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# Robust Fault Diagnosis for Discrete-Time Switched System with Unknown State Delays Subject to Component Faults

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This paper investigates the problems of fault detection and diagnosis in a class of discrete-time switched systems with unknown state delay against component faults. The component fault, also called the multiplicative fault or loss of actuator effectiveness, is a common type of operation fault for switched systems, which has been paid more attention. In this paper, the learning observers (LOs) technology are proposed to reconstruct both the system state and component faults. The proposed LOs technique is also extended to the system with unknown time delay and external disturbance. The sufficient conditions for the convergence of the output of LOs and the uniformly ultimately boundedness of the reconstructed errors are proved by the switched Lyapunov function and  $H_\infty$  technique. The proposed fault reconstruction strategy has certain tolerance for fault types, which is effective for both slow and fast time-varying faults by adjusting the iteration step size. In the simulation parts, three kinds of fault signals are taken into consideration to verify the effectiveness and robustness of the proposed method.

**KEYWORDS:** Fault Diagnosis, Switched System, Unknown State Delay, Lyapunov Function.

## 1. Introduction

With the increasing complexity of modern industrial systems and higher pursuit of system performance, fault diagnosis is an important and challenging problem in many practical systems, such as chemical en-

gineering, nuclear engineering and aerospace engineering [9, 12]. Faults may occur at any time, and then destroy the system stability or even make the system tend to collapse. Nowadays, fruitful model-based fault

diagnosis works have been published in several excellent papers and books [3, 5, 14, 15, 17, 23, 26]. Among these approaches, the most commonly used one is first to design the observers or filters to reconstruct the fault signal or residual signal, and then generate an alarm when the residual function is larger than the threshold or the fault reconstruction signal is not zero [18].

Recently, several physical systems present hybrid behaviors [6, 19]. Wherein, the switched systems play an important role in industrial process and have drawn more and more attention for the stability and controllability research [5]. A generalized switched system is composed of subsystems with their own parameters subject to a specific rule orchestrating the switching law among the various subsystems. The motivation of studying switched systems drives from the facts that many physical scenes exhibit the switching features among multi-models.

Because of the increasing demand for the operational safety refer to switched systems, the research about fault diagnosis of switched systems has been a hot topic [7]. Hwang et al. [12] introduced three types of fault in the field of model-based systems, including actuator fault, sensor fault and component fault or loss of effectiveness fault. The actuator fault and sensor fault are also called the additive fault, because they have an additional relationship with the original system state. There are many excellent fault diagnosis methods for additive faults in switched systems. In [4], a multiple Lyapunov function approach was employed for fault detection and diagnosis in switched systems. Then the switched Lyapunov function approach was first applied in stability analysis of switched systems [7]. Wang et al. [20] designed a fault filter for the actuator fault and sensor fault based on  $H_\infty$  performance index. Hespanha et al. [11] applied the average dwell time technology to conduct stability analysis for switched systems, while Zhang et al. [23] converted the fault detection problems into an  $H_\infty$  filtering problem by using the average dwell time technology. Du et al. [8] adopted the asynchronous switched approach in switched systems to reconstruct the additive faults.

Compared with the actuator and sensor faults, the component faults or loss of effectiveness faults can also be called the multiplicative faults, which can weaken system input performance, and destroy the stability. It is necessary to design fault diagnosis strategy for component faults in switched systems. There

are relatively fewer research results for the switched systems with component faults. He et al. [10] designed a fault detection filter for a more general sensor fault, including the loss of effectiveness, the outage and the drifted fault in switched systems. The effectiveness of sensor was decayed in a certain proportion, and the attenuation rate was assumed to be a constant. Rodrigues et al. [16] proposed a fault tolerant control model for switched systems with component faults. However, the fault signal was set to be constant or slow time-varying. The component faults are multiplied with the actuator and sensor, which is the main reason why it is difficult to design the fault diagnosis method. In this paper, the learning observer technology is applied to make reconstruction for the component faults in switched systems, which is a new attempt.

As is well-known, time-delay phenomenon is very common in many real physical systems, such as air pollution system and chemical process etc. [23]. It would often cause the instability and poor performance of the switched systems. There are some works about fault diagnosis taking time-delay phenomenon into consideration [20, 22]. Wang et al. [20] proposed a robust fault detection method for switched linear systems with state delay. Yu et al. [22] presented a fault detection frame for discrete-time switched systems with distributed delays. However, the existing models were only for additive faults. In addition, the delay time was known in advance. It should be mentioned that in our previous work [25], we designed an adaptive fault diagnosis model for continuous time-delay repetitive systems subject to sensor fault, where the delay time was known in advance. In this paper, the unknown time-delay element is taken into consideration. A relatively slack restriction for delay time with known upper and lower bounds improves the generalization performance of fault reconstruction model.

In this work, a fault diagnosis method based on LOs against the component faults for switched systems is proposed. It can handle the task of fault diagnosis for more complex systems, even with unknown delay time and external disturbance. Compared with the existing works for the constant faults or slowly time-varying faults, the proposed method is effective for relatively fast time-varying faults. By using the switched Lyapunov function and  $H_\infty$  technique, the sufficient conditions for the convergence of the output of LOs are presented in term of linear matrix

inequalities (LMIs). Finally, the simulation results for three forms of loss of actuator effectiveness faults demonstrate the feasibility of this scheme. To simplify, the contributions of this paper can be summarized as follows:

- 1 The LOs are employed in switched systems against component faults, which is a new attempt.
- 2 The time-delay element and external disturbance are taken into consideration in the fault diagnosis model. A more relaxed bounded constraint for delay time is assumed, which leads to the proposed LOs having better generalization performance.
- 3 The proposed fault reconstruction strategy has certain tolerance for fault types. It is still valid for fast time-varying faults.

The rest of the paper is organized as follows. In Section 2, the description for a generalized switched system is presented and the problems are formulated. Section 3 is the main part of this paper, which presents the robust design of LOs. Simulation results are carried out in Section 4 and conclusions are drawn in Section 5. Section 6 is the appendix, where the proofs of the theorems are presented.

**Notations:** The notations used throughout the paper are normative.  $A^T$ ,  $A^{-1}$ ,  $\lambda(A)$  and  $\|A\|$  are the symbols to denote, respectively, the transpose, the inverse, the eigenvalues and the Euclidean norm of any square matrix  $A$ . Particularly,  $\lambda_{\max}(A)$  and  $\lambda_{\min}(A)$  present the maximum and minimum eigenvalues of  $A$ . We use  $A > 0$  ( $A < 0$ ) to denote that  $A$  is positive (negative) defined and  $I$  to denote the identity matrix with an appropriate dimension.  $\|\cdot\|_{\infty}$  represents the infinity norm of any vector. The symbol  $*$  means the symmetric terms in some matrices.

## 2. System Description and Problem Formulation

We consider the following discrete-time switched system

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}f(k)u(k) \\ y(k) = C_{\sigma(k)}x(k) \end{cases}, \quad (1)$$

where  $x(k) \in R^m$ ,  $u(k) \in R^n$  and  $y(k) \in R^p$  are the

state, control input and output, respectively. Diagonal matrix  $f(k) = \text{diag}\{f_1(k), \dots, f_n(k)\}$  represents the component faults with  $f_i(k) \in (0, 1]$ . Specifically, when  $f_i(k) = 1$ , the  $i$ th actuator runs fault-free; if  $0 < f_i(k) < 1$ , partial effectiveness of the actuator is lost; and  $f_i(k) = 0$  means that the actuator has no effectiveness in the system totally.  $\sigma$  is the switching rule which takes the values in the finite set  $\mathcal{N} : \{1, \dots, N\}$ , where  $N$  is the number of subsystems. It is assumed that  $\sigma(k)$  is dependent on  $k$  or  $x_k$ , or other switching rules at an arbitrary discrete time  $k$ .  $A_{\sigma}$ ,  $B_{\sigma}$ , and  $C_{\sigma}$  are of appropriate size constant known matrices. Since  $f(k)$  is a diagonal matrix, the switched system can be rewritten as follows

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}U_k F_k \\ y(k) = C_{\sigma(k)}x(k) \end{cases}, \quad (2)$$

where  $U_k = \text{diag}\{u_1, \dots, u_n\}$  and  $F_k = [f_1, \dots, f_n]^T$ . The fault signals are presented by a column vector, which is suitable for the following analysis and the proofs of the convergence theorems.

**Assumption 1:** The switching rule  $\sigma(k)$  is available at each sampling time  $k$ . Additionally, only one subsystem is active each time.

In most case, the switching signal is not known beforehand, and that means one can not know in advance the distribution of conversion signals. However, the instantaneous value of switching signals is available. Thus, the design of observer modes will be modified with the change of switching signals. Some similar operations are shown in [2, 20].

Here we introduce the switching signal function as  $\xi(k) = [\xi_1(k), \dots, \xi_n(k)]^T$ , where  $\xi_i(k) : Z^+ \rightarrow \{0, 1\}$  and  $\sum_{i=1}^N \xi_i(k) = 1$ ,  $k \in Z^+$ .  $\xi_i(k) = 1$  means that the  $i$ th subsystem is active at  $k$ th sampling step, while it is out of operation when  $\xi_i(k) = 0$ . Then one can rewrite the switched system as

$$\begin{cases} x(k+1) = \sum_{i=1}^N \xi_i(k) (A_i x(k) + B_i U_k F_k) \\ y(k) = \sum_{i=1}^N \xi_i(k) C_i x(k) \end{cases}. \quad (3)$$

**Assumption 2:** The system control input  $u(k)$  is norm-bounded,  $\|u(k)\| \leq u_{\max}$ .

Assumption 2 says that the control input is bounded, which is reasonable for a system running in the normal operating environment. Additionally, the observability is another reasonable hypothesis for the design of LOs in the following sections.

This paper aims to establish a fault diagnosis model to reconstruct the component faults  $F_k$ . The model should not only guarantee the convergence speed, but also be applied in many kinds of fault types. In addition, it should be successful to deal with time-delay phenomenon and external disturbance.

### 3. Fault Reconstruction based on LOs

#### 3.1. Design of Learning Observers

In this subsection, a novel fault reconstruction method for the above switched system is presented. The corresponding LOs are established as

$$\left\{ \begin{array}{l} \hat{x}(k+1) = \sum_{i=1}^N \left[ \xi_i(k) (A_i \hat{x}(k) + B_i U_k \hat{F}_k) \right] \\ \quad + \sum_{i=1}^N \left[ \tilde{L}(y(k) - \hat{y}(k)) \right] \\ \hat{y}(k) = \sum_{i=1}^N \xi_i(k) C_i \hat{x}(k) \\ \hat{F}_k = K_1 \hat{F}_{k-\tau} + \tilde{K}_2 (y(k) - \hat{y}(k)) \end{array} \right. , \quad (4)$$

where  $\hat{x}(k)$  and  $\hat{y}(k)$  are the estimations of system state and output, respectively.  $\hat{F}_k$  represents the reconstructed fault of loss of actuator effectiveness updated by both its previous information at the sampling step  $k-\tau$  and current output error.  $\tau$  is the updating interval.  $K_1$  is a diagonal matrix, that is,  $K_1 = \text{diag}\{k_1, \dots, k_n\}$  with  $k_i \in (0, 1]$ ,  $i \in \{1, \dots, n\}$ , and  $\tilde{L} = \sum_{i=1}^N \xi_i(k) L_i$ ,  $\tilde{K}_2 = \sum_{i=1}^N \xi_i(k) K_{2i}$ .  $K_1$ ,  $L_i$  and  $K_{2i}$  would be selected later.

Define the system state-estimation error, output-estimation error and fault-estimation error as  $e_x(k) = x(k) - \hat{x}(k)$ ,  $e_y(k) = y(k) - \hat{y}(k)$  and

$e_F(k) = F_k - \hat{F}_k$ . In order to drive the fault reconstruction error less restrictively and reasonably, the following assumption is necessary.

**Assumption 3:** Denote  $\tilde{F}_k = F_k - K_1 F_{k-\tau}$  and assume  $\tilde{F}_k$  is norm-bounded, such that  $\|\tilde{F}_k\| \leq \kappa_f$ .

Assumption 3 shows a relatively loose restriction compared with other works for fault diagnosis, where norm-bound condition of fault signal or its derivative is necessary. The restriction of norm-bound of the derivative of faults means that the corresponding faults are time-varying slowly, which is removed in this paper. The proposed LOs can deal with broader fault signals, including slowly and fast time-varying ones. Since  $0 < f_i \leq 1$ ,  $i = 1, 2, \dots, n$ , the upper bound of  $\|\tilde{F}(k)\|$  can be calculated easily, such as  $\kappa_f = 1 + \|K_1\|$ .

**Remark 1:** The choice of  $K_1$  and  $\tau$  is generally based on human experience.  $K_1$  approximates the identity matrix with proper dimension. It presents the influence level of the historical reconstructed signal on the current one. A relatively larger value of  $K_1$  means more information in the historical reconstructed signal is preserved, while less information is retained with relatively smaller value of  $K_1$ . In general, the value of  $\tau$  is determined according to the fault signal. When dealing with the fast time-varying signal,  $\tau$  should be sufficiently small. On the contrary,  $\tau$  can be chosen as a larger interval if the fault signal is slow time-varying.

Define the state error  $e_x(k)$ , output error  $e_y(k)$  and fault error  $e_F(k)$  as

$$\left\{ \begin{array}{l} e_x(k) = x(k) - \hat{x}(k) \\ e_y(k) = y(k) - \hat{y}(k) \\ e_F(k) = F_k - \hat{F}_k \end{array} \right. \quad (5)$$

Then one can get the following dynamic equations

$$\left\{ \begin{array}{l} e_x(k+1) = \sum_{i=1}^N \xi_i(k) \begin{bmatrix} (A_i - L_i C_i) e_x(k) \\ + B_i U_k e_F(k) \end{bmatrix} \\ e_y(k) = \sum_{i=1}^N \xi_i(k) C_i e_x(k) \\ e_F(k) = K_1 e_F(k-\tau) - \\ \quad \sum_{i=1}^N \xi_i(k) K_{2i} C_i e_x(k) + \tilde{F}_k \end{array} \right. \quad (6)$$

### 3.2. Parameter Configuration of LOs

Here the analysis of LOs is presented. In order to facilitate and simplify the proof, the matrix  $B_i$  is omitted. The convergence condition is presented in the following theorem, whose proof is shown in Part A in the appendix.

**Theorem 1:** Considering the switched system in (3) with Assumptions 1-3, if there exist positive definite symmetric matrices  $P_i \in R^{m \times m}$  and  $Q_i \in R^{n \times n}$ , and matrices  $K_1$ ,  $L_i$ ,  $K_{2i}$ , such that the following LMIs hold:

$$\begin{bmatrix} -(1+\gamma_1)P_j^{-1} & A_i - L_i C_i & 0 \\ * & -\alpha \rho_2 I & K_{2i} C_i \\ * & * & -P_i \end{bmatrix} < 0 \quad (7)$$

and

$$\alpha \rho_1 K_1^T K_1 - Q_l < 0, \quad (8)$$

then state estimation error  $e_x(k)$  and fault reconstruction error  $e_F(k)$  are uniformly bounded, where  $\alpha = \left(1 + \frac{1}{\gamma_1}\right) \lambda_{\max}(P_j) u_{\max}^2 + (1+\delta) \lambda_{\max}(Q_l)$  with  $\gamma_1 > 0$  and  $\delta \geq 0$ .

### 3.3. Robust Learning Observers Design with Delay Time

In the previous subsection, the LOs subject to the component faults are designed. Here, based on the Lyapunov stability theory and  $H_\infty$  technique, the robust LOs are presented taking time-delay and external disturbance into consideration. The corresponding system model is

$$\begin{cases} x(k+1) = \sum_{i=1}^N \xi_i(k) \begin{pmatrix} A_i x(k) + A_{di} x(k-d_k) \\ + U_k F_k + E_i \eta_k \end{pmatrix}, \\ y(k) = \sum_{i=1}^N \xi_i(k) C_i x(k) \end{cases}, \quad (9)$$

where  $d_k$  represents the unknown state delay and  $\eta_k$  is the external disturbance signal with norm boundary,  $\|\eta_k\| \leq \bar{\eta}$ . Additionally,  $A_{di}$  and  $E_i$  are known constant matrices with appropriate dimensions.

**Assumption 4:** We suppose that  $d_k$  is unknown time

delay variable with known lower and upper bounds:  $\underline{d} \leq d_k \leq \bar{d}$ .

The robust LOs are designed as follows:

$$\begin{cases} \hat{x}(k+1) = \sum_{i=1}^N \left( A_i \hat{x}(k) + A_{di} \hat{x}(k-d_k) + U_k \hat{F}_k \right) \\ \quad - \tilde{L} (y(k) - \hat{y}(k)) \\ \hat{y}(k) = \sum_{i=1}^N \xi_i(k) C_i \hat{x}(k) \\ \hat{F}_k = K_1 \hat{F}_{k-\tau} + \tilde{K}_2 (y(k) - \hat{y}(k)) \end{cases} \quad (10)$$

where the symbols are defined above.

Then the state estimation error, output estimation error and fault reconstruction error are rewritten as

$$\begin{cases} e_x(k+1) = \sum_{i=1}^N \xi_i(k) \begin{bmatrix} (A_i - L_i C_i) e_x(k) + E_i \eta_k \\ A_{di} e_x(k-d_k) + U_k e_F(k) \end{bmatrix} \\ e_y(k) = \sum_{i=1}^N \xi_i(k) C_i e_x(k) \\ e_F(k) = K_1 e_F(k-\tau) - K_{2i} C_i e_x(k) + \tilde{F}_k \end{cases}. \quad (11)$$

The convergence condition for the robust LOs is presented in the following theorem. Its proof is shown in Part B in the appendix.

**Theorem 2:** Consider the switched system in (9) with Assumptions 1-4. For a given positive scalar  $\gamma$ , if there exist positive definite symmetric matrices  $P_i \in R^{m \times m}$ ,  $Q_i \in R^{m \times m}$  and  $R_i \in R^{n \times n}$ , and matrices  $K_1$ ,  $L_i$ ,  $K_{2i}$ ,  $\forall i \in [1, 2, \dots, N]$ , such that the following LMIs hold

$$\begin{bmatrix} -\zeta_1 P_j^{-1} & 0 & 0 & A_i - L_i C_i & 0 \\ * & -P_i & K_{2i} C_i & 0 & I \\ * & * & -I & C_i & 0 \\ * & * & * & -\alpha \rho_2 I & 0 \\ * & * & * & * & -\sigma Q_i^{-1} \end{bmatrix} < 0 \quad (12)$$

$$\begin{bmatrix} -\zeta_2 P_j^{-1} & 0 & 0 & A_{di} \\ * & -\zeta_3 P_j^{-1} & A_{di}^T P_j E & 0 \\ * & * & -\gamma^2 I & E_i^T \\ * & * & * & -\lambda_{\min}(Q) I \end{bmatrix} < 0 \quad (13)$$

and

$$\alpha \rho_1 K_1^T K_1 - R_i < 0, \tag{14}$$

the proposed LOs can make the state estimation error, output estimation error and fault reconstruction error uniformly ultimately bounded. Additionally, the output estimation error satisfies  $\|e_y(k)\| \leq \gamma \bar{\eta} + \sqrt{\alpha \rho_3} \kappa_f$ , where  $\alpha$ ,  $\sigma$  and  $\zeta_i$ ,  $i = 1, 2, 3$ , would be defined in the proof.

**Theorem 3:** Consider the switched system in (9) with Assumptions 1-4. For a given positive scalar  $\gamma$ , (12) and (13) are feasible if there exist the positive  $P_i$  and  $Q_i$ , and matrix  $G_i$ ,  $i \in \mathbb{N}$ , such that the following LMIs hold:

$$\begin{bmatrix} -\zeta_1 \hat{P}_j & 0 & 0 & G_i^T A_i - Y_i C_i & 0 \\ * & -G_i^T P_i G_i & M_i C_i & 0 & G_i^T G_i \\ * & * & -I & C_i & 0 \\ * & * & * & -\alpha \rho_2 I & 0 \\ * & * & * & * & -\sigma \hat{Q}_i \end{bmatrix} < 0 \tag{15}$$

and

$$\begin{bmatrix} -\zeta_2 \hat{P}_j & 0 & 0 & G_i^T A_{di} \\ * & -\zeta_3 \hat{P}_j & G_i^T A_{di} P_j E & 0 \\ * & * & -\gamma^2 I & E_i^T \\ * & * & * & -\lambda_{\min}(Q) I \end{bmatrix} < 0, \tag{16}$$

where  $\hat{P}_j = -P_j + (G_i + G_i^T)$ ,  $\hat{P}_j = -P_j + (G_i + G_i^T)$ ,  $Y_i = G_i^T L_i$  and  $M_i = G_i^T K_{2i}$ .

**Remark 2:** In Part C of the appendix, the strict proof of Theorem 3 is shown. It is also the necessary condition proven in [1, 15]. The introduction of the auxiliary slack matrix  $G_i$  can couple the product term between  $P_j$  and  $(A_i - L_i C_i)$ , which is less conservative for constrained problems.

### 4. An Illustrative Example

In this section, some simulation studies are presented to verify the effectiveness of the proposed LOs method. Consider the following discrete-time switched

system consisting of two subsystems with parameters as

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.2 & 0.1 \\ 0 & -0.5 \end{bmatrix} & A_{d1} &= \begin{bmatrix} -0.1 & \\ & -0.1 \end{bmatrix} \\ B_1 &= \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} & E_1 &= \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} & C_1 &= \begin{bmatrix} 1 & 0.5 \\ -0.2 & 0.6 \end{bmatrix} \\ A_2 &= \begin{bmatrix} -0.3 & 0.1 \\ 0.2 & -0.6 \end{bmatrix} & A_{d2} &= \begin{bmatrix} -0.1 & \\ & -0.1 \end{bmatrix} \\ B_2 &= \begin{bmatrix} 1 & 0.2 \\ 0.3 & 0.5 \end{bmatrix} & E_2 &= \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} & C_2 &= \begin{bmatrix} 1 & 0.5 \\ 0 & 0.5 \end{bmatrix}. \end{aligned}$$

Other parameters are chosen as  $\gamma_i = 0.5$ ,  $\zeta_i = 0.5$  with  $i = 1, 2, \dots, 5$ ,  $\delta = 0.06$  and  $\gamma = 1.2$ . The delay time  $d_k$  is variable, which changes with the switching signal. The upper and lower bounds of  $d_k$  are known as  $\bar{d} = 6$  and  $\underline{d} = 2$ . The system input  $u(k)$  is set as

$$u(k) = \begin{bmatrix} 0.3 * \sin(k/10) \\ 0.2 * \sin(k/8) + 0.2 \end{bmatrix}.$$

The component fault is assumed as  $F(k) = [\tilde{f}(k) \ \tilde{f}(k)]^T$ , where  $\tilde{f}(k) = 1$  represents the normal actuator without fault and  $\tilde{f}(k)$  stands for the loss of effectiveness fault in the second actuator component. For verifying the generalization performance of the proposed LOs, three kinds of fault modes are considered.

$\tilde{f}_1(k)$  represents the constant fault, which is the most common type of component faults. In general, the actuator loses the effectiveness with a fixed degree.  $\tilde{f}_1(k)$  is defined as follows:

$$\tilde{f}_1(k) = \begin{cases} 0.5 & 60 \leq k \leq 120 \\ 1 & \text{others} \end{cases}.$$

$\tilde{f}_2(k)$  is a slowly time-varying signal, which means that the actuator gradually loses its effectiveness. It will seriously affect system performance if no action exists in a timely manner.  $\tilde{f}_2(k)$  is defined as follows:

$$\tilde{f}_2(k) = \begin{cases} 1 - 0.0005k & 80 \leq k \leq 140 \\ 1 & \text{others} \end{cases}.$$

$\tilde{f}_3(k)$  represents an irregular and fast time-varying fault signal, which is defined as follows:

$$\tilde{f}_3(k) = \begin{cases} 0.15 \sin(k/40) + 1.1e^{-k/3000} - 0.1 & k \geq 80 \\ 1 & \text{others} \end{cases}$$

For the design of LOs, the parameter  $K_1$  can be chosen as  $0.9500I_2$  and the other parameters will be calculated using Matlab LMI toolbox as follows:

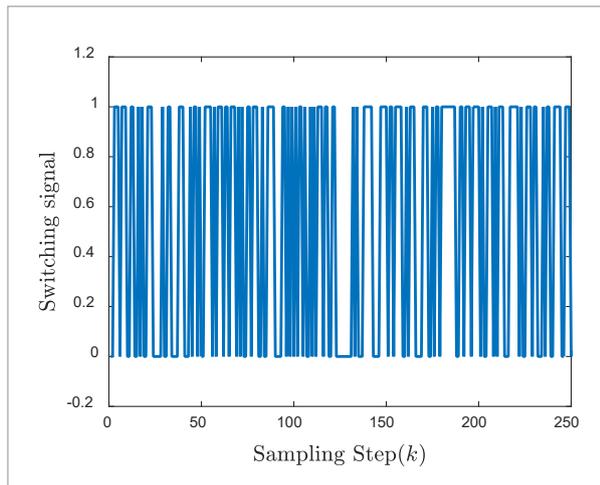
$$\begin{bmatrix} L_1 \\ K_{21} \end{bmatrix} = \begin{bmatrix} 0.0254 & 0.1026 \\ -0.0356 & 0.0698 \\ -0.0052 & 0.0065 \\ -0.2589 & -0.3025 \end{bmatrix}$$

$$\begin{bmatrix} L_2 \\ K_{22} \end{bmatrix} = \begin{bmatrix} 0.1283 & -0.1563 \\ -0.0895 & -0.0956 \\ -0.0084 & 0.0105 \\ 0.1986 & -0.2344 \end{bmatrix}$$

The switching signal is generated randomly and shown in Figure 1. The value 0 of switching signal

**Figure 1**

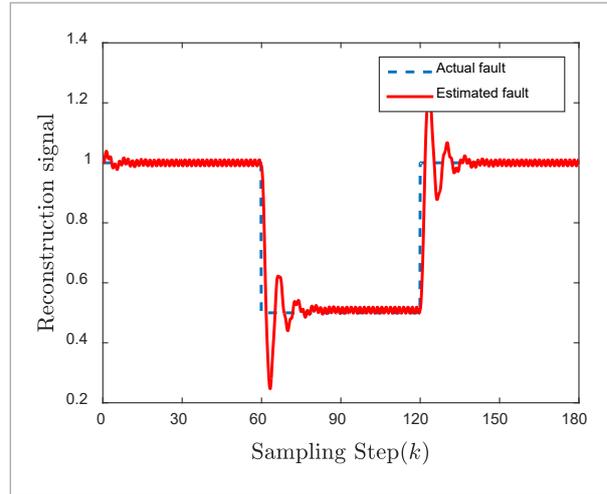
The switching signal



represents the first subsystem is active, while the other subsystem is on running when the switching signal is 1. Simulation results of LOs, which refer to above three component faults, are listed in Figures 2-4. One can see that the proposed LOs can reconstruct the fault signals effectively.

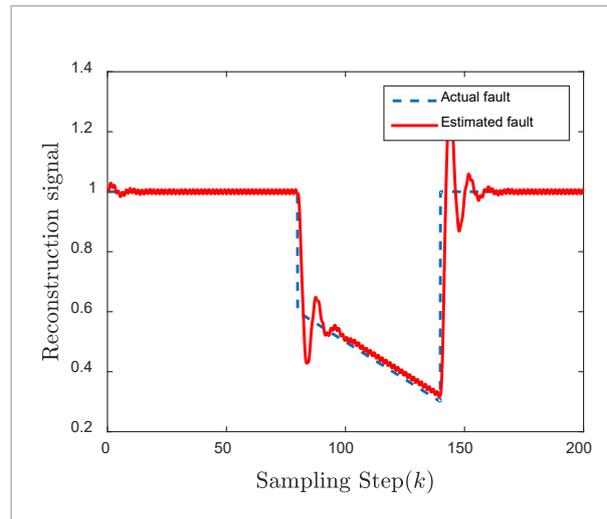
**Figure 2**

The signals of loss control effectiveness ( $\tau = 6$ ):  $\tilde{f}_1(k)$  and its estimation:  $\hat{f}_1(k)$



**Figure 3**

The signals of loss control effectiveness ( $\tau = 5$ ):  $\tilde{f}_2(k)$  and its estimation:  $\hat{f}_2(k)$



Before the occurrence of any fault, the LOs can identify the effectiveness of actuators very accurately, which means it can also be applied for fault detection applications. When the loss of actuator effectiveness happens, the LOs can reconstruct any form of faults including the constant fault in Figure 2, slowly time-varying fault in Figure 3 and fast time-varying fault in Figure 4.

**Figure 4**

The signals of loss control effectiveness ( $\tau = 2$ ):  $\tilde{f}_3(k)$  and its estimation:  $\hat{f}_3(k)$

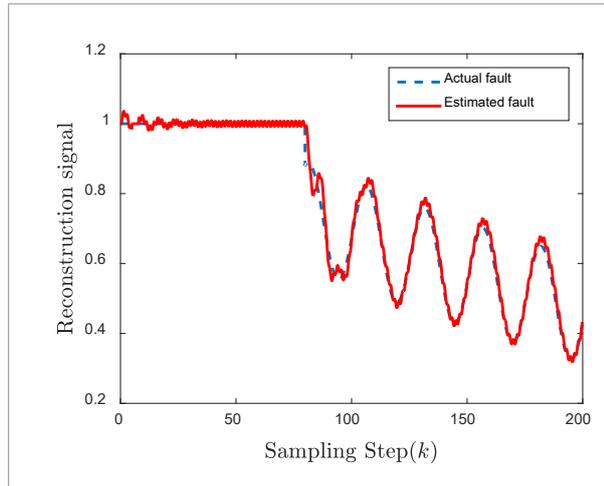
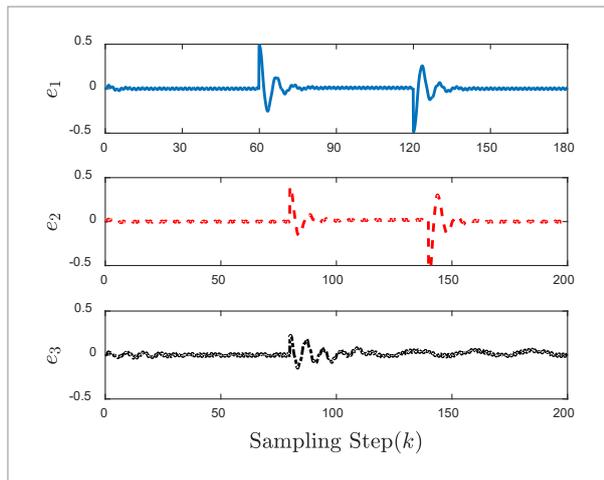


Figure 5 shows the fault reconstruction errors. The error tends to converge after small fluctuations within a limited time, which proves once again the effectiveness of the proposed scheme.

**Figure 5**

The fault reconstruction errors



tiveness of the proposed scheme. Table 1 presents the simulation errors for the reconstruction of three component faults from a quantitative perspective, while  $E_{max}$ ,  $E_{ave}$  and  $Std$  represent the max value, average value and standard deviation of the errors. The average error is within the allowable range, while the standard deviation meets the real requirements.

**Table 1**

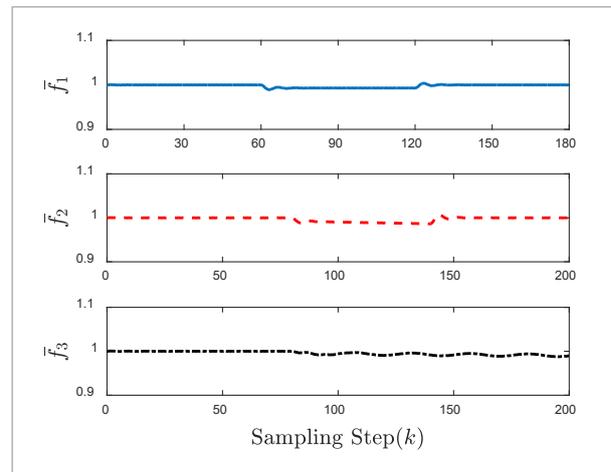
Simulation errors for three component faults

| Error | $E_{max}$ | $E_{ave}$ | $Std$  |
|-------|-----------|-----------|--------|
| $e_1$ | 0.4902    | 0.0031    | 0.0641 |
| $e_2$ | 0.3944    | 0.0042    | 0.0706 |
| $e_3$ | 0.2248    | 0.0119    | 0.0361 |

The first actuator is on normal running with  $\bar{f}(k) = 1$  for all the time. Figure 6 represents the influence of LOs on the normal actuator, where one can see that the fault reconstruction for a faulty actuator will affect another normal one. However, the influence is within the system permissibility.

**Figure 6**

The effect on normal actuator

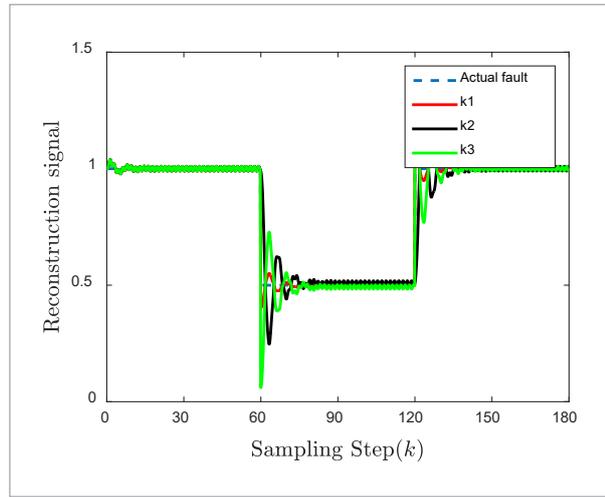


$K_1$  plays an important role of “Memory” for the reconstructed fault signal. In general,  $K_1$  is a diagonal matrix with elements close to one. One can determine the value of  $K_1$  based on the degree of fluctuation of fault signals. A relatively smaller value of  $K_1$  can help the LOs easily ignore the historical signal and keep track of the fault changes. A simulation experiment to verify the influence of  $K_1$  is established subject to  $\tilde{f}_1(k)$ . Three values are assigned to  $K_1$ :  $k_1 = 0.92I$ ,  $k_2 = 0.95I$ ,  $k_3 = 0.98I$ . Figure 7 presents the simulation results.  $\tilde{f}_1(k)$  has relatively large changes at the sampling point  $k = 60$  and  $k = 120$ . Based on the selection criteria of the value of  $K_1$ , one should choose a relatively smaller value

of  $K_1$ , which can easily ignore the historical signal and keep track of the fault changes. Figure 7 shows that three simulation results all converge to the true fault signal, and the first  $k_1$  value is the best choice with relatively small fluctuations.

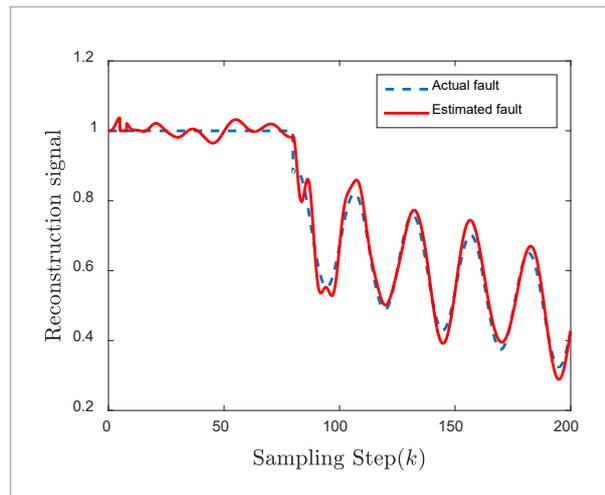
**Figure 7**

Simulation results for  $\tilde{f}_1(k)$  based on different  $K_1$



**Figure 8**

Simulation results for  $\tilde{f}_3(k)$  with external noise



In addition, in order to verify the robustness of the proposed component fault diagnosis scheme, an external disturbance  $\eta_k$  is added in system (9). There are two actuators in the simulation system. We consider two different external noises as follows:

$$\eta_k = \begin{bmatrix} 0.02 * \sin(3.8 * k / 10) \\ 0.01 * \sin(2.5 * k / 10) + 0.01 * \cos(k / 8) \end{bmatrix}.$$

Figure 8 presents the simulation result for  $\tilde{f}_3(k)$  with the external noise. Compared with the simulation result for  $\tilde{f}_3(k)$  without external noise in Figure 4, there exist some fluctuations for the output of the LOs during the first 100 steps. Then it tends to vary gently. It can also well reconstruct the component faults under the disturbance of external noise.

## 5. Conclusion

There are relatively few research works for the component fault diagnosis of switched system. In this paper, an active fault detection and estimation scheme based on LOs is investigated against component faults in a class of switched systems. The proposed LOs can guarantee the uniformly ultimately boundedness of fault reconstruction error with external disturbance using switched Lyapunov function and  $H_\infty$  technique. The design of the LOs is simple and easy to implement, which can deal with different types of fault. In the simulation parts, three kinds of fault reconstruction examples have demonstrated the effectiveness of the proposed method.

The research of this paper is a supplement to the field of fault diagnosis of switched systems. It can also be extended to other control systems. Future works are as follows:

- 1 The system involved in this paper is a linear switched system. However, most switched systems exhibit nonlinear characteristics. It is necessary to consider the effects of nonlinearity.
- 2 The choice of  $K_1$  and  $\tau$  in the LOs are generally based on human experience. It should be designed an adaptive change rule for the parameters in the LOs.

## Appendix

There are three parts in the appendix for the proofs of the theorems presented above.

**Part A: Proof for Theorem 1:** First two lemmas are listed which are useful in the following proof.

Lemma 1 [24]: For a positive defined symmetric ma-

trix  $P$ , and two arbitrary matrices  $M$  and  $N$  with appropriate dimensions, the following inequality holds:

$$2MPN \leq \gamma MPM^T + N^T PN / \gamma, \tag{17}$$

where  $\gamma > 0$ .

Lemma 2 [13, 21]: For the fault updating observer established in (4), the following inequality for fault reconstruction error is satisfied

$$e_F^T(k)e_F(k) \leq \rho_1 e_F^T(k-\tau)K_1^T K_1 e_F(k-\tau) + \rho_2 e_x^T(k)(K_{2i}C_i)^T (K_{2i}C_i)e_x(k) + \rho_3 \tilde{F}_k^T \tilde{F}_k, \tag{18}$$

where  $\rho_1 = 1 + \mathfrak{g}_1 + \mathfrak{g}_2$ ,  $\rho_2 = 1 + 1/\mathfrak{g}_1 + \mathfrak{g}_3$  and  $\rho_3 = 1 + 1/\mathfrak{g}_2 + 1/\mathfrak{g}_3$  with  $\mathfrak{g}_i > 0, i = 1, 2, 3$ .

Consider the following Lyapunov function

$$V(k) = e_x^T(k) \left( \sum_{i=1}^N \xi_i(k) P_i \right) e_x(k) + \sum_{j=k-\tau}^{k-1} e_F^T(j) \left( \sum_{i=1}^N \xi_i(j) Q_i \right) e_F(j), \tag{19}$$

where  $P_i$  and  $Q_i$  are the positive defined symmetric matrices. Then the difference of Lyapunov function is expressed as

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= e_x^T(k+1) \left( \sum_{i=1}^N \xi_i(k+1) P_i \right) e_x(k+1) \\ &\quad - e_x^T(k) \left( \sum_{i=1}^N \xi_i(k) P_i \right) e_x(k) \\ &\quad + e_F^T(k) \left( \sum_{i=1}^n \xi_i(k) Q_i \right) e_F(k) \\ &\quad - e_F^T(k-\tau) \left( \sum_{i=1}^N \xi_i(k-\tau) Q_i \right) e_F(k-\tau) \end{aligned} \tag{20}$$

The difference equation (20) above corresponds to any mode of switching rules. As declared in Assumption 1, only one subsystem is active each time. So we assume  $\xi_i(k) = 1, \xi_{\varphi \neq i}(k) = 0, \xi_j(k+1) = 1,$

$\xi_{\varphi \neq j}(k+1) = 0, \xi_l(k-\tau) = 1, \xi_{\varphi \neq l}(k-\tau) = 0$ . Then formula (20) can be simplified into

$$\begin{aligned} \Delta V(k) &= e_x^T(k+1)P_j e_x(k+1) - e_x^T(k)P_i e_x(k) \\ &\quad + e_F^T(k)Q_i e_F(k) - e_F^T(k-\tau)Q_l e_F(k-\tau) \\ &= e_x^T(k) \left[ (A_i - L_i C_i)^T P_j (A_i - L_i C_i) - P_i \right] e_x(k) \\ &\quad + 2e_x^T(k)(A_i - L_i C_i)^T P_j U_k e_F(k) \\ &\quad - e_F^T(k-\tau)Q_l e_F(k-\tau) \\ &\quad + e_F^T(k) \left[ U_k^T P_j U_k + Q_i \right] e_F(k) \end{aligned} \tag{21}$$

According to Lemma 1, one can get

$$\begin{aligned} 2e_x^T(k)(A_i - L_i C_i)^T P_j U_k e_F(k) &\leq \\ \gamma_1 e_x^T(k)(A_i - L_i C_i)^T P_j (A_i - L_i C_i) e_x(k), &\tag{22} \\ + 1/\gamma_1 e_F^T(k)U_k^T P_j U_k e_F(k) & \end{aligned}$$

where  $M = e_x^T(k)(A_i - L_i C_i)^T, P = (A_i - L_i C_i)^T$  and  $N = U_k e_F(k)$ .

Based on Assumption 2, the formula (21) satisfies the following inequality

$$\begin{aligned} \Delta V(k) &\leq e_x^T(k) \left[ (1 + \gamma_1)(A_i - L_i C_i)^T P_j \right. \\ &\quad \left. \cdot (A_i - L_i C_i) - P_i \right] e_x(k) \\ &\quad - \delta e_F^T(k)Q_l e_F(k) + \alpha e_F^T(k)e_F(k) \\ &\quad - e_F^T(k-\tau)Q_l e_F(k-\tau) \end{aligned} \tag{23}$$

where  $\alpha = (1 + 1/\gamma_1)\lambda_{\max}(P_j)u_{\max}^2 + (1 + \delta)\lambda_{\max}(Q_l)$  with  $\delta \geq 0$ .

Applying Lemma 2, the above inequality can be further expressed as

$$\begin{aligned} \Delta V(k) &\leq e_x^T(k) [(1 + \gamma_1)(A_i - L_i C_i)^T P_j (A_i - L_i C_i) \\ &\quad - P_i + \alpha \rho_2 (K_{2i}C_i)^T (K_{2i}C_i)] e_x(k) - \delta e_F^T(k)Q_l e_F(k) \\ &\quad + e_F^T(k-\tau) (\alpha \rho_1 K_1^T K_1 - Q_l) e_F(k-\tau) + \alpha \rho_3 \tilde{F}_k^T \tilde{F}_k \end{aligned} \tag{24}$$

From inequality (7) with Schur complement lemma, one can get

$$\begin{aligned} (A_i - L_i C_i)^T P_j (A_i - L_i C_i) - P_i \\ + \alpha \rho_2 (K_{2i}C_i)^T (K_{2i}C_i) < 0 \end{aligned} \tag{25}$$

Let  $\Lambda$  represent the left-hand side of the above inequality, and define  $\lambda_1 = \lambda_{\min}(-\Lambda) > 0$  and  $\lambda_2 = \delta\lambda_{\min}(-Q_i) > 0$ . Together with Assumption 3, one can get

$$\Delta V(k) \leq -\lambda_1 \|e_x(k)\|^2 - \lambda_2 \|e_F(k)\|^2 + \alpha\rho_3\kappa_f^2. \quad (26)$$

Thus

$$\max[\|e_x(k)\|, \|e_F(k)\|] \leq \sqrt{\frac{\alpha\rho_3}{\lambda_1 + \lambda_2}} \kappa_f. \quad (27)$$

Hence, the uniform ultimate boundedness of  $e_x(k)$  and  $e_F(k)$  can be guaranteed.

**Part B: Proof for Theorem 2:** Consider the following Lyapunov function:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k), \quad (28)$$

where

$$\begin{aligned} V_1(k) &= e_x^T(k) \left( \sum_{i=1}^N \xi_i(k) P_i \right) e_x(k) \\ V_2(k) &= \sum_{j=k-d}^{k-1} e_x^T(j) \left( \sum_{i=1}^N \xi_i(j) Q_i \right) e_x(j) \\ V_3(k) &= \sum_{j=k-\bar{d}}^{k-d} \sum_{s=j}^{k-1} e_x^T(s) \left( \sum_{i=1}^N \xi_i(s) Q_i \right) e_x(s). \quad (29) \\ V_4(k) &= \sum_{j=k-\tau}^{k-1} e_F^T(j) \left( \sum_{i=1}^N \xi_i(j) R_i \right) e_F(j) \end{aligned}$$

Then

$$\begin{aligned} \Delta V_1(k) &= e_x^T(k+1) P_j e_x(k+1) \\ &\quad - e_x^T(k) P_i e_x(k), \quad (30) \end{aligned}$$

$$\begin{aligned} \Delta V_2(k) + \Delta V_3(k) &\leq \sigma e_x^T(k) Q_i e_x(k) \\ &\quad - e_x^T(k-d) Q_{k-d} e_x(k-d) \\ &\quad - e_x^T(k-d) Q_{k-d} e_x(k-d) \\ &\quad - e_x^T(k-\bar{d}) Q_{k-\bar{d}} e_x(k-\bar{d}) \end{aligned} \quad (31)$$

$$\begin{aligned} \Delta V_4(k) &= e_F^T(k) R_i e_F(k) \\ &\quad - e_F^T(k-\tau) R_i e_F(k-\tau), \quad (32) \end{aligned}$$

where  $\sigma = (\bar{d} - d + 2)$ . Then setting  $\lambda_{\min}(Q) = \min\{\lambda_{\min}(Q_j), j = k-d, k-d-1, \dots, k-\bar{d}\}$ , or  $\lambda_{\min}(Q) = \min\{\lambda_{\min}(Q_i), i \in \mathbb{N}\}$ , one can get

$$\begin{aligned} \Delta V_2(k) + \Delta V_3(k) &\leq \sigma e_x^T(k) Q_i e_x(k) \\ &\quad - \lambda_{\min}(Q) e_x^T(k-d_k) e_x(k-d_k) \end{aligned} \quad (33)$$

Combining the state estimation error in (11), one can get

$$\begin{aligned} S &= e_x^T(k+1) P_j e_x(k+1) \\ &= e_x^T(k) (A_i - L_i C_i)^T P_j (A_i - L_i C_i) e_x(k) \\ &\quad + e_x^T(k-d_k) A_{di}^T P_j A_{di} e_x(k-d_k) \\ &\quad + e_F^T(k) U_k^T P_j U_k e_F(k) + \eta_k^T E_i^T P_j E_i \eta_k \\ &\quad + 2e_x^T(k) (A_i - L_i C_i)^T P_j A_{di} e_x(k-d_k) \\ &\quad + 2e_x^T(k) (A_i - L_i C_i)^T P_j U_k e_F(k) \\ &\quad + 2e_x^T(k) (A_i - L_i C_i)^T P_j E_i \eta_k \\ &\quad + 2e_x^T(k-d_k) A_{di}^T P_j U_k e_F(k) \\ &\quad + 2e_x^T(k-d_k) A_{di}^T P_j E_i \eta_k + 2e_F^T(k) U_k^T P_j E_i \eta_k \end{aligned} \quad (34)$$

We employ Lemma 1 to simplify the above equation. For the term  $2e_x^T(k) (A_i - L_i C_i)^T P_j A_{di} e_x(k-d_k)$ , one can define  $M = e_x^T(k) (A_i - L_i C_i)^T$ ,  $N = A_{di} e_x(k-d_k)$ ,  $P = P_j$ , then

$$\begin{aligned} 2e_x^T(k) (A_i - L_i C_i)^T P_j A_{di} e_x(k-d_k) &\leq \\ \gamma_1 e_x^T(k) (A_i - L_i C_i)^T P_j (A_i - L_i C_i) e_x(k) &+ \\ + \frac{1}{\gamma_1} e_x^T(k-d_k) A_{di}^T P_j A_{di} e_x(k-d_k) \end{aligned} \quad (35)$$

where  $\gamma_1 > 0$ .

Similarly, one can get

$$\begin{aligned} 2e_x^T(k) (A_i - L_i C_i)^T P_j U_k e_F(k) &\leq \\ \gamma_2 e_x^T(k) (A_i - L_i C_i)^T P_j (A_i - L_i C_i) e_x(k) &+ \\ + \frac{1}{\gamma_2} e_F^T(k) U_k^T P_j U_k e_F(k) \end{aligned} \quad (36)$$

$$\begin{aligned}
& 2e_x^T(k)(A_i - L_i C_i)^T P_j E_i \eta_k \leq \\
& \gamma_3 e_x^T(k)(A_i - L_i C_i)^T P_j (A_i - L_i C_i) e_x(k) \\
& + \frac{1}{\gamma_3} \eta_k^T E_i^T P_j E_i \eta_k
\end{aligned} \quad (37)$$

$$\begin{aligned}
& 2e_x^T(k-d_k) A_{di}^T P_j U_k e_F(k) \leq \\
& \gamma_4 e_x^T(k-d_k) A_{di}^T P_j A_{di} e_x(k-d_k) \\
& + \frac{1}{\gamma_4} e_F^T(k) U_k^T P_j U_k e_F(k)
\end{aligned} \quad (38)$$

$$\begin{aligned}
& 2e_F^T(k) U_k^T P_j E_i \eta_k \leq \gamma_5 \eta_k^T E_i^T P_j E_i \eta_k \\
& + \frac{1}{\gamma_5} e_F^T(k) U_k^T P_j U_k e_F(k)
\end{aligned} \quad (39)$$

where  $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5 > 0$ .

Based on Assumption 2, we can get

$$\begin{aligned}
S \leq & \left[ e_x^T(k)(1 + \gamma_1 + \gamma_2 + \gamma_3)(A_i - L_i C_i)^T \cdot \right. \\
& \left. P_j (A_i - L_i C_i) e_x(k) \right. \\
& + e_x^T(k-d_k) \left( 1 + \frac{1}{\gamma_1} + \gamma_4 \right) A_{di}^T P_j A_{di} e_x(k-d_k) \\
& + e_F^T(k) \left( 1 + \frac{1}{\gamma_2} + \frac{1}{\gamma_4} + \frac{1}{\gamma_5} \right) u_{\max}^2 P_j e_F(k) \\
& + \eta_k^T \left( 1 + \frac{1}{\gamma_3} + \gamma_5 \right) E_i^T P_j E_i \eta_k \\
& \left. + 2e_x^T(k-d_k) A_{di}^T P_j E_i \eta_k \right].
\end{aligned} \quad (40)$$

Thus

$$\begin{aligned}
\Delta V(k) \leq & e_x^T(k) \left[ \begin{array}{c} (1 + \gamma_1 + \gamma_2 + \gamma_3)(A_i - L_i C_i)^T \\ P_j (A_i - L_i C_i) + \sigma Q_i - P_i \end{array} \right] e_x(k) \\
& + e_x^T(k-d_k) \left[ \begin{array}{c} \left( 1 + \frac{1}{\gamma_1} + \gamma_4 \right) A_{di}^T P_j A_{di} \\ -\lambda_{\min}(Q) \end{array} \right] e_x(k-d_k) \\
& + \eta_k^T \left( 1 + \frac{1}{\gamma_3} + \gamma_5 \right) E_i^T P_j E_i \eta_k - \delta e_F^T(k) R_i e_F(k) \\
& + \alpha e_F^T(k) e_F(k) - e_F^T(k-\tau) R_i e_F(k-\tau) \\
& + 2e_x^T(k-d_k) A_{di}^T P_j E_i \eta_k
\end{aligned} \quad (41)$$

where  $\alpha = \left( 1 + \frac{1}{\gamma_2} + \frac{1}{\gamma_4} + \frac{1}{\gamma_5} \right) u_{\max}^2 \lambda_{\max}(P_j) + (1 + \delta) \lambda_{\max}(R_i)$ .

Using Lemma 2, we obtain

$$\begin{aligned}
\Delta V(k) \leq & e_x^T(k) \left[ \begin{array}{c} \zeta_1 (A_i - L_i C_i)^T P_j (A_i - L_i C_i) \\ + \sigma Q_i - P_i \\ + \alpha \rho_2 (K_{2i} C_i)^T (K_{2i} C_i) \end{array} \right] e_x(k) \\
& + e_F^T(k-\tau) (\alpha \rho_1 K_1^T K_1 - R_i) e_F(k-\tau) \\
& - \delta e_F^T(k) R_i e_F(k) + \zeta_3 \eta_k^T E_i^T P_j E_i \eta_k \\
& + e_x^T(k-d_k) \left[ \zeta_2 A_{di}^T P_j A_{di} - \lambda_{\min}(Q) \right] e_x(k-d_k) \\
& + 2e_x^T(k-d_k) A_{di}^T P_j E_i \eta_k + \alpha \rho_3 \tilde{F}_k^T \tilde{F}_k
\end{aligned} \quad (42)$$

where  $\zeta_1 = 1 + \gamma_1 + \gamma_2 + \gamma_3$ ,  $\zeta_2 = 1 + \frac{1}{\gamma_1} + \gamma_4$  and  $\zeta_3 = 1 + \frac{1}{\gamma_3} + \frac{1}{\gamma_5}$ .

In order to make the proposed LOs robust to external disturbance, consider the following  $H_\infty$  performance index

$$J = \sum_{k=0}^K \left[ e_y^T(k) e_y(k) - \gamma^2 \eta_k^T \eta_k \right], \quad (43)$$

where  $K$  is an arbitrary positive integer.

Under zero-initial conditions, one has

$$\begin{aligned}
J & = \sum_{k=0}^K \left[ e_y^T(k) e_y(k) - \gamma^2 \eta_k^T \eta_k + \Delta V(k) \right] - V(K) \\
& \leq \sum_{k=0}^K \left[ e_y^T(k) e_y(k) - \gamma^2 \eta_k^T \eta_k + \Delta V(k) \right]
\end{aligned} \quad (44)$$

It is noted that

$$\begin{aligned}
& e_y^T(k) e_y(k) - \gamma^2 \eta_k^T \eta_k + \Delta V(k) \leq e_x^T(k) \Xi_1 e_x(k) \\
& + e_F^T(k-\tau) \Xi_3 e_F(k-\tau) + \alpha \rho_3 \tilde{F}_k^T \tilde{F}_k \\
& + \left[ e_x^T(k-d_k) \eta_k^T \right] \Xi_2 \begin{bmatrix} e_x(k-d_k) \\ \eta_k \end{bmatrix}
\end{aligned} \quad (45)$$

Based on Theorem 2 together with Schur complement lemma, one can get  $\Xi_1 < 0$ ,  $\Xi_2 < 0$ ,  $\Xi_3 < 0$ . Thus

$$\sum_{k=0}^K \left[ e_y^T(k) e_y(k) - \gamma^2 \eta_k^T \eta_k \right] \leq \sum_{k=0}^K \alpha \rho_3 \tilde{F}_k^T \tilde{F}_k. \quad (46)$$

Thus, the uniform ultimate boundedness of system state estimation error and fault reconstruction error are guaranteed and output estimation error satisfies

$$\|e_y(k)\| \leq \gamma \bar{\eta} + \sqrt{\alpha \rho_3} \kappa_f.$$

**Part C: Proof for Theorem 3:** Here we give the proof that LMI (12) is equivalent to the formula (15). Similarly, LMI (13) can be transformed into the inequality (16).

Assume that (15) is feasible. Then  $G_i + G_i^T - P_j > 0$  which means that  $G_i$  is nonsingular. For strictly positive matrix  $P_j$ , one can get

$$(P_j - G_i)^T P_j^{-1} (P_j - G_i) \geq 0. \quad (47)$$

Thus

$$G_i^T P_j^T G_i \geq G_i + G_i^T - P_j. \quad (48)$$

Then (15) can be transformed into

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$$\begin{bmatrix} -\zeta_1 G_i^T P_j^{-1} G_i & 0 & 0 & G_i^T A_i - Y_i C_i & I \\ * & -G_i^T P_j G_i & M_i C_i & 0 & G_i \\ * & * & -I & C & 0 \\ * & * & * & -\alpha \rho_2 I & 0 \\ * & * & * & * & -\sigma G_i^T Q_i^{-1} G_i \end{bmatrix} < 0. \quad (49)$$

Then, pre multiplying inequality (47) by  $\text{diag}\{G_i^{-T}, I, I, I, G_i^{-T}\}$  and post multiplying by  $\text{diag}\{G_i^{-1}, I, I, I, G_i^{-1}\}$ , one can obtain (12). So the proof is finished and the parameters of LOs are calculated by  $L_i = G_i^{-T} Y_i$  and  $K_{2i} = G_i^{-T} M_i$ .

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