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Controlling Hyperchaotic Finance System with Combining Passive and Feedback Controllers

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In this paper, a novel control method that combines passive, linear feedback, and dislocated feedback control methods is proposed and applied to the control of the four-dimensional hyperchaotic finance system which has been introduced and controlled with the linear feedback and speed feedback control methods by Yu, Cai, and Li (2012). The stability of the hyperchaotic finance system at its equilibrium points is ensured on the basis of a Lyapunov function. Computer simulations are used for verifying all the theoretical analyses visually. In the simulations, the proposed control method is also compared with the speed feedback and linear feedback control methods to observe its effectiveness. Finally, the comparative findings are discussed.

KEYWORDS: Hyperchaotic finance system, passive control, speed feedback control, linear feedback control, dislocated feedback control, chaos control.

1. Introduction

This paper investigates the control of a hyperchaotic finance system which was proposed by Yu *et al.* [46]. Almost all financial systems have nonlinear factors, multiplicity and ambiguity in their inward structure. They may cause undesired chaotic trajectories which had better be eliminated. Therefore, it has become important to inquire the control of chaos in the financial systems. Since the pioneer study of Ott *et al.* in 1990 [29], several effective methods for the control of chaotic systems have been proposed. The chaos control approaches include linear feedback control [13, 15, 16, 18, 22, 24, 41, 44, 47], nonlinear feedback control [7], time-delayed feedback control [34], adaptive control [2, 40], sliding mode control [22, 12], passive control [8, 22, 28, 33, 45], backstepping design [12, 32], and intelligent control [20] methods. Among them, the linear feedback control has been widely used due to its simplicity of implementation. It stabilizes the chaotic system with the negative feedback gains of related state variables. In recent years, it has been applied for the control of Lorenz [47], Chua [18], Rössler [16], Chen [13], Lü [24], Liu [41], four-dimensional Rabinovich [22], and many other chaotic systems [15, 44]. If another feedback state variable gain is used, then it is named as dislocated feedback control method. The control of Lorenz [36], Liu [51], and Lü [31] chaotic systems to their equilibrium points is achieved with the dislocated feedback control. The speed feedback control method uses the derivative of independent variable by multiplying with a coefficient as a feedback gain. The Lorenz [36], Liu [51], Lü [31], unified [37], Rössler [37], and Tigan [9] chaotic systems are successfully controlled with the speed feedback control method. Passive control, the other significant chaos control method, has been applied for the control of Lorenz [45], Chen [33], unified [8], four-dimensional Rabinovich [22], and some other chaotic systems [28]. The goal of the passivity theory is to keep a system asymptotically stable. It is done by a controller which makes the closed loop system passive upon the specificities of the system. The methodology of passivity can be accessed in a number of papers [8, 28, 33, 45].

In 2001, the first chaotic finance system was introduced [25, 26]. Then, some new chaotic finance systems were proposed [3, 4, 35]. Afterwards, a hyperchaotic finance system was shown [10]. In 2012, Yu *et al.* presented a new hyperchaotic finance attractor from the first chaotic finance system [46]. Several control methods were used for the control of chaos and hyperchaos in finance systems. Linear feedback controllers [5, 42], speed feedback controllers [5, 42], adaptive controllers [5, 35], selection of gain matrix controllers [42], revision of gain matrix controllers [42], positive feedback gain matrix [49], time-delayed feedback controllers [6, 43, 48], nonlinear feedback controllers [1], linear controllers [23], a passive controller [11], H_∞ controllers [50], active controllers [30], and neural controllers [21] were assigned for the control of chaotic finance equations. The control of the former hyperchaotic finance system was reported with the speed feedback [10], linear feedback [39], time-delayed feedback [14], and sliding mode [38] control methods. The control of the latter hyperchaotic finance system was achieved with the speed feedback [46], linear feedback [46], nonlinear recursive backstepping [17], and sliding mode [17] controllers. According to the literature review, although the synchronization of chaotic finance system is applied with the passive control method [19], there has not been any published papers which have specifically exploited the control of a hyperchaotic finance system by using a passivity based control method.

In this paper, a further approach on the control of the new hyperchaotic finance system is analysed. The remainder of this paper is organized in the following order. In Section 2, a brief representation of the related chaotic and hyperchaotic finance systems is given. In Section 3, in order to achieve the control of new hyperchaotic finance system, the hybrid controllers including passive controllers, a linear feedback controller, and a dislocated feedback controller are employed. In Section 4, simulation results are obtained numerically and presented graphically to confirm the efficiency of the proposed control method by comparing with the speed feedback and linear feedback control results. Finally, the paper is concluded in Section 5.

2. The Hyperchaotic Finance System

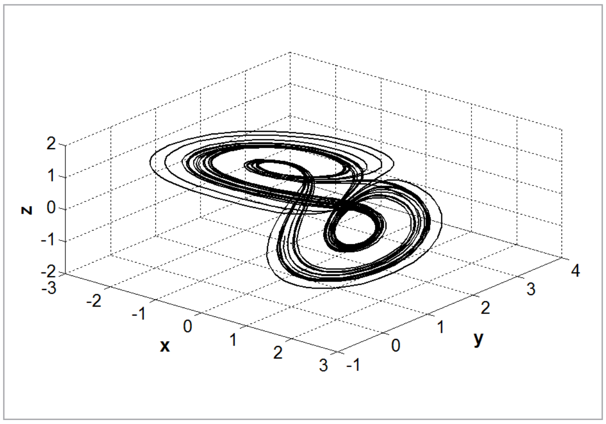
The following differential equations define the three-dimensional chaotic finance system:

$$\dot{x} = z + (y - a)x,$$
$$\dot{y} = 1 - by - x^2, \quad (1)$$
$$\dot{z} = -x - cz.$$

In system (1), the state variables x, y, z represent the interest rate, investment demand, price exponent; the positive real constants a, b, c represent saving amount, per-investment cost, elasticity of demands of commercials, respectively [25, 26]. When the parameter values are $a = 0.9$, $b = 0.2$, and $c = 1.2$, the nonlinear finance system (1) displays chaotic motions [42]. Under the initial values $x(0) = 1$, $y(0) = 2$, and $z(0) = -0.5$, the three-dimensional phase plane of the chaotic finance system is shown by using MATLAB's ode45 function in Fig. 1.

Figure 1

3D phase plane of the chaotic finance system



The following first-order differential equations define the new four-dimensional hyperchaotic finance system:

$$\dot{x} = z + (y - a)x + w,$$
$$\dot{y} = 1 - by - x^2, \quad (2)$$
$$\dot{z} = -x - cz,$$
$$\dot{w} = -dxy - kw,$$

where w is the new state variable that represents average profit margin, and d, k are new positive real constants [46]. The hyperchaotic varies of system (2) are analysed with demonstrating the bifurcation diagrams versus parameters c and k [46]. It displays hyperchaotic behaviour for the parameter values $a = 0.9$, $b = 0.2$, $c = 1.5$, $d = 0.2$, and $k = 0.17$ [46]. Under the initial values $x(0) = 1$, $y(0) = 2$, $z(0) = 0.5$, and $w(0) = 0.5$, the time series and three-dimensional phase planes of the hyperchaotic finance system are shown by using MATLAB's ode45 function in Fig. 2 and Fig. 3, respectively.

Figure 2

Time series of hyperchaotic finance system for (a) x signals, (b) y signals, (c) z signals, and (d) w signals

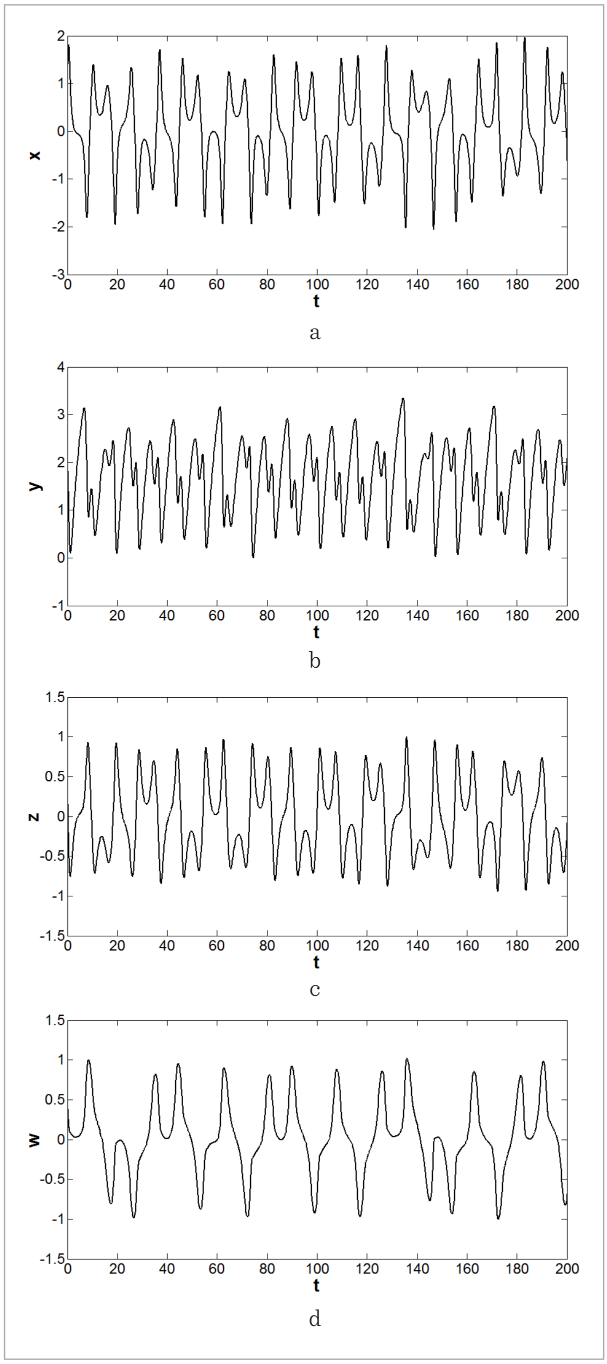
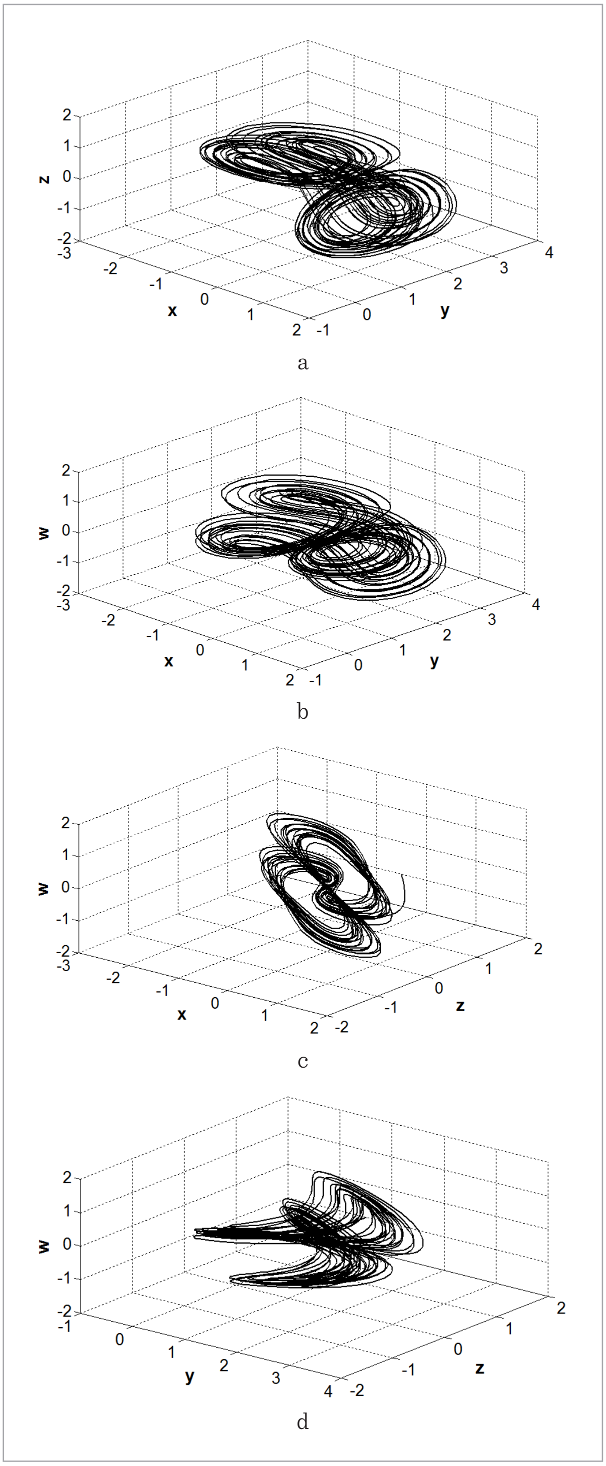


Figure 3

3D phase planes of hyperchaotic finance system for (a) x-y-z phase plane, (b) x-y-w phase plane, (c) x-z-w phase plane, and (d) y-z-w phase plane



The system (2) has three equilibrium points: $E_1(0, 1/b, 0, 0)$, $E_2(-\theta, (k + ack)/(k - d), \theta/c, -\theta d(1 + ac)/(cd - ck))$, and $E_3(\theta, (k + ack)/(k - d), -\theta/c, \theta d(1 + ac)/(cd - ck))$ where $\theta = \sqrt{(bk + abck)/(cd - ck) + 1}$. According to the above-given parameter values, the equilibrium points of hyperchaotic finance system are calculated as: $E_1(0, 5, 0, 0)$, $E_2(1.66, -8.87, -1.11, 17.4)$, and $E_3(-1.66, -8.87, 1.11, -17.4)$.

3. The Control of Hyperchaotic Finance System

In this section, the control of new hyperchaotic finance system is applied with a passivity based feedback control method. In addition, the controlled hyperchaotic finance system via speed feedback and linear feedback control methods are described.

3.1. The Passive Feedback Control

For the control of hyperchaotic finance system to its equilibrium points, u_1, u_2, u_3, and u_4 controllers are added to the system (2). Then, the controlled system becomes

$$\begin{aligned} \dot{x} &= z + (y - a)x + w + u_1, \\ \dot{y} &= 1 - by - x^2 + u_2, \\ \dot{z} &= -x - cz + u_3, \\ \dot{w} &= -dxy - kw + u_4. \end{aligned} \tag{3}$$

An equilibrium point of the system can be defined as $(\bar{x}, \bar{y}, \bar{z}, \bar{w})$, then the trajectory error states are determined as $e_1 = x - \bar{x}$, $e_2 = y - \bar{y}$, $e_3 = z - \bar{z}$, and $e_4 = w - \bar{w}$. Thus, the state variables are $x = e_1 + \bar{x}$, $y = e_2 + \bar{y}$, $z = e_3 + \bar{z}$, and $w = e_4 + \bar{w}$. The error state dynamic equations of system (3) can be denoted as

$$\begin{aligned} \dot{e}_1 &= (e_3 + \bar{z}) + ((e_2 + \bar{y}) - a)(e_1 + \bar{x}) + (e_4 + \bar{w}) + u_1, \\ \dot{e}_2 &= 1 - b(e_2 + \bar{y}) - (e_1 + \bar{x})^2 + u_2, \\ \dot{e}_3 &= -(e_1 + \bar{x}) - c(e_3 + \bar{z}) + u_3, \\ \dot{e}_4 &= -d(e_1 + \bar{x})(e_2 + \bar{y}) - k(e_4 + \bar{w}) + u_4. \end{aligned} \tag{4}$$

Then, the error dynamics in system (4) become

$$\begin{aligned} \dot{e}_1 &= (\bar{y} - a)e_1 + \bar{x}e_2 + e_3 + e_4 + e_1e_2 - a\bar{x} + \bar{z} + \bar{w} + \overline{xy} + u_1, \\ \dot{e}_2 &= -e_1^2 - 2\bar{x}e_1 - be_2 - \bar{x}^2 - b\bar{y} + 1 + u_2, \\ \dot{e}_3 &= -e_1 - ce_3 - \bar{x} - c\bar{z} + u_3, \\ \dot{e}_4 &= -d\bar{y}e_1 - d\bar{x}e_2 - ke_4 - de_1e_2 - k\bar{w} - d\overline{xy} + u_4. \end{aligned} \tag{5}$$

Because of $-a\bar{x}+\bar{z}+\bar{w}+\overline{xy}=0$, $-\bar{x}^2-b\bar{y}+1=0$, $-\bar{x}-c\bar{z}=0$, and $-k\bar{w}-d\overline{xy}=0$, they do not affect the error dynamics. Thus, system (5) can be simplified as

$$\begin{aligned}\dot{e}_1&=(\bar{y}-a)e_1+\bar{x}e_2+e_3+e_4+e_1e_2+u_1,\\ \dot{e}_2&=-e_1{}^2-2\bar{x}e_1-be_2+u_2,\\ \dot{e}_3&=-e_1-ce_3+u_3,\\ \dot{e}_4&=-d\bar{y}e_1-d\bar{x}e_2-ke_4-de_1e_2+u_4.\end{aligned}\tag{6}$$

The state variable e_1 is considered as the output of the system by taking $u_2=0$, $u_3=0$, $u_4=0$. $Z_1=e_2$, $Z_2=e_3$, $Z_2=e_4$, $Y=e_1$, and $Z=[Z_1\ Z_2\ Z_3]$ are assumed. Then, system (6) can be rewritten as

$$\begin{aligned}\dot{Z}_1&=-Y^2-2\bar{x}Y-bZ_1,\\ \dot{Z}_2&=-Y-cZ_2,\\ \dot{Z}_3&=-d\bar{y}Y-dYZ_1-d\bar{x}Z_1-kZ_3,\\ \dot{Y}&=(\bar{y}-a)Y+YZ_1+\bar{x}Z_1+Z_2+Z_3+u_1.\end{aligned}\tag{7}$$

The passivity definition has the following generalized form [45]:

$$\begin{aligned}\dot{Z}&=f_0(Z)+p(Z,Y)Y,\\ \dot{Y}&=b(Z,Y)+a(Z,Y)u,\end{aligned}\tag{8}$$

where system (7) can be written in the normal form of system (8) as follows:

$$f_0(Z)=\begin{bmatrix}-bZ_1\\ -cZ_2\\ -d\bar{x}Z_1-kZ_3\end{bmatrix},\tag{9}$$

$$p(Z,Y)=\begin{bmatrix}-Y-2\bar{x}\\ -1\\ -d\bar{y}-dZ_1\end{bmatrix},\tag{10}$$

$$b(Z,Y)=(\bar{y}-a)Y+YZ_1+\bar{x}Z_1+Z_2+Z_3,\tag{11}$$

$$a(Z,Y)=1.\tag{12}$$

Let a storage function be selected as

$$V(Z,Y)=W(Z)+\frac{1}{2}Y^2,\tag{13}$$

where

$$W(Z)=\frac{1}{2}(Z_1^2+Z_2^2+Z_3^2)\tag{14}$$

is the Lyapunov function of $f_0(Z)$, and $W(0)=0$. The zero dynamics of system (7) describe the internal dynamics which are consistent with the external constraint $\dot{Z}=f_0(Z)$. It implies

$$\begin{aligned}\frac{d}{dt}V(Z,Y)&=\frac{\partial W(Z)}{\partial Z}\dot{Z}+Y\dot{Y}\\ &=\frac{\partial W(Z)}{\partial Z}f_0(Z)+\frac{\partial W(Z)}{\partial Z}p(Z,Y)Y\\ &\quad+b(Z,Y)Y+a(Z,Y)Yu.\end{aligned}\tag{15}$$

The error system can be obtained with a minimum phase

$$\frac{d}{dt}W(Z)f_0(Z)\le 0.\tag{16}$$

Then, Equation (15) becomes

$$\frac{d}{dt}V(Z,Y)\le\frac{\partial W(Z)}{\partial Z}p(Z,Y)Y+b(Z,Y)Y+a(Z,Y)Yu.\tag{17}$$

The zero dynamics of system (8) describe the internal dynamics that is relevant with the external constraint $Y=0$, i.e., $\dot{Z}=f_0(Z)$. $\dot{W}(Z)$ according to Equation (9) is

$$\begin{aligned}\dot{W}(Z)&=\frac{\partial W(Z)}{\partial Z}f_0(Z)=[Z_1\quad Z_2\quad Z_3]\begin{bmatrix}-bZ_1\\ -cZ_2\\ -d\bar{x}Z_1-kZ_3\end{bmatrix}\\ &=-bZ_1^2-cZ_2^2-d\bar{x}Z_1Z_3-kZ_3^2\end{aligned}\tag{18}$$

which is not exactly negative definite. This implies that $f_0(Z)$ is not globally asymptotically stable. Thus, the classical passive control method does not ensure the control of system (6). For a solution, control signal u_4 is assumed as a dislocated feedback controller and linear feedback gains are also added to the states for better control performance. They are taken as

$$\begin{aligned}u_2&=-k_1e_2,\\ u_3&=-k_1e_3,\\ u_4&=d\bar{x}e_2-k_1e_4,\end{aligned}\tag{19}$$

where k_1 is a positive real constant. Now, system (6) becomes

$$\begin{aligned}\dot{e}_1&=(\bar{y}-a)e_1+\bar{x}e_2+e_3+e_4+e_1e_2+u_1,\\ \dot{e}_2&=-e_1{}^2-2\bar{x}e_1-(b+k_1)e_2,\\ \dot{e}_3&=-e_1-(c+k_1)e_3,\\ \dot{e}_4&=-d\bar{y}e_1-(k+k_1)e_4-de_1e_2.\end{aligned}\tag{20}$$

The state variable e_1 is again considered as the output of the system. $Z_1 = e_2$, $Z_2 = e_3$, $Z_2 = e_4$, $Y = e_1$, and $Z = [Z_1\ Z_2\ Z_3]$ are assumed. Then, system (20) can be rewritten as

$$\begin{aligned}\dot{Z}_1 &= -Y^2 - 2\bar{x}Y - (b+k_1)Z_1,\\ \dot{Z}_2 &= -Y - (c+k_1)Z_2,\\ \dot{Z}_3 &= -d\bar{y}Y - dYZ_1 - (k+k_1)Z_3,\\ \dot{Y} &= (\bar{y}-a)Y + YZ_1 + \bar{x}Z_1 + Z_2 + Z_3 + u_1.\end{aligned} \tag{21}$$

System (21) can be written in the normal form of system (8) as follows:

$$f_0(Z) = \begin{bmatrix} -(b+k_1)Z_1 \\ -(c+k_1)Z_2 \\ -(k+k_1)Z_3 \end{bmatrix}, \tag{22}$$

$$p(Z,Y) = \begin{bmatrix} -Y-2\bar{x} \\ -1 \\ -d\bar{y}-dZ_1 \end{bmatrix}, \tag{23}$$

$$b(Z,Y) = (\bar{y}-a)Y + YZ_1 + \bar{x}Z_1 + Z_2 + Z_3, \tag{24}$$

$$a(Z,Y) = 1. \tag{25}$$

If the same storage and Lyapunov functions are taken as in Equations (13) and (14), then the derivative of $W(Z)$, according to Equation (22), becomes

$$\begin{aligned}\dot{W}(Z) &= [Z_1 \quad Z_2 \quad Z_3]\begin{bmatrix} -(b+k_1)Z_1 \\ -(c+k_1)Z_2 \\ -(k+k_1)Z_3 \end{bmatrix}\\ &= -(b+k_1)Z_1^2 - (c+k_1)Z_2^2 - (k+k_1)Z_3^2,\end{aligned} \tag{26}$$

which is negative definite. This implies that $f_0(Z)$ is globally asymptotically stable. Thus, the zero dynamics of the controlled hyperchaotic finance system (21) is stable with the Lyapunov's method and it is a minimum phase system.

According to the passive control method, the controlled system (21) will be equivalent to a passive system and globally asymptotically stabilized at its zero equilibrium if the state controller is considered as in the following equation [27]:

$$u = a(Z,Y)^{-1}\left[-b(Z,Y) - \left(\frac{\partial W(Z)}{\partial Z}p(Z,Y)\right)^{\mathrm{T}} - \alpha Y + v\right], \tag{27}$$

where $\alpha > 0$ is a positive real constant and v is an external signal which is connected to the reference input. The signal v provides an alternative solution for adjusting the control of system to its non-zero equilibrium points. v equals to zero if the equilibrium points are already considered.

According to Equation (27), the passive control function becomes

$$u_1 = -(\bar{y}-a)Y + \bar{x}Z_1 - Z_3 + d\bar{y}Z_3 + dZ_1Z_3 - \alpha Y + v. \tag{28}$$

If the conversions $Z_1 = e_2$, $Z_2 = e_3$, $Z_2 = e_4$, and $Y = e_1$ are taken back, the passive controller is rewritten as

$$u_1 = -(\bar{y}-a)e_1 + \bar{x}e_2 - e_4 + d\bar{y}e_4 + de_2e_4 - \alpha e_1 + v. \tag{29}$$

Hence, the control of hyperchaotic finance system (3) with uncertain parameters by using the passivity based feedback control method is provided with Equations (19) and (29).

3.2. The Speed Feedback Control

Yu *et al.* achieved the control of hyperchaotic finance system by means of a speed feedback controller [46]. The controlled hyperchaotic finance system is constructed by

$$\begin{aligned}\dot{x} &= z + (y-a)x + w,\\ \dot{y} &= 1 - by - x^2,\\ \dot{z} &= -x - cz,\\ \dot{w} &= -dxy - kw - k_2\dot{x},\end{aligned} \tag{30}$$

where the control gain is calculated as $k_2 = 3.5$ for the E_1 equilibrium point [46].

3.3. The Linear Feedback Control

Yu *et al.* applied the control of hyperchaotic finance system using linear feedback controllers [46]. It is designed by

$$\begin{aligned}\dot{x} &= z + (y-a)x + w - k_3(x-\bar{x}),\\ \dot{y} &= 1 - by - x^2 - k_3(y-\bar{y}),\\ \dot{z} &= -x - cz,\\ \dot{w} &= -dxy - kw - k_3(z-\bar{z}),\end{aligned} \tag{31}$$

where the control gain is evaluated as $k_3 = 1.5$ for the E_2 equilibrium point [46].

4. The Numerical Simulations

In this section, computer simulations are performed to demonstrate the controlled hyperchaotic finance systems in Equations (3), (30), and (31). In all numerical simulations, the fourth-order Dormand–Prince method is used with variable-time step. MATLAB® software is used. The parameter values of hyperchaotic finance system are taken as $a = 0.9$, $b = 0.2$, $c = 1.5$, $d = 0.2$, and $k = 0.17$ with the initial values $x(0) = 1$, $y(0) = 2$, $z(0) = 0.5$, and $w(0) = 0.5$ to ensure the chaotic behaviour [46].

For obtaining the control to all equilibrium points, the coefficients of the speed feedback and linear feedback controllers are taken as $k_2 = 5.5$ and $k_3 = 3.5$, respectively. For providing the same conditions, the passive control parameters are taken as $\alpha = 3.5$, $v = 0$; and the linear feedback controller gain in the proposed control method is considered as $k_1 = 3.5$. The controllers are activated at $t = 40$ for showing both the chaotic trajectories and the control in the simulations. The results for the speed feedback, linear feedback, and proposed passive feedback control of hyperchaotic finance system towards E_1, E_2, and E_3 equilibrium points are demonstrated comparatively in Fig. 4, Fig. 5, and Fig. 6, respectively.

As expected, the related Figs. 4–6 show that the outputs of hyperchaotic finance system converge to its equilibrium points after the controllers are activated. Hence, the computer simulations have confirmed all the theoretical analyses. Figs. 4–6 include comparative results for the control of hyperchaotic finance system. While control is provided at $t \geq 41.5$ by using the passive feedback controllers, it is observed when $t \geq 46.5$ with the linear feedback controllers and $t \geq 68$ with the speed feedback controller for $E_1(0, 5, 0, 0)$ equilibrium point. The signals x and z play significant role in the control performance of linear feedback controllers, and the signal y plays significant role in the control performance of speed feedback controller. Furthermore, the control is firstly observed with the passive feedback controllers for both $E_2(1.66, -8.87, -1.11, 17.4)$ and $E_3(-1.66, -8.87, 1.11, -17.4)$ equilibrium points. The speed feedback control method provides the worst control performance again. The signal z designates the effectiveness of linear feedback controllers in overall. To summarize, the comparisons show that the proposed control method, which combines

Figure 4

The controlled hyperchaotic finance system towards $E_1(0, 5, 0, 0)$ equilibrium point when the controllers are activated at $t = 40$ for (a) x, (b) y, (c) z, and (d) w time series

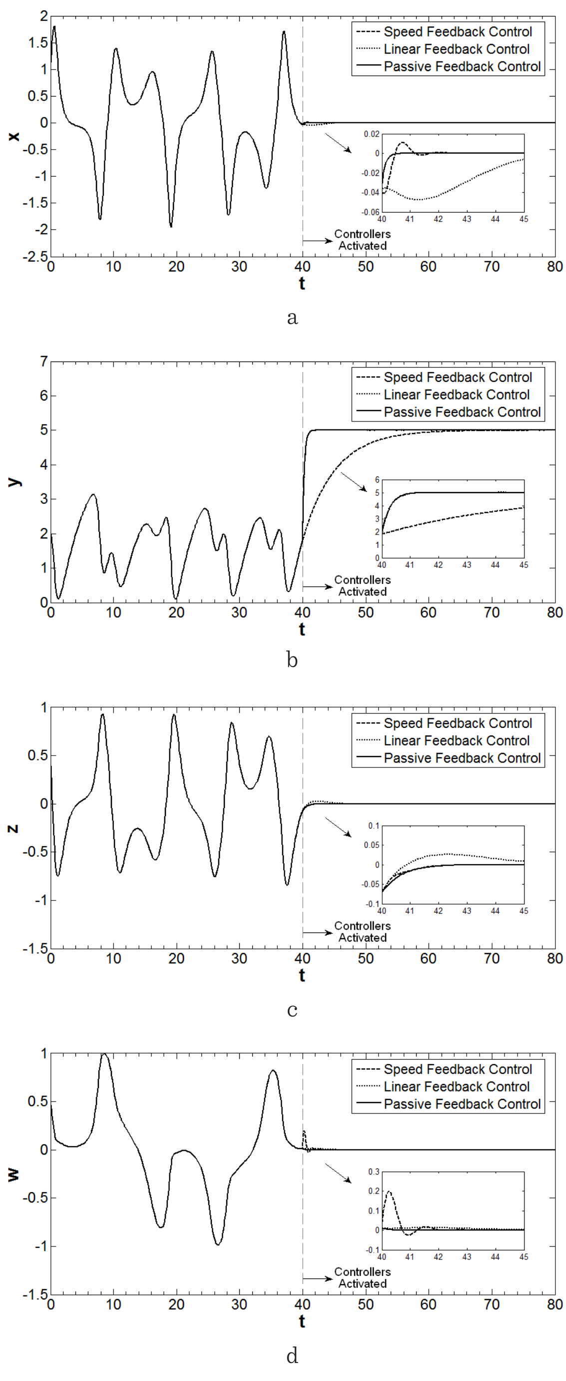


Figure 5

The controlled hyperchaotic finance system towards E_2(1.66, -8.87, -1.11, 17.4) equilibrium point when the controllers are activated at t = 40 for (a) x, (b) y, (c) z, and (d) w time series

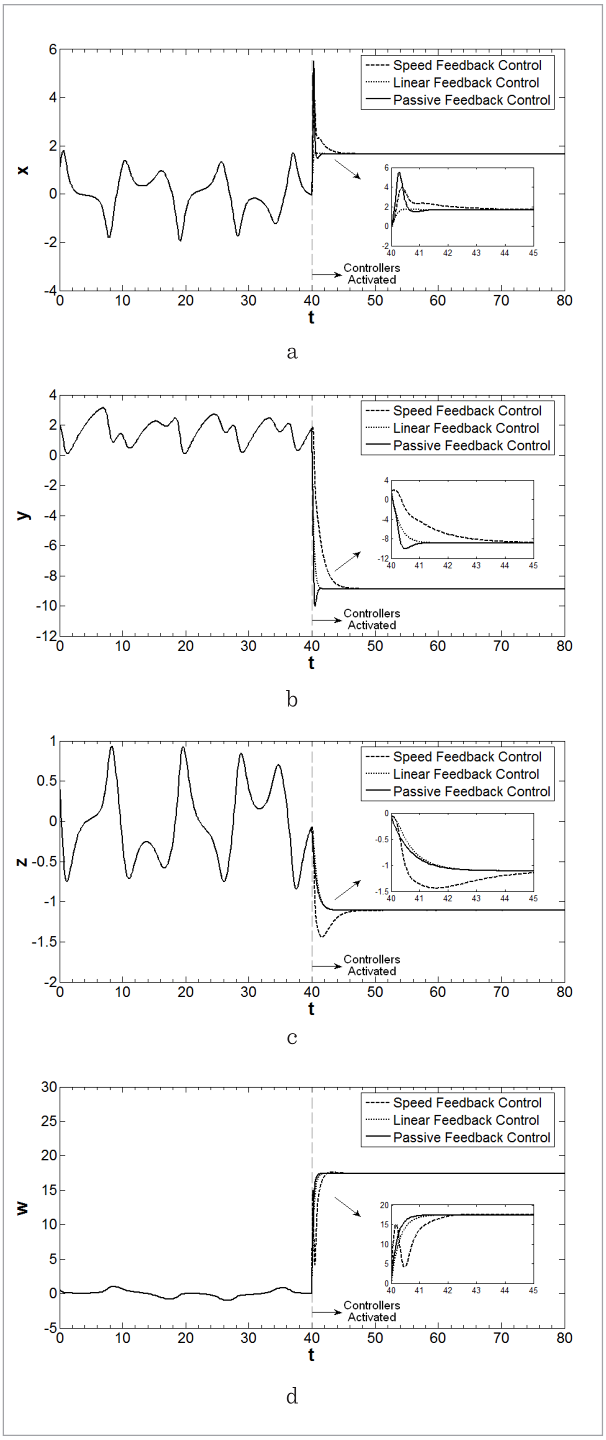
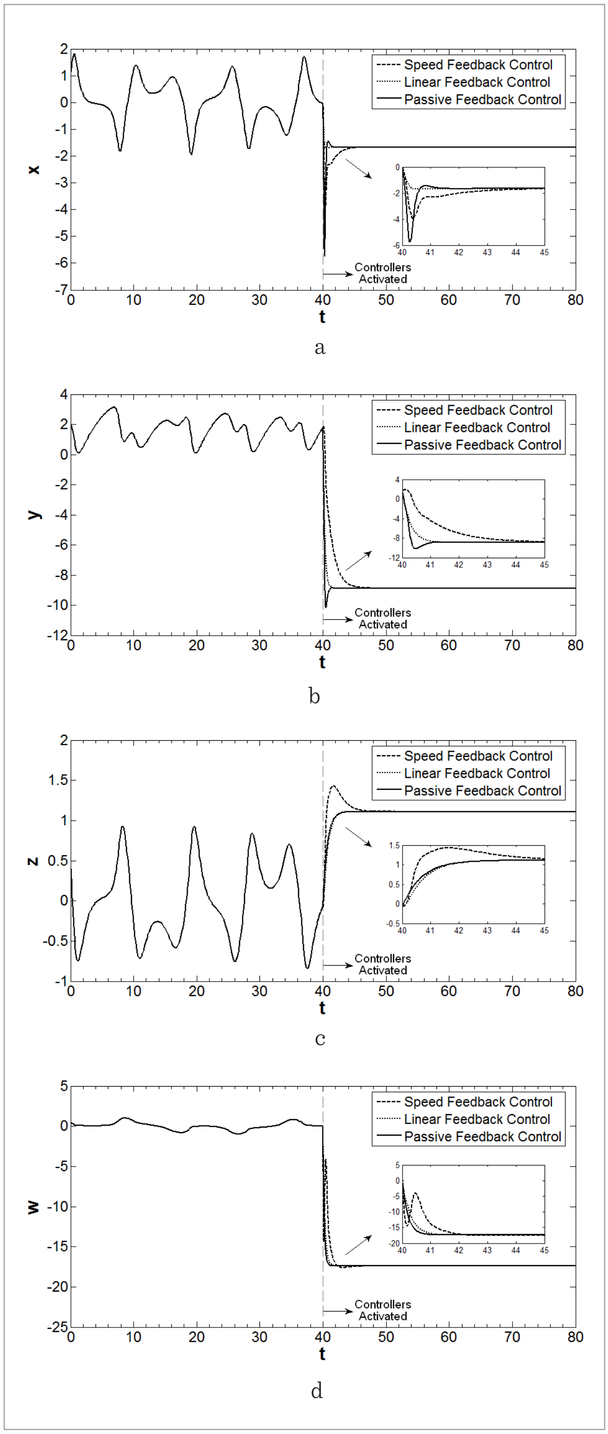


Figure 6

The controlled hyperchaotic finance system towards E_3(-1.66, -8.87, 1.11, -17.4) equilibrium point when the controllers are activated at t = 40 for (a) x, (b) y, (c) z, and (d) w time series



passive and feedback control methods, performs better than the speed feedback and linear feedback control methods for the control of the hyperchaotic finance system.

5. Conclusions

In this paper, the control of a hyperchaotic finance system is realized with a hybrid control approach. The finance systems are very sophisticated nonlinear systems that are interested in market and cover many unpredictable factors. The control of finance systems neutralizes many undesired factors in the economic systems and utilizes some benefits to regular growth. For this purpose, the passive control method is investigated for the control of a hyperchaotic finance system which was proposed by Yu *et al.* in 2012, but the classical passive control theory does not maintain its control. As a solution, passivity based feedback controllers have been designed to achieve the asymptotic stability of the continuous time hyperchaotic finance system towards its equilibrium points. Numerical simulations have confirmed the theoretical analysis of the proposed passive feedback controllers in Equations (19) and (29). Simulation results also show that the proposed controllers regulate the hyperchaotic finance system to its equilibrium points faster than the speed feedback and linear feedback controllers. As a result, the proposed method is more appropriate for the control of new hyperchaotic finance system. Future researches may be applied on the control, synchronization, and stabilization of chaotic and hyperchaotic systems by using the proposed passive feedback control method.

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