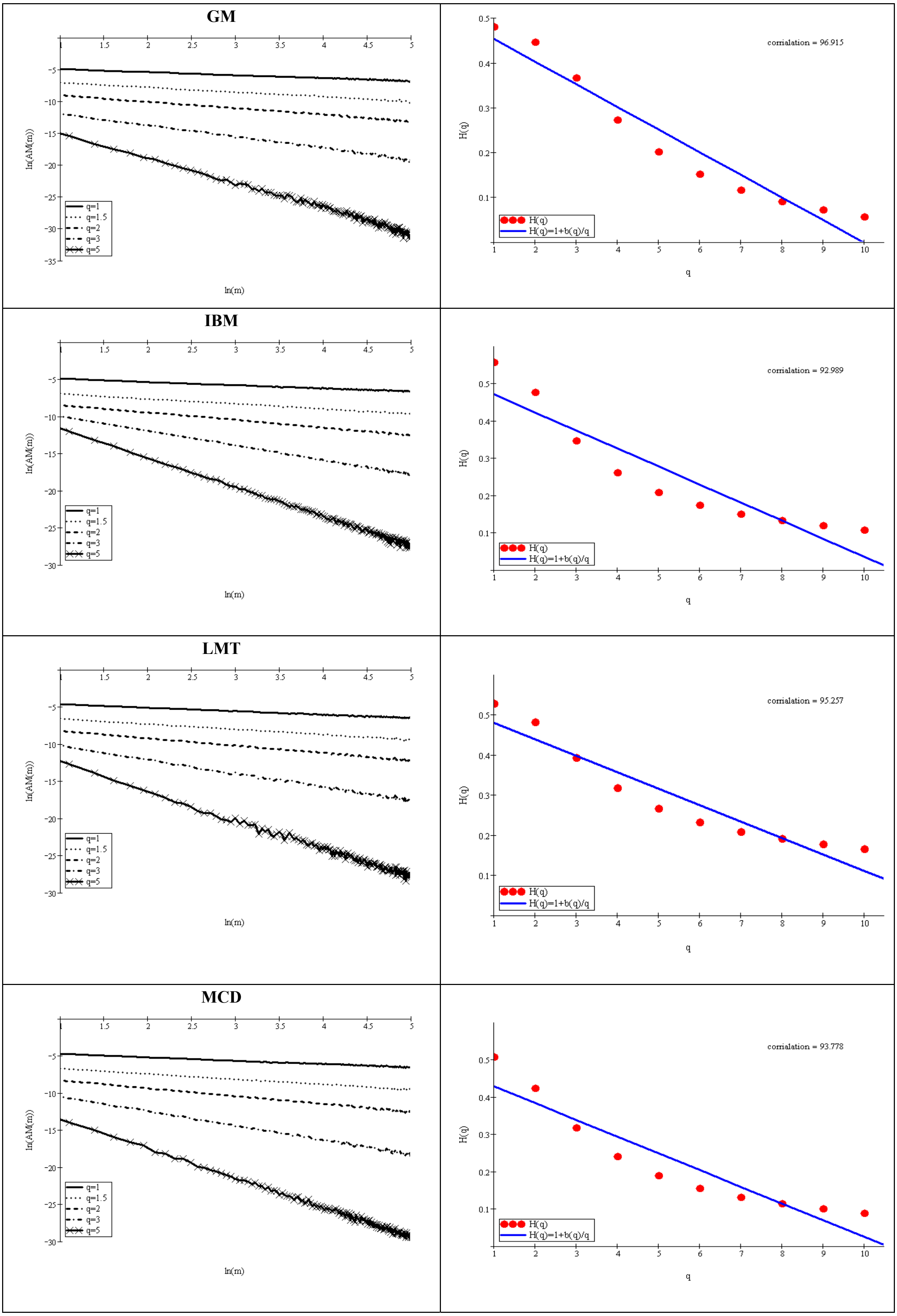
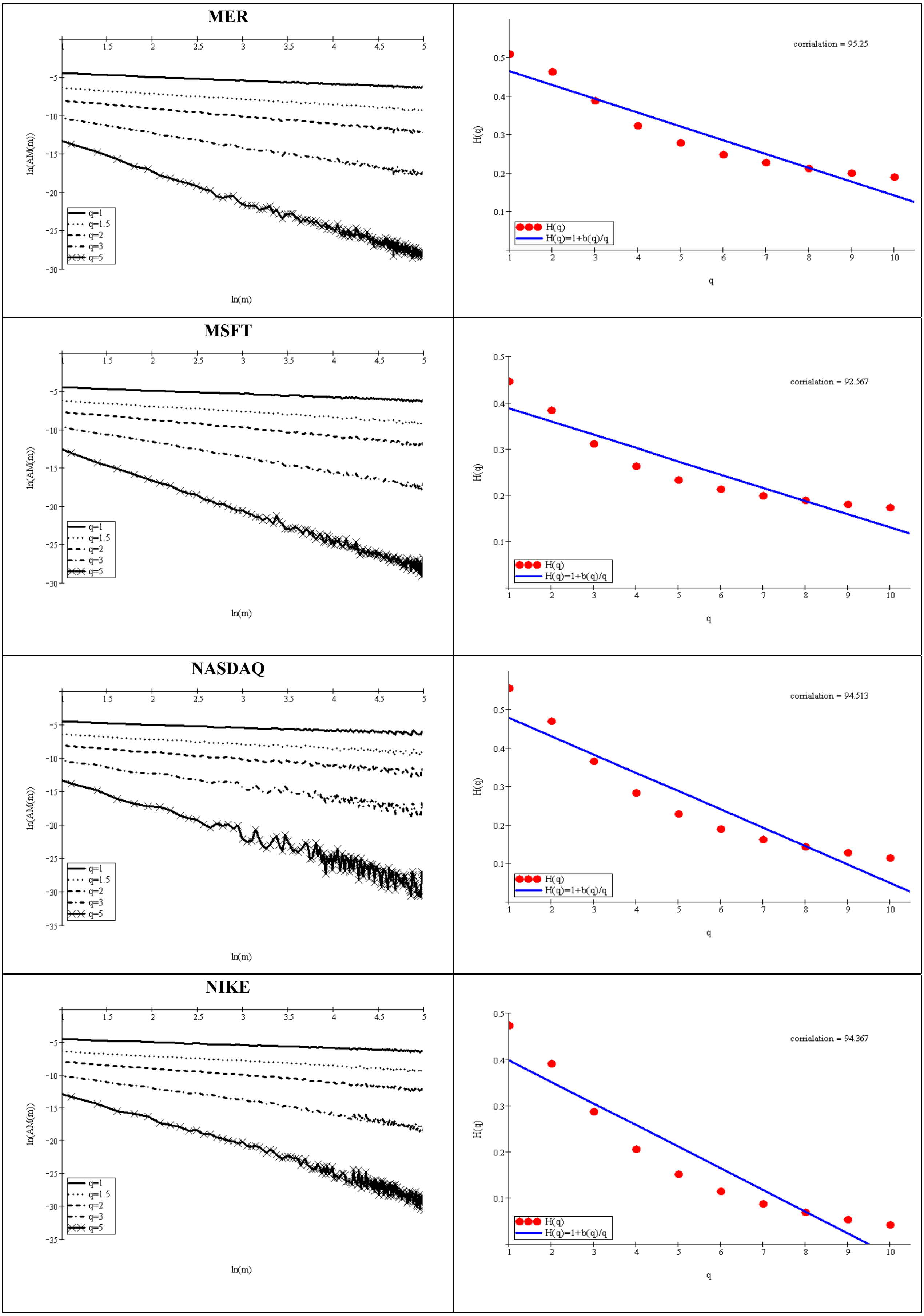





GM
ln(AM(m))
ln(m)
q=1
q=1.5
q=2
q=3
q=5
H(q)
q
corrialation = 96.915
H(q)=1+b(q)/q
IBM
corrialation = 92.989
LMT
corrialation = 95.257
MCD
corrialation = 93.778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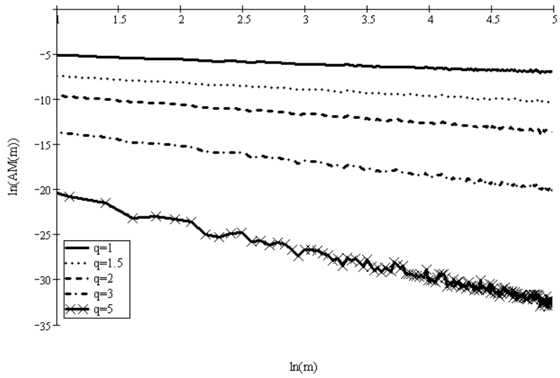
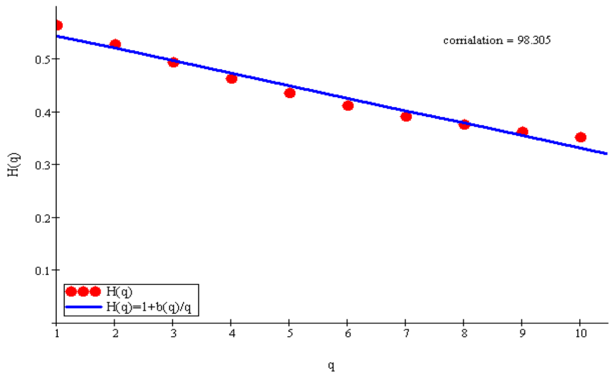
corrialation = 95.25
MSFT
corrialation = 92.567
NASDAQ
corrialation = 94.513
NIKE
corrialation = 94.367
ln(AM(m))
ln(m)
q=1
q=1.5
q=2
q=3
q=5
H(q)
H(q)=1+b(q)/q
q

NIKKEI





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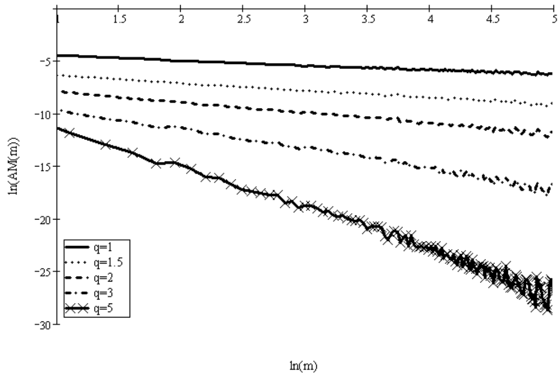

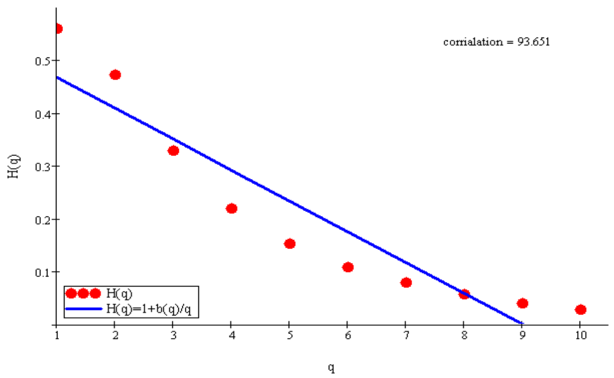

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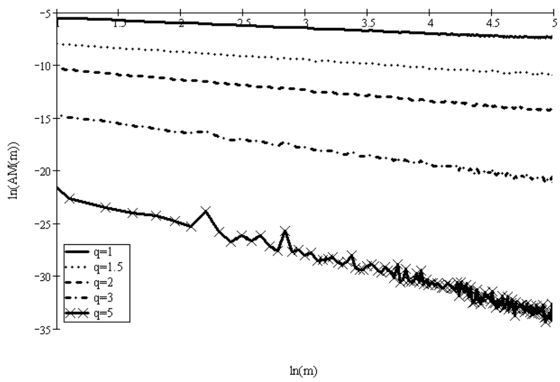

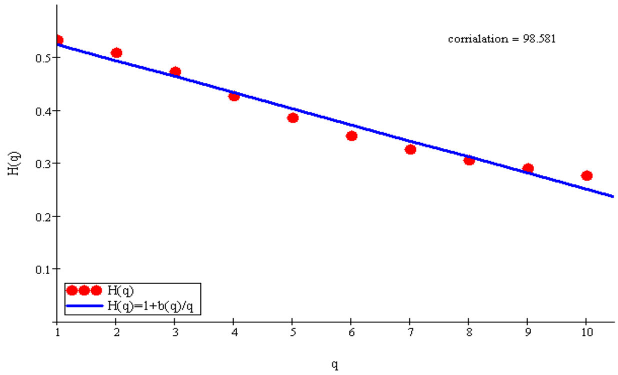

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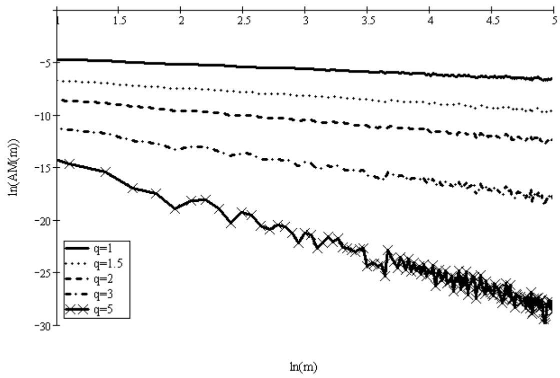

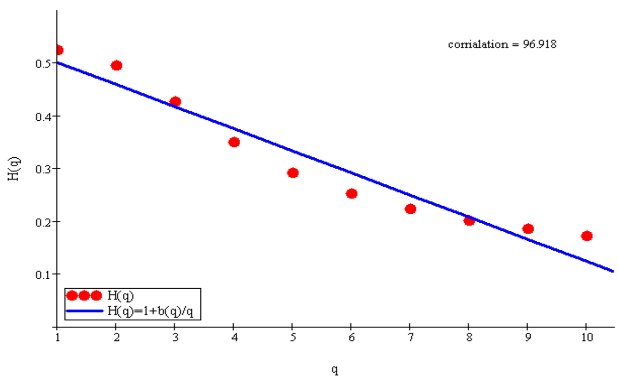

Appendix C

The relationship of stability tests. The results marked by value "0" indicates clearly stable financial series, and the results marked by value "1" are probably stable (one of the methods shows the stability, another does not), "2" marks non-stability.

Code	Acceptable (+) by A-D criterion	Acceptable (+) by K-S criterion	Acceptable (+) by sum by *m* = 10	Acceptable (+) by sum by *m* = 15	Self-similar (+) or Multifractal (-)	Result
ISPIX	+	+	+	+	+	0
AMEX	+	+	+	+	+	0
AT&T	-	-	+	-	-	1
BP	-	-	+	+	-	1
FCHI	+	+	+	+	+	0
CAC	-	-	-	-	-	2
COCA	-	-	+	+	-	1
GDAXI	+	-	+	+	+	0
DJC	+	-	+	+	+	0
DJ	+	-	+	+	+	0
DJIA	-	-	-	-	-	2
DJTA	-	-	+	+	+	1
FIAT	-	-	-	-	-	2
GE	-	-	+	+	-	1
GM	-	-	+	+	-	1
IBM	+	-	+	+	-	1
LMT	-	-	+	+	-	1
MCD	+	-	+	+	-	1
MER	+	-	+	+	-	1
MSFT	+	-	+	+	-	1
NASDAQ	+	+	-	+	-	1
NIKE	+	-	+	+	-	1
NIKKEI	-	-	-	-	+	1
PHILE	-	-	+	+	-	1
S&P	-	-	+	+	+	1
SONY	-	-	+	+	-	1