Reconfiguration Schemes of SC Biquad Filters for Oscillation Based Test

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Abstract. Transformation rules for oscillation based test for different types of switched-capacitor (SC) biquad filter stages based on Fleischer-Laker biquad SC structure are proposed. In our earlier work presented at the 9th European Test Symposium, a solution for all-pass SC biquads has been reported. In this paper we generalize the approach to other classes of SC filter biquads. Theoretical background of the proposed approach is described and a set of practical design-for-test rules for each of the addressed SC structure is provided. Proposed oscillation based test structures can be included into a built-in self-test design with minimum hardware overhead.

Keywords: Mixed-signal test; Oscillation based test; Switched-capacitor filters; Test structures.

1. Introduction

Oscillation-based test (OBT) strategy is a promising approach for testing analog and mixed-signal circuits [1-8]. Considering OBT does not require a stimulus generator and the test result analyzer can be realized with a simple pulse counter circuit the method is especially suitable for built-in self-test designs. Reported implementations show that the technique allows obtaining reliable test results [4] and high fault coverage [5]. In addition, the technique requires low area overhead. An overview of oscillation based test principles and applications is given in [7]. In oscillation-based test, the circuit is transformed into an oscillator and the frequency of oscillation is measured. Fault detection is based on the comparison of the measured oscillation frequency of the circuit-under-test with a reference value obtained from a fault-free (i.e. golden sample) circuit, operating under the same test conditions. Discrepancy between the oscillation frequency of a circuit-under-test and the reference value indicates possible faults. The technique of putting a given circuit into oscillation as well as the relation between oscillation frequency and relevant circuits parameters however depends on the original circuit topology and the employed test strategy.

Switched-capacitor (SC) devices are frequently used for implementing analog filtering functions in mixed-signal ICs. Earlier work on oscillation-based test of monolithic SC filters has been reported in [8]. The authors proposed a comprehensive OBT strategy based on partitioning complex filters into second-order (biquad) filter stages. Single biquads, combined with a nonlinear feedback circuit, form a filter-based oscillator producing controlled oscillations [9,10]. Since oscillation frequency and amplitude are determined by biquad transfer function coefficients, measuring both values can detect faults in most biquad components. The applied oscillator model presumes that the SC biquad provides for necessary band-pass filtering of the nonlinear feedback signal. Although specific biquad configurations may already comply in their original form, most will require circuit transformations in order to implement a band-pass transfer function during test. A novel low-overhead approach using complex oscillation regimes has recently been reported [11].

This paper presents transformation rules for different types of SC biquad filter stages based on the generic Fleischer-Laker biquad SC structure [12]. A solution for the specific class of all-pass SC biquads has been reported in our earlier work [13]. In this paper we generalize the approach to other classes of SC filter biquads, describing the theoretical background and providing a set of practical design-for-test rules for each of the addressed SC structures. In comparison with general guidelines for the design of OBT structures of SC biquad filters [7] the proposed solutions can be regarded as a step forward towards the application in practice. They are specially suited for built-in self-test design since they require relatively low hardware overhead. They also comply with the test strategy recently reported in [14].
2. OBT transformation rules for SC biquad structures

2.1. Achieving oscillation conditions in generic Fleischer-Laker SC biquad

A generic Fleischer-Laker biquad SC stage with minimum switch configuration is shown in Figure 1 and its discrete-time model in Figure 2.

The values of capacitors determine the frequency characteristics (i.e., lowpass, bandpass, or highpass). The transfer function poles are determined by the integrator loop including capacitors A through D and a damping capacitor (E or F) and the transfer function zeros are determined by the input branches including capacitors G through J. Capacitors E and F are mutually redundant, hence in practical applications only one of them is employed. Depending on the

![Figure 1. A generic Fleischer-Laker biquad SC stage](image1)

![Figure 2. Discrete-time model of Fleischer-Laker biquad SC stage](image2)

The above discrete-time model is described by the following two transfer functions:

\[ H(z) = \frac{V_{out}}{V_i} = \frac{D_1 - (D_1 + D_1 - A_1)z^{-1} + (D_1 - A_1)z^{-2}}{D(F + B) - (2DB + DF - AC - AE)z^{-1} + (DB - AE)z^{-2}} = \frac{N(z)}{D(z)} \]  

\[ H'(z) = \frac{V_{out}}{V_i} = \frac{(F_1 + G_1 - I_1z^{-1}) - (G_1 + BH + FH - I_1z^{-1} + (BH - JE)z^{-2}}{D(F + B) - (2DB + DF - AC - AE)z^{-3} + (DB - AE)z^{-2}} = \frac{N'(z)}{D(z)} \]
required frequency characteristics one or more input branches may be omitted from the FLB structure.

In order to construct a filter based oscillator as described in [10] using the given SC biquad, the later has to implement a bandpass transfer function at any of its two functional outputs. For a continuous time bandpass transfer function:

\[
H(s) = \frac{k \alpha_h}{Q_p} \frac{s^2 + \omega_b s + \omega_0}{s^2 + \omega_b s + \omega_0} = \frac{N(s)}{D(s)} \tag{3}
\]

the equivalent SC biquad discrete-time transfer function is given by:

\[
H(z) = \frac{k' (1 - z^{-1})(1 + z^{-1})}{1 - 2r_p \cos\Theta_p z^{-1} + r_p z^{-2}} = \frac{N(z)}{D(z)} \tag{4}
\]

The zeros of the discrete-time transfer function at \(z=1\) and \(z=-1\) correspond to the frequencies \(\omega_0 = 0\) and \(\omega_0 = \omega_0/2\). For typical sampling frequencies where \(\omega_0 << \omega_0\) we can neglect the impact of the zero at \(z = -1\). Consequently, the term \((1 + z^{-1})\) in the numerator of the transfer function can be replaced by a constant or a unit-delay term:

\[
N(z) = 2k' \left(1 - z^{-1}\right) \tag{5}
\]

or

\[
N(z) = 2k' z^{-1} \left(1 - z^{-1}\right) \tag{6}
\]

If any of the two transfer functions of a SC biquad contains the numerator of the form (5) or (6), self-sustained oscillation can be produced by applying an external nonlinear block between its terminals [9, 10]. When this is not the case, the original \(N(z)\) or \(N'(z)\) should be modified for the purpose of OBT in such a way that a bandpass form is obtained. The numerator can be modified independently from the denominator by adjusting biquad input branches (capacitors \(G\) through \(J\)). Although it is possible to change actual capacitor sizes (e.g. by using programmable capacitor arrays) this would require major modifications of the original SC design as well as substantial analog area overhead. On the other hand, minor modifications of switching resources can allow performing efficient transformations of input capacitors based on rules illustrated in Figure 3.

In accordance with the above rules, oscillation based test reconfiguration schemes of a low-pass, high-pass, all-pass and band-reject SC biquad can be derived.

2.2. Low-pass SC biquad OBT reconfiguration scheme

The discrete-time transfer function of a low-pass SC biquad is given by

\[
H(z) = \frac{k' (1+z^{-1})^2}{1-2r_p \cos\Theta_p z^{-1} + r_p z^{-2}} = \frac{N(z)}{D(z)} \tag{7}
\]

The term \((1+z')\) in the numerator of the transfer function can be replaced by a constant, which yields the design options given in Table 1.

For the oscillation test, the SC stage has to implement a bandpass transfer function, which requires some circuit reconfiguration (Table 2).

The condition \(G=H\) can be satisfied by transforming the input branch phase switched capacitor into a non-switched capacitor as illustrated by the transformation rule in Figure 3. This is achieved by modifying the control of capacitor \(G\) (in configuration \(LP1\) and \(LP3\)) or capacitor \(H\) (in configuration \(LP2\)) switches during OBT. The introduction of an additional test mode signal and
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Table 1. Design equations for a low-pass SC biquad

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$N(z)$</th>
<th>Design equations</th>
<th>Trivial solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP1</td>
<td>$k'(1 + z^{-1})^2$</td>
<td>$I =</td>
<td>k'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I + J - G = -2</td>
<td>k'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$J - H =</td>
<td>k'</td>
</tr>
<tr>
<td>LP2</td>
<td>$4k'z^{-2}$</td>
<td>$I = 0$</td>
<td>$I = J = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I + J - G = 0$</td>
<td>$G = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$J - H = -4</td>
<td>k'</td>
</tr>
<tr>
<td>LP3</td>
<td>$4k'z^{-1}$</td>
<td>$I = 0$</td>
<td>$I = J = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I + J - G = -4</td>
<td>k'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$J - H = 0$</td>
<td>$H = 0$</td>
</tr>
</tbody>
</table>

Table 2. Applied transformations in OBT

<table>
<thead>
<tr>
<th>Condition</th>
<th>Applied transformations</th>
<th>Implemented $N(z)$</th>
<th>Oscillation frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I = J = 0$</td>
<td>$K</td>
<td></td>
<td></td>
</tr>
</tbody>
</table><p>ightarrow 0$ | $k'(1 - z^{-1})$ | $\omega_{osc} = \frac{1}{T} \arccos \left( \frac{1 - AC}{2D(B + F)} \right)$ |
| $G = H$ | $K \rightarrow L = 4|k'|$ | |</p>

![Figure 4. (a) low-pass SC biquad, (b) its OBT reconfiguration scheme](image)

switch control decode logic allows us to set selected input branch switches to a constant ON or OFF state during the test. Furthermore, the minimum switch biquad configuration must be augmented such to provide independent switching resources for capacitor $C$ in the integrator loop. Figure 4 presents the resulting low-pass SC biquad OBT reconfiguration scheme.

2.3. High-pass SC biquad OBT reconfiguration scheme

The discrete-time transfer function of a high-pass SC biquad is given by

$$H(z) = \frac{k'(1 - z^{-1})^2}{1 - 2r_p \cos \theta_p z^{-1} + r_p z^{-2}}.$$  \hspace{1cm} (8)

Similarly, we can write the design equations and the corresponding solution as shown in Table 3.

Table 3. Design equations for a high-pass SC biquad

<table>
<thead>
<tr>
<th>Config.</th>
<th>$N(z)$</th>
<th>Design equations</th>
<th>Trivial solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP</td>
<td>$k'(1 - z^{-1})^2$</td>
<td>$I =</td>
<td>k'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I + J - G = -2</td>
<td>k'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$J - H =</td>
<td>k'</td>
</tr>
</tbody>
</table>

The conditions for oscillation can be fulfilled by reconfiguring the non-switched capacitor $K$ into a phase switched capacitor $I = K$, which gives the following transformations (Table 4):
Table 4. Applied transformations in OBT

<table>
<thead>
<tr>
<th>Condition</th>
<th>Applied transformations</th>
<th>Implemented $N(z)$</th>
<th>Oscillation frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J = 0$</td>
<td>$K \rightarrow I = k$</td>
<td>$k'(1 - z^{-1})$</td>
<td>$\omega_{osc} = \frac{1}{T} \arccos \left( 1 - \frac{AC}{2(BD + AE)} \right)$</td>
</tr>
</tbody>
</table>

The high-pass SC biquad shown in Figure 5a is modified by introducing additional switches into capacitor $K$ input branch, which allow for its transformation into a phase switched capacitor $I$ during oscillation based test (shown in Figure 5b).

![Figure 5. (a) high-pass SC biquad, (b) its OBT reconfiguration scheme](image)

2.4. All-pass biquad OBT reconfiguration scheme

The discrete-time transfer function of an all-pass SC biquad is given by

$$H(z) = \frac{k'(1 - \frac{2}{r_p} \cos \Theta z^{-1} + \frac{1}{r_p} z^{-2})}{1 - 2r_p \cos \Theta z^{-1} + r_p^2 z^{-2}}.$$  \hspace{1cm} (9)

According to FLB design rules [15], the following set of design equations can be compiled (Table 5):

Table 5. Design equations for an all-pass SC biquad

<table>
<thead>
<tr>
<th>Config.</th>
<th>$N(z)$</th>
<th>Design equations</th>
<th>Trivial solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP</td>
<td>$k'(1 - \frac{2}{r_p} \cos \Theta z^{-1} + \frac{1}{r_p} z^{-2})$</td>
<td>$I = k\left(1 - \frac{2}{r_p}\cos \Theta \right)$</td>
<td>$G = k\left(1 - \frac{2}{r_p}\cos \Theta \right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I + J - G = 2k\left(1 - \frac{1}{r_p}\cos \Theta \right)$</td>
<td>$H = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$J - H = k\left(1 - \frac{1}{r_p}\right)$</td>
<td>$J = k\left(1 - \frac{1}{r_p}\right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K = k\left(1 - \frac{1}{r_p}\right)$</td>
<td>$K = k\left(1 - \frac{1}{r_p}\right)$</td>
</tr>
</tbody>
</table>

Although the trivial solution proposed above omits the $H$ input capacitor, the latter is usually implemented in order to reduce total capacitors area. Figure 6a illustrates a typical all-pass FLB with minimum switch configuration and input capacitors relations $G \neq H \neq 0$ and $I \neq J \neq 0$. Input SC pairs $\{G, H\}$ and $\{I, J\}$ are actually replaced by equivalent structures $\{G, L\}$ and $\{J, K\}$, according to the rules in Figure 3.
By comparing the all-pass SC biquad transfer function with possible band-pass \( N(z) \) implementations we can determine necessary transformations for the circuit under test (CUT) as shown in Table 6.

### Table 6. Applied transformations in OBT

<table>
<thead>
<tr>
<th>Condition</th>
<th>Applied transformations</th>
<th>Implemented ( N(z) )</th>
<th>Oscillation frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H = 0 )</td>
<td>( L \rightarrow 0 )</td>
<td>( J \rightarrow 0 )</td>
<td>( k'(1 - z^{-1}) )</td>
</tr>
<tr>
<td>( I = J = 0 )</td>
<td>( J \rightarrow 0 )</td>
<td>( K \rightarrow 0 )</td>
<td></td>
</tr>
</tbody>
</table>

In order to achieve band-pass characteristics, input branches \( H(L), I(K) \) and \( J \) should be disabled. As it was the case with previous biquad designs \((LP, HP)\), the original minimum switch configuration in Figure 6a does not provide sufficient resources to implement the above transformations, therefore additional test switches are required. Furthermore, some of the existing switches need to behave differently in the test mode than in the normal operation, requiring additional clocking signals control logic. Nevertheless the required test area overhead remains small since original capacitor layouts are not affected.

One major drawback of the proposed transformation is that disconnecting capacitors \( L, J \) and \( K \) during OBT will prevent any faults in those devices to be manifested through deviations of the output signal frequency and/or amplitude. We explored the possibility to apply an additional biquad transformation, which would allow us to test the frequencies of the all-pass transfer function zeros. By re-arranging the original biquad as illustrated in Figure 7 the following transfer functions become available at biquad functional outputs:

\[
H(z) = \frac{V_o}{V_i} = \frac{ACz^{-1}}{D(I + Jz^{-1})} = \frac{N(z)}{D(z)} \quad (10)
\]

\[
H(z) = \frac{V_o}{V_i} = \frac{C(I - Jz^{-1})}{D(I + Jz^{-1})} = \frac{N(z)}{D(z)} \quad (11)
\]

Comparing the above expressions with expression (1), one can determine that the original all-pass transfer function numerator is identical to the re-arranged biquad function denominator. However further transformations are required in order to produce a suitable OBT biquad configuration since:

- none of the above expressions features a band-pass \( N(z) \) form,
- simply converting the original zeros into poles would produce an unstable circuit with poles laying outside the \( Z \)-plane unit circle.
Considering the above requirements, we can compile the set of transformation rules given in Table 7.

Table 7. Design equations for the improved version

<table>
<thead>
<tr>
<th>Condition</th>
<th>Applied transformations</th>
<th>Implemented $N(z)$</th>
<th>Oscillation frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G &gt; H &gt; 0$</td>
<td>$L + E</td>
<td></td>
<td></td>
</tr>
</tbody>
</table><p>ightarrow L'$  | $J' + K' = (1 - z^{-1})$ |
| $I = J$          | $J + Kightarrow K'$  | $Bightarrow 0$  | $\omega_{osc} = \frac{1}{T} \arccos \left( \frac{1 + \frac{DK' - AG'}{DK'' - AH''}}{2} \right)$ |</p>

where $G' = G + L'$ and $H' = L'$.

A set of test switches is required to reconfigure biquad components into the desired layout. Furthermore, the capacitors pair $\{J, K\}$ switching is modified such to produce an equivalent non-switched capacitor $K'$ and capacitor $B$ is disabled during “zeros” OBT. Removing capacitor $E$ from the circuit would require additional test switches to be inserted into the loop, however this is not necessary as we can simply take into account its impact on the CUT oscillation frequency. It should be noted that circuit stability is guaranteed as long as $G' > H' > 0$.

Figure 7. Biquad reconfiguration for zeros frequency test

Figure 8. Minimum all-pass FLB configuration (a) and circuit with test resources (b)
Figure 8b illustrates the all-pass biquad scheme complete with test resources necessary to implement the above transformations. Greyed areas denote required modifications in the SC switching network: dark grey fields represent switches added to the original configuration while light grey fields represent existing switches with clocking signals that require additional control logic. Adequate logic expressions are given for each controlled switch, where $T_P$ and $T_Z$ denote control signals active during “poles” OBT and during “zeros” OBT, respectively. Switches $T_{Sw1}$ through $T_{Sw5}$ are required by the “poles” OBT set-up while the remaining test switches ($T_{Sw6}$ to $T_{Sw13}$) are only activated during “zeros” OBT. The total number of switches in the modified biquad is about twice that of the original circuit.

### 2.5. Band-reject SC biquad OBT reconfiguration scheme

The discrete-time transfer function of a band-reject SC biquad is given by

$$H(z) = \frac{k'(1-2\cos\Theta_n z^{-1} + z^{-2})}{1 - 2r_p \cos\Theta_n z^{-1} + r_p z^{-2}}.$$  \hspace{1cm} (12)

The design equations with the corresponding solutions are given in Table 8.

#### Table 8. Design equations for a band-reject SC biquad

<table>
<thead>
<tr>
<th>Config.</th>
<th>$N(z)$</th>
<th>Design equations</th>
<th>Trivial solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BR$</td>
<td>$k'(1-2\cos\Theta_n z^{-1} + z^{-2})$</td>
<td>$I = \abs{k'}$</td>
<td>$G = 2\abs{k'}(1 + \cos\Theta_n)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I + J - G = -2\abs{k'}\cos\Theta_n$</td>
<td>$H = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$J - H = \abs{k'}$</td>
<td>$I = J = K = \abs{k'}$</td>
</tr>
</tbody>
</table>

The conditions for oscillation can be achieved by the following transformations (Table 9):

#### Table 9. Applied transformations in OBT

<table>
<thead>
<tr>
<th>Condition</th>
<th>Applied transformations</th>
<th>Implemented $N(z)$</th>
<th>Oscillation frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G = H = 0$</td>
<td>$G \to 0$</td>
<td>$K \to I$</td>
<td>$k'(1 - z^{-1})$</td>
</tr>
<tr>
<td>$J = 0$</td>
<td>$\omega_{osc} = \frac{1}{2\pi} \arccos\left(1 - \frac{AC}{2(BD - AE)}\right)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Reconfiguration of capacitor $K$ into a phase switched device can be achieved by introducing additional switching resources as it was done in case of the high-pass SC biquad. Furthermore, capacitor $G$ can be disconnected by controlling the corresponding switches during OBT.

Similar to the all-pass SC biquad configuration, the band-reject frequency characteristics are determined by the placement of transfer function complex zeros on the $Z$-plane unit circle, as illustrated in Figure 9.

![Figure 9. Band-reject frequency characteristics and poles/zeros placement](image)

Again, we can re-arrange the original biquad layout into the circuit illustrated in Figure 7 obtaining the transfer function described by (11). Note however that the poles of the reconfigured biquad will be placed on the $Z$-plane unit circle, producing a marginally stable circuit. In order to shift the poles within the unit circle it is necessary to introduce a “test” capacitor $H$, which should observe the condition $G > H > 0$. For typical designs it should be possible to use the existing capacitor $E$ as a replacement instead of introducing a dedicated test capacitor $H$. The following set of transformation rules can then be compiled (Table 10):
3. Conclusions

The oscillation-based test approach has gained considerable popularity and has been applied in testing different classes of mixed-signal circuits. The main issue in the oscillation-based test is the design of such testability structures and circuit-reconfiguration schemes, which provide for efficient test implementation. OBT test solutions of SC biquad filters based on internal reconfiguration [10] require implementation of a precise feedback control in a monolithic SC stage, which is rather difficult due to the lack of mechanisms for internal tuning of circuit components. On the other hand, transformation rules of generic Fleischer-Laker biquad SC structure presented in this paper offer simple and efficient test solutions, which can be easily implemented in practice. Our proposed approach is different from other reported solutions in that it tries to validate actual filter parameters against the designed ones. Furthermore, our goal was to develop OBT structures that can be included into a built-in self-test design with minimum hardware overhead.

Like in digital world, mixed-signal logic operation is challenged by relentlessly increasing performance and complexity. As pointed out in [16], the rapidly shrinking design increases the impact of random defects on circuit operation. The integration of digital, analog, power management, mixed signal, and RF/microwave circuitry in a single package calls for a combined effort in developing efficient test solutions. The described OBT structures could serve as a basis for the development of low-cost built-in self-test implementation.

References

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